Development a New Concept of Redemption Point Model towards Genuine Profit Sharing

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Redemption point occurs when customers with the membership cards redeem their accumulated points with voucher, cash or discounts based on a reward chart pre-established by the firm. It is the most effective tool to attract customers to visit and shop at a business premises. However, many customers are lured by the promise of reward, but in fact it is in favour of the owners of business premises only. Not only that, even offered rewards points expire if not redeemed. Therefore we proposed a new model of redemption points that will give benefit not only to owner of the premise but benefit to the employee of the premises as well as to the customers. The proposed of redemption model is built by maximizing the value creation for the owner of the premise, employee and customers. Through these concepts, it can provide analytical solutions and sensitivity analysis to support Loyalty Reward Programme host’s decision making on rewards supply planning. Also, can lead to a universal redemption reward program.

Keywords: Redemption point; membership cards; Decision Making; Loyalty Reward Programme

Introduction

Loyalty reward programs are the programs to attract the customer. This programs have been used in various industries such as airlines, hotel, supermarkets, telecommunication, entertainment, banking and also gaming (Diaby and Nsakanda 2008; Yuheng Cao 2010). As the members of the premises, the customer will gain some points for every purchases. For example, each purchase of RM1 the customer will get 1 point. After a certain period, the customers will accumulate several points according to their purchases. Then, the customer may choose what need to be redeem and how much to redeem (Dorotic, Verhoef et al. 2014). Loyalty reward programs play an important role to increase the number of customers comes to the premises.

Basically, the rewards program sponsors provide the future goods, value or services to the member in exchange to the promises by purchasing goods or services (Liu and Brock 2009; Gault, Llaguno et al. 2012). There are several types of rewards such as coupons, points, cashbacks or air miles (Leone and Srinivasan 1996; Cao 2011). In this study, we consider the loyalty reward programme based on points.

Model Formulation

Previous Model

Loyalty reward program systems can be seen as the rewards points supply chains which involving the host, accumulation partner and also the redemption partner, (Cao 2011; Gandomi and Zolfaghari 2011). From the previous model, the customers buy the goods from the accumulation partners. As a reward, the accumulation partners gave a point.
to the customer. The points given are purchased from the host which is the service provider. After a period of time, the customers will accumulate several numbers of points. Then, the customers redeem the points from the host. Host will purchased the points from the redemption partners. The supply chain between the host, accumulation partners and redemption partners are as follows,

![Figure 1](Conceptual Model of Loyalty Reward Program)

All the entities in this rewards points supply chain will acquire the value as the system worked. The values obtained by the members are through the points that are collected. For the accumulation partner, value obtained from the generated revenue by providing the accumulation services. The redemption partner obtained the value from the purchases of the reward from the host. The host obtained the value from the accumulation partners when they buy points from the host. In this model, the host gets the profit when the accumulation partner purchases the points. Besides, the host also benefited value by the difference between the actual prices of the reward and the cost of the reward purchased by the host.

According to (Cao 2011), the accumulation partners are denoted as \( A_i \), \( i=1,2, ..., I \) in the system, the redemption partners as \( R_j \), \( j=1,2, ..., J \), the ordering quantity of points of the accumulation and redemption partner as \( q^A_i \) and \( q^R_j \) respectively, the member’s redemption and accumulation demand are \( D^R_j \) and \( D^A_i \) respectively. In the previous model, in order to maximize the host profitability, the main decision problems were the decision of the ordering quantities of points from the accumulation partners as well as the host’s optimal ordering quantity decisions of rewards to the redemption partner.

(Cao, Nsakanda et al. 2012) proposed the model in two different sides which are from the accumulation side and the redemption side. At the accumulation side, the profits obtained by the host were the revenue through selling points to the accumulation partner and also the extra revenue gains through when the demand is higher than the ordering quantity. The formulation at the accumulation side is as follows

\[
\pi_{H(A)}(;D^A) = \sum_{i=1}^{I} \left( w^A_i \times q^A_i + w^A_i \times \left[ D^A_i - q^A_i \right] \right)
\]

The host profitability function at the redemption side can be written as

\[
\pi_{H(R)}(;D^R) = \sum_{j=1}^{J} \left[ p^R_j \times \min\{ q^R_j, D^R_j \} - q^R_j \right] - \sum_{j=1}^{J} \left[ w^A_j \times q^R_j \right]
\]

The first component, \( p^R_j \times \min\{ q^R_j, D^R_j \} \) defines the value of rewards offered by each partner \( R_j \). The second component, \( w^A_j \times q^R_j \) indicates the host purchasing cost of rewards. The under-
stock cost of rewards and the salvage value of over stocking rewards are given by the third and fourth part respectively.

In this modelling problem, the maximization of the host profitability function is subject to the liability control constraints, the redemption partners’ capacity limitations on offering rewards, the LRP host’s budget constraint on purchasing rewards and also the non-negative constraints given in (3-5) respectively.

\[
L_{LB} \leq \frac{t}{l_0} \leq L_{UB}, \quad \text{where}
\]

\[
l = l_0 + \sum_{i=1}^{I} (q_i^A + [D_i^A - q_i^A]) - \sum_{j=1}^{J} (\min\{q_j^R, D_j^R\})
\]

\[
q_j^R \leq Q_j^R
\]

\[
\sum_{j=1}^{J} (w_j^R \times q_j^R) \leq W_R
\]

**Proposed Model**

The entities from the previous model by (Gandomi and Zolfaghari 2012) consist of the host, members, accumulation and redemption partners. The supply reward supply chain in this proposed model is excluding the accumulation partners. Therefore, the supply chain consists of the host, members and the redemption partners only. This is because this model insists to ensure the host obtain the profit directly from the customers. Figure 2 illustrates the new reward supply chain.

![Figure 2: The New Reward Supply Chain](image)

In order to view the model in further, we choose a simple example. Assume that, a member purchases a product amounted to RM100, then that member will obtain 100 points since for every RM1 spending at the premise, the member can collect one point. Assume that, the wholesale price for the product is RM85 including all other expenses. Then, the profit obtain by the premise is RM15.

From the previous model, the accumulation partner will buy the points from the host. Assume that, the price for each point is RM0.05, therefore the host will only gain RM5 profit and the accumulation partner will obtain RM10 profit. For the redemption side, the host buy the reward from the redemption partner for RM0.008 per unit point resulting RM0.80 profit for
RM100 purchase. For the member, the profit gains from the purchase of RM100 are only RM1. The details of the profit are shown in Table 1.

Table 1
The Profit Obtained by the Host, Members and The Partners.

<table>
<thead>
<tr>
<th></th>
<th>Previous Model</th>
<th>New Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price (RM)</td>
<td>RM100</td>
<td>RM100</td>
</tr>
<tr>
<td>Points obtained</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Host (RM)</td>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>Member (RM)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Redemption partner (RM)</td>
<td>0.80</td>
<td>0.80</td>
</tr>
<tr>
<td>Accumulation partner (RM)</td>
<td>10</td>
<td>-</td>
</tr>
</tbody>
</table>

This proposed model also considering the two processes in reward program which are the accumulation side and the redemption side. The different is at the accumulation side, the accumulation partner is removed. Therefore, the profit obtained by the host is no longer from the purchasing of the points from the accumulation partner. The host will obtain profit through the product amount of purchase by the member and the price per unit of points and the unredeemed points. Let \( w_k \) denotes the price per unit of points, \( q_k \) denotes the number of points obtained by the member, \( r_k \) is the number of point redeem by the members and \( w_k \) unit price per point for the unredeemed points. The new formulation at the accumulation side is as follows

\[
\pi_{H(A)}(;r_k^M) = \sum_{j \in J} \left( 2 \times w_k^M \times q_k^M + w_k^M \times [q_k^M - r_k^M] \right)
\]

(6)

The host profitability at redemption side is the same as the previous model since the members can redeem their point at any premises.

Combining the host profit from the accumulation and redemption subject to the constraints (3-5) the problem of supply of rewards can be formulated as follows:

\[
\max \Pi_H(q^*, D^*, r^*)
\]

(7)

Subject to

(3),(4),(5)

The model above is according to the linear programming model which consists of the objective function subject to the constraints. In this model, we maximize the profit gained by
the host from both accumulation and redemption sides. The host profitability is measured by
the value creation at each supply chain.

According to (Cao, Nsakanda et al. 2014), the value is subject to the redemption partners’
capacities on offering rewards, the host’s overall budget for purchasing rewards and also the
host’s control on points liability. In this model, there are a few uncertainties that need to be
taken care which are the members’ points’ redemption demand and the number of points
redeem by the members.

In order to deal with the uncertainties, the problem is remodelled as a two stage stochastic
with recourse while considering random members’ points’ redemption demand and the
quantity of points redeemed and consequently random processes.

The first stage of the model is including the parameter with certainties where as in the second
stage, the parameters are uncertain.

Then the two stage stochastic linear programming with recourse is as follows:

\[
\max \Pi_q(q^r; D^r, t^M) = \sum_{r=1}^r w^r \times q^r + \sum_{j=1}^J \{ p^j \times q^r - w^r \times q^r \} + E[g(x, w)] \\
\text{Subject to} \\
q^r \leq Q^r \\
\sum_{j=1}^J \{ w^r \times q^j \} \leq W^r \\
q^r, q^M \geq 0
\]

where

\[
g(x, w) = g(q^r, D^r, t^M) = \sum_{i=1}^i w^i \times l^i + \sum_{j=1}^J \{ p^j \times l^j \}
\]

Subject to

\[
L_{\text{aq}} \leq \frac{1}{l_j} \leq L_{\text{up}}, \text{ where } l = l_i + \sum_{i=1}^i \{ q^r \times l^i \} - \sum_{j=1}^J \{ q^r \times l^j \}
\]

Methodology

(Zanjani, Kadi et al. 2007) stated that the two stage stochastic model can be solved by using
the linear programming solvers such as CPLEX LP Solver. In this model, we need to generate
the uncertainty data randomly by using Monte Carlo sampling techniques to obtain the
approximate solutions. Therefore the solution approach to solve this model is by selecting the
sample average approximation (SAA) scheme. In this scheme, the expectation of second stage
objective function is approximated by the sample average function which is

\[
E[g(x, w)] = \frac{1}{N} \sum_{n=1}^N g(x, w)
\]

Therefore, the two stage stochastic linear programming with recourse with sample average
approximation (SAA) is given by
max $\Pi_N(q_i^*; D_i^*, r_i^*)$  

\[
= \sum_{i=1}^{k} \left( r_i^* \times q_i^* - w_i^* \times q_i^* \right) + \frac{1}{N} \sum_{i=1}^{k} w_i^* I_i^* - \\
+ \frac{1}{N} \sum_{i=1}^{k} (-v_i^* \times I_i^* + s_i^* \times I_i^*)
\]

Subject to

(4),(5),(9),(11),(12),(13),(14)

The main idea of this solving procedure consists of generating the approximate solution of the number of replications, $M$ of the SAA problems with $N$ sampled scenarios. Then, we tested the quality of the candidate solution by bounding the optimality gap between the true objective value and the expected objective value through statistical procedures. The solving procedure is as follows:

Step 1: Generate independent identical distributed for samples of $N=5, 10, 15, 20, 25$ and with $m$ replications, $M=1, 2, \ldots, 5$. The random parameter generated are the demand parameter which are the number of redemption demands, $D_j$ and quantity of points redeem by the members, $r_k^M$.

Step 2: The model of SAA problem is solved for each replication. The problem is solved by using Excel Solver.

Step 3: Choose $N'$ which is larger than $N$.

3.1: Solve the SAA problem for $N'$ larger than $N$ as in step 2.

3.2: The true objective value is estimated for all replications of samples with sample size $N$ as follows

\[
\hat{\pi}_N = \frac{1}{N} \left( \sum_{j=1}^{N} \left( D_j \times q_j^* - w_j^* \times q_j^* \right) \right) - \frac{1}{N} \left( \sum_{j=1}^{N} \left( D_j \times I_j^* + v_j^* \times I_j^* \right) \right)
\]

3.3: Compute the optimality gap for the candidate solution as follows

$G_N^m (\bar{x}) = \hat{\pi}_N^m - \hat{\pi}(\bar{x})$ for $m= 1, \ldots, m$.

Step 4: Compute the sample mean and sample variance for the optimality gap, $G_N^m (\bar{x})$ as follows.

Mean:

\[
\bar{G}_N^m (\bar{x}) = \frac{1}{M} \sum_{m=1}^{M} G_N^m (\bar{x})
\]

Variance:

\[
S_G^2 (\bar{x}) = \frac{1}{M-1} \sum_{m=1}^{M} (G_N^m (\bar{x}) - \bar{G}_N^m (\bar{x}))^2
\]

Step 5: Compute the approximate $(1-\alpha)$ level confidence interval for the optimality gap of $G_N^m (\bar{x})$ as

\[
\left[ \left( G_N^m (\bar{x}) \pm \frac{t_{M-1,\alpha} S_G}{\sqrt{M}} \right) \right]
\]

where $\alpha = \frac{1}{M-1,\alpha} S_G^2 (\bar{x})$
Computational Results

The first numerical solution is to compare the profit obtained by the host from the previous model and the proposed model. The solution of the model is conducted by using Excel Solver. In order to solve the model, there are a few parameters that need to be considered. Table 2 and Table 3 summarize the main parameters considered to generate the test problems.

Table 2
Problem Generation Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average redemption cost of rewards in terms of ringgit value per unit points (AvgMC)</td>
<td>8.2</td>
</tr>
<tr>
<td>Salvage value factor (SR)</td>
<td>0.5</td>
</tr>
<tr>
<td>Shortage cost factor (VR)</td>
<td>1.2</td>
</tr>
<tr>
<td>Average wholesale price of points in terms of ringgit per unit points (AvgMP)</td>
<td>12</td>
</tr>
<tr>
<td>Range for capacity factor of rewards (CA)</td>
<td>1-1.5</td>
</tr>
<tr>
<td>Range of unredeemed price factor of points (BF)</td>
<td>1.5-2</td>
</tr>
<tr>
<td>Range of market/retailing price factor of rewards (MR)</td>
<td>2-10</td>
</tr>
<tr>
<td>Upper bound of liability ratio (LUB)</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 3
Problem Generation Parameters Per Partner Type

<table>
<thead>
<tr>
<th>Parameter</th>
<th>PARTNER TYPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability distribution</td>
<td>I</td>
</tr>
<tr>
<td>Redemption cost factor (FR)</td>
<td>0.2</td>
</tr>
<tr>
<td>Range of wholesale price factor per unit of points ordered (FA)</td>
<td>0.4</td>
</tr>
<tr>
<td>Range of wholesale price factor per unit of points ordered (FA)</td>
<td>0.4-1</td>
</tr>
<tr>
<td>Range for redemption (DR) / accumulation (DA) demand (uniform distribution)</td>
<td>40-50</td>
</tr>
<tr>
<td>Mean and standard deviation for redemption (DR) / accumulation (DA)demand (normal distribution)</td>
<td>(45, 6)</td>
</tr>
<tr>
<td></td>
<td>II</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>1.0-1.2</td>
</tr>
<tr>
<td></td>
<td>20-30</td>
</tr>
<tr>
<td></td>
<td>(25, 3)</td>
</tr>
<tr>
<td></td>
<td>III</td>
</tr>
<tr>
<td></td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>1.2-1.4</td>
</tr>
<tr>
<td></td>
<td>5-10</td>
</tr>
<tr>
<td></td>
<td>(7.5, 1)</td>
</tr>
</tbody>
</table>

The additional inputs required according to the scheme described below:

a) Wholesale prices of rewards are the product of the redemption cost factor (FR) and the average redemption cost of rewards (AvgMC).

b) Retailing prices are given by the product of wholesale price of rewards and the market price factor (MR).
c) Salvage values of rewards are the product of wholesale price of rewards and salvage value factor (SR).

d) Shortage costs of rewards are the product of wholesale price of rewards and shortage cost factor (VR).

e) Wholesale price for per unit points given to the members are the product of wholesale price factor of points (FA) and average wholesale price of points (AvgMP).

f) Price per unit point of the unredeem points are the product of the wholesale price and the unredeemed price factor of points (BF).

g) The right hand side of the capacity constraint for each redemption partner is generated based on the redemption demand parameter and the capacity factor of the rewards.

h) The available budgets for purchasing reward are generated according to the capacity of rewards offered by the corresponding redemption partner and also the wholesale price.

i) The initial liability is determined based on the information of accumulation points.

In this research, we only consider a redemption partner and an accumulation partner. The accumulation partner is actually the host. The sizes of the problem changes accordingly to the sample sizes (N). The number of sample sizes are set to N=5, 10, 15, 20, 25 and M=5. In all cases we did set N'=30. The tested values are used to solve both previous and proposed model. The profit for both model are shown in Table 4.

As it can be observed in Table 4, we have solved the model in (15) subject to constraints stated in (4), (5), (9), (11), (12), (13), (14). Since some of the parameters are generated randomly the solving procedure is repeated by 5 times. After five replications of solving procedure, it is shown that the profit for both previous model and proposed model are increase as the sample size increase. This shows that the solutions by sample average approximation (SAA) scheme are valid.

From Table 4, for N=5 the profit for the proposed model is higher than the previous model. This results are applicable for N=10, 15, 20, 25. The proposed model creates more values in terms of the profitability to the host as compared to the existing model. As the number of samples increases from time to time the profit gains by the host also will increase. Before the average of the solutions is calculated, the difference between the current solutions with the solutions for N’ which is 30 is computed. This step is repeated for each N=5, 10, 15, 20, 25.Then, the mean is calculated by using the formula in (17) which is by finding the average of the difference solution from 5 replications of solving procedure. Next, the variance for the optimality gap for each N is computed by using formula in (18). Then the standard deviation is just the root of the variance. The value in standard deviation will be used to find the confidence interval.

In constructing the confidence interval, assume that \( \alpha = 0.05 \) in order to compute the approximate \((1-\alpha)\) level confidence interval for the optimality gap. The formula for the confidence interval is stated in (19). From the formula, the value for \( t \) distribution for M=5 and \( \alpha = 0.05 \) is equals to 0.5754424.
## Table 4
Profit of the Previous Model and the New Model

<table>
<thead>
<tr>
<th>N</th>
<th>m</th>
<th>Previous model</th>
<th>Proposed Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1</td>
<td>4501.94</td>
<td>4898.76</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3066.61</td>
<td>3443.38</td>
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<tr>
<td></td>
<td>3</td>
<td>3057.41</td>
<td>3511.43</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>3104.89</td>
<td>3550.12</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>2935.48</td>
<td>3399.39</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>4532.54</td>
<td>9044.16</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>4739.59</td>
<td>9453.68</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>4664.10</td>
<td>9290.80</td>
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<tr>
<td></td>
<td>4</td>
<td>4412.10</td>
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<td></td>
<td>5</td>
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<td>15</td>
<td>1</td>
<td>4502.76</td>
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<td></td>
<td>2</td>
<td>4426.01</td>
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<tr>
<td></td>
<td>3</td>
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<td></td>
<td>5</td>
<td>4613.58</td>
<td>9088.35</td>
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<tr>
<td>20</td>
<td>1</td>
<td>6549.65</td>
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<td>2</td>
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<td></td>
<td>3</td>
<td>5421.18</td>
<td>10700.04</td>
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<tr>
<td></td>
<td>4</td>
<td>5212.26</td>
<td>10279.79</td>
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<td></td>
<td>5</td>
<td>5251.56</td>
<td>10368.11</td>
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<tr>
<td>25</td>
<td>1</td>
<td>6932.29</td>
<td>13717.07</td>
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<td></td>
<td>2</td>
<td>6566.38</td>
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<td>6712.78</td>
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<td>4</td>
<td>7725.13</td>
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<td></td>
<td>5</td>
<td>6746.83</td>
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<td>30</td>
<td>1</td>
<td>7867.02</td>
<td>15712.30</td>
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<td></td>
<td>2</td>
<td>8039.98</td>
<td>16060.07</td>
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<td>3</td>
<td>7666.39</td>
<td>15319.12</td>
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<td></td>
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<td>8969.01</td>
<td>17973.53</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>7802.81</td>
<td>15586.73</td>
</tr>
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</table>
In Table 5, it is shown that, by increasing the sample size, the value for the mean, variance, standard deviation and also the length of the confidence interval are decrease. These show that the quality of approximate solutions improves and the tighter confidence intervals for the optimality gaps of candidate solutions are constructed. The length of the confidence interval from N equals to 5 up to N equals to 25 decreases. This means that, the solutions of the both model are close to the true solutions of the problem when the sample size increase. The solutions follow the rule in the sample average approximation (SAA) scheme which is the higher the number of sample size the closer the approximate solution to the true solutions of the problems.

Finally, we can conclude that the proposed model gives more profit than the existing model. We also can conclude that, by considering a moderate number of scenarios among the potential enormous number of scenarios, we obtain an approximate solution in a reasonable amount of time with an interval closer to the true solutions. Thus, this solution can be accepted as a good approximation to the optimal solution. The problem with two stage stochastic linear programming with recourse and remodel using sample average approximation (SAA) scheme are useful to solve the model with uncertainty and known with specified distribution for example normal or uniform distribution.

Conclusions

In this paper, we have proposed a new concept of redemption point model. From the previous model, we have known that, there are four entities exist in rewards supply chain which is the host, accumulation partner, redemption partner and the host. The previous model focus on maximize value creation in terms of profitability for the host. The host gains profit from the accumulation partner by selling the point to the accumulation partner.

In the proposed model, we eliminate the accumulation partner so that the host gains profit directly from the members. The new model maximizes the value creation by the host subject to the liability control, budget and capacity limitations and demand uncertainties. Due to the uncertainties parameter, we solved the model by using sample average approximation (SAA) approach. This approach ensures that as the sample sizes increase, the approximate solutions are closer to the true optimal solutions. The model is solved by using Excel Solver.

There are two main conclusions in this study. Firstly, the comparison between the previous model and the proposed model, the results show that the profit obtained from the proposed
model is higher than the previous model for each of the tested sample sizes, N. This shows that, the proposed model gives a better approach to the host to obtain higher value creations in terms of profitability. Secondly, the model with uncertainty can be solved by using sample average approximation (SAA). The results show that as the sample size increase the approximate confidence interval is decrease. This means, the current solutions is close to the actual solutions.

References


