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Investigating the Role of Modeling Practices on Mathematical Literacy

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ABSTRACT

The mathematical literacy or competence notion in PISA deals with the capacity of students to analyze, reason and communicate efficiently as they pose, formulate, solve and interpret mathematical problems in a variety of situations. The best way to improve mathematics literacy is that students have the necessary mathematical knowledge and different problem solving strategies, know when and how to use these strategies, and work with activities that involve different contexts of interest. When considered in this respect, teachers have an important role in the development of students' mathematical literacy. The aim of this study was to investigate the mathematics literacy status of pre-service mathematics teachers through PISA questions. The mathematical literacy status of pre-service teachers was examined within the scope of conceptual, operational and contextual questions and in terms of gender, academic grade point average and mathematical modeling. At the same time, semi-structured interviews have examined the difficulties that pre-service teachers experienced. Fully mixed concurrent equal status design was used. The participants were 113 pre-service mathematics teachers. Independent samples t-test and covariance analysis was used for the comparisons. Qualitative data analysis was conducted with content analysis. The research results, together with the suggestions, reveal important points for future studies.

Keywords: Mathematical literacy, knowledge types, pre-service education, PISA

Modelleme Uygulamalarının Matematik Okuryazarlığı Üzerindeki Rolünün İncelenmesi

ÖZ

Uluslararası Öğrenci Değerlendirme Programı’nı (PISA) yürüten Ekonomik İşbirliği ve Kalkınma Teşkilatı (OECD) tarafından yapılan matematik okuryazarlığının tanımı; matematiğin dünyada oynadığı rolü anlama, sağlam temellere dayanan matematiksel hükümler verme, yapıcı-ilgili düşünce üreten kişiler olarak bireysel yaşamların gereksinimlerini karşılarken matematiği kullanma ve matematikle meşgul olma kapasitesi şeklindedir. Matematik okuryazarlığını geliştirmenin en iyi yolu öğrencilerin gerekli matematiksel bilgiye ve farklı problem çözme stratejilerine sahip olmaları, bu stratejileri ne zaman ve nasıl kullanacaklarını bilmeleri ve ilgilerini çeken farklı bağlamları barındıran etkinliklerle çalışmaları olarak ifade edilmektedir. Bu anlamda öğretmenler, öğrencilerin matematiksel okuryazarlığının gelişmesinde önemli bir role sahiptirler. Bu tespitten yola çıkılarak çalışmada Ortaokul Matematik Öğretmeni Adaylarının matematik okuryazarlık durumlarının PISA soruları üzerinden incelenmesi amaçlanmıştır. Öğretmen adaylarının matematiksel okuryazarlık durumları kavramsal, işlemsel ve bağlamsal sorular kapsamında cinsiyet, akademik not ortalaması ve matematiksel modelleme değişkenleri ile incelenmiştir. Aynı zamanda yarı yapılandırılmış görüşmelerde öğretmen adaylarının yaşadıkları zorluklar ele alınmıştır. Çalışmada tamamen karma eşzamanlı eşit statülü tasarım kullanılmıştır. Katılımcılar 113 matematik öğretmeni adayıdır. Karşılaştırmalar, bağımsız örneklem t testi ve kovaryans analizi ile gerçekleştirilmiştir. Nitel veriler ise içerik analizine tabi tutulmuştur. Araştırma sonuçları, önerilerle birlikte ileriki çalışmalar için önemli noktaları ortaya koymaktadır.

Anahtar kelimeler: Matematiksel okuryazarlık, bilgi türleri, hizmet öncesi eğitim, PISA

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INTRODUCTION

Many teachers in mathematics classrooms are likely to be asked, "Why are we learning maths?" When students were unable to recognize the repercussions of the subject matter and its relationship to the real world, they asked, "What will maths accomplish for us in everyday life?" In truth, the answer lies in mathematical literacy, which is defined as the ability to employ mathematical knowledge to the greatest extent feasible when confronted with problems in daily life and in generating solutions to problems (Niss & Jablonka, 2020; Steen, Turner & Burkhardt, 2007). The Organization for Economic Cooperation and Development (OECD) which has been administering the Programme for International Student Assessment (PISA) defined mathematical literacy as understanding the role of mathematics in the world, making mathematical judgments based on solid foundations, the capacity to use mathematics and engage in mathematics while meeting the needs of individual lives. As a result of PISA, mathematical literacy has become the primary concern of educational reforms in many countries. In PISA 2012, the framework for evaluating mathematical literacy was addressed from three perspectives. These are mathematical content, mathematical processes, and mathematical contexts (OECD, 2014). PISA studies enable the education systems of the participating countries to be described, followed, and compared. Considering these results, the need to increase students' mathematical literacy levels constitutes the focus of reform movements in education systems.

According to Bransford, Brown, and Cocking (2001), learners with organized knowledge can solve new issues and remember more relevant information than those who have merely memorized individual mathematical facts or methods. According to Rittle- Johnson and Koedinger (2005), appropriately structured knowledge necessitates people's contextual, conceptual, and procedural knowledge to be integrated within a domain. Contextual knowledge is linked to real-world problems and is introduced into the classroom through the presentation of the problem in a context with its own story. When the problems are described using solely mathematical symbols with numbers, operators, and variables, that they do not evoke any images (Rittle- Johnson & Koedinger, 2005). Conceptual knowledge was defined as a connected web of knowledge, a cognitive network in which node-to-node relationships are as significant as the discrete pieces of information that make up these nodes (Groth & Bergner, 2006; Lenz et al.,2020). According to Hiebert and Lefevre (1986), conceptual knowledge is required to comprehend challenges and to develop new strategies or adapt existing techniques to handle new problems. Procedures are included in procedural knowledge, which entails step-by-step actions and concurrent learning of all components. Knowledge of the formal language or system of symbolic representation of mathematical ideas, as well as knowledge of the rules or algorithms used to perform mathematical tasks, were identified as two categories of procedural knowledge (Hiebert & Lefevre, 1986).

Teacher quality is one of the most critical variables in student performance in international tests, according to studies (An, Kulm & Wu, 2004; Fauth, 2019). The best way to improve mathematics literacy is providing the students with the necessary mathematical knowledge and different problem-solving strategies, ensuring they know when and how to use these strategies, and facilitating activities that involve different contexts of interest. When considered in this respect, teachers have an important role in the development of students' mathematical literacy. Teachers can assist students in developing different mathematical knowledge and skills regarding mathematical literacy by using different teaching methods and techniques that guide students to understand and reason mathematically. In addition to exposing the teacher factor to succeed in international comparative exams, the importance of mathematics teachers' ability to support students' mathematical literacy skills in the field of mathematics education is also very important (Kilpatrick, 2001). For this reason, it is extremely important to determine the mathematics literacy status of pre-service teachers. Researchers have shown that mathematical performance varies across cultures (Chiu & Xihua, 2008; Kriegbaum, Jansen, & Spinath, 2015), but few studies have compared pre-service teacher performance in the specific released items in their analyses or explored categorizations of these items regarding the type of knowledge they require (e.g., Olande, 2014; Sáenz, 2009), that is, school mathematical knowledge (contextual, conceptual, and procedural). Indeed, little is known about the challenges that aspiring teachers face when completing PISA problems (e.g., Sáenz, 2009). There are also a few international studies where pre-service teachers' difficulties regarding PISA questions are investigated. In a study evaluating the mathematics skills of pre-service teachers while solving PISA questions, Sáenz (2009) states the requirement of assessments which employ school mathematics in contextual, conceptual and operational manners. The study showed that contextual knowledge about the real world is important in terms of establishing a relationship between the use of skills needed to solve PISA questions and traditional school mathematics expressed by concepts and processes.

Working with activities that involve many contexts that appeal to students' interests is the greatest strategy to enhance mathematics literacy. Students form permanent associations with mathematics and the world in which they live because modeling problems provide examples with a wide range of linkages (Swan, Turner, Yoon, 2006). Adaptation of modeling activities to school lessons paves the way for students to become social citizens and develop high-level mental skills that are critical in the community. Activities for daily life problems which seek solutions in the learning-teaching process (that progresses with the modeling method) allow many students to develop practice-based beliefs (Maab, 2005). Mathematical modeling makes the world out of mathematics and mathematics bilaterally efficient (Bonotto, 2007; Pollak, 1979). In other words, it is possible to utilize modeling activities to improve mathematics literacy, which means the capacity to transfer mathematics to daily life. Based on these findings, the aim of this study was to investigate the mathematics literacy status of Elementary School Pre-Service Mathematics Teachers through PISA questions. The mathematical literacy status of pre-service teachers was examined within the scope of conceptual, operational, and contextual questions and in terms of gender, academic grade point average, and mathematical modeling. At the same time, semi-structured interviews have examined the difficulties that pre-service teachers experienced with PISA questions.

METHOD

In this study, a fully mixed concurrent equal status design was used (Johson & Onwuegbuzie, 2004). It is a design in which the qualitative and quantitative stages are of equal weight and they are mixed in one or more components of the research at the same time. When quantitative results cannot adequately explain outcomes and qualitative data can help overcome the difficulty by strengthening and explaining the quantitative results, mixed methods research is strongly suggested (Teddlie & Tashakkori, 2003).

Sample

The participants were 113 pre-service mathematics teachers from a state university in the central part of Turkey. They were 57 juniors and 56 seniors studying elementary mathematics education and had no prior experience with PISA items. The third and fourth-grade students were selected as the sample because the current research focused on the role of modeling in mathematical literacy and the mathematical modeling course was given as an elective in the fall semester of the third year. Among the participants, 45 students took mathematical modeling courses whereas the remaining 68 students did not. The study also investigated the gender differences in mathematical literacy so the comparisons were conducted based on the data of 82 females and 31 males.

Instruments

The participants took a computer-based test containing 21 released items of the PISA 2012 mathematical literacy test. Information regarding the tasks of assessment items, type of knowledge, and maximum scores was given in Table 1.

Table 1. Mathematical Literacy Item Descriptions

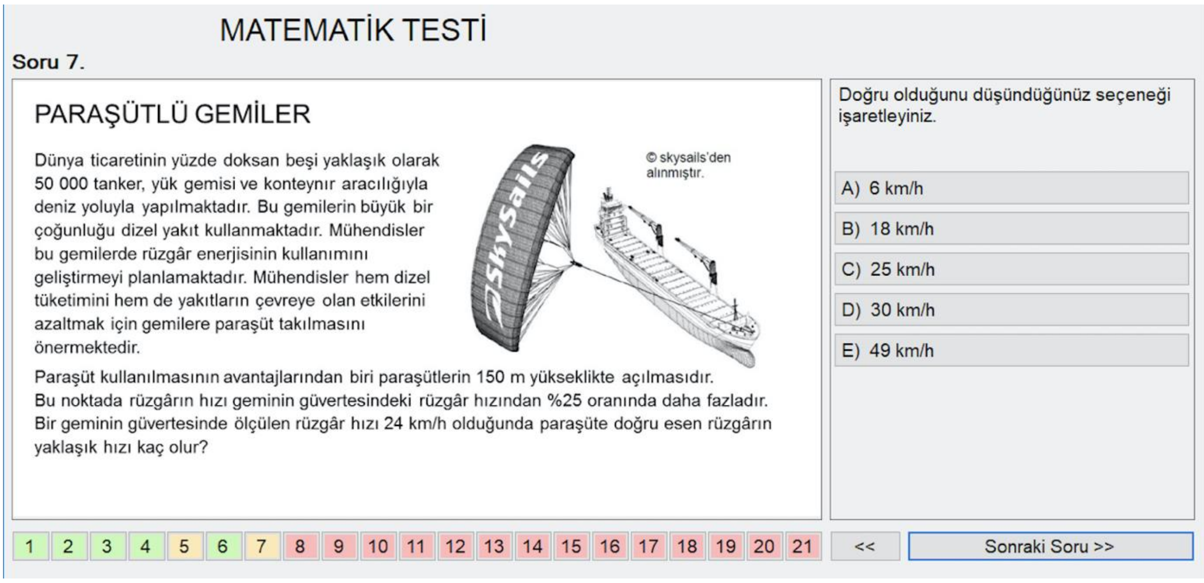
Task and item	Max. score	Type of knowledge*
Charts (item 1)	1	PR
Charts (item 2)	1	
Sauce (item 1)	1	
Ferris wheel (item 1)	1	
Which car? (item 2)	1	
Which car? (item 3)	1	
Revolving door (item 1)	1	
Charts (item 3)	1	PR, CC
Sailing ships (item 2)	1	
Ferris wheel (item 2)	1	
Climbing Mount Fuji (item 1)	1	
Climbing Mount Fuji (item 3)	2	
Helen the cyclist (item 1)	1	
Helen the cyclist (item 2)	1	
Revolving door (item 3)	1	PR, CC, CT
Sailing ships (item 1)	1	
Sailing ships (item 3)	1	
Climbing Mount Fuji (item 2)	1	
Helen the cyclist (item 3)	1	
Garage (item 2)	2	
Revolving door (item 2)	1	

*: The abbreviation PR stands for procedural knowledge, CC stands for conceptual knowledge and CT stands for contextual knowledge.

The categorization of the mathematical literacy items according to the type of knowledge was conducted by the two experts in mathematics education. Then, the percentage of the fit index was calculated as 91.3% by considering the structure in Aydın and Özgeldi (2019). Afterwards, the experts discussed the items which were categorized into different knowledge types and decided to adhere to the structure as indicated in Aydın and Özgeldi's (2019) study.

There were 7 items measuring procedural knowledge, 8 items measuring procedural and conceptual knowledge, and finally 6 items measuring procedural, conceptual, and contextual knowledge. The testing environment was developed by the researchers on Visual Studio and C# programming language was used with the MSSQL database. Before answering the items in the test, the participants were asked to provide descriptive information about their gender, academic grade point average (GPA), and whether or not they took a mathematical modeling course. A screenshot of the testing environment was given in Figure 1.

Figure 1. *The Design of the Testing Environment*



The testing time for the entire test was 60 minutes. After administering the test, the scoring of the responses was completed based on the PISA scoring rubric. Two items were scored out of 2 points so partial credit was applicable in these items whereas the remaining items were worth 1 point. So, the test form measuring procedural knowledge was out of 7 points. On the other hand, the test form measuring both procedural and conceptual knowledge was out of 9 points and the test form measuring all three knowledge types was out of 7 points.

Moreover, semi-structured interviews were conducted to focus on the tests in detail with twenty pre-service teachers who received the lowest total scores and ten randomly selected pre-service teachers who took an elective mathematical modeling course, i.e. 30 pre-service teachers in total. In semi-structured interviews, pre-service teachers were asked to solve and interpret the solutions as if they were performing the application again in the computer-based testing environment, and they were asked to indicate the issues while responding to each item. So, it was possible to focus on the critical points which made the item challenging. During the interviews, the following questions were posed to the participants.

1. Could you explain how you reached your response?
2. What were the item characteristics making the (specific) item challenging?
3. Which items were challenging? How did you struggle with these items? Do you have any specific methods?
4. Did you leave any items unanswered? If yes, what was the reason behind these omitted items?

Data Analysis

After testing the assumptions, independent samples t-test and covariance analysis (known as ANCOVA) were used for the comparisons . The skewness and kurtosis values and the frequency distribution indicated that the dependent variables were approximately normally distributed. Also, the homogeneity of variance assumption was checked by Levene's test for each distribution. These values were not significant indicating that the variances of each group were equal. For ANCOVA, alongside these assumptions, we observed that the covariate was independent of the treatment effects. Moreover, the tests of between-subjects effects indicated non-significant results. Therefore, the homogeneity of the regression slopes was met for the dependent variables.

Finally, Levene's test of equality of error variances showed that error variances of each dependent variable were equal across groups.

The independent samples t-test was conducted to investigate whether the procedural (PR) knowledge scores differ in terms of the variables status of taking elective mathematical modeling course, gender, and academic grade point average. However, the comparisons of conceptual (CC) knowledge scores for these variables were not explicitly set. Instead, covariance analysis (ANCOVA) was conducted by setting procedural and conceptual knowledge (PR-CC) scores as the dependent variable and PR scores as the covariate. In the same manner, to compare contextual (CT) knowledge scores in terms of the fixed factors, another ANCOVA was conducted by taking procedural, conceptual, and contextual knowledge (PR-CC-CT) scores as the dependent variable and PR-CC scores as the covariate. A brief summary of the variables in data analysis was given in Table 2.

Table 2. The Types of Variables in Data Analysis

Fixed factor	Dependent variable	Covariate
Gender	PR score	-
	PR-CC score	PR score
	PR-CC-CT score	PR-CC score
Academic grade point average	PR score	-
	PR-CC score	PR score
	PR-CC-CT score	PR-CC score
Status of taking elective mathematical modeling course	PR score	-
	PR-CC score	PR score
	PR-CC-CT score	PR-CC score

Researchers have proposed several categories of knowledge, including conceptual and procedural knowledge (e.g., de Jong & Ferguson-Hessler, 1996; Hiebert & Lefevre, 1986). Rittle-Johnson and Koedinger (2005) added contextual knowledge and proposed three complementary categories of mathematical knowledge: contextual, conceptual, and procedural. They described contextual knowledge as understanding how things operate in real-world settings, conceptual knowledge as an integrated understanding of key principles, and procedural knowledge as an understanding of the sub-components of a correct technique. A group of experts in mathematics education categorized each released item according to the type of knowledge the solution requires in a preliminary analytical stage prior to data collection.

In the analysis of the semi-structured interviews, Miles and Huberman (1994) used a qualitative data analysis method consisting of three stages: "data reduction", "data representation" and "revealing and verification of results". In the data reduction phase, the raw data were extracted for the purpose of the study, and then the categories and themes were created by encoding the data. Afterwards, the data were visualized with the help of a table or figure. In the stage of revealing and verifying the results, the relationships were interpreted, compared, and contrasted with the literature. In order to ensure reliability, the data obtained from the interview were independently coded by the researchers. For scorer reliability, the number of "agreement" and "disagreement" statuses was determined and Miles and Huberman's (1994) "Reliability = (Agreement) / [(Agreement) + (Disagreement)]" formula was used and the reliability of data analysis of semi-structured interviews was found to be 94%.

Research Ethics

In planning the study researchers had the responsibility to evaluate carefully any ethical concerns. Three issues were addressed in the study by the researchers. First of all, there were not any situations that had to be handled about the protection of participants from harm. Secondly, the confidentiality of research data was ensured. Finally, researchers conducted the study using methods that do not require deception.

FINDINGS

The results would be expressed in terms of the fixed factors: gender, academic grade point average, and the status of taking an elective mathematical modeling course.

Gender

The results of the independent samples t-test analysis showed that there was no significant difference between the PR knowledge scores, t(111)=.536, p=.593, of the female students (M=5.74, SD=1.08) and male students (M=5.61, SD=1.36).

In comparing PR-CC knowledge scores, a one-way ANCOVA was conducted to determine a statistically significant difference between the female and male students on PR-CC knowledge scores controlling PR knowledge scores. The results pointed out that there was no significant difference in mean PR-CC knowledge scores [F(1,110)=3.900, p=0.051] between female (M=6.11, SD=1.74) and male students (M=6.68, SD=1.89).

Another one-way ANCOVA was conducted to determine a statistically significant difference between the female and male students on PR-CC-CT knowledge scores controlling PR-CC knowledge scores. The results of this analysis indicated that there was also no significant difference in mean PR-CC-CT knowledge scores [F(1,110)=1.253, p=0.266] between female (M=1.99, SD=1.09) and male students (M=2.39, SD=1.31).

Academic Grade Point Average

The results of the analysis showed that there was no significant difference between the PR knowledge scores, t(111)=.152, p=.879 of the students having higher academic success (M=5.72, SD=1.14) and the students having lower academic success (M=5.69, SD=1.18).

In comparing PR-CC knowledge scores, a one-way ANCOVA was conducted to determine a statistically significant difference between the students having higher and lower academic success on PR-CC knowledge scores controlling PR knowledge scores. The results pointed out that there was no significant difference in mean PR-CC knowledge scores [F(1,110)=1.099, p=0.297] between the students having higher (M=6.43, SD=1.75) and lower (M=6.09, SD=1.84) academic success.

Another one-way ANCOVA was conducted to determine a statistically significant difference between the students having higher and lower academic success on PR-CC-CT knowledge scores controlling PR-CC knowledge scores. The results showed that there was also no significant difference in mean PR-CC-CT knowledge scores [F(1,110)=.148, p=0.701] between the students having higher (M=2.09, SD=1.11) and lower (M=2.10, SD=1.22) academic success.

Mathematical Modeling

The results of the analysis showed that there was no significant difference between the PR knowledge scores, t(111)=1.530, p=.129 of the students who took the mathematical modeling course (M=5.91, SD=1.04) and the students who did not (M=5.57, SD=1.21).

In comparing PR-CC knowledge scores, a one-way ANCOVA was conducted to determine a statistically significant difference between the students who took the mathematical modeling course and who did not on PR-CC knowledge scores controlling PR knowledge scores. The results pointed out that there was no significant difference in mean PR-CC knowledge scores [F(1,110)=1.977, p=0.160] between the students who took the mathematical modeling course (M=6.67, SD=1.85) and the students who did not (M=6.00, SD=1.72).

Another one-way ANCOVA was conducted to determine a statistically significant difference between the students having higher and lower academic success on PR-CC-CT knowledge scores controlling PR-CC knowledge scores. The results showed that there was a statistically significant difference in mean PR-CC-CT knowledge scores [F(1,110)=4.861, p=0.030] between the students who took the mathematical modeling course (M=2.47, SD=1.24) and the students who did not (M=1.85, SD=1.06).

Difficulties in PISA Mathematical Literacy Items

Before the interviews, item analysis was conducted to observe the item-specific difficulties. Table 3 indicated full credit, partial credit (if applicable), and no credit percentages.

Table 3. Full Credit, Partial Credit, and No Credit Percentages of Mathematical Literacy Items

Task and item	Full credit	Partial credit	No credit	Type of knowledge
Charts (item 1)	92.0%	-	8.0%	
Charts (item 2)	89.4%	-	10.6%	
Sauce (item 1)	91.2%	-	8.8%	
Ferris wheel (item 1)	70.8%	-	29.2%	PR
Which car? (item 2)	88.5%	-	11.5%	
Which car? (item 3)	64.6%	-	35.4%	
Revolving door (item 1)	74.3%		25.7%	
Charts (item 3)	87.6%	-	12.4%	
Sailing ships (item 2)	61.9%	-	38.1%	
Ferris wheel (item 2)	76.1%	-	23.9%	
Climbing Mount Fuji (item 1)	77.9%	-	22.1%	PR, CC
Climbing Mount Fuji (item 3)	42.5%	12.4%	45.1%	
Helen the cyclist (item 1)	86.7%	-	13.3%	
Helen the cyclist (item 2)	65.5%	-	34.5%	
Revolving door (item 3)	73.5%	-	26.5%	
Sailing ships (item 1)	85.8%	-	14.2%	
Sailing ships (item 3)	18.6%	-	81.4%	
Climbing Mount Fuji (item 2)	58.4%	-	41.6%	PR, CC, CT
Helen the cyclist (item 3)	24.8%	-	75.2%	
Garage (item 2)	7.1%	6.2%	86.7%	
Revolving door (item 2)	1.8%	-	98.2%	

Looking at the items that preservice teachers have had the most difficulties with, it is seen that there are questions used in three types of knowledge. The codes were obtained from the opinions of the preservice teachers who were asked why they had difficulties, especially in the questions that they have difficulty with: "not translating the contextual knowledge from a real-world setting to the mathematical model"," lacked reasoning about the context", "not understanding the context of the problem", "not creating arguments on these matters" constitutes the type of contextual knowledge theme. This theme is supported by the sample pre-service teacher views below. Preservice teachers were coded as PT01, PT02,…PT113.

"It was difficult to express the answer to the questions by supporting them with reasons for writing, and it was difficult to express them because the questions that required explanation in a linguistic logical cycle were challenging because we were used to explaining them with mathematical symbols." PT34

"We are used to mathematically using ready-made algorithms in our minds that I had a hard time interpreting according to the problem situation given to us" PT23

"For example, in the revolving door, maybe the math we will use is easy, but trying to interpret it with what is explained in the question was really challenging." PT48

It was observed that randomly selected preservice teachers taking elective mathematical modeling lessons made mostly operational errors in problems. When asked how they interpreted the questions that the majority of pre-service teachers could not do, it was found that they presented familiarity with "mathematical reasoning", "communication" and "modeling process". These issues are supported by pre-service teachers' representative views below.

"… It was not challenging to write a description for questions that contain the context of this daily life because I was able to interpret it more easily because we made these explanations in the presentation sections for each question in the modeling lesson." PT07

"The transition from real life to mathematical model in modeling questions, perhaps the dominance in comparing the model we created with real life situation helped me solve these questions" PT52

"The explanation part was not challenging, because I supported and presented our mathematical arguments by discussing in the modeling lesson, I can say that I have run similar processes in these questions." PT19

DISCUSSION & CONCLUSION

The mathematical literacy or competence notion in PISA (OECD, 2014) deals with the capacity of students to analyze, reason, and communicate efficiently as they pose, formulate, solve and interpret mathematical problems in a variety of situations. In this study, pre-service mathematics teachers' mathematical literacy skills were analyzed according to gender, academic grade point average, and mathematical modeling within the scope of procedural, conceptual, and contextual items. At the same time, semi-structured interviews were held to reveal their difficulties in solving PISA mathematical literacy items.

The results of this analysis indicated that there was no significant difference in mean PR-CC-CT knowledge scores, PR-CC knowledge scores, and PR knowledge scores between female and male students. The results showed that there was also no significant difference in mean PR-CC-CT knowledge scores, PR-CC knowledge scores, and PR knowledge scores between the students having higher and lower academic success. When it is examined according to whether or not to take an elective mathematical modeling course, the results of the analysis showed that there was no significant difference between the PR knowledge scores. Also, the results pointed out that there was no significant difference in mean PR-CC knowledge scores between the students who took the mathematical modeling course and the students who did not. However, the results showed that there was a statistically significant difference in mean PR-CC-CT knowledge scores between the students who took the mathematical modeling course and the students who did not.

The findings are consistent with Rittle-Johnson and Koedinger's (2005) theoretical approach, but they also show that categorizing PISA items according to mathematical knowledge types can aid in understanding the links between contextual, conceptual, and procedural knowledge. Contextual knowledge is shown to be a vital link between conceptual and procedural knowledge in the current study and thus serves as an important tool for solving the most difficult items, particularly those that require more than a simple reproduction of definitions and algorithms. In Rittle-Johnson and Koedinger (2005)'s study, it is argued that the benefits of real-life circumstances come because they give alternate approaches and/or informal solution methodologies, and therefore elicit improved problem comprehension. Furthermore, Saenz (2009) investigated the difficulties pre-service teachers experienced while solving PISA questions and reached the conclusion that evaluating the mathematics skills of pre-service teachers requires assessment of school mathematics as contextual, conceptual, and operational. The study showed that contextual knowledge about the real world is important in terms of establishing a relationship between the use of skills needed to solve PISA questions and traditional school mathematics expressed by concepts and processes. In the study, it was observed that pre-service teachers had difficulties in expressing their mathematical arguments about the solution and instead used their mother tongue. In addition, pre-service teachers, who were found to have difficulties in interpreting graphics, expressed their personal opinions about the context of the problem as a solution without any mathematical judgment about the question.

This contextual knowledge plays a key role in the following ways. It is possible to say that it provides vital guidance to students in the selection of several alternative techniques. Contextual information can help learners choose between alternate procedures that function in the same settings as procedural knowledge. Furthermore, contextual knowledge can aid learners in evaluating a cognitive network in a challenge in which the relationships between nodes are as significant as the discrete pieces of information that make up these nodes concerning the relevance of known procedures to real-world circumstances. It was discovered in this study that preservice teachers who did not take the mathematical modeling course and who have limited contextual knowledge about how things work in everyday life are more likely to choose incorrect procedures and fail to transform the known procedure for use in the specific situation. On the other hand, it was discovered that preservice teachers who took the mathematical modeling course gained experience in questions involving contextual knowledge about how things work in everyday life, which may have led them to choose correct procedures more frequently and successfully transform the known procedure for use in the specific situation. The result of this study was highlighted in Widjaja (2011)'s study with pre-service teachers in which the potential of PISA questions for pre-service teachers were investigated. It has been concluded that such contextual questions offer various strategies and that such questions enable pre-service teachers to experience the power of mathematics in their daily life contexts. It was also stated that giving contextual questions and mathematical modeling as part of the pre-service teachers' training will increase the capacity of future teachers to plan and apply lessons to improve mathematics literacy and these kinds of problems should be included in teacher education programs. As a result, it was stated that providing pre-service teachers with such learning experiences in their education would better equip them to use their mathematical knowledge and skills in real life. In light of these studies, the fact that the mathematical modeling course is a compulsory course in Turkey's renewed program in elementary mathematics education can be considered a positive development in this respect. In this sense, teaching through comprehension and promoting the achievement of new educational goals such as the development of higher-order thinking skills, particularly problem-solving skills, should be at the forefront in the classrooms, rather than teaching based on the transmission of knowledge. Mathematics instructors' knowledge and practices in terms of issues, results, and viewpoints should be founded on contextual knowledge. Effective teachers are required to help students employ techniques and build their own strategies that can help them solve many types of problems.

Findings from semi-structured interviews highlight the challenges, particularly in argumentation and communication and modeling competencies. In the studies in the literature, this situation was handled from different perspectives. It is seen that the difficulty of expressing mathematical expressions arising in this study is handled in different studies. According to the studies, linguistic challenges appear to be one of the most important factors contributing to students' low performance on specific mathematical tasks (Movshovitz-Hadar, Zaslavsky & Inbar, 1987; Radatz, 1980). Since PISA poses challenges by presenting a situation of a personal, employment, or scientific nature, it appears normal to believe that linguistic difficulties can have a substantial impact on PISA competencies such as argumentation and communication, which require a command of the natural language. Furthermore, PISA is based on certain competencies evaluating students' total ability (mathematical literacy). Competencies are cognitive processes that need to be triggered in order to connect the actual world with mathematics and solve the problem at hand. These issues, according to the pre-service instructors in this study, are related to the difficulty in formulating and articulating mathematical reasoning, as well as expressing oneself in a variety of ways on matters of mathematical content, and monitoring and managing the modeling process. These signs point to argumentation, communication, and modeling skills. Knowing what mathematical proofs are and how they differ from other sorts of mathematical reasoning, following and evaluating chains of different types of mathematical arguments, developing intuitive processes, and inventing and articulating mathematical arguments are all part of argumentation. Communication entails expressing oneself in a number of ways, both orally and in writing, on subjects with a mathematical element, as well as understanding others' written or oral remarks on such matters. Modeling entails structuring the situation to be modeled, translating "reality" into mathematical structures, working with a mathematical model, validating the model, reflecting, analyzing, and criticizing the model and its results, effectively communicating about the model and its results, and monitoring and controlling the modeling process. Taking into account the content of these competencies, which includes the problems that were mentioned by preservice teachers in this study.

Statements of Publication Ethics

Ethical permission for the research was approved by Niğde Ömer Halisdemir University Ethics Committee. The ethics committee document number is 2021/15-01.

Researchers' Contribution Rate

All authors contributed equally rate to the research.

Conflict of Interest

The authors confirm that there are no conflicts of interest associated with this study.

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