Ekim 2013 Cilt:21 No:4 (Özel Sayı) Kastamonu Eğitim Dergisi 1317-1326

INTERPRETATION OF UNKNOWN AND VARIABLE PRIOR TO FORMAL ALGEBRAIC INSTRUCTION

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İlk Kayıt Tarihi: 02.09.2012 ***Yayına Kabul Tarihi: 03.01.2013***

Abstract

The purpose of the study is to investigate the sixth grade students' use of unknowns and variables prior to any formal algebraic instruction. Three students were presented with a questionnaire including the manipulation of unknowns and variables. The students showed the ability to interpret algebraic expressions such as solving equations and finding patterns. It is a crucial point that they defined both of the term of unknown and variable as unknown. In this study it was examined the grounds of this use.

Keywords: *algebraic instruction, transition from arithmetic to algebra, sixth grade level*

CEBİR ÖĞRETİMİNE GEÇİŞTE BİLİNMEYEN VE DEĞİŞKEN KAVRAMLARININ YORUMLANMASI

Özet

Bu çalışmanın amacı, altıncı sınıf öğrencilerinin cebir öğretimine geçiş aşamasında bilinmeyen ve değişken kavram kullanımlarını nasıl kullandığını incelemektir. Üç öğrenciye, bilinmeyen ve değişken kavramlarını içeren bir ölçek sunulmuştur. Sonuçlar, denklem çözme ve örüntü bulma gibi cebirsel ifadelerde, öğrencilerin bilinmeyen ve değişken kavramlarını bilinmeyen olarak algıladıklarını göstermektedir.

Anahtar Kelimeler: *cebir öğretimi, aritmetikten cebre geçiş, altıncı sınıf düzeyi*

1. Introduction

Taking into account the transition from arithmetic to algebra; variables and unknowns, equations and equation solving, and the algebra word problems have been research areas from 1977 on the learning and teaching algebra (1). Several researches have reported that there have been many learning difficulties in algebraic equation solving (2; 1), the conception of variables (3; 4), and the meaning of unknowns (5; 6). These overall researches show that understanding of algebra relies on generally students' experience in arithmetic.

The cognitive gap between arithmetic and algebra, "the students' inability to operate spontaneously with or on the unknown" (p. 59), focuses on students' arithmetic background that points to some of the problems in early algebra (2). Even though students' prior experience with arithmetic problems enables to understand the meaning of unknown (6); the interpretation of unknown leads to its function. In essence to understand unknown, the relation with variable should be considered.

(4) intended that purposes of algebra are defined by conceptions of algebra and uses of variables. According to him, uses of variables are varied in their different conceptions. For instance, when conception of algebra means to solve certain problems; variables are unknowns and constants. Also, the notion of variable is defined by (7) as something that can vary. He noticed that the term of variable is used in an equation when students are first introduced to find the unknown. For example, x is called as variable in the equation 3x +4=19 where there is only one value of variable (7). It is important to say that students learn the term of variable when solving the equation with one unknown that does not vary actually.

(8) defined the variable as a basic tool for representing generalization. They concluded that pattern approach could facilitate introducing the variable. In this way, pattern activities might provide symbolic representation including variables. Therefore, seeing pattern becomes essential to establish algebra and generalization (9). However, activities provide students to study with letters as hidden numbers; the generalization let them use letters as variables (10). At this point, (11) integrated together the meaning of unknown, variables, equations and pattern approach, "…The algebraic concepts of unknown and equation appear to be intrinsically bound to the problem-solving approach, and that the concept of variable and formula appear to be intrinsically bound to be the pattern generalization approach" (p.111).

1.1. Purpose of the Study

This study is aimed to explore how the sixth graders might respond to the task of manipulating unknowns and variables before having any formal algebraic instruction. It is presented equations and patterns that are required to understand the uses of unknown and variables. In this conception, the following research questions are investigated:

1. How do the sixth grade students interpret unknowns and variables in an algebraic expression prior to formal algebraic instruction?

2. What kind of prior knowledge do the students have to solve algebraic equations?

3. Are the curriculum and the mathematics textbooks related to the interpretation of unknowns and variables?

It is assumed that the content of the mathematics textbooks and the curriculum standards would be related to the students' definitions and uses of unknowns and variables.

2. Method

2.1. Participation and Data Collection

(12) proposed that a particular subgroup could give comprehensive information in the qualitative study. The interview was conducted with three 6th grade students who did not have any formal algebraic instruction. They were in the same class which was thought as the particular group. As their mathematics teacher reported, they were above-average students in mathematics course. To sum up, it was expected that it could be acquired the detailed information about their thinking.

The questionnaire was designed that was to be solved by the students. The questionnaire consisted of equations and patterns that were representative of examples and problems in the 6th grade mathematics textbook. Mathematics textbook was taken into the consideration in this study because the textbook is primarily teaching material to decide the lesson objectives and the instructional activities (13).

2.2. Data Analysis

The verbal data was gathered from video recording and then compared with the written data in the students' documents. The students were asked to think aloud while responding the questions. Also, the interviews gave insights into the students' thinking and explained the underlying the written solutions that were collected from the documents.

Data analysis consisted of two steps; in the first step of the analysis, it was transcribed the verbal data and discussed the process, that is, to understand how students gave the information while responding the questions in the questionnaire. Since all questions were solved by all three students, it was focused on the uses of unknowns and variables as well as on the content of the textbooks. Next, the codes were developed that described the content in the interviews. Therefore, the data was analyzed in three parts which are based on the uses of unknown, the uses of variables, and the pattern approach in turn.

3. Results

3.1. Part 1: Uses of Unknown

The first example (see Appendix A) describes an equation solving approach that emphasizes using letters as unknowns. The students were said to "inverse operation" in the solution of this example. They reviewed their arithmetic facts about solving the equation while responding. The students' answers indicated that their arithmetical background was sufficient to solve this question. The following is the excerpts from the interview:

Student1

Student1: Here, we are going to get to 6, by multiplying this number with 3 and then subtracting it from 30.

Instructor: What are you thinking about?

Student 1: I am thinking about how I can get to 6 by multiplying and subtracting from this number.

Student2

Student2: In order to define the square, we need to subtract 4 from 25, then divide it by four, and the result is 7.

Student2: Here we are going to add 8 to 36. We will subtract 6 from 30. This is the way I followed in all three questions, look for the equanimity. We perform inverse operations. Number 2 is the same, number3 looks a little different. What number do we need to subtract to get to 6? Actually, it is the same pattern.

Student3

Student3: Square multiplied by three and then 3 added equals to 25. First, I subtract 4 from 25, which equals to 21. Then, there is the 3, I divide 21 by 3.

I will add 36 to 8, which equals to 44. Then, there is 4 multiply. That is why, I divide 44 by 4. The triangle is 11, and the square is 7. Now to the last one. If I multiply 3 with 2, I get to 6. But there is the subtraction from 30. Therefore, I will subtract 6 from 30, which equals to 24. Then I will divide 24 by 3, which equals to 8. That means "a" is 8.

Instructor: What did you use when solving all these?

Student3: I performed inverse operations in all three equations and I got to the results.

This result does not seem surprising because students' experience points out the prior arithmetical knowledge. Essentially, (2) appraised that it can be handled as algebra because the place holder is used as an unknown.

Interviews show that the students have already faced linear equations with one unknown of the types ax+b=c. However, the solution process seems as arithmetic; (14) supposed that it is algebra because students have some understanding of unknown and equation. When the students were asked the meaning of some terms such as square, triangle, and the letters "a and x", and also they were required to compare the uses of these shapes and letters; they answered that these were unknown. For example, Student1 explained in the first question:

Student1

Instructor: You have used some terms such as square, triangle, and a. What are they called?

Student1: They are the term of unknown.

In the second question of finding the value for $\Delta=3$, Students2 clarified that this was unknown. Also, when students compared the question 1 and 2, Students3 explained that unknowns were used in these questions.

Student2

Student2: Are we going to use 3 in this one? Yes, I think so. Then it is 11.

Instructor: What does it mean? The square equals to 3.

Student2: It means the term of unknown here is 3. When we use 3 in this one, it equals to 11. If we put 3 next to 3, it is 33.

Instructor: Does it make any difference if it is "x" or square or triangular?

Student2: It doesn't make any difference since it is the term of the unknown.

Student3

Instructor: Which of these questions are similar?

Student3: 1 and 2

Instructor: Why?

Student3: In 1 and 2 the term of the unknown is symbolized with shapes such as square or triangle, therefore they may be similar

The students were aware of the uses of shapes and letters because they could represent that there was no difference between the uses of the letter "x" and the square or triangle. However, such kinds of uses are related to the uses of variables; the students claimed that they were unknowns.

3.2. Part 2: Uses of Variables

During the interview it was not observed that the students used the term of variable; even though they used the term of unknown frequently. They felt the uses of variable but they could not declare the discrimination of them. They explained in the following:

Student1

Instructor: In this question (points to question 4), "a" is 5 in one, 15 in the other, and 24 in the other. What does it mean?

Student1: It does not use a certain number for "a", that is, it is different.

Instructor: OK. Which of the above questions do you think question 4 can be considered as similar from the usage point?

Student1: From the usage point, I think it is question 1.

Student2

Instructor: If we compare the questions which ones look alike?

Student2: Question 1, 2 and 3 look alike. The terms of the unknown are symbolized with shapes. Questions 2 and 3 look alike, and questions 1 and 4 look alike.

Instructor: Why do you think questions 1 and 4 are alike?

Student2: In these questions it asks the numbers to be used instead of the square, triangle and "a", that is the similarity.

Students3

Instructor: Does it make a difference if we used "a" or square or triangle to symbolize the term of the unknown? In question 3 we used "x".

Students3: Well, it is the same. But in comparison, they look more similar. "x" is used for the term of the unknown. It is the same here. I think there is a common point in all four questions, because the term of the unknown is symbolized by different shapes such as "a", "x" triangle and square.

Instructor: Good.

When the students compared the question 1 and 4, they might have been allocated according to the letters' different value because in the question 2 and 3 there was a specific value. Student3 pointed out that these all four questions have a general sense; that is, it might be indicated the uses of variable. These are showed that unknowns and variables are particularly treated as facets of a single concept of variable without any explicit representation (3).

When the curriculum is analyzed from this perspective, the term of variable is not used from first grade through fifth grade. There is often used the expression of "hidden numbers" (15, p.145, p.146, p.200, p.254). They are indicated to the use of unknown. For example; in the 3rd grade mathematics program is given: "Symbols □, Δ, □ e.g. are used as hidden numbers" (15, p.145), and the 4th grade program is addressed: "Different shapes and letters are used as hidden numbers" (15, p.200).

3.3. Part 3: Pattern Approach

When it is analyzed the 5th grade mathematics textbook, there are several examples about pattern finding. Therefore, the students seemed more comfortable while finding the pattern. (16) suggested that finding pattern and generalizing a rule from situation has a positive effect on investigating process and thinking algebraically throughout the middle grades. Therefore, it can be predicted that they would enhance their algebraic thinking in the following grades. They reflected their generalization with this example:

Student1

Instructor: There is a, say, 10th or 20th step ahead. I am calling it "step a" without giving it any number.

Student1: Let's skip the in-between numbers. The number of the shapes for the 20th step house is 3 plus 20.

Instructor: What does it mean then to fill the table for "step a"?

Student1: It means, is the number of shapes that form the house according to the step number, plus filling the number of the clouds and the total number of shapes in the table.

Student2

Instructor: How can I define it in table? How would I fill it if I said "step a"?

Student2: Then, I could the draw the shapes on my own. I would draw the house and the chimney, and put the clouds according to the increase rate.

Instructor: Which is?

Student2: 7

Instructor: According to what?

Student2: 7 clouds for 7th step, which totals to 10 for total number of shapes.

Instructor: In accordance with the increase rate, how would you fill the table for 20th step?

Student2: This is 3, plus 20, which is 23.

Student3

Instructor: Let's call a step "step a".

Student3: What it means with "step a" is a step after 5th. Since it is one step after 5th, it will be the 6th step. The 6th step, or "step a", it is 6 clouds; total number of shapes is 9.

Instructor: So you say, "step a" means the 6th step?

Student3: I think it does. Any among these could be "step a".

Instructor: Could 1st or 2nd step, for instance, be "step a"? Or can I call a step after the 5th "step a"?

Student3: Since it is "a step", it can be the 1st or 2nd or 4th. I considered "step a" as the 6th step because it comes after the 5th. That is why it is 9, since we would have 6 clouds plus the house shapes – the body, roof and the chimney.

Instructor: What would you say if it was the 20th step?

Student3: If it was the 20ts step... There is a correlation between the step and the cloud numbers since they increase accordingly. That is, 20th step will have 20 clouds, the shapes that form the house are 3, which totals to 23.

From these excerpts, the students could not display the generalization for any step whatsoever. They offered specific values for the step. It is suggested that they are not ready for such generalization. Essentially, they would learn and use this in the following semester. In addition, they could verbalize their generalizations easier than write them symbolically. (8) observed that it is not surprising because students are more experienced in such patterning activities.

4. Discussion

This study indicated how the sixth grade students used unknowns and variables in equations and patterns. It was observed that the students' prior learning experience could facilitate the transition from arithmetic to algebra. The students' arithmetical background helped them interpret the algebraic expressions, however; the algebraic expressions were new to them. On the other hand, we should consider that such kind of experiences could not be always useful for students' interpretations of algebraic symbolism because the uses of letters are changed with respect to their context (17).

It is remarkable that the term of unknown is not used in the curriculum from first grade through fifth grade, however; the students almost used it in all their responses. The term of unknown could be seen in sixth grades but the first introduction is not so clear. It is given an explanation about it in the sixth grade mathematics textbook; "a is defined as unknown or variable in the expression of a+3" (18, p.126). The limitation of the definition can cause difficulties in learning algebra over the long term. Therefore, to keep from the lapses in attention to its meaning, the transition from arithmetic to algebra should be coordinated with the arithmetical language (19).

Another point to consider is the uses of variable. The students had knowledge about patterns which they have learned at fifth grades. In contrast, they did not mention the term of variable, however; the concept of variables generally appears in the pattern approach. In addition, patterns are taken place in the curriculum from first grade through eighth grades but the term of variable is not well characterized in them.

While the students could not manipulate the uses of variable, they could not represent the generalization of pattern. To support the understanding of the generalization, different conception of variables might be helpful. (4) considered the variable as pattern generalizers in the conception of generalized arithmetic and as unknowns in the conception of solving certain problem. From this way, the uses of different conceptions of variable might facilitate the understanding of unknowns, variables as well as generalization.

5. References

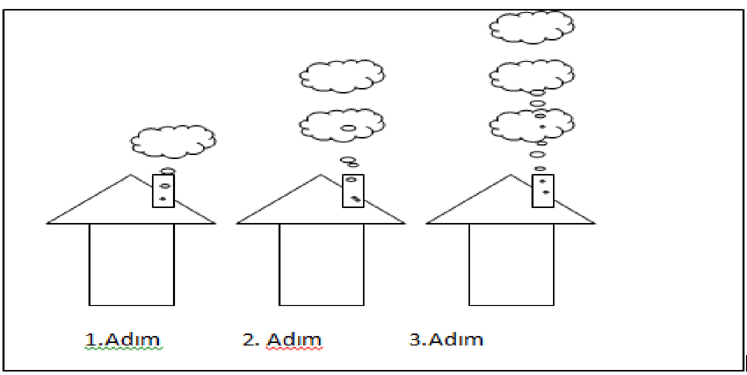
1. Kieran,C.,(2006). Research on the learning and teaching of algebra. In A.Gutierrez & P.Boero (Eds.), Handbook of Research on the Psychology of Mathematics Education: Past, Present and Future (pp. 11-49). Rotterdam: Sense Publisher.
2. Herscovics,N., Linchevski,L.(1994).A cognitive gap between arithmetic and algebra. Educational Studies in Mathematics, 27(1), 59–7.
3. Trigueros, M, Ursini, S. (2003). First-year undergraduates' difficulties in working with different uses of variable. In E. Dubinsky, A. H. Schoenfeld, J. Kaput, (Eds.) Research in Collegiate Mathematics Education (CBMS Issues in Mathematics Education, Vol. 12, pp.1-29). Rhode Island: American Mathematical Society.

4. Usiskin, Z. (1988). Conceptions of school algebra and uses of variables. In A. Coxford & A. P. Schulte (Eds.), The ideas of algebra, K-12 (1988 Yearbook: pp. 8-19). Reston, VA: NCTM.
5. Chalouh,L. & Herscovics, N. (1988). Teaching algebraic expressions in a meaningful way. In A.E.Coxford (Ed.), The ideas of algebra, K-12(1988 Yearbook, pp.33-42). Reston, VA: NCTM.
6. Stacey, K. & MacGregor, M. (2000). Learning the algebraic method of solving problems. Journal of Mathematical Behavior, 18 (2), 149-167.
7. Foster, D. (2007). Making meaning in algebra examining students' understandings and misconceptions. In A.H. Schoenfeld (Ed.), Assessing Mathematical Proficiency (MSRI Publications , Vol. 53, pp163-176). NY: Cambridge University Press.
8. English, L. D. & Warren, E. A. (1999). Introducing the variable through pattern exploration. In B. Moses (Ed.), Algebraic Thinking, grades K-12: readings from NCTM's school-based journals and other publications, (pp. 141-145). Reston, VA: NCTM.
9. Zazkis,R. & Liljedahl, P. (2002). Generalization of patterns: the tension between algebraic thinking and algebraic notation. Educational studies in mathematics, 49, 379-402.
10. Lee, L. (1996). An initiation algebraic culture through generalization activities. In N. Bednarz, C. Kieran & L. Lee (Eds.), Approaches to Algebra. Perspectives for research and teaching (pp. 87-106). Dordrecht : Kluwer.
11. Radford, L. (1996). Some reflections on the teaching algebra through generalization. In N. Bednarz, C. Kieran & L. Lee (Eds.), Approaches to Algebra. Perspectives for research and teaching (pp. 107-111). Dordrecht: Kluwer.
12. Patton,M .Q.(2002).Qualitative research and evaluation methods(6th ed.), Thousand Oaks, CA : Sage.
13. Borko,H. & Livingston,C. (1989). Cognition and Improvisation: Differences in Mathematics Instruction by Expert and Novice Teachers .American Educational Research Journal, 26(4).pp. 473-498.
14. Herscovics,N. & Kieran, C. (1999). Constructing meaning for the concept of equation. Introducing the variable through pattern exploration. In B. Moses (Ed.), Algebraic Thinking, grades K-12: readings from NCTM's school-based journals and other publications, (pp. 181-188). Reston, VA: NCTM.
15. Milli Eğitim Bakanlığı Talim ve Terbiye Kurulu Başkanlığı. (2005). Ilköğretim matematik dersi öğretim programı ve kılavuzu (1-5. sınıflar). Ankara: Devlet Kitapları Müdürlüğü.
16. Herbert, K. & Brown, R.H. (1999). Pattern as tools for algebraic reasoning. In B. Moses (Ed.), Algebraic Thinking, grades K-12: readings from NCTM's school-based journals and other publications, (pp. 123-128). Reston, VA: NCTM.
17. Stacey,K. & MacGregor, M. (1999). Ideas about symbolism that students bring to algebra. In B. Moses (Ed.), Algebraic Thinking, grades K-12: readings from NCTM's school-based journals and other publications, (pp. 308-312). Reston, VA: NCTM.
18. Aktaş,Ş., Atalay,A., Aygun,S., Aynur,N., Bilge,O, Celik,M., Çuha, Karaman,U., Öcal,İ., Öncü,F., Özçelik,U., Uluğbay,M., Ünsal,N. (2006). İlköğretim matematik 6 ders kitabı, Ankara: Devlet Kitapları.

19. Wagner, S. (1999). What are these things called variables? Ideas about symbolism that students bring to algebra. In B. Moses (Ed.), Algebraic Thinking, grades K-12: readings from NCTM's school-based journals and other publications, (pp. 316-320). Reston, VA: NCTM.

6. Appendix A

1. Aşağıdaki ifadelerde , Δ , a yerine ne gelmesi gerekir?
   3. □+ 4= 254. Δ - 8 = 36
   30 – 3.a = 6
2. Aşağıdaki her bir ifadenin Δ=3 için değerini bulunuz.
   Δ+9=
   3Δ=
   2(Δ-1)=
   Δ^2 =
3. Aşağıdaki her bir ifadenin x=2 için değerini bulunuz.
   4x=
   3(x-1)+(x+2)=
   x^3 =
4. Aşağıdaki sayı sisteminde "a" yerine gelebilecek sayılar nelerdir?
   1 , 3 , a , 7 , 9
   3 , 9 , a , 21 , 27
   12 , a , 36 , 48 , 60



Tabloda evi meydana getiren geometrik şekiller ve bacadan çıkan bulut sayısı verilmektedir.

Adım	Evi oluşturan şekillerin sayısı + bulut sayısı	Toplam şekil sayısı
1		
2		
3		

a) 4. ve 5. adım için tabloyu doldurunuz.
b) Herhangi bir adımımıza "a" diyelim. Bu durumda a .adım için tabloyu doldurunuz.
c) Adım ve toplam şekil sayısı arasındaki ilişkiyi yazarak ifade ediniz.