A STOCHASTIC-OPTIMIZATION MODEL FOR DETERMINING THE OPTIMAL MICRO-SITING OF WIND TURBINES

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Abstract: We propose a general model for the placement of wind turbines in a rectangular grid formation over a flat area. For better realism, we consider stochastic wind speeds and directions, in conjunction with the wake effects that upstream turbines impose on downstream ones. The objective is to pack as many turbines as economically optimal in a given area, i.e. to maximize the expected MW output per dollar of capital investment and O&M costs per meter square. Due to the complex structure of the mathematical model, we apply a hybrid approach of Monte-Carlo sampling of wind speeds and directions together with the Nelder-Mead heuristic method to search for the optimal horizontal and vertical spacing of the turbines. Results of a case study based on a real dataset of wind speeds and directions, a selected commercial turbine's approximated power curve, and industry estimates of costs is discussed.

Keywords: wind energy, wind power, wind investment, micro-siting, wake effect

Introduction

Wind energy has gained great attention because it represents an important option for reducing the reliance on hydrocarbons for energy production, especially for electricity. With the current technology, one challenge faced in wind farm design is the appropriate placement of the individual wind turbines in order to optimally harvest the energy from the wind. Grouping the turbines leads to a reduction of the power produced by the downstream wind turbines due the so-called "wake" effect. That is, if a turbine is within the area of the turbulence caused by another turbine, or the area behind another turbine, the wind speed suffers a reduction, and therefore, there is a decrease in its electricity production. On the other hand, spacing the turbines too far apart requires a larger land area for the wind farm, which may not only bring added costs but also be just infeasible.

Because of the considerable amount of power loss due to the wake effects in a large wind farm, it is considered one of the main issues that should be focused on when optimizing the wind turbine position. Several studies have been conducted in recent years in order to maximize energy production and the efficiency of the turbines. These studies have focused primarily on obtaining the optimal placement of turbines within a wind farm, based on their output efficiency.

To state this problem, we define two objectives: (i) maximization of energy production, and (ii) minimization of the total cost and land area requirements. In the present research we use an objective function that represent the expected MW output per dollar of capital investment and O&M costs per meter square of land (i.e. MW/\$m^2). By use of the Nelder-Mead heuristic method adapted to stochastic wind speeds and directions, we search for the optimal vertical and horizontal spacing (respectively in the North-South and East-West coordinate axes) of a rectangular grid of wind turbines. That is, given a fixed number of turbines in a rectangular formation, the goal is maximize the area-density of the expected power return on the investment.

The remainder of the paper is organized as follows. Section two formulates the model of the wake model used in the wind farm. Section three presents describes the wind farm layout optimization model, including such components and parameters as the power curves, the wind direction and speed distribution, and cost estimation. Section four presents the two case scenarios and discusses the results of the optimization process. Finally, in section five we present our conclusions and discuss futures avenues of research on the problem.

The Wake Model

Wind turbine wakes are generally divided into three different regions, as described in Figure 1. These are the near wake region, the transition region, and the far wake region. This model assumes the wake region is an empirical linear expansion region, Jensen Model, starting from behind the wind turbine, as shown in Figure 2. Here the model is characterized by a uniform velocity profile at any distance x in the downstream behind the turbine.

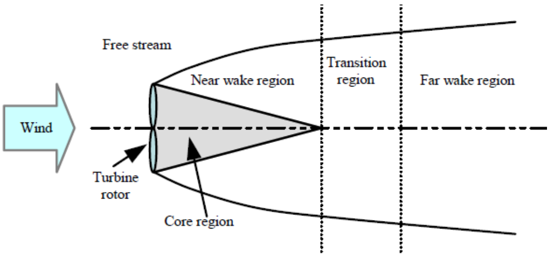

Figure 1: Wake Regions Behind a Wind Turbine

To start analyzing the model we first write the wind speed at a distance x behind the turbine. Let us consider Figure 2 again, where we have a turbine T_0 generating a wake region. We can write the equation of the wake radius at distance x from turbine i as:

$$r_x = r_0 + \alpha x \tag{1}$$

Where r_0 is the rotor radius of turbine T_0, and α is a decay factor expressing how fast the wake breaks down. An analytical equation is given for α concerning the height z of the turbine generating the wake and the constant surface roughness z_0, which depends on the terrain characteristics in the form of

$$\alpha = \frac{0.5}{\ln\left(\frac{z}{z_0}\right)}. \tag{2}$$

If we consider v_0 as the ambient wind speed and v_1 as the wind speed at distance x, then we can write a balance of mass equation as

$$\pi r_0^2 u + \pi(r_x^2 - r_0^2)v_0 = \pi r_x^2 v_1, \tag{3}$$

where u is the wind speed just behind turbine T_0.

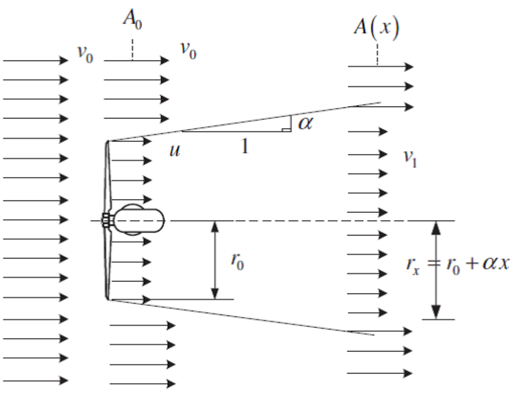

Figure 2: Single Wind Turbine Wake Model

A study of the aerodynamics of wind turbines provides us with an analytical equation connecting the ambient speed with the wind speed right behind the turbine. From this we can write

$$u = \sqrt{1 - C_T}\, v_0,$$

Where the term C_T is the thrust coefficient of the turbine. Solving the previous equation for v_1, we obtain

$$v_1 = v_0\left(1 - \left(-1\sqrt{1 - C_T}\right)\frac{r_0^2}{r_x^2}\right). \tag{4}$$

The previous equation expresses the wind speed at distance x behind a turbine T_0 when the radius of the wake at that distance is r_x. In case of a wind farm, where we have two turbines, i and j, and Turbine j is in the wake region of Turbine i, then the wind speed affecting Turbine j is given by

$$v_j = v_0\left(1-\left(1-\sqrt{1-C_T}\right)\frac{r_i^2}{r_j^2}\right). \tag{5}$$

From Equation 5 we can see how the speed received by Turbine j is reduced due to the wake effect of Turbine i. Therefore, the power output of Turbine j will be reduced too.

Equation 5 gives the resulting wind speed at a given turbine when we considered that the turbine is totally covered and affected by the wake effects. Which means that the wind speed is the same over all the turbine surface. But in wind farm we can find partial shadowing which means the turbine surface is partially in the wake area and not totally. Therefore, the turbine will be affected by different wind speed at its rotor's sweep area and the calculation of the power output needs more attention. To this end, the speed from the wake effects affecting the turbine can be rewritten to take the partial shadowing into account:

$$v_x = v_0\left(1+\left(\sqrt{1-C_T}-1\right)\frac{r_i^2}{r_j^2}\frac{A_{jshadowed}}{A_j}\right), \tag{6}$$

where A_j is the total area of the turbine and $A_{shadowed}$ is the shadowed area by the affecting turbine. Here we must note that there are different shadowing possibilities between the two turbines: complete shadowing, quasi-complete, partial shadowing and no shadowing. Therefore before applying the formula we must determine the type of shadowing depending on the horizontal and vertical distances between the two turbines and the direction of the wind. This study considers these detailed geometric calculations, whose details are not presented here.

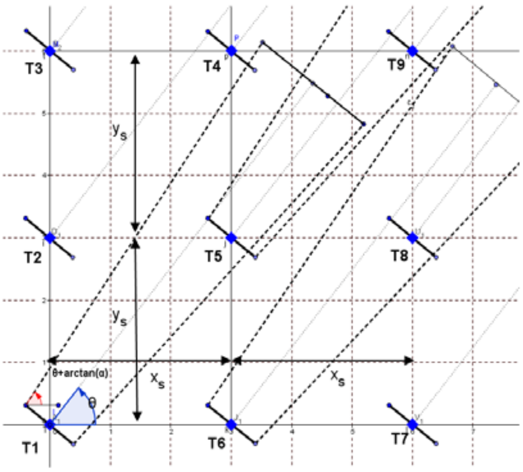

Figure 3: Bird's Eye View Illustration of Multiple Wakes within a Wind Farm

In a wind farm each turbine will produce a wake area, as shown in Figure 3. This will cause a turbine to possibly be under the effect of multiple wakes caused by different turbines. Therefore, a method to combine the different single wakes effects is required. One of the used methods is the sum of squares of velocity deficit, and this method is useful here since it provides a formula for the deficit wind speed as

$$\delta v_{xjdiff} = 1-\frac{v_x}{v_0} = \left(1-\sqrt{1-C_T}\right)\frac{r_i^2}{r_j^2}\frac{A_{jshadowed}}{A_j}. \tag{7}$$

And combining the multiple wakes effects, we obtain

$$\delta v_{xidiff} = \sqrt{\sum_{j\in U_i}\left(\delta v_{xjdiff}\right)^2}, \tag{8}$$

Where U_i is the collection of turbines affecting turbine i by its wake. Then the total speed at Turbine i will be given by

$$v_i = v_0\left(1-\delta v_{xidiff}\right). \tag{9}$$

Combining our calculations, we conclude that the wind speed affecting Turbine j is given by

$$v_i = v_0\left(1 - \sqrt{\sum_{j\in U_i}\left(\left(1-\sqrt{1-C_T}\right)\frac{r_i^2}{r_j^2}\frac{A_{jshadowed}}{A_j}\right)^2}\right). \tag{10}$$

Wind Farm Layout Optimization Model

We consider a rectangular grid of a certain number of turbines where they are separated by a distance of x on the horizontal dimension and a distance of y on the vertical dimension (see Figure 4). For this study, we simply take the horizontal axis to be the exact East-West direction and the vertical axis to the North-South direction. While it is possible to choose the axes in a arguably more efficient fashion (e.g., setting the vertical axis to be the speed-weighted mean of directions), or even defining an additional optimization variable for the "tilt" angle of the rectangular layout, these generalizations are left as a topic for further study. The objective is to find the most optimal values of x and y with respect to the objective of maximizing the economic return per grid area as explained previously.

As also shown in Figure 4, the stochastic wind blows from the direction θ and with a speed of $v(\theta)$ (i..e whose distribution depends on θ) at a prespecified height equal to the turbines' hubs. We also assume here that the wind's direction and magnitude applied uniformly all over the wind farm in the vertical and horizontal planes. In other words, each turbine is assumed to receive a uniform wind direction and speed over its entire rotor speed area and all turbines receive the same direction and speed regardless of their position on the grid. In reality wind's speed usually increases with height (i.e. upper sections of the turbines sweep area will receive higher speeds) and, due to the considerably high volatile nature of wind, distant locations in a given wind farm can be facing significantly different speeds (and perhaps directions) as wind travels throughout the farm's considerable large area. Nevertheless, the uniformity assumption that we make are not just for mathematical simplicity, but, rather, because of the limited time- and height- resolution of typical wind data and also since the errors should partly cancel each other out. In our model, the turbines still receive different speeds due to the complex wake effects occurring within the wind farm. Given the "adjusted" speed for each turbine, we use the turbine's specific "power curve" to estimate its power output of the turbine.

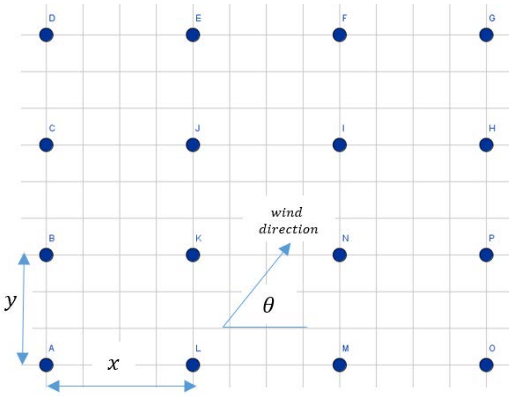

Figure 4: Wind Farm Grid Layout

The power curve of a wind turbine describes the turbine's generated power versus the wind speed, $p = f(v)$, when the wind is perpendicular to the rotational plane. When the speed of wind reaches a threshold of so-called "cut-in" value, the turbine starts generating power, and as the speed increases the power generated increases non-linearly to its maximum value at the so-called "saturation" speed (or, rated speed). Due to structural stability and other concerns, the turbine is regulated to generate a steady maximum (or, rated) output between the saturation speed and the so-called "cut-out" speed, beyond which the blades are stopped as operating the turbine at such "storm" winds poses mechanical and safety hazards. Based on the theory laid out in Betz's Law and also empirical evidence, the increasing portion of the power curve between the cut-in and saturation speeds is often modeled as a cubic function of the form cv^3, where c is a constant depending on the turbine's mechanical properties and of the air flow (density, temperature, etc.) and v is the wind speed. In reality, however, the increasing portion begins deviating from the cubic behavior and curls rather smoothly the maximum plateau level. The shape of the increasing section of the power curve is suitably described by a (approximately cubic) convex lower section and a concave and flattening upper-section (see Figure 5).

For this study, we considered several different models of Vestas turbines and we approximated their power curves by fitting suitable polynomials to their discrete speed-power data that was available from the manufacturer. Two-piece polynomials were fitted (using Mathematica 9) to the increasing portion of the data –one to the convex lower half and a another to the concave upper half. The polynomials vary between 4 and 8 in degrees in order to yield an exact fit to the data points and some degree of subjectivism was involved in identifying the two separate portions in order to obtain the best fit. The power curve fitting results for 8 Vestas models of various rated powers and other parameters are shown in Figure 5. For our case study, chose to use the Vestas V90 with 3MW maximum power output.

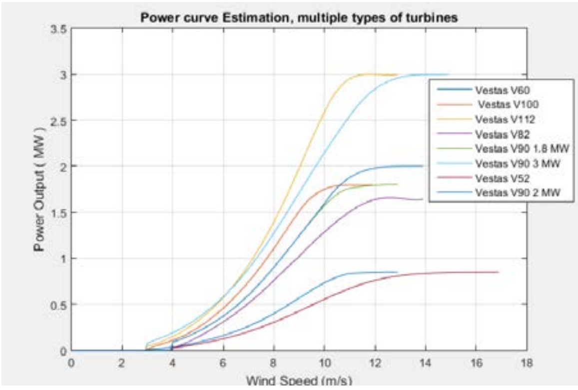

Figure 5: Estimated Power Curves for Several Vestas Wind Turbine Models

For a better realism of the optimization process, we consider stochastic wind speeds and directions. Wind speed and direction measurements collected by the YEGM (Turkish Renewable Energy General Directorate) at various sites Turkey during 2003-2011 were obtained. The wind data is comprised of 10-minute apart measurements of speed and direction at 30 meters height. The total length of the measurement period (i.e. number of years) vary between sites, and the data had to be cleaned rigorously for missing or obviously erroneous values. To assure continuity of the used data, two suitable sites were chosen to be used in the case study. One is the Biga site, which is near the same-named town of the Çanakkale province in the Marmara region of Turkey. The second is the Tavas site, which is a town of the Denizli Province in the inner Aegean region of Turkey. The Biga site was selected partly because it is one of the strongest-wind sites in Turkey. The Tavas site, on the other hand, can be classified as a moderate-to-low wind site. This choice of this particular site pair is motivated by comparison purposes. We use one year's length of data for each site, yielding a total of 52704 (10 minutes data over 366 days) speed-direction data points for each site. Moreover, since the data were taken at a 30 meters height, they were converted to a more meaningful hub height of 100 meters using the standard Equation 13 given below. In Equation 13, α is an empirically derived coefficient that varies dependent upon the stability of the atmosphere. For this study, we adopt the standard value of 0.143 for this parameter.

$$\mathrm{v}_{100} = v_{30}\left(\frac{100}{30}\right)^{\alpha} \tag{11}$$

Figure 6 below presents a more detailed picture of the two sites' wind properties in the form of a modified "wind rose", i.e. a joint circular histogram of the distribution of the wind direction and average wind speed at that site. The histograms consist of 5° bins for the direction, where the color of the bin corresponds to the average speed of the wind in that direction according to the given color scale.

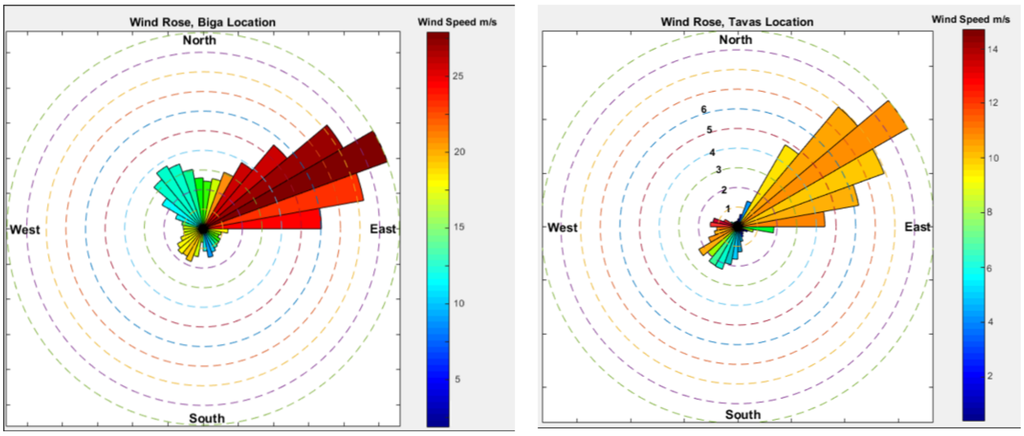

Figure 6: Annual Wind Roses for the Biga and Tavas Sites

It is once apparent that the Biga site is considerably windier than the Tavas site and that the wind at both sites blow predominantly in the Northeast direction. Other observations from Figure include that for both locations wind almost never blows in the South-East quadrant of the direction coordinates (and for Tavas, also almost never in the North-West quadrant) and that there's very moderate wind occurrence in the opposite direction of the predominant North-East. These other details of the presented Wind roses could be explained better in a more-informed context of the geography and topography of the sites.

To verify the commonly encountered Weibull behavior of the wind speeds, a fitting process of the Weibull distribution over the whole data was performed. Based on Figure 7, we can justify that the wind speed data agrees strongly with a Weibull distribution, and, therefore, we can safely perform random variate generation using the estimated Weibull parameters in order to use for our simulated samples.

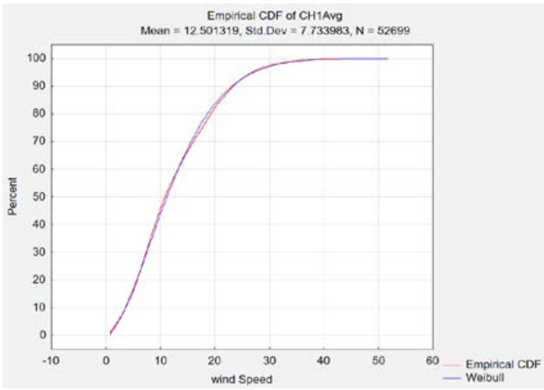

Figure 7: Weibull Distribution Fit for the Biga Site Wind Speed Data

Based on the analyses of Wiser and Bolinger (2013). for the wind farm development project carried out in the USA in 2012, the average capital cost for projects with more than 5 MW output is around 1,900 $ /kW. In our case, we take the cost to be around 2,000 $ /KW. In addition to this initial capital cost, we also take the O&M costs into account. Because O&M costs are realized over time, they are customarily reported in dollars per energy produced, i.e. $/MWh. To get an estimate for this rather elusive cost type, we again refer the issues of the D.O.E. reports (Wiser and Bolinger, 2012, 2013), which give the example of two wind farm operators reporting that the total operating costs are around 20 $/MWh. Assuming an economic life span of the turbines of 20 years and using a discount rate of 6%, the present worth of the total O&M costs can be calculated as

$$cost = (\$2{,}000{,}000/MW) * P_{max}(MW) + (11.469yr) * (\$20/MWh) * (8{,}784hr/yr * m(MW)), \quad (14)$$

where P_{max} is the total power output and m is the mean power production.

Optimization Scenarios and Results

Performing the optimization using the whole data set (52704 point) for every *x-y* pair of spacing of the turbines require a huge amount of calculation time. Therefore, make use of a hybrid approach of Monte-Carlo sampling of wind directions combined with a Weibull random-variate generation of the speed from the corresponding Weibull fit.

We provide an itemized description of our approach for simulating one direction-speed pair as follows:

- We divide the wind direction into 360 discrete categories (i.e. slices) in increments of 1° Celsius and calculate the frequency of the direction data in each category.
- For each category we fit a Weibull distribution to the wind speed data corresponding to the direction data in that slice.
- For each sample to be simulated, we first generate a random discrete direction via Monte-Carlo simulation using the categories' relative frequencies (we take the simulated direction to be the middle of that slice, i.e. 0.5°, 1.5°,…., 359.5°).
- Next we generate a wind-speed random variate using the estimated Weibull distribution for that slice.
- Based on the simulated direction and speed, we calculate the wake effects and the effective speed facing each individual turbine on the grid.
- We next calculate each turbine's output using the power curve and average the results over all the objective function.
- We finally obtain the value of the objective function by dividing the average output by the total cost of a turbine and by the total area of the grid.

The histogram in Figure 8, which is actually a "flattened-out" wind rose, shows the distribution of the wind direction and the average wind speed for the 360 1-degree slices for the Biga site. Each slice has its individual estimated Weibull speed distribution, which are not presented here.

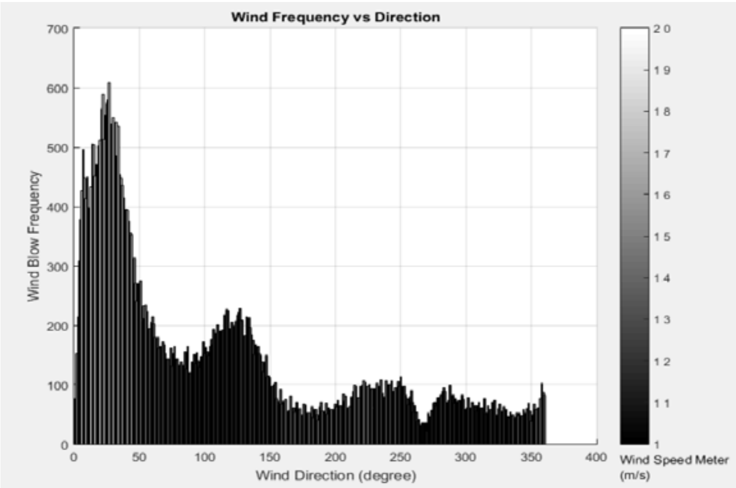

Figure 8: distribution of wind speed over direction

Due to the complex nature of the model, mostly because of the incorporation of the wake effects, a differential calculus formulation of the objective function is not trivial. Due to the low variable-dimensionality of the problem and the considerable computational effort required to perform the calculations for each single simulated sample, the Nelder-Mead heuristic method presents itself as a suitable algorithm. With two decision variables (*x* and *y*) the method requires only 3 points in the *x-y* plane to implement. For our stochastic model, however, the objective function value for each Nelder-Mead point needs to be defined as the average of many simulations. Deciding that 1000 simulations for each point is a reasonable compromise between representative accuracy and computational power, we implemented the algorithm with the stopping criterion that the objective function value has not improved by more than 0.01% for the past 100 iterations.

In this study, we present the results for two square cases of the grid, a 10x10 grid and a 30x30 grid. The optimization results for the Biga and Tavas sites are presented in, respectively, Table 1 and Table 2 below.

Table 1: Case Study Results for the Biga Site (Vestas V90 – 90 m Rotor Diameter).

Number of turbines in x-axis	Number of turbines in y-axis	Spacing distance in the x-axis	Spacing distance in the y-axis
10	10	584.3 m	211.4 m
30	30	590.6 m	221.9 m

From Table 1, we see that the *x*-spacing is considerable larger than the *y*-spacing for both grid sizes (the ratio *x*/*y* is 2.77 for the 10x10 grid and 2.67 for the 30x30 grid). This result is consistent with the predominant wind directions at the Biga site as evident from Figures 6 and 7 –winds are stronger and more frequent in the East-North-East direction (first octant of the compass). A larger *x*-spacing would thus help reduce wake effects more than a larger *y*-spacing. Given 90-meter rotor diameter of the Vestas V90 – 3MW turbine, this indicates a spacing of 6.49 diameters along the *x*-axis and a spacing of 2.34 diameters along the *y*-axis for the 10x10 grid and very close values for the 30x30 grid. These numbers also compare reasonably with the industry rule of thumb of spacing the turbines by 2 diameters in the perpendicular line to the prevailing wind direction and by 7 diameters along this direction (note that our grids is not oriented to face the "prevalent" wind direction perpendicularly, but are in the fixed East-West and North-South axes). The result that turbines are slightly more packed in a smaller grid can be explained by the "boundary" effect as follows: Turbines at certain borders of the grid will be impacted much less (the East and South borders in the case of Biga) from the wake effects as compared to turbines in the interior and on the opposite borders. The fact that the "border" turbines constitute a greater fraction of the total number of turbines in a smaller grid as compared to a larger grid will compensate better for internal wake losses, and, thus, it may allow tighter optimal packing of the whole turbine set.

Results for the Tavas site in Table 2 are quite close to the values for the Biga site and bear largely similar interpretations. However, comparing the two sites is somewhat confusing. Both the *x*- and y-spacing for the Tavas 10x10 grid is slightly larger than their counterparts for the Biga site, possibly owing to the fact that the prevailing winds at the Tavas site are slightly more in the North direction as compared to Biga. However, both of spacing values are for the 30x30 grid are slightly less at the Tavas site than at Biga, thus, running counterintuitive to the previous explanation. It is also noteworthy that while the *y*-spacing at Tavas increases slightly from the 10x10 to the 30x30 grid, as consistent with the "border turbines" effect, the x-spacing decreases slightly. Given the complex nature of the model and the non-deterministic behavior of a heuristic algorithm, we caution from over-comparing the two sites and postulating unsupported conclusions.

Table 2: Case Study Results for the Tavas Site (Vestas V90 – 90 m Rotor Diameter)

Number of turbines in x-axis	Number of turbines in y-axis	Spacing distance in the x-axis	Spacing distance in the y-axis
10	10	591.5 m	215.6 m
30	30	583.4 m	218.8 m

Conclusion and Future Research

The general problem of the optimal micro-siting of wind turbines a complicated one due to several reasons. This paper aims to develop a mathematical for a special case of the model where turbines of the same type are placed at uniformly spaced grid points over a flat wind farm area in the form of a rectangle whose edges sit on the East-West and North-South directions. The model's main focus is to incorporate the complex wake effects that may occur within a wind farm that critically impact the total output. This paper also differs from other studies that consider only the total output in that we consider an economic objective, the return on investment per area required. The wake effects are modeled in an idealized –yet mathematically quite complicated– formulation a la Jensen (Katic et al., 1986). The consideration of stochastic wind speeds and directions, and the use in the case study of real measurement data from 2 sites in Turkey, is yet another aspect of the study that increases its realism. The numerical and analytical complexities introduced by the incorporation of wake effects and the use of real data are dealt with by adopting a hybrid approach of using the heuristic Nelder-Mead optimization method based on Monte-Carlo simulation of random samples of wind direction-speed pairs.

Despite the fixed orientation of the grid, the model's result seem to validate the industry' simplistic rule of thumb practice of spacing the turbines by 2 and 7 rotor diameters along, respectively, the perpendicular and parallel direction of the "prevalent" winds. Introducing a third decision variable corresponding to the "tilt" angle of the rectangular grid, which would be optimized together with the horizontal and vertical turbine spacings, could yield even-more confirming results, but this is the subject of future generalizations of the model. The case study results also indicate –albeit with an exception– that a larger wind farm (i.e. having more turbines) requires larger spacing between the turbines in order to operate optimally, possibly owing to the fact that a greater portion of the turbines in a larger wind farm are in the farm's interior region where the wake effects are stronger than on the boundaries. Nevertheless, we offer these interpretations with some degree of caution as the aforementioned observations and possible other ones ask for a more generalized model as well as many more site cases to be analyzed, which again, are among planned extensions of the present study.

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