EFFECT OF TWO LOBE WAVE SQUEEZE FILM DAMPER IN THE SUPPORT OF AN UNBALANCED RIGID ROTOR

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Abstract: A possible improvement of the performances of squeeze film damper (SFD) for supporting the rotors of high-speed turbomachinery has been sought adopting a two lobe, wave (2LW) geometry of the bearing bore. A statically unbalanced, symmetrical, rigid rotor supported with 2LW-SFD has been theoretically examined through numerical continuation, assuming laminar, isoviscous oil flow within the damping film and incomplete centering action of the retainer springs mounted parallel to the film. Despite nonlinearity which still affects the system, the obtained results highlight the potential of the unconventional geometry as a mean for a possible reduction of the typical drawbacks in the response with conventional SFD, mainly consisting in undesired whirling motions with too large journal orbits and/or nonsynchronous character, so as to assure more safe conditions for the rotor operation. Yet, further theoretical and experimental investigation is desirable in order to confirm such an outcome.

Key words: lubrication, squeeze film damper, rotordynamics, bifurcation.

Introduction

The poor quality and drawbacks which affect the dynamic behaviour of turbomachinery on rigid supports equipped with simple rolling bearings, in terms of forces transmitted to the frame and vibrations, fostered the concept of combining the bearing to an oil film since from the Thirties of the past century (Birmann, 1933). As a device capable to work in this way, in order to assure enough damping to the rotor-system, the squeeze film damper (SFD) has received substantial and systematic scientific attention in the last five decades (Della Pietra & Adiletta, 2002). This huge research work has been mainly addressed to fluid dynamic and structural aspects of the damping device, like oil cavitation and inertia or oil feeding and seals effects, which highly influence the pressure distribution in the film and hence the film forces. The said factors, together with the basic nonlinear character of the film forces, represent a crucial issue as regards the dynamic response of the supported rotor. Here the nonlinearity, possibly exalted under some circumstances, can give rise to undesirable or unexpected whirling motions of the rotor with very large orbits described by the journals within the relative bearings and/or characterized by nonsynchronous nature. General consequences of these effects are high levels of vibrations and force transmissibility, shortening in the life of the bearings, damage of the damper bearing, possible failure of the rotor due to the fatigue mechanism connected to the nonsynchronous whirl. Therefore, a consistent part of the research from literature has been just focused onto the dynamic behaviour of the rotor-support system with SFD, represented by practical or experimental real rotors, often theoretically characterized by very simple flexible or rigid models, provided with supports of the uncentralized or centralized type (Adiletta & Della Pietra, 2002). Regarding to this last option, it is worth mentioning that the rotor journal is let free to whirl within the damping film, i.e. subjected to the only hydrodynamic forces of the squeeze film, or it is further supported by springs which work in parallel to the film and allow a preliminary centering of the journal within the bearing clearance when the rotor is at rest. Nevertheless, the registration of the static position can be carried out in practice with higher or lesser accuracy and the complete, full centering turns out to be only theoretical. Bistable conditions with jump in rigid rotors were investigated, among others, by White (1970) and Hahn (1979). Mohan & Hahn (1974) analyzed the horizontal centralized rotor, where the flexibility of supports is due the spring elements mounted in parallel to the damping film, and pointed out the importance of a critical value of the unbalance beyond which the squeeze film bearing with flexible mount behaved worse than a rigid support. The influence of the initial conditions, when numerical integration is carried out for solving the nonlinear equations of the rotor-support system, was investigated by Taylor & Kumar (1980). They observed on this basis the dominant character of one of the two solutions present in intervals of speed with bistable behaviour. Li & Taylor (1987) showed the coexistence of nonperiodic and synchronous solutions after numerical study of the unbalanced rigid rotor on SFD with centralizing springs. Also, they investigated into the effects of incomplete centering and pointed out the importance of the complex characteristic roots about the degree of attraction of the different coexistent motions. Similar analyses were carried out by Zhao & Hahn (1993) and Zhao et al. (1994) who analyzed the bifurcating behaviour of the rotor system. In the latter paper, successive bifurcations along rotor run up, respectively representing changes from sysnchronous to quasi-periodic, sub-harmonic and again to synchronous whirl, were reported and explained with the study of Floquet multipliers. More recent examples of research works about nonlinear behaviour of rigid rotors on SFD are represented by Bonello, Brennan, & Holmes (2002), focused onto receptance harmonic balance method that was adopted to determine periodic solutions, and Inayat-Hussain, Kanki, & Mureithi (2003), where the bifurcating dynamics of the nonlinear systems was dealt with by means of a numerical case study and recourse to continuation technique. Besides the rigid rotor case, flexible rotor systems have been extensively study too, especially in the simple symmetrical model with rigid disk, flexible shaft and SFD end supports (Inayat-Hussain, 2009). Furthermore, particular research efforts have been addressed to innovative design of the squeeze damper, in order to improve the dynamic performances of the supported rotor (de Santiago et. al. 1999, El-Shafey & El-Hakim, 2000). In this regard, the present work is aimed to test the adoption of a two lobe, wave (2LW) geometry of the bearing bore, in place of the common circular profile. Such a concept, with adoption of different shapes, is well known in journal bearing design (Pinkus, 1956). The three lobe wave profile, in particular, has been thoroughly investigated for gas bearings application (Dimofte, 1995) putting in evidence its advantages and the present authors have recently focused their attention onto the 2LW geometry for oil lubricated journal bearings (Adiletta, Mancusi, & Strano, 2011). In this case, wave amplitude and angular phase of the profile represent, in respect of the conventional circular geometry, two further parameters that influence the behaviour of the rotor-support system and are possibly at hand to optimize the dynamic response. On this basis, a statically unbalanced, symmetrical, rigid rotor supported with 2LW-SFD has been theoretically examined under the hypotheses of laminar, isoviscous oil flow within the damping film and incomplete centering action of the retainer springs mounted parallel to the film. The analysis has been carried out with use of numerical integration of system equations and continuation algorithm.

The Equations of Motion and the Fluid Film Forces ın the SFD

A statically unbalanced, rigid rotor is in horizontal disposition and supported at each end by roller bearing plus squeeze film damper (SFD) with retaining springs. The whole system is assumed to be symmetrical, so that cylindrical whirl can be assumed for the rotor motion and the following equations are written for a single half of the rotor-support system:

$$\begin{cases} m\ddot{\bar{x}} + \sigma\dot{\bar{x}} + k\,\bar{x} = m\rho\omega^2 \cos\omega t + F_{SFx} \\ m\ddot{\bar{y}} + \sigma\dot{\bar{y}} + k\left(\bar{y} - \bar{y}_S\right) = m\rho\omega^2 \sin\omega t + F_{SFy} \end{cases} \tag{1}$$

Equations (1) are written assuming a reference fixed frame placed in the bearing center, with the $\bar{y}$ axis parallel to the gravity force, upward directed, and $\bar{z}$ direction parallel to the rotor axis. A gravity residual $\bar{y}_S$, due to hypothesized incomplete centering of the springs, is defined as:

$$\bar{y}_S = \bar{y}_0 + f_S \ ,$$

where f_S represents the static deflection of the spring system, i.e. $f_S = -mg/k$, and $\bar{y}_0$ is the coordinate of journal center of the rotor in the absence of weight. Substitutions:

$$x = \frac{\bar{x}}{C},\ y = \frac{\bar{y}}{C},\ y_S = \frac{\bar{y}_S}{C},\ \omega t = \tau,\ \dot{x} = \frac{dx}{d\tau}\frac{d\tau}{dt} = x'\omega,\ \dot{y} = \frac{dy}{d\tau}\frac{d\tau}{dt} = y'\omega,\ \zeta = \frac{\bar{z}}{L},$$

$$\omega_R = \sqrt{\frac{k}{m}},\quad \Omega = \frac{\omega}{\omega_R},\quad \omega_B = \frac{\mu R L^3}{m C^3},\quad \mathrm{f} = \frac{\omega_B}{\omega_R},\quad U = \frac{\rho}{C},\quad q = \frac{\sigma}{2\sqrt{m k}},$$

$$\lambda = \frac{L}{D},\quad F_{SFx} = \frac{\mu\omega_R L R^3}{C^2} f_{SFx},\quad F_{SFy} = \frac{\mu\omega_R L R^3}{C^2} f_{SFy}, \tag{2}$$

$$f_{SFx} = -\int_{-1/2}^{1/2}\int_{\delta 1}^{\delta 2} \gamma \cos\delta\, d\delta\, d\zeta,\qquad f_{SFy} = -\int_{-1/2}^{1/2}\int_{\delta 1}^{\delta 2} \gamma \sin\delta\, d\delta\, d\zeta,\ \gamma = p\Big/\left(\mu\omega_R \frac{R^2}{C^2}\right),$$

$$w_1 = x,\ w_2 = y,\ w_3 = x',\ w_4 = y',\ w_{2,S} = y_S,$$

yield the following system of first order ODEs equivalent to (1):

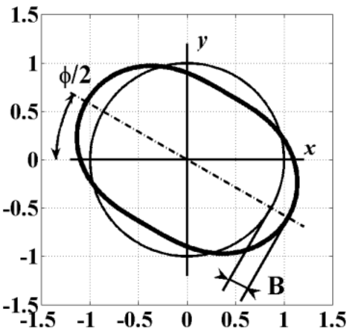
$$\begin{cases} w_1' = w_3 \\ w_2' = w_4 \\ w_3' = -\dfrac{2q}{\Omega} w_3 - \dfrac{1}{\Omega^2} w_1 + \dfrac{\mathrm{f}}{4\Omega\lambda^2} f_{SFx} + U\cos\tau \\ w_4' = -\dfrac{2q}{\Omega} w_4 - \dfrac{1}{\Omega^2}\left(w_2 - w_{2,\mathrm{S}}\right) + \dfrac{\mathrm{f}}{4\Omega\lambda^2} f_{SFy} + U\sin\tau \end{cases} \tag{3}$$

Components F_{SFx} and F_{SFy} of the fluid film force were obtained from the pressure distribution determined solving the Reynolds equation written for finite SFD, in the presence of non circular bearing profile and incompressible, isothermal, isoviscous, laminar flow:

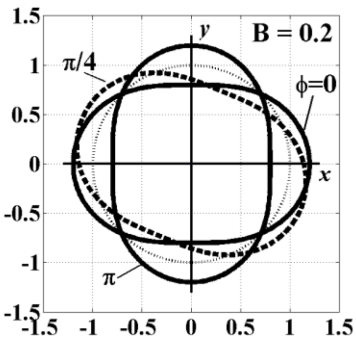
$$\frac{1}{R^2}\frac{\partial}{\partial\vartheta}\left(\frac{\bar{h}^3}{\mu}\frac{\partial p}{\partial\vartheta}\right) + \frac{\partial}{\partial \bar{z}}\left(\frac{\bar{h}^3}{\mu}\frac{\partial p}{\partial \bar{z}}\right) = -12\left(\dot{x}\cos\delta + \dot{y}\sin\delta\right) \tag{4}$$

The fluid film region with $\delta \in [0, 2\pi]$, $\zeta \in$[-1/2, 1,/2] was discretized by means of a two-dimensional mesh of N+1 columns by M rows, with cell dimension $\Delta\delta$ x $\Delta\zeta$ and $\Delta\delta=2\pi/N$, $\Delta\zeta=1/(M-1)$. Finite differences solution of the Reynolds equation was carried out by means of a forward-time, centered-space FD scheme, with M=5, N=81, and a subsequent SOR algorithm for solving the algebraic system. Ambient pressure was adopted at film borders and to replace sub-ambient values obtained from the FD routine. Equations (3) were numerically integrated with a Runge-Kutta routine. The determination of trajectories and Poincaré sections was carried out together with numerical continuation of periodic solutions after suitable choice about the set of the system parameters.

According to (2), Fig. 1 and Adiletta, Mancusi, & Strano, (2011) the bearing of the damper presented a two-lobe, wave profile characterized by dimensionless amplitude *B* and angular orientation φ. In the present investigation, *B* was given values 0, 0.05, 0.1, 0.15 and 0.2 (the null value was included in the set in order to represent the circular bearing case) while φ was assigned in the set $\pi, -3\pi/2, -\pi/2, -\pi/4, 0, \pi/4, \pi/2, 3\pi/2$. Figure 1a illustrates the modification of the original circular bearing clearance as an effect of *B* and φ . Fig. 1b shows three examples with *B* = 0.2 and $\varphi = \pi$, 0 and $\pi/4$. The geometry of the bearing was further specified by means of a length to diameter ratio λ that was assumed equal to 0.125, as typical of short-bearing aspect.



a)



b)

Fig. 1

The effects of damper geometry and orientation onto the dynamic behaviour of the rotor were tested once suitable set-up conditions of the system had been fixed. In this regard, the results from Zhao and Hahn (1995), relative to a common circular bearing case in the presence of severe unbalance, suggested to assign: f = 0.072, *U* = 0.3, w_{2s} = -0.7, given the remarkable nonlinearity which affected the rotor response under these conditions. A specific, dimensional case study was conceived in congruence with the above parameters, fixing *k* = 929752 N/m, *m* = 50 kg, *R* = 0.04 m, *C* = 0.00015 m and *q* = 0.005. The angular speed was given in the interval Iω = [100, 1200] rad/s. Then the remaining quantities ω_R , *L*, μ and Ω could be inferred:

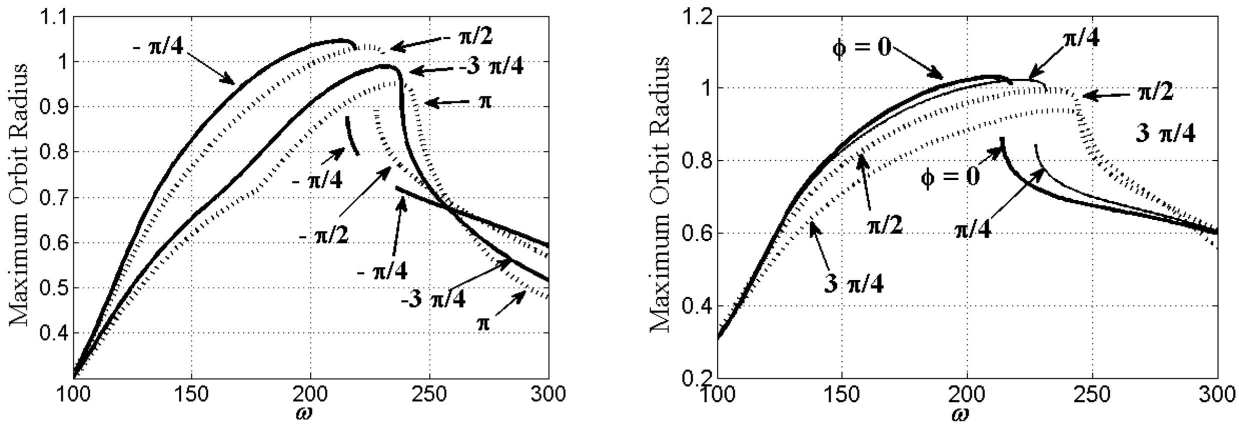
$$\omega_R = \sqrt{\frac{k}{m}} = 136.36 \text{ rad/s}, \quad L = 2R\lambda = 0.01 \text{ m}, \quad \mu = \mathrm{f}\,\frac{m\,\omega_R\,C^3}{RL^3} = 0.0414 \text{ Pa s}, \; \Omega = [0.733, 8.80] \; .$$

Results of Numerical Analysis

Bifurcation analysis of periodic and *k*-periodic solutions was carried out by means of a continuation algorithm, based onto AUTO 97 (Doedel *et al.*, 1997), which traced the fixed point locus of the discrete-time "equivalent" system constructed via the Poincaré sections. The branches of the fixed points were computed together with Floquet multipliers. These quantities made it possible to check the stability of the branches. On the other hand, numerical integration of Eq. (3) based on Runge-Kutta method made it possible to check some of the results from continuation through direct observation of the journal orbits. Only two significant examples of the obtained numerical results have been reported here below:

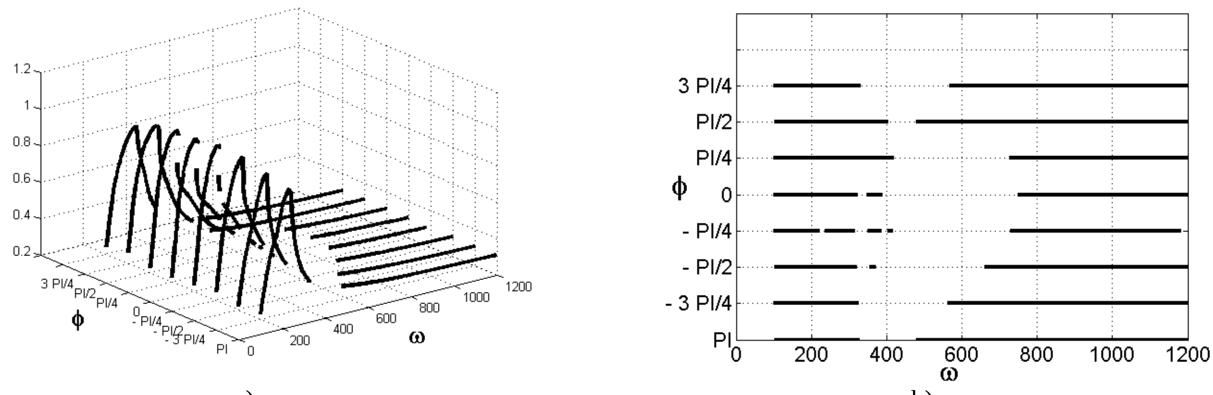
a) The resonance curves of the synchronous response (1*T*), respectively obtained for different φ values, when the bifurcation parameter ω varies in the interval Iω. The angular positioning of the damper bearing not only modifies the resulting orbit magnitude, here expressed by means of the maximum orbit radius, but determines too the presence or absence of the saddle-node bifurcation connected to the well known jump effect. In Fig. 2a, the curves obtained with $\varphi = \pi$ and $- 3\pi/4$ are deprived of the "nose" followed by instable branch (not reported) that characterized the curves with $\varphi = - \pi/2$ and $- \pi/4$. In Fig. 3a waterfall picture of the resonance curves has been plot in the whole interval Iω. Fig. 3b shows a from-the-top view of the same plot, evidencing the sub-intervals of Iω where the synchronous motion turns out to be stable (absence of line represents instability).Three examples of the journal orbits about resonance are put in comparison in Fig. 4a-c.

b) Two continuation diagrams of the 1*T*, obtained at given values of *B* and ω, assuming φ as the bifurcation parameter which varies in the interval $[0, 2\pi]$. In Fig. 5a, b the angular speed has been assumed equal to 600 and 500 rad/s, respectively, i.e within a zone of Iω that is generally characterized by instability of the synchronous solution (compare Fig. 3b). Fig. 6 illustrates a particular from Fig. 5b. The plots point out that the existence of the stable motion, at given speed, is conditional on the choice of suitable values for *B* and φ. In the diagrams the response of the circular bearing case has been further reported to allow comparison.



a) b)

Fig. 2



a) b)

Fig. 3

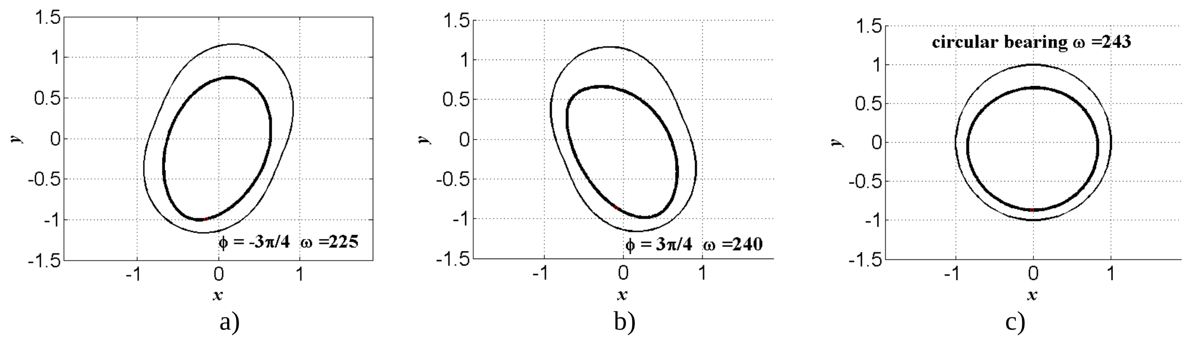


Fig. 4

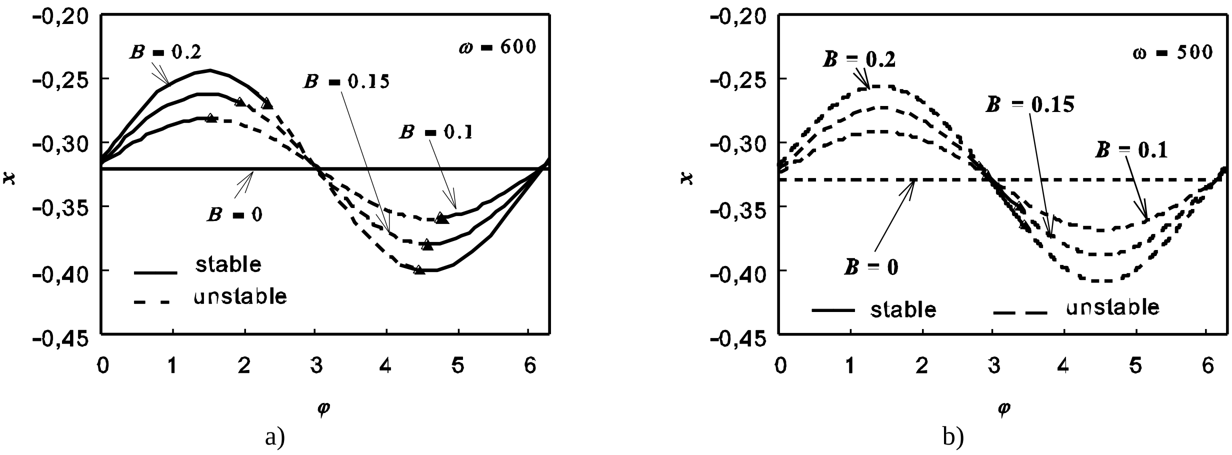


Fig. 5

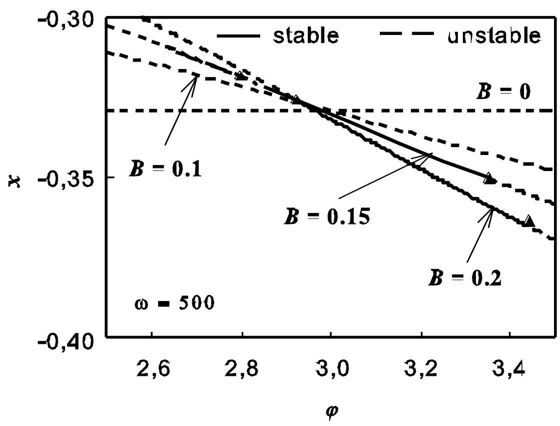


Fig. 6

Conclusions

The use of 2LW-SFD for supporting a rigid unbalanced rotor has been investigated through numerical continuation method, with particular attention to the effects of the bearing geometry onto the synchronous response. A case study with given unbalance, gravity residual, lubricant viscosity, wave amplitude of the bearing profile, and different values for the angular position of the 2LW bearing has been selected for the analysis. The obtained results, despite the restriction of the investigation to the synchronous response, put in evidence the substantial effect of the choice of such a bearing geometry, with particular regard to the jump phenomenon and the recover of stable solutions. Further investigation, both theoretical and experimental, is desirable in order to assess possible practical benefits of the device.

Nomenclature

C,L,D,R	= reference radial clearance, axial length, reference diameter and radius of the bearing
F_{SFx}, F_{SFy}	= fluid force component, in function of dimensional coordinates and velocities
m	= half rotor mass
φ	= angular orientation of the 2-lobe bearing (2LB)
U	= (ρ/C) dimensionless static unbalance
ω	= angular speed of rotor
k	= stiffness of the SFD restraining springs
μ	= lubricant viscosity

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