Independent Transversal Domination Number of Corona and Join Operation in Path Graphs

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Abstract. A dominating set of a graph *G* which intersects every independent set of a maximum cardinality in *G* is called an independent transversal dominating set. The minimum cardinality of an independent transversal dominating set is called the independent transversal domination number of *G* and is denoted by $\gamma_{it}(G)$. In this paper we investigate the independent transversal domination number of the path graph P_n with the star graph $S_{1,m}$, the wheel graph $W_{1,m}$ and the complete graph K_n under neihgbourhood corona, edge corona and join operation providing $\beta(P_n) > \beta(G)$.

1. Introduction

In this paper, we consider simple finite undirected graphs without loops and multiple edges. Let G = (V(G), E(G)) be a graph. For a vertex x of G, N(x) denotes the set of all neighbours of x in G. The *distance* d(u, v) between two vertices u and v in G is the length of a shortest path between them. The *diameter* of G, denoted by diam(G) is the largest distance between two vertices in V(G). The number of the neighbor vertices of the vertex v is called degree of v and denoted by $deg_G(v)$. The minimum and maximum degrees of a vertex of G are denoted by $\delta(G)$ and $\Delta(G)$. A vertex v is said to be pendant vertex if $deg_G(v) = 1$. A vertex u is called support if u is adjacent to a pendant vertex [7]. The eccentricity e(u) of a vertex u in G is the distance from u to a vertex farthest from u. The minimum eccentricity of the vertices of the graph G is the *radius* of G denoted by rad(G), while the *diameter* of G is the greatest eccentricity[4].

Let *G* be a graph and $S \subseteq V(G)$. We denote by $\langle S \rangle$ the subgraph of *G* induced by *S*. A set *S* is said to be an *independent set* of *G*, if no pair of vertices of *S* are adjacent in *G*. The *independence number* of *G*, denoted by $\beta(G)$, is the cardinality of a maximum independent set of *G*. We denote by $\Omega(G)$ the set of all maximum independent sets of *G*. A vertex and an edge are said to *cover* each other if they are incident. A set of vertices which cover all the edges of a graph *G* is called a *vertex cover* for *G*, while a set of edges which covers all the vertices is an *edge cover*. The smallest number of vertices in any vertex cover for *G* is called its *vertex covering number* and is denoted by $\alpha(G)$ [7]. For any graph *G* of order *n*, $\alpha(G) + \beta(G) = n$.

A *dominating set S* in a graph *G* is a set of vertices of *G* such that every vertex in V(G) - S is adjacent to at least one vertex in *S*. The domination number of *G*, denoted by $\gamma(G)$, is the minimum cardinality of a

Received: 30 November 2021; Accepted: 20 January 2022; Published: 30 April 2022

2010 Mathematics Subject Classification. 05C69, 05C40, 68M10, 68R10

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Keywords. Domination number, Independent transversal domination number, Independence number.

Cited this article as: Atay Atakul B. Independent Transversal Domination Number of Corona and Join Operation in Path Graphs, Turkish Journal of Science. 2022, 7(1), 7-13.

dominating set of *G* [8, 9]. It is clear that for a graph *G*, $\gamma(G) \leq \beta(G)$, and If *G* has no isolated vertices, $\gamma(G) \leq \alpha(G)$.

Given a graph *G* and a collection of subsets of its vertices, a subset of V(G) is called a *transversal* of *G* if it intersects each subset of the collection. If we think of the graph as modeling a communication network, many graph theoretical parameters have been used to describe the stability of communication networks including connectivity, toughness, integrity, binding number, domination, exponential domination, independent transversal domination etc. The independent transversal domination number is one of the measures of the graph vulnerability. A transversal of a collection of sets is a set of distinct representatives of the elements in the collection. It is possible to find transversals regarding several types of vertex sets in graphs such that the domination number, the chromatic number and the independence number of a graph. In [2], the concept "partition domination number" was defined as the largest integer *k* such that given any partition of the vertex set of the graph having at most *k* elements in every set of the partition, there is transversal of the partition being a dominating set. Some complexity results regarding the associated desicion problems and some bounds or exact values for some specific families of graphs were presented in [2]. A recent work in new style of transversal-type concepts has been presented in [6]: the independent transversal domination number[1].

A dominating set of *G* which intersects every independent set of maximum cardinality in *G* is called an *independent transversal dominating set*. The minimum cardinality of an independent transversal dominating set is called the *independent transversal domination number* of *G* and is denoted by $\gamma_{it}(G)$. An independent transversal dominating set of cardinality $\gamma_{it}(G)$ is called a $\gamma_{it}(G) - set$. Thus, if *D* is an ITD-set of *G*, then *D* is a dominating set of *G* and $\beta(G) > \beta(G - D)$. The notion of independent transversal domination was first introduced by Hamid [3, 6].

In this paper, firstly known results are given. Then, we investigate the independent transversal domination number for the neighbourhood corona, the edge corona of the path graph with the star graph $S_{1,m}$, the wheel graph $W_{1,m}$, the complete graph K_m and join operation of the path graph with some graphs G providing $\beta(P_n) > \beta(G)$. Lastly, the conclusion section is presented.

2. Known Results

Theorem 2.1. [6] If G is a complete multipartite graph having r maximum independent sets, then

$$\gamma_{it}(G) = \begin{cases} 2 & , r = 1 \\ r & , otherwise \end{cases}$$

Theorem 2.2. [6] For complete graph with order *n* and complete bipartite graph with order m + n, $\gamma_{it}(K_n) = n$ and $\gamma_{it}(K_{m,n}) = 2$, respectively.

Theorem 2.3. [6] For any path P_n of order n, we have

$$\gamma_{it}(P_n) = \begin{cases} 2 & , n = 2,3 \\ 3 & , n = 6 \\ \lceil \frac{n}{3} \rceil & , otherwise \end{cases}$$

Theorem 2.4. [6] For any cycle C_n of order n, we have

$$\gamma_{it}(C_n) = \begin{cases} 3 & , n = 3, 5\\ \lceil \frac{n}{3} \rceil & , otherwise \end{cases}$$

Theorem 2.5. [6] If W_n is a wheel on *n* vertices, then

$$\gamma_{it}(W_n) = \begin{cases} 2, & \text{if } n = 5\\ 3, & \text{if } n \ge 7 \text{ and is odd or } n = 6\\ 4, & \text{otherwise} \end{cases}$$

Theorem 2.6. [6] If G is a disconnected graph with compenents $G_1, G_2, ..., G_r$, then $\gamma_{it}(G) = \min_{1 \le i \le r} \{\gamma_{it}(G_i) + \sum_{j=1, j \ne i}^r \gamma(G_j)\}$.

Theorem 2.7. [6] If G has an isolated vertex, then $\gamma_{it}(G) = \gamma(G)$.

Theorem 2.8. [6] For any graph G, we have $1 \le \gamma_{it}(G) \le n$. Further $\gamma_{it}(G) = n$ if and only if $G = K_n$.

Theorem 2.9. [6] Let G be a graph on n vertices. Then $\gamma_{it}(G) = n - 1$ if and only if $G = P_3$.

Theorem 2.10. [6] Let G be a non-complete connected graph with $\beta(G) \ge \frac{n}{2}$. Then $\gamma_{it}(G) \le \frac{n}{2}$.

Theorem 2.11. [6] If G is bipartite, then $\gamma_{it}(G) \leq \frac{n}{2}$.

Theorem 2.12. [6] Let a and b be two positive integers with $b \ge 2a - 1$. Then there exists a graph G on b vertices such that $\gamma_{it}(G) = a$.

Theorem 2.13. [6] If G is a non-complete connected graph on n vertices, then $\gamma_{it}(G) \leq \lceil \frac{n}{2} \rceil$.

Theorem 2.14. [6] For any graph G, we have $\gamma(G) \leq \gamma_{it}(G) \leq \gamma(G) + \delta(G)$.

Corollary 2.15. [6] If T is a tree, then $\gamma_{it}(T)$ is either $\gamma(T)$ or $\gamma(T) + 1$.

Theorem 2.16. [6] If G is a graph with diam(G) = 2, then $\gamma_{it}(G) \leq \delta(G) + 1$.

Theorem 2.17. [3] If G is a connected graph and u is a vertex of minimum degree in G, then

$$\gamma_{it}(G) \leq \begin{cases} \delta(G) + 1 & if \ ecc_G(u) \leq 2\\ \frac{n(G)}{2} + 1, & if \ ecc_G(u) \geq 3 \end{cases}$$

and these bounds are tight.

Theorem 2.18. [3] If G is a graph with $\beta(G) \ge \frac{n(G)}{2}$, then $\gamma_{it}(G) \le \gamma(G) + 1$, and this bound is tight.

3. Independent Transversal Domination Number for the Neighbourhood Corona of the Path Graph

Definition 3.1. [6] A dominating set $S \subseteq V$ of a graph G is said to be an independent transversal dominating set if S intersects every maximum independent set of G. The minimum cardinality of an independent transversal dominating set of G is called the independent transversal domination number of G and is denoted by $\gamma_{it}(G)$. An independent transversal dominating set S of G with $|S| = \gamma_{it}(G)$ is called a $\gamma_{it} - \text{set}$.

The following figure shows the independent transversal domination number of a graph G.

where, $\beta(G) = 4$, $\gamma(G) = 4$. The maximum independent set of the graph consists of four pendant vertices or two two support vertices on cycle having deg(v) = 2. The dominating set of the graph consists of four pendant vertices or four support vertices on cycle having deg(v) = 2. Let *S* be an independent transversal dominating set. If we pick the support vertices on cycle for *S*, then all vertices of the graph *G* are dominated. But the independence number of the graph doesn't decrease. V - S contains at least one $\beta - set$. So, we must also add any pendant vertex to *S*. Hence, $\beta(G) > \beta(G - S)$ and $\gamma_{it}(G) = 5$.

Definition 3.2. [5]

The graph $G_1 * G$ which is obtained by neighbourhood corona operation of a connected graph G_1 and graph G is formed as follows: Every vertex u_i of graph G_1 correspond to a graph G and every vertex v_{ij} of G is adjacent to every neighbour vertex of the corresponding vertex u_i of G_1 , where $i = \overline{1, |G_1|}$ and $j = \overline{1, |G|}$.

The neighbourhood corona of the graph $P_6 * P_2$ can be depicted as in the following figure:

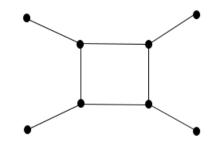


Figure 1: The graph G

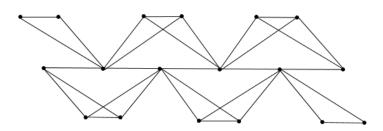


Figure 2: The graph $P_6 * P_2$

Theorem 3.3. Let P_n and P_m be any path graphs with order n and m, respectively. Then,

 $\gamma_{it}(P_n * P_m) = \begin{cases} \frac{n}{2} + 1, & \text{if } n \equiv 0 \pmod{4} \text{ and } m \text{ is odd,} \\ \frac{n}{2} + 2, & \text{if } n \equiv 0 \pmod{4} \text{ and } m \text{ is even} \\ \lfloor \frac{n}{2} \rfloor + 2, & \text{if } n \equiv 1, 2, 3 \pmod{4} \text{ and } m \text{ is odd,} \\ \lfloor \frac{n}{2} \rfloor + 3, & \text{if } n \equiv 1, 2, 3 \pmod{4} \text{ and } m \text{ is even.} \end{cases}$

Proof. We denote the vertices of P_n with u_i , $i = \overline{1, n}$ and the corresponding vertices of P_m with v_j , $j = \overline{1, m}$. Let D be a γ – *set* of the graph $P_n * P_m$. So, for $k \in \{0, 1, ..., \lfloor \frac{n}{4} \rfloor - 1\}$,

 $D = \{u_{4k+2}, u_{4k+3}\}$ and $|D| = \frac{n}{2}$, if $n \equiv 0 \pmod{4}$;

 $D = \{u_{4k+2}, u_{4k+3}, u_{n-1}\}$ and $|\overline{D}| = \lfloor \frac{n}{2} \rfloor + 1$, *if* $n \equiv 1 \pmod{4}$;

 $D = \{u_{4k+2}, u_{4k+3}, u_{n-1}, u_n\}$ and $|D| = \lfloor \frac{n}{2} \rfloor + 1$, if $n \equiv 2, 3 \pmod{4}$.

The vertices of the maximum independent set of $P_n * P_m$ consist of the maximum independent sets of every P_m . Then, $\beta(P_n * P_m) = n\lceil \frac{m}{2}\rceil$. Independence number of $\langle V(P_n * P_m) - D \rangle$ is the same as the independence number of $V(P_n * P_m)$. Let *S* be the independent transversal dominating set of the graph $P_n * P_m$. $S = D \cup \{v_{11}\}$ if *m* is odd and $S = D \cup \{v_{11}, v_{12}\}$ if *m* is even. So, we have $\beta(V(P_n * P_m) - S) < \beta(P_n * P_m)$ and this means that $\langle V(P_n * P_m) - S \rangle$ doesn't contain any β – set of $P_n * P_m$. So,

$$\gamma_{it}(P_n * P_m) = \begin{cases} \frac{n}{2} + 1, & \text{if } n \equiv 0 \pmod{4} \text{ and } m \text{ is odd,} \\ \frac{n}{2} + 2, & \text{if } n \equiv 0 \pmod{4} \text{ and } m \text{ is even} \\ \lfloor \frac{n}{2} \rfloor + 2, & \text{if } n \equiv 1, 2, 3 \pmod{4} \text{ and } m \text{ is odd,} \\ \lfloor \frac{n}{2} \rfloor + 3, & \text{if } n \equiv 1, 2, 3 \pmod{4} \text{ and } m \text{ is even.} \end{cases}$$

The proof is completed. \Box

Theorem 3.4. Let P_n be any path graph with order n and $S_{1,m}$ be a star graph with order m + 1. Then,

$$\gamma_{it}(P_n * S_{1,m}) = \begin{cases} \frac{n}{2} + 1, & \text{if } n \equiv 0 \pmod{4} \\ \lfloor \frac{n}{2} \rfloor + 2, & \text{if } n \equiv 1, 2, 3 \pmod{4} \end{cases}$$

Proof. The proof is similar to the proof of Theorem 3.1. \Box

Theorem 3.5. Let P_n be any path graph with order n and $W_{1,m}$ be a wheel graph with order m + 1 for m > 3 and $m \neq 9$. Then,

$$\gamma_{ii}(P_n * W_{1,m}) = \begin{cases} \frac{n}{2} + 2, & \text{if } n \equiv 0 \pmod{4} \text{ and } m \text{ is even}, \\ \frac{n}{2} + 3, & \text{if } n \equiv 0 \pmod{4} \text{ and } m \text{ is odd}, \\ \lfloor \frac{n}{2} \rfloor + 3, & \text{if } n \equiv 1, 2, 3 \pmod{4} \text{ and } m \text{ is even}, \\ \lfloor \frac{n}{2} \rfloor + 4, & \text{if } n \equiv 1, 2, 3 \pmod{4} \text{ and } m \text{ is odd}. \end{cases}$$

Proof. The proof is similar to the proof of Theorem 3.1. \Box

Theorem 3.6. Let P_n be any path graph with order n and K_m be any complete graph with order m. Then,

$$\gamma_{it}(P_n * K_m) = \begin{cases} \frac{n}{2} + m, & \text{if } n \equiv 0 \pmod{4}, \\ \frac{n}{2} + 1 + m, & \text{if } n \equiv 2 \pmod{4}, \\ \lceil \frac{n}{2} \rceil, & \text{if } n \equiv 1, 3 \pmod{4}, \end{cases}$$

Proof. Let *D* be a γ -set of the graph $P_n * K_m$. $|D| = \frac{n}{2}$ if $n \equiv 0 \pmod{4}$, $|D| = \lfloor \frac{n}{2} \rfloor + 1$ otherwise. $\langle V(P_n * K_m) - D \rangle$ contains a maximum independent set so, $\gamma_{it}(P_n * K_m) > \gamma(P_n * K_m)$. We denote the vertices of P_n with u_i , $i = \overline{1, n}$ and the corresponding vertices of K_m with v_{ij} , $j = \overline{1, m}$. We have three cases: **Case 1.** $n \equiv 0 \pmod{4}$

In this case $D = \{u_{4k+2}, u_{4k+3}\}, k = \overline{0, \lfloor \frac{n}{4} \rfloor - 1}$ is a dominating set and the maximum independent sets are $\beta_1 = \{u_1, v_{1j}, u_3, v_{3j}, u_5, v_{5j}, ..., u_{n-1}, v_{n-1j}\}, \beta_2 = \{u_2, v_{2j}, u_4, v_{4j}, u_6, v_{6j}, ..., u_n, v_{nj}\} \text{ and } \beta_3 = \{v_{1j}, v_{2j}, v_{3j}, ..., v_{nj}\},$ where each *j* is related to exactly one member of $\{1, 2, ..., m\}$. So, $\beta(P_n * K_m) = n$. Let *S* be an independent transversal dominating set of $P_n * K_m$. We must add $\frac{n}{2}$ vertices from the graph P_n and *m* vertices from any graph K_m to S so that $\langle V(P_n * K_m) - S \rangle$ doesn't contain any β – set. So, we have $\gamma_{it}(P_n * K_m) = \frac{n}{2} + m$. **Case 2.** $n \equiv 2 \pmod{4}$

In this case $D = \{u_{4k+2}, u_{4k+3}, u_{n-1,u_n}\}, k = 0, \lfloor \frac{n}{4} \rfloor - 1$ is a dominating set and the maximum independent sets are $\beta_1 = \{u_1, v_{1j}, u_3, v_{3j}, u_5, v_{5j}, ..., u_{n-1}, v_{n-1j}\}, \beta_2 = \{u_2, v_{2j}, u_4, v_{4j}, u_6, v_{6j}, ..., u_n, v_{nj}\}$ and $\beta_3 = \{v_{1j}, v_{2j}, v_{3j}, ..., v_{nj}\}, \beta_2 = \{u_2, v_{2j}, u_4, v_{4j}, u_6, v_{6j}, ..., u_n, v_{nj}\}$ where each j is related to exactly one member of $\{1, 2, ..., m\}$. So $\beta(P_n * K_m) = n$. For the independent transversal dominating set selected as $S = D \cup K_m$, $\langle V(P_n * K_m) - S \rangle$ doesn't contain any β – set and all vertices of the graph $P_n * K_m$ are dominated. So, we have $\gamma_{it}(P_n * K_m) = \frac{n}{2} + 1 + m$. **Case 3.** $n \equiv 1, 3 \pmod{4}$

In this case the maximum independent set of $P_n * K_m$ is $\{u_1, v_{1j}, u_3, v_{3j}, ..., u_n, v_{nj}\}$, where each j is related to exactly one member of $\{1, 2, ..., m\}$. The vertex set that occurs one vertex from every graph K_m isn't a maximum independent set. For $k = 0, \lfloor \frac{n}{4} \rfloor - 1$, $D = \{u_{4k+2}, u_{4k+3}, \dots, u_{n-1}\}$ if $n \equiv 1 \pmod{4}$ and D = 1 $\{u_{4k+2}, u_{4k+3}, \dots, u_{n-1}, v_{nj}\}$ if $n \equiv 3 \pmod{4}$ is a dominating set. So, $\langle V(P_n * K_m) - D \rangle$ doesn't contain any β - set and $\gamma_{it}(P_n * K_m) = \gamma(P_n * K_m) = \lceil \frac{n}{2} \rceil$.

The proof is completed. \Box

4. Independent Transversal Domination Number for the Edge Corona of the Path Graph

Definition 4.1. [10]

The graph $G_1 \diamond G$ which is obtained by edge corona operation of a connected graph G_1 and graph G is formed as follows: Every edge e_i of graph G_1 correspond to a graph G and every vertex vij of G is adjacent to two end vertices of the corresponding edge e_i of G_1 , i = 1, $|E(G_1)|$ and j = 1, |V(G)|.

The edge corona of the graph $P_6 \diamond P_2$ can be depicted as in the following figure:

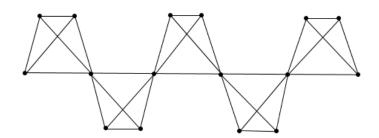


Figure 3: The graph $P_6 \diamond P_2$

Theorem 4.2. Let P_n and P_m be any path graphs with order n and m, respectively. Then,

$$\gamma_{it}(P_n \diamond P_m) = \begin{cases} \lfloor \frac{n}{2} \rfloor + 1, & \text{if } m \equiv 1 \pmod{2} \\ \lfloor \frac{n}{2} \rfloor + 2, & \text{if } m \equiv 0 \pmod{2} \end{cases}$$

Proof. The domination set of $(P_n \diamond P_m)$ is $D = \{u_{2k}\}, k = \overline{1, 2, ..., \lfloor \frac{n}{2} \rfloor}$. Also, the maximum independence number $\beta((P_n \diamond P_m) = (n-1)\beta(P_m)$. Let *S* be the independent transversal dominating set of $P_n \diamond P_m$. $< V(P_n \diamond P_m) - D >$ contains β – *set*. So, $\gamma_{it}((P_n \diamond P_m) > \gamma((P_n \diamond P_m))$. If we also add the vertex v_{11} in case $m \equiv 1 \pmod{2}$ and the vertices v_{11}, v_{12} in case $m \equiv 0 \pmod{2}$ to the *S* with V(D), then $< V(P_n \diamond P_m) - S >$ doesn't contain any β – *set*. Hence, we have

$$\gamma_{it}(P_n \diamond P_m) = \begin{cases} \lfloor \frac{n}{2} \rfloor + 1, & if \ m \equiv 1 \pmod{2} \\ \lfloor \frac{n}{2} \rfloor + 2, & if \ m \equiv 0 \pmod{2} \end{cases}$$

The proof is completed. \Box

Theorem 4.3. Let P_n be any path graph with order n and $S_{1,m}$ be a star graph with order m + 1. Then,

$$\gamma_{it}(P_n \diamond S_{1,m}) = \lfloor \frac{n}{2} \rfloor + 1.$$

Proof. The proof is similar to the proof of Theorem 4.1. \Box

Theorem 4.4. Let P_n be any path graph with order n and $W_{1,m}$ be a wheel graph with order m + 1. Then,

$$\gamma_{ii}(P_n \diamond W_{1,m}) = \begin{cases} \lfloor \frac{n}{2} \rfloor + 2, & if \ m \ is \ even, \\ \lfloor \frac{n}{2} \rfloor + 3, & if \ m \ is \ odd, \end{cases}$$

Proof. The proof is similar to the proof of Theorem 4.1. \Box

Theorem 4.5. Let P_n be any path graph with order n and K_m be any complete graph with order m. Then,

$$\gamma_{it}(P_n \diamond K_m) = \lfloor \frac{n}{2} \rfloor + m.$$

Proof. The proof is similar to the proof of Theorem 4.1. \Box

5. Independent Transversal Domination Number for the Join Operation of the Path Graph

Definition 5.1. [7] *Graphs* G_1 *and* G_2 *have disjoint vertex sets* V_1 *and* V_2 *and edge sets* E_1 *and* E_2 *respectively. Their union* $G = G_1 \cup G_2$ *has, as expected,* $V = V_1 \cup V_2$ *and* $E = E_1 \cup E_2$. *Their join is denoted* $G_1 + G_2$ *and consists of* $G_1 \cup G_2$ *and all edges joining* V_1 *with* V_2 .

The join operation of the graph $P_2 + P_3$ can be depicted as in the following figure:

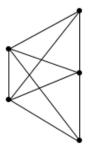


Figure 4: The graph $P_2 + P_3$

Theorem 5.2. Let G be any graph with order m and P_n be any path graph with order n. If $\beta(P_n) > \beta(G)$ then,

$$\gamma_{it}(G+P_n) = \begin{cases} 2, & if \ n \equiv 1(mod \ 2), \\ 3, & if \ n \equiv 0(mod \ 2). \end{cases}$$

Proof. We label the vertices as $u_i \in G$ and $v_j \in P_n$ of the graph $G + P_n$, for $i = \overline{1, m}$ and for $j = \overline{1, n}$. We can dominate all vertices of P_n with any vertex u_i and all vertices of G with any vertex v_j since $d(u_i, v_j) = 1$ $\forall u_i, v_j$. So, $\gamma(G + P_n) = 2$. Let S be any independent transversal domination set. If $n \equiv 1 \pmod{2}$ then, $\beta(P_n) = \beta(P_{n-1})$. In this case $S = \{u_i, v_1\}$ doesn't contain any β – *set*. If $n \equiv 0 \pmod{2}$ then, $\beta(P_n) = \beta(P_{n-1}) + 1$. In this case $S = \{u_i, v_1, v_2\}$ doesn't contain any β – *set*, where each i is related to exactly one member of $\{1, 2, ..., m\}$ So, we have

$$\gamma_{it}(G+P_n) = \begin{cases} 2, & \text{if } n \equiv 1 \pmod{2}, \\ 3, & \text{if } n \equiv 0 \pmod{2}. \end{cases}$$

The proof is completed. \Box

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