RISK PREMIUM IN MOTOR VEHICLE INSURANCE

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ABSTRACT

The pure premium or risk premium is the premium that would exactly meet the expected cost of the risk covered ignoring management expenses, commissions, contingency loading, etc. Claim frequency rate and mean claim size are required for estimation in calculating risk premiums.

In this study, we discussed to estimate claim frequency rate and mean claim size with several methods and calculated risk premiums. Data, which supported our study, is provided by insurance company involving with motor vehicle insurance.

Key words: *Number of Claims, Claim Sizes, Claim Frequency Rate, Risk Premium.*

1. Introduction

Insurance is an important part of risk management programs for organisations and individuals. Insurance is a risk financing transfer under which an insurer agrees to accept financial burdens arising from loss. More formally, insurance can be defined as a contractual agreement between two parties: the insurer and the insured.

Insurance may be classified into three groups – insurance of person, insurance of property and pecuniary interest, and insurance of liability. General insurance, or Non-Life insurance or Property/Casualty insurance as it is sometimes called, normally relates to the insurance of property and liability.

In this study, we are interested in motor vehicle insurance. Motor vehicle insurance contracts are schedule contracts that the permit the insured to purchase both property and liability insurance under one policy. The contract can be divided, however, in two separate contracts – one providing insurance against physical damage to automobiles, and the other protecting against potential liability arising out of the ownership, maintenance, or use of an automobile. Some automobile insurance

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contracts, notably those issued by insurers associated with automobile finance companies, cover only physical damage insurance.

2. The Risk Premium

The pure premium or risk premium is the premium that would exactly meet the expected cost of the risk covered ignoring management expenses, commissions, contingency loading, etc. To compute it, we need to estimate the claim frequency rate and mean claim size. Multiplying claim frequency rate by mean claim size yields the risk premium.

A claim is an assertion of a right to payment, as when a customer notifies a manufacturer of an injury from a defective product and expenses a belief that the injury justifies compensation. Usually, the payment is to occur at some future point in time. It should not be assumed that all accidents become claims, nor that the amount of the claim will always be the same as the amount of the damage.

The claim size is the sum, which the insurer has to pay on the occurrence of a fire, an accident, death or some other insured event. The sum of the individual claims constitutes the aggregate claim amount, which is one of the key concerns, both in the practical management of an insurance company and in theoretical consideration.

The claim frequency rate is a rate which can be estimated as the number of claims divided by the number of units of exposure.

It is important that the two components, claim frequency rate and claim size be considered separately. Only in exceptional circumstances can the use of 'rate of payment of claims' be considered satisfactory for the assessment of a risk premium.

3. Method of Study

Analysis is based on simple formulations as follows (Hossack, 1989: 123-125):

(1) Depending on yearly total claim sizes and number of claims, mean claim sizes for each year can be calculated by the formula

$$MCS_i = \frac{TCS_i}{c_i}$$

$$(1)$$

where the notation is

MCS_i : mean claim size for year i

TCS_i : total claim size for year i

C_i : number of claims in year i

(2) Depending on yearly total amount of premiums and number of policies, average office premium per policy for each year can be calculated by the formula

$$AOP_i = \frac{TOP_i}{N_i}$$

$$(2)$$

where the notation is

AOP_i : average office premium per policies for year i.

TOP_i : total amount of premiums for year i.

N_i : number of policies in year i.

(3) Depending on yearly total claim sizes and number of policies, risk premium for each year can be calculated by the formula

$$RP_i = \frac{TCS_i}{N_i}$$

$$(3)$$

where RP_i denotes risk premium for year i.

(4) Depending on yearly office premium and risk premium, the ratio can be calculated by the formula

$$k_i = \frac{AOP_i}{RP_i}$$

(4)

where k_i denotes the ratio of office premium to risk premium for year i.

(5)   A fixed claim frequency rate can be calculated depending on the total number of policies and the total number of claims.

In fact, claim frequency rate is the ratio of the number of claims in a period to the exposure to risk for this period. The main problem in calculating claim frequency rate is to find the exposure to risk can be calculated by various methods. One of them is the Census Method.

Let $N(t)$ denote the number of policies booked by the insurance company at time t. During the short interval of time δt immediately following time t, the change in the number of policyholders a premium will be negligible. Each of the $N(t)$ premium policyholders currently on the books will contribute δt years of exposure during that small time interval so that the total number of years' exposure during the interval of time δt following time t will be $N(t)\delta t$. The total exposure during 1997-1998 will,

therefore, be approximately

$$\sum_{\substack{t=0 \ \ to \ \ 1 \\ in \ \ steps \ \ of \\ \delta t}} N(t)\delta t$$

the calculation becomes more and more accurate the smaller δt becomes, and exact when δt becomes too small. Then in mathematical terms dt is written instead of δt, and becomes
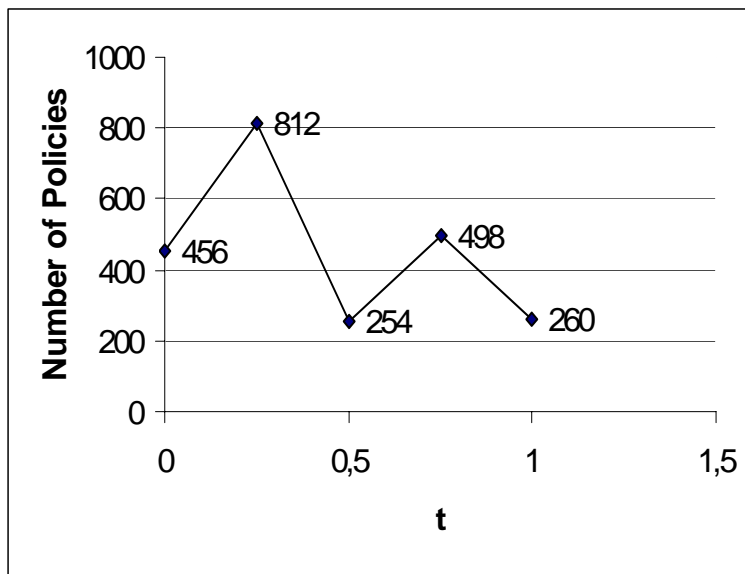
$$\int_0^1 N(t)dt$$

i.e. the area under the *N(t)* curve between *t=0* and *t=1*.

For example, when data given in Table 4.2 is considered Figure 4.1 is the approximate graph of *N(t)*.

Figure 1. Available number of policies during October 1,1997 to October 1,1998

The area under *N(t)* curve is

$$\sum_{t=0}^{3} \frac{1}{2}\left[N\left(\frac{t}{4}\right) + N\left(\frac{t+1}{4}\right)\right] \frac{1}{4}$$

and is calculated to be 481, since the number of claims during this period is reported to be 183, the claim frequency rate is said to be 183/481 and equals 0.38.

When the period is extended to k years and total number of claims booked each year are assumed to be exposure to risk for that year, then total exposure to risk becomes $N_1+N_2+…+N_k$. The total number of claims given by

$C_1+C_2+\ldots+C_k$. therefore yields the claim frequency rate to be calculated by the formula

$$m = \frac{\sum_{i=1}^{k} C_i}{\sum_{i=1}^{k} N_i}$$

(5)

where m is the claim frequency rate which is assumed to be constant during k years.

(6) On the implications of the mean claim sizes and total number of claims booked by the company the growth rate can be calculated. In the growth rates geometric progression can be assumed or both geometric and arithmetic progressions can be assumed.

4. Application

The insurance company is established in 1992 and the available data comprises of

- the number of policies booked each year

- total amount of the premiums collected for each year

- yearly number of claims

- total claim size for each year

beginning from 1993 to 1997. These data are given in Table 1.

Table 1. Yearly Income and Outgo

Years	Number of Policies	Total Amount of Premium ($)	Number of Claims	Total Claim Size ($)
1993	614	52 190	325	28 340
1994	785	78 500	402	42 150
1995	1 044	96 048	328	51 140
1996	1 472	114 816	499	62 539
1997	2 178	141 570	592	79 856
Total	**6 093**	**483 124**	**2 146**	**264 025**

Besides, the numbers of policies booked by the company October 1, 1997 to October 1, 1998 are also available. This data is presented in Table 2.

Table 2. Available number of policies during October 1, 1997 to October 1, 1998

Dates	Number of Policies
October 1, 1997	456
January 1, 1998	812
April 1, 1998	254
July 1, 1998	498
October 1, 1998	260

Number of claims during the time between October 1, 1997 to October 1, 1998 is 183.

In this section the results will be obtained by applying each step explained in section are presented.

(1) Mean claim sizes for each year are presented in Table 3.

Table 3. Mean Claim Sizes

Years	Number of Claims	Total Claim Sizes ($)	Average Claim Sizes ($)
1993	325	28 340	88
1994	402	42 150	105
1995	328	51 140	156
1996	499	62 539	125
1997	592	79 856	135

(2) Average office premium for each year are presented in Table 4.

Table 4. Average Office Premiums

Years	Total Office Premiums	Number of Policies	Average Office Premiums
1993	52 190	614	85
1994	78 500	785	100
1995	96 048	1 044	92
1996	114 816	1 472	78
1997	141 570	2 178	65

(3) Risk premium for each year are presented in Table 5.

Table 5. Risk Premiums

Years	Total Claim Sizes ($)	Number of Policies	Risk Premiums
1993	28 340	614	47
1994	42 150	785	54
1995	51 140	1 044	50
1996	62 539	1 472	42
1997	79 856	2 178	36

(4) *k* for each year are presented in Table 6.

Table 6. Yearly ratios of office premiums to risk premiums

Years	Average Office Premiums	Risk Premiums	k
1993	85	47	1.81
1994	100	54	1.85
1995	92	50	1.84
1996	78	42	1.86
1997	65	36	1.81

(5) Claim frequency rate depending on data given Table 1 and calculated by formula (5) is

$$m = \frac{2146}{6093} = 0,35$$

and this can be assumed to be a crude estimate.

(6) The most reasonable yearly mean claim size figures constitute a geometric progression. The first mean claim size in Table 3 is $88 and four year after it becomes $135.

Let *r* denote the growth rate in mean claim size and with the assumption of geometric growth

$$MCS_{i+1} = (1+r)MCS_i$$

Therefore, r can be calculated from

$$135 = (1+r)^4 88$$

and is found to be 11.29%.

The number of policies booked by the insurance company which are given in Table 1 on yearly basis implies not only a geometric growth but also an arithmetic growth.

The relation between the successive number of total policies is found to be as follows by trial and error.

$$n_2=1.2785n_1$$

$$n_3=(1.2785+0.063)n_2$$

$$n_4=(1.2785+2\times0.063)n_3$$

$$n_5=(1.2785+3\times0.063)n_4$$

or expressed by a general formula as follows

$$n_{i+1} = \left[1.2785 + (i-1)\times0.063\right]n_i$$

We begin the work with the following assumptions:

(1) In 1993, 614 policyholders are to be booked, and mean claim size is to be $88.

(2) Claim frequency rate during the following 5 years is to be constant and equal to 35%.

(3) Mean claim size increases by a constant rate which is equal to 11.29%.

(4) Number of policyholders increases by the relation

$$n_{i+1} = \left[1.2785 + (i-1)\times0.063\right]n_i$$

(5) Only half of the claims reported in a year are paid in that year, and the other half of the claims are paid in the succeeding year.

With respect to the above assumptions mean claim sizes, number of policies, number of claims and total claim size paid each year can be calculated. The calculated values are showed in Table 7.

Table 7. Number of policies and claim amount under the assumptions

i	Years	Average Claim Sizes ($)	Number of Policies	Number of Claims	Total Claim Sizes($)
1	1993	**88**	**614**	215	$\frac{215}{2}(88+98) = 19\ 995$
2	1994	98	785	275	$\frac{275}{2}(98+109) = 28\ 462$
3	1995	109	1 053	369	$\frac{369}{2}(109+121) = 42\ 435$
4	1996	121	1 479	518	$\frac{518}{2}(121+135) = 66\ 304$
5	1997	135	2 170	760	$\frac{760}{2}(135+150) = 108\ 300$

Calculation of claim frequency rate as a ratio of number of claims to the exposure to risk yields a relation which is very useful to estimate the number of claims for each year. The number of claims which are denoted by c_i in Table 7 are calculated by

$$c_i = m_i \times n_i \qquad (6)$$

Total claim size for each year is to be calculated by considering the number of claims produced that year and half of them to be paid mean claim size of that year, and the other half to be paid mean claim size of the succeeding year with respect to assumption (5) stated above:

$$TCS_i = \frac{c_i}{2}\left(MCS_i + MCS_{i+1}\right) \qquad (7)$$

The numerical figures presented in Table 7 can be used in order to calculate the expected mean claim sizes by applying formula (1), and risk premiums by formula (3).

The risk premiums obtained by these calculations are presented in Table 8.

Table 8. Risk premiums under the assumptions

I	Years	Number of Claims	Number of Policies	Total Claim Sizes	Risk Premium
1	1993	215	614	19 995	32.55
2	1994	275	785	28 462	36.22
3	1995	369	1 053	42 435	40.25
4	1996	518	1 479	66 304	44.80
5	1997	760	2 170	108 300	49.87
		2 137	**6 101**	**265 496**	

The total row in Table 8 contains the most important numerical figures in our analysis. As a result of the analysis total number of policies booked in 5 years is to be 6101, total number of claims in this period is to be 2137, and the total amount of claim sizes to be paid in this period is $ 265 496.

If the calculated risk premiums in Table 8 are to be applied during the years 1993-1997, the number of policies realized (data given in Table 1) would give rise to collection of total amount of net premiums as presented in Table 9.

Table 9. Total amount of net premiums

i	Years	Realized Number of Policies	Planned Risk Premium	Total amount of premium
1	1993	614	32.55	19 985.70
2	1994	785	36.22	28 436.62
3	1995	1 044	40.25	42 021.00
4	1996	1 472	44.80	65 945.60
5	1997	2 178	49.87	108 627.75
Total		**6 093**		**265 016.67**

The total rows both in Table 1 and 9 show that 6093 policy are booked in five years, total number of claims in this period in 2146 (Table 1) and the total amount of risk premiums to be collected by the proposed plan is $265 016.67 (Table 9).

5. Conclusion

We see that the realized values of the five years totals of number of claims and number of policies are very close to the values obtained under the planning assumptions. Besides, the total amount of risk premiums which would be collected under the proposed plan would cover the realized total amount of claim sizes.

In this paper, we studied the data provided by an insurance company involving with motor vehicle insurance in the light of theory of insurance risk model and risk premiums.

First of all, available data is analysed. After words, reasonable assumptions on the growth of number of policies, increase of claim frequency rate, increase of mean claim size by a constant rate, a risk premium payment plan for five years is proposed. In conclusion, the proposed plan is found to give rise to very close values the realized ones, and be capable to cover the realized expenses.

Özet

Risk primi, bir sigorta şirketinde yönetim masrafları, komisyonlar göz önüne alınmadan riskin beklenen maliyetini karşılayan primlerdir. Risk primlerinin hesaplanabilmesi için hasar sıklık oranı ve ortalama hasar büyüklüğünün tahmin edilmesi gerekir.

Bu çalışmada, hasar sıklık oranı ve ortalama hasar büyüklüğünün çeşitli yöntemlerle tahmin edilmesi tartışılmış ve risk primleri hesaplanmıştır. Çalışmamızı destekleyen veriler kasko sigortalarını içeren sigorta şirketi tarafından elde edilmiştir.

Anahtar Kelimeler: hasar sayısı, hasar büyüklüğü, hasar sıklık oranı, risk primi.

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