International Journal of Educational Studies in Mathematics, 2022, 9(1), 49-66

International Journal of Educational Studies in Mathematics

IJESIM

EDUGARDEN

Geometric Habits of Mind: The Meaning of Quadrilaterals for Elementary School Student Teachers

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ABSTRACT

The primary purpose of this study is to determine the geometric habits of mind of classroom teacher candidates and the level of these habits. The study was conducted with 55 elementary education program students studying at an education faculty in the 2017-2018 spring term. The criterion used in the determination of the study group is that the teacher candidates should have knowledge about almost all the courses in the classroom education program and have taken courses I and II in mathematics teaching. Three worksheets were distributed to the teacher candidates as a data collection tool. Since the research is like a case study, the mathematical ideas developed by the teacher candidates about quadrilaterals and their written answers were summarized and interpreted according to the predetermined themes. When the findings obtained as a result of the research are examined in general, it can be said that teacher candidates are not sufficient in finding relationships between quadrilaterals, generalizing the relationships they find by using strategies, associating the feature valid for a quadrilateral with quadrilaterals that may have that feature, namely the whole, and making judgments.

ARTICLE INFO

Article History:
Received: 06.12.2021
Received in revised form: 24.03.2022
Accepted: 24.03.2022
Available online: 28.03.2022
Article Type: Standard paper
Keywords: geometric habits of mind, elementary school student teachers, rectangles, exclusion definition, inclusive definition.

1. Introduction

Humanity's effort to make sense of the universe and life has experienced a process that has evolved till today's information society. Today, the groundbreaking development of knowledge continues its advance at an almost untraceable speed. However, the proliferation of information and easy accessibility brought a different challenge in this development process. It is seen that a process in which the acquisition of many basic thinking structures, which are daily life skills, is not sufficient, awaits us. For example, someone who is trying to buy a house thinks about his/her belongings and the architecture of the house, being unable to establish a geometric relationship between these two, and someone who is preparing to go on vacation having difficulty in preparing a package according to the luggage size of the car. When these examples are examined, it is seen that recognizing geometric shapes, creating models, spatial thinking, and associating these with daily life are essential. In brief, it is necessary to have basic skills related to geometry, a sub-branch of mathematics that has many places in daily life.

The meaning of the word geometry is expressed as "the branch of mathematics that examines the relations, measurements and properties of points, lines, angles, surfaces, and objects, in the dictionary

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DOI: https://doi.org/10.17278/ijesim.1033078

of the Turkish Language Association (2019). Geometry as a term makes it easier for us to understand the earth as shape, place, and direction; It is a field taught in schools as a part of the mathematics lesson, which includes the relations with the object and shape information (Türnüklü, Gündoğdu Alaylı, and Akkaş, 2013). The aim of mathematics education in schools is to equip the individuals with basic knowledge about arithmetic, algebra, and geometry and to enable them to possess the sensitivity of being consistent between the results obtained by guiding them to different ways of thinking (Yıldırım, 2000). As one of the easiest subfields of mathematics to embody, Geometry is full of concepts and shapes that we frequently encounter daily. Therefore, the primary purpose of teaching geometry in schools is to enable students to think and reason by establishing relationships between subject areas and to gain skills such as being consistent in the results they reach.

While mathematics requires analytical thinking, establishing relationships between geometric shapes, concepts and principles are at the forefront to solve geometry problems in geometric thinking (Van de Walle, Karp & Bay-Williams, 2016). In order to establish a relationship between geometric shapes such as squares, rectangles, parallelograms, it is necessary to define geometric shapes and express their properties. According to the Van Hiele geometric thinking levels, which follow a hierarchical order and develop not with age, but with the geometry experience of the individual, at the visual level, the individual cannot realize that the geometric shapes consist of parts and cannot perceive their features. At the descriptive level, although he/she knows that the geometric shapes are composed of parts, he/she cannot see the relationship between the features. Therefore, instead of making a necessary and sufficient conditional definition of a shape, it defines it with sentences that include the features of the shape. For example, in the definition of a parallelogram, too many features can be inferred from each other, such as "opposite sides equal, opposite sides parallel, opposite angles equal, quadrilateral, four-angled...". At the superficial inference level, the individual can make a short and concise definition by stating the sufficient and necessary features to define a concept, since he can establish relationships between shape classes and see the relationships between features (Duatepe Paksu, 2016; Van de Walle, Karp, & Bay-Williams, 2016). Usiskin, Griffin, Witonsky, and Willmore (2008) have a different approach to definition. They express the definitions based on the relationship between special quadrilaterals as inclusive definitions. In the inclusive definition, one quadrilateral includes the definition of the other quadrilateral. The relationship between special quadrilaterals is ignored, and only one quadrilateral is defined as an external definition. The external definition is the definition of only one quadrilateral, ignoring the relationship between special quadrilaterals. In the external definition, a definition does not include the definition that belongs to another quadrilateral.

Geometric thinking habits are another important concept in understanding the conceptual aspect of geometric shapes and showing these relationships by judging the relationship between geometric shapes and establishing relationships between them. Geometric thinking habits are the repertoire of an individual who encounters a geometry problem to solve the problem (Erşen, 2017). Geometric habits of mind (GHM) are expressed by Cuoco Goldenberg and Mark (2010) as the habits of reasoning, investigating geometric invariants, thinking extreme cases, visualizing, and manipulating. Driscoll, DiMatteo, Nikula, and Egan (2007) mentioned a similar thinking habit. According to Driscoll et al. (2007), geometric habits of mind (GHM), a productive state of thinking that supports learning and applying geometry and mathematics, consists of four sub-components. These are:

Associating: In this component, which can be defined as the investigation of geometric relationships within geometric shapes and objects, relationships can be between discrete shapes, parts of the same shape, or between concepts (Driscoll et al., 2007; Driscoll, DiMatteo, Nikula, Egan, June and Kelemanik, 2008). Identifying critical features in objects involves investigating the relationships between these features and the geometric shapes in the problem (Wiles, 2013). In the component, relationships can be revealed with the help of questions such as “How are the similar aspects of the shapes?”, “What are the differences of the shapes?”, “What are the other shapes that fit the definition?” (Driscoll et al., 2007).

Generalizing: It can be defined as "the desire to understand and explain most of the time and always statements about geometric concepts and operations." In this component, the results and properties of geometric shapes and objects are generalized to the cluster to which that shape or objects belong (Driscoll et al., 2008 and 2007). Generalization can be seen when students explore a geometric problem and use insights to understand better problems related to the related problem or more complex problems. Students ask themselves how they can generalize assumptions about a particular situation to larger classes of geometric objects through questions such as "Does this happen in every situation?", "If it does, why?", "Can I find different examples suitable for this definition?".

Examining invariants: It can be defined as "analyzing which properties of a geometric shape are affected as a result of a transformation (such as reflection, translation, rotation)" (Driscoll et al., 2008). Features that change geometrically and do not change due to the transformation are analyzed by asking, "What is changing or unchanging? Why?" (Driscoll et al., 2007).

Discoveries with Reflections: It is the final component of the geometric habits of the mind. The component can be expressed as "Trying out different approaches, usually chosen as a result of proposed hypotheses, and regularly considering what has been learned" (Driscoll et al., 2008).

Among the studies conducted in Turkey on the geometric habits of the mind are conducted with mathematics teacher candidates (Bülbül, 2021; Bülbül and Güler, 2021; Koç & Bozkurt, 2012), classroom teacher candidates (Yavuzsoy-Köse & Tanışlı, 2014), high school students (Erşen, 2017), mathematics teachers (Tolga & Cantürk-Günhan, 2019). In addition, studies investigating features such as definitions, representations, and reasoning of quadrilaterals with each other were conducted with teacher candidates (Bütüner & Filiz, 2006; Erşen & Karakuş, 2013; Fujita & Jones, 2006; Kartal & Çınar, 2017; Türnüklü, Gündoğdu Alaylı & Akkaş). , 2013), with secondary school students (Aktaş & Cansız Aktaş, 2012; Akuysal, 2007; Ay & Başbay, 2017), with teachers (Akkaş and Türnüklü, 2014). When all the studies are examined in general, it is seen that the geometric habit levels of the mind are low, and there is a significant correlation between geometric attitude and habits in all age groups participating in the study; it has been found that they mostly have difficulties in making correct definitions of geometric shapes, in hierarchical classification that requires establishing relationships with each other, and in expressing their diagonal features. For example; In the study in which the performances and geometric reasoning skills of prospective mathematics teachers were examined, it was determined that while most of the teacher candidates could not identify the geometric habits they used to complete the study and their geometric thinking skills were limited (Koç & Bozkurt, 2012). In a study conducted by Yavuzsoy Köse and Tanışlı (2014), which reached similar results; classroom teacher candidates do not have multiple ways of thinking in the context of reasoning, discovering and investigating invariants according to the geometric habits of the mind; the majority of them generalize at an underdeveloped level; It was also found that the geometric habits of mind were not at the desired level in teacher candidates. On the other hand, Bülbül and Güven (2020) concluded that the geometric habits of the mind developed and changed, based on the results of their longitudinal study with pre-service teachers.

Erşen and Karakuş (2013) concluded in their research with teacher candidates that they made incorrect drawings due to the lack of notation representation in quadrilateral drawings, not knowing the properties of the shape, and not being able to classify the relationships between quadrilaterals. Teacher candidates acted more intuitively in the drawing of quadrilaterals within the scope of the study. It has been determined that teacher candidates have a single quadrilateral image in their minds.

In the study in which the geometric thinking habits of the tenth-grade students of science high school and their attitudes towards geometry were investigated, it was concluded that the geometric thinking habits of the students were moderate, there was a high level of relationship between the attitudes towards geometry and thinking habits, and the attitude towards geometry predicted their thinking habits (Erşen, 2017). In the study in which the relationship between the geometric habits of thinking of pre-service mathematics teachers and their academic success was investigated, it was concluded that there was a high level of positive and significant relationship between the academic success and their geometric thinking habits of prospective mathematics teachers (Bülbül & Güven, 2019). In the study conducted to determine the misconceptions of seventh-grade students about geometric concepts, the students thought that quadrilaterals such as square, rectangle, trapezoid would not have parallelogram properties; Since they memorized the properties of special quadrilaterals, they could not establish the relationship between geometric shapes; It has been determined that they remember the shapes with the form and name they first learned, and they could not associate them with the geometric concepts learned later (Akuysal, 2007). In the study, in which the mathematics teacher candidates were tried to determine the exceptional cases of quadrilaterals and their hierarchical classification performances, it was concluded that the majority of the mathematics teacher candidates could not detect the exceptional cases of the trapezoid, deltoid, and rectangle (Bütüner & Filiz, 2006). In their study, Türnüklü, Gündoğdu Alaylı and Akkaş (2013) concluded that the definitions of quadrilaterals by mathematics teacher candidates were generally formed by themselves and their definitions were shaped within the framework of their perceptions.

In addition to various research results, it is seen that while the average of the geometry section consisting of 30 questions in the Undergraduate Placement Exam (UPE/LYS), which was applied as a transitional exam to secondary education until recently, was 10.5 in 2010, and it decreased over the years reaching to 3.8 in 2015. In this sense, it is clear that the geometric habits of the mind are a subject that should be emphasized.

1.1. Purpose and importance of the study

Students are required to classify geometric shapes to develop their geometric knowledge, skills, and thoughts. It is possible to consider geometric shapes with their functional aspects and see and teach geometry as a network of relations (Olkun & Aydoğdu, 2003). In order to achieve the goal of developing geometric thinking in classrooms, teachers need to develop their understanding of geometric thinking (Driscoll et al., 2007). Because their understanding will affect the learning-teaching process, it is one of the aims of mathematics education to enable students to become aware of the approaches that can help them both in daily life and while doing mathematics and to make these behaviors a habit (Körükcü & Şengül, 2017). It is essential for the development of mathematical thinking that the habits of the mind related to classroom practices (Lim & Selden, 2009) are taught to students from an early age (Korkmaz, Dündar & Yaman, 2016).

According to Cuoco, Goldenberg and Mark (1996), habits of mind enable developing different solution approaches to newly encountered problem situations. Bass (2008) expressed the habits of mind as the practices of mathematicians and stated that these practices include asking questions, searching for structures and examples, consulting relevant literature or experts, making connections, using mathematical language carefully, searching and analyzing evidence, and generalizing. In addition, he claims that children can develop these practices starting from their primary school years (Lim & Selden, 2009). In mathematics education, it can be said that the effects of the classroom teachers on the success of the students in the mathematics lesson are high. Therefore, teachers are expected to have some skills to act like mathematicians in their classroom practices.

Shulman (1987) gathered the knowledge that a teacher should have within the scope of teacher competence in seven groups. These are content, pedagogy, curriculum, students and learning, educational content, philosophy, purpose and objectives, pedagogical content knowledge. Within these seven different groups, content knowledge and pedagogical content knowledge come to the fore. Shulman (1987) states that content knowledge includes the knowledge of concepts and facts in the teacher's field, and pedagogical content knowledge includes showing and formulating a subject that makes it understandable for students. Studies have shown that teachers' subject matter knowledge and pedagogical content knowledge are important factors affecting students' mathematics achievement (Dursun & Dede, 2004; Hill et al., 2005; Romberg & Carpenter; 1986). Therefore, practical teaching of the mathematics course within the scope of the stated competencies requires the teacher to have sufficient field knowledge and to be able to effectively transfer this knowledge to the students.

Teachers need to have habits of mind in general, and geometric habits of mind in particular, to compare the transformation, translation, and similar situations of geometric shapes and reveal the similarities and differences of geometric shapes with special statuses other shapes.

Giving students the geometric habits of mind will increase their geometry and success in other courses. For students to acquire and internalize these habits, teachers need to be active in the process. Teachers must work on geometric problems with their students, to help students express their thoughts, and to enable them to have teaching experiences that will contribute to the development of their geometric thinking (Yavuzsoy Köse & Tanışlı, 2014). This situation is closely related to the "Zone of Proximal Development" theory that Vygotsky put forward in the cognitive development process (Vygotsky, 1998, cited in Fer, 2014). According to this theory, which is also referred to as "Area for Development," it is the difference between the potential that the learner can reach alone and the potential that the learner can reach with the support of the teacher at the current level of development (Koç & Demirel, 2004). This situation is expressed as the stage in which learning takes place. In order to fulfill all these important tasks, teachers must first have the geometric habits of the mind and exhibit these habits.

It was necessary to carry out this research in order to investigate the geometric habits of mind possessed by prospective elementary school teachers who will guide the acquisition of geometric thinking skills at primary school (1-4th grade), which is a critical period in the teaching of basic geometric concepts and to determine the habits of teacher candidates. Teachers need to have a broad and deep knowledge of geometry in order to be able to provide sound guidance while teaching geometry subjects, and they must also be aware of the relationship between geometry concepts (Ma, 1999; cited in Kartal & Çınar, 2017). Furthermore, teachers also need to be prepared in this regard (Özen, 2015). In the light of the studies in the literature, this study aims to determine the geometric habits of mind of classroom teacher candidates and the level of these habits. For this purpose, answers to the following questions were sought:

1. What are the definitions of the classroom teacher candidates regarding quadrilaterals?
2. Which characteristics of quadrilaterals do the definitions of the quadrangles by the classroom teacher candidates?
3. What are classroom teacher candidates' geometric habits about quadrilaterals?

2. Methodology

In this part of the study, the type and design of the research, the research group, the data collection tool used, the methods used to analyze the data, and the reliability of the research are explained. In addition, this study was approved by the ethics committee of Süleyman Demirel University with the number E-87432956-050.99-125461.

2.1. Type and Design of The Research

This study is a qualitative research conducted to examine classroom teacher candidates' geometric habits of mind (GHM) processes. This study, it is aimed to examine in-depth the cognitive knowledge processes of classroom teacher candidates on quadrilaterals according to GHM and to reveal the difficulties they encounter, if any. For this reason, it was aimed to determine the Geometric Habits of Mind and at which points they had difficulties related to the subject of quadrilaterals by giving a structured form developed by the researchers and handled in two dimensions. First, the written data were analyzed and interpreted by qualitative methods. Therefore, this study is a case study that can explain and analyze one or more cases (Karasar, 2003; Yin, 2011; Creswell, 2016). The primary purpose of this type of study is to analyze one or more situations holistically in a limited way (Yıldırım & Şimşek, 2013). For this purpose, an internal case study was planned, that is, a study used primarily to understand a particular case (Demir et al., 2014, p.395).

2.2. Study group

The study was carried out with 55 classroom teacher candidates studying their last year at an education faculty. In order to obtain an answer to the research problem, criterion sampling, one of the purposive sampling methods, was used (Yıldırım & Şimşek, 2013). The criterion used in the determination of the study group is that the teacher candidates have knowledge about almost all the courses in the elementary education program and have taken the Mathematics Teaching I and II courses.

Table 1 displays the age distribution of the students.

Table 1. Study group and their features

	Female	Male	Total
4th Grade	44	11	55

Teacher candidates who will participate in the study volunteered to participate. In addition, it was thought that the closeness of the study group to obtaining the teacher qualification and completing their education would impact the validity and reliability of the data obtained in the study.

2.3. Data collection tool

The research data collection tool is a form developed by the researchers, and this form consists of two parts. In the first part of the form, demographic characteristics were asked. In the second part, two questions aimed at revealing the teacher candidates' knowledge about the quadrilaterals and the geometric habits of the mind were asked. In the first question in the second part, they were asked to define six quadrilaterals using their properties. In the second question, at least one of these six shapes was printed out, and the pre-service teachers were asked to cut out these shapes, group the quadrilaterals with the shapes they associate, and write down which shapes they associated why. In order to ensure content validity, opinions were obtained from three experts who have a doctorate in the field of mathematics education experts examined the questions in terms of language, content, and compatibility with the components. Similarly, to ensure its validity, a pilot study was conducted with ten teacher candidates studied at the same department and were excluded from the study group regarding the status of the questions in the test to represent the area to be measured. Finally, in line with the data obtained, the suitability of the questions to the components of the geometric thinking habits of the mind was independently examined by the researchers, and the final form of the form was created in line with the opinions.

2.4.Data collection method

When the studies on geometry are examined, it is seen that data are collected by asking questions such as defining geometric shapes, drawing or classifying geometric shapes, explaining what the given shape is and the reason for it, and similar questions. In this study, data collection was done differently. The classroom environment where the application was made is shown in Figure 1.

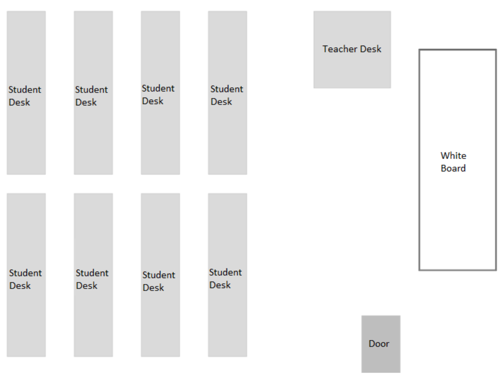

Figure 1. The Structure of the Class in which the Study is Conducted

In the study, firstly, general information about the quadrilaterals and the study was provided to the classroom teacher candidates, and then the data collection tool was distributed. Three worksheets were distributed to the teacher candidates as a data collection tool. Another sheet of tables was given to the teacher candidates to group the geometric shapes given in the worksheet according to their similarities and express their thoughts on the grouping they made. Then, the candidates were asked to cut the rectangles from the given worksheets and paste them to the relevant places in the tables. In addition, they were asked to explain why they attached the rectangles to the relevant places in the tables, and they were allowed to express their thoughts. For this, scissors and glue were provided to the teacher candidates in the classroom. Thus, it was aimed that the teacher candidates answer the questions by doing and living in the process.

There are eight geometric shapes in the leaf, consisting of geometric shapes. Due to translation and rotation, some of these shapes are given more than one place.

The study was carried out outside the classroom hours at the end of the spring semester of the 2017-2018 academic year. There was no time limit for answering the form. All of the participants completed the information in the form within a total of 57 minutes. The obtained data were documented in writing and analyzed qualitatively.

2.5. Validity and reliability of the study

If the reliability of research is defined as the fact that a study repeated each time yields the same results, the accuracy or suitability of the results obtained can be expressed as the validity of the research (Whittemore, Chase, & Mandle, 2001; Houser, 2015). While the human factor is the most critical (Merriam, 2013) in the stages of data collection and analysis in educational science research, we can reach the facts about events and phenomena with the help of observations and interviews. Therefore, the researcher should objectively convey all the information he/she has obtained as a result of observations and interviews and should provide the reader with the opportunity to evaluate the results of the research with the results obtained (Merriam, 2013; Yıldırım & Şimşek, 2013; Creswell, 2016). In this sense, the essential duty and responsibility of the researcher in qualitative research are to act objectively and convince the reader that the study is reliable and valid (Merriam, 2013). For this, researchers can talk about the concepts of credibility instead of internal validity, transferability instead of external validity, consistency instead of internal reliability, and repeatability instead of external reliability (Lincoln & Guba, 1985) by the nature of qualitative research. However, Creswell (2003) mentions eight features to ensure reliability in qualitative studies. It is possible to count them as follows: (1) long-term interaction and continuous observation, (2) diversification (triangulation), (3) peer review and questioning, (4) negative situation analysis, (5) explanation of the researcher's prejudices, (6) participant confirmation, (7) rich and detailed description, (8) external audit (examination). Creswell (2003) stated that the presence of at least two of these eight features in a qualitative study would be sufficient for the study to be reliable and valid.

In this study, the stages of providing three of the eight features mentioned above in order to ensure reliability and validity are presented as follows:

Ensuring an appropriate and sufficient interaction with the participants during the data collection phase of the research is essential to increase the trust and credibility during the study (Creswell, 2016). Therefore, the researchers interacted with teacher candidates by informing them about GHM before applying. Another way to increase credibility is triangulation. Accordingly, more than one researcher should use a data collection method with more than one data source to control the accuracy and authenticity of the findings (Merriam, 2013). For this purpose, during the implementation of the study, the researchers were in the classroom, and various data sources in the form of environment observation notes and worksheets of teacher candidates were consulted. The teacher candidates' worksheets and observation notes were analyzed using data diversity at the end of the process.

Instead of external validity, which means the generalizability of research results, the concept of transferability is used in qualitative studies. In qualitative studies, transferability means that the research results cannot be directly generalized to similar environments, but tentative judgments and hypotheses to be tested regarding the applicability of the results to such environments (Yıldırım & Şimşek, 2011). In this research, a detailed description strategy was used to ensure transferability. The environment and participants were defined in a rich and in-depth manner. The findings were presented by supporting direct quotations from research notes and analyses. In addition, a study group was formed from teacher candidates who took mathematics teaching I and II courses by using purposive sampling.

2.6. Data analysis

Data were collected from the group suitable for this case study. Analysis in a case-study depends on a detailed description of the situation and setting (Cresswell, 2003). For this reason, a descriptive analysis process was developed in which the teacher candidates' mathematical ideas about quadrilaterals and their written answers were summarized and interpreted according to predetermined themes. The primary purpose of descriptive analysis is to present the findings to the reader in an organized and interpreted way. To achieve this goal, a four-stage path was followed: (1) creating a general framework for the analysis, (2) processing the data according to the created framework, (3) defining the findings, and (4) interpreting the findings (Yıldırım & Şimşek, 2013). In the analyses related to the first and second primary purpose of the research, the definitions of quadrilateral were examined by the content analysis method. In content analysis, (Türnüklü, Gündoğdu-Alaylı & Akkaş, 2013) and (Öztoprakçı & Çakıroğlu, 2013) were used as criteria for definitions. Accordingly, the definitions made by the teacher candidates without giving place to the necessary and sufficient characteristics and misconceptions were accepted as academic definitions. The missing definition included in the data analysis is how the quadrilaterals' necessary and sufficient properties cannot be made but do not contain any errors or misconceptions. Definitions that specify extra features make more than necessary and sufficient features. Inability to make a definition is not answering the definition question or incomplete sentences, while wrong definitions are incorrect definitions. The geometric habits developed by the teacher candidates about quadrilaterals were analyzed using the theoretical framework of Yavuzsoy Köse and Tanışlı (2014), which is explained in detail in the literature. During the analyses, the thinking processes of the teacher candidates about the quadrilaterals and the difficulties they encountered in these processes were determined.

3. Findings

The data obtained in the research are discussed in two parts, namely the definitions of quadrilaterals and the relationship between quadrilaterals according to the geometric habits of the mind. The quadrilateral definitions and the geometric habits of the mind of the classroom teacher candidates were interpreted within the framework of the determining themes and by giving frequency-percentage tables. In the study's first aim, the definitions of rectangle, square, trapezoid, parallelogram, deltoid, and rhombus were asked from teacher candidates. The teacher candidates' knowledge about the definitions of quadrilaterals is essential in determining whether they can reveal the quadrilateral images in their minds and whether they can make inferences. Therefore, the frequency and percentage values of the teacher candidates (TC) regarding the definitions of quadrilaterals are given in Table 2.

Table 2 shows that teacher candidates cannot define the deltoid, while in terms of academic definition, they are most successful in the quadrilateral square and parallelogram. The most deficiencies of the pre-service teachers were in the definition of square and then the rectangle, and the most wrong definition was in the trapezoid. Examples of quadrilateral definitions of classroom teacher candidates are given in Table 3.

Table 2. Frequency and Percentage Values of the Classroom Teacher Candidates for the Definitions of Quadrilaterals

Quadrangles	In. Ab. Ma. Def*.		Acad. Def*.		Mis. Def.*.		Def. Spec. Ext. Fe*.		In. Def*.		Total	
	f	%	f	%	f	%	f	%	f	%	f	%
Rectangle	2	3,6	3	5,5	26	47,3	6	10,9	18	32,7	55	100
Square	1	1,8	12	21,8	30	54,5	2	3,6	10	18,2	55	100
Trapezoid	5	9,1	10	18,2	14	25,5			26	47,3	55	100
Parallelogram	5	9,1	12	21,8	21	38,2	5	9,1	12	21,8	55	100
Deltoid	16	29,1	3	5,5	17	30,9			19	34,5	55	100
Rhombus	4	7,3	27	49,1			7	12,7	17	30,9	55	100

*In. Ab. Ma. Def.: Inability to make a definition, *Acad. Def.: Academic definitions, *Mis. Def.: Missing definition, *Def. Spec. Ext. Fe.: Definitions that specify extra features, *In. Def.: Incorrect definitions.

Table 3. The Answers of the ClassroomTeacher Candidates Regarding the Definitions of Quadrilaterals

Quadrangles	Acad. Def*.	Mis. Def*.	Def. Spec. Ext. Fe*.	In. Def*.
Rectangle	It is a polygon with four sides and four corners, whose opposite sides are equal, and the angle between the sides is 90° (TC 55).	It is a geometric figure whose opposite sides are equal to each other (TC 5)	A quadrilateral consisting of two right triangles whose opposite sides are equal and parallel to each other and whose interior angles add up to 360° is called a rectangle (TC 20).	Its opposite sides are equal to each other. Each angle of it is 90° (TC 36).
Square	It is a quadrilateral whose sides are equal in length and whose interior angles are 90 ° (TC 50).	It has four equal-length sides (TC 29).	They are quadrilaterals with four sides, whose sides are congruent and parallel, and intersect at an angle of 90° (TC 3).	It is a geometric solid with all sides equal. Its angle is 90 °. The sum of all angles is 360 ° (TC 23).
Trapezoid	A geometric shape whose lower base and upper base are parallel, and the other two sides are not parallel (TC 13).	It is a quadrilateral with two parallel sides (TC 22).	----------------------	They are trapezoidal, non-perpendicular shapes with unequal side lengths (TC 2).
Parallelogram	It is a quadrilateral whose opposite sides are equal (TC 40).	The sum of the angles of the sides facing each other is 180 ° (TC 41).	It is a type of quadrilateral whose two sides are parallel, whose bottom and topsides are parallel, and whose sides are mutually equal. (TC 45).	It is the non-perpendicular form of the rectangle (TC 39).
Deltoid	The figure is formed by combining two isosceles triangles with equal base lengths (TC 26).	They are quadrilaterals with four sides whose side lengths are equal and parallel (TC 18).	---------------------	It is a geometric object with four equal sides (TC 27).
Rhombus	It is a parallelogram with equal sides (TC 55).	--------------------	They are quadrilaterals with four sides whose side lengths are equal and parallel (TC 18).	It is a definition of Quadrilaterals with congruent sides. The lengths of their diagonals are equal to each other (TC 42).

*Acad. Def.: Academic definitions, *Mis. Def.: Missing definition, *Def. Spec. Ext. Fe.: Definitions that specify extra features, *In. Def.: Incorrect definitions.

As shown in Table 3, the teacher candidates used the expressions polygon, geometric shape, shape, geometric object, object, and quadrilateral in their definitions. While polygon, geometric figure, and figure are interpreted academically correctly in quadrilateral definitions, the geometric body expression is not acceptable for quadrilateral definitions. For this reason, the expression of an object and the geometric object was evaluated as a wrong definition.
In the second aim of the study, the definitions of quadrilaterals made by the teacher candidates were evaluated according to the side, symmetry, and diagonal properties of the quadrilaterals. The frequency values of the definitions of teacher candidates according to these characteristics are given in Table 4.

Table 4. Distribution of teacher candidates' definitions according to the side, symmetry, and diagonal properties of quadrilaterals

Quadrangles	P. W. Def*.		Def. Ac. Ed. Pr*.		Def. Ac. Di. Pr*.		Def. Sym. Pr*.	
	f	%	f	%	f	%	f	%
Rectangle	2	3,6	53	96,4	-	-	-	-
Square	1	1,8	54	98,2	-	-	-	-
Trapezoid	5	9,1	50	90,9	-	-	-	-
Parallelogram	5	9,1	50	90,9	-	-	-	-
Deltoid	16	29,1	39	70,9	-	-	-	-
Rhombus	4	7,3	50	90,9	1	1,8	-	-

*P. W. Def.: People who defined, *Def. Ac. Ed. Pr.: Definition According to Edge Properties, *Def. Ac. Di. Pr.: Definition According to Diagonal Properties, *Def. Sym. Pr.: Definition by Symmetry Properties

When the definitions of the teacher candidates are examined, it is seen that they define the quadrilaterals according to their edge properties. Only 1 (one) person mentioned the diagonal property in the definition of a rhombus.

When the definitions of the teacher candidates were evaluated in general, it was seen that they specified the features of sides and angles in the definitions of rectangles. The edge properties included statements that opposite sides are equal, there are two short and two long sides, the angles are 90 o, or the sides intersect perpendicularly. Defining the sides as short and extended according to their lengths shows that the teacher candidate has a prototype rectangle concept. In addition, this definition is not suitable for the hierarchical order of the quadrilaterals. It reveals that the square is not accepted as a particular rectangle form. When the definitions of the square are examined in general, definitions have been made that the four sides are congruent and the angles are 90 o. It is seen that the teacher candidates have misconceptions about the terms side and angle in the definitions of rectangle and square. Teacher candidates use the terms side and angle interchangeably. When the answers given for trapezoidal shape, which is the most misidentified geometric shape by the teacher candidates, were examined, it was seen that they did not have a conceptual definition of a trapezoid. In trapezoidal definitions, definitions such as "It is a geometric shape with five sides.” (TC 32), “The non-perpendicular form of the rectangle.” (TC 2). When the definitions of parallelograms are examined, it is seen that the definition “A quadrilateral (shape / geometric figure) with opposite sides parallel.” is quite frequent. In the definition of a parallelogram, the teacher candidates expressed their edge and angle properties. In their definitions, it was observed that they used expressions such as “The opposite sides are congruent, the opposite angles are congruent.”. The deltoid is the first quadrilateral that the teacher candidates cannot be defined. It was found that the teacher candidates used the concept of the isosceles triangle intensively in the definition of the deltoid. It was also seen in the definitions that they confused the deltoid with the rhombus. When the definitions of rhombus were evaluated, it was seen that they generally expressed the rhombus as "a geometric shape with four equilateral sides (shape, quadrilateral)." In addition, a teacher candidate used the definition of a rhombus as “The lengths of their diagonals are congruent.”. In his/her definition, (TC 42) used the term diagonal. It was also observed that teacher candidates made definitions that opposite sides are parallel and of the same length.

Table 5. Frequency and percentage values of teacher candidates' geometric thinking of mind about quadrilaterals

	Undefined		Associating by reasoning				Generalizing geometric ideas								Examining Invariants				Balancing discoveries with reflections			
Quadrangles			FMS*		FPSS*		USRS*		Underdeveloped		Transition		Developed		DT*		CEE*		OD*		PEG*	
	n	f	n	f	n	f	n	f	n	f	n	f	n	f	n	f	n	f	n	f	n	f
Rectangle	38	69,1	15	27,3		1,8	-	-	-	-	-	-	-	-	1	1,8	-	-	-	-	-	-
Square	39	70,9	13	23,7	2	3,6	-	-	-	-	-	-	-	-	1	1,8	-	-	-	-	-	-
Trapezoid	43	78,2	11	20,0	1	1,8	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Parallelogram	35	63,6	17	30,9	1	1,8	-	-	-	-	-	-	-	-	2	3,6	-	-	-	-	-	-
deltoid	48	87,3	7	12,7		-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Rhombus	32	58,2	22	40		-	-	-	-	-	-	-	-	-	1	1,8	-	-	-	-	-	-

*FMS: Focusing on Multiple Shapes, *FPSS: Focusing on Parts of a Single Shape, *USRS: Using Special Reasoning Skills, *DT: Dynamic Thinking, *CEE: Checking for Evidence of Effects, *OD: Outstanding Discovery, *PEG: Prominent End Goals

Table 5 shows the distribution of teacher candidates according to the geometric habits of mind regarding the relationships between geometric shapes. When the table is examined, it is seen that teacher candidates cannot associate the associations by making explanations that reflect the geometric habits of the mind. Instead, it is seen that they intensively used the geometric habits of the mind in the sub-theme of focusing on multiple shapes within the scope of reasoning with relations. For example, teacher candidates evaluated each quadrilateral only in its defining features, used these features, and associated them with each other by creating new structures. Therefore they went to reasoning to associate a quadrilateral with other quadrilaterals.

As an example of the undefined one among the answers given by the teacher candidates, the association of the square with a square of a different size can be given. The justification for this association (TC 22) said, "Having the equal length." However, it is not stated what the feature with equal length is. Therefore, it is considered undefined. Another undefined statement can prove that quadrilaterals are used as prototypes in schools. Teacher candidate 9's answer to the definition of the trapezoid and related quadrilaterals is as follows:

“Since it was called trapezoid, I always thought of a rough-drawn skirt. Later I learned that this shape is stereotyped. But again, this trapezoidal shape comes to my mind directly.”. It is seen in the statement of the teacher candidate that there is no association between this expression and geometric shapes and that the concepts are taught with a single figure representation.

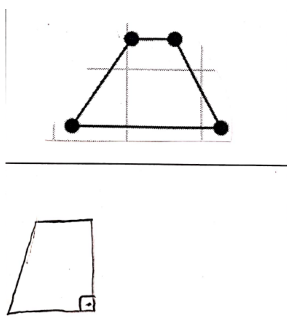

Figure 2. Representation of TC 9

As an example of focusing on multiple shapes in the theme of reasoning with relationships, "A rectangle and parallelogram have two short sides and two long sides. In both, opposite sides are parallel, and opposite side lengths are equal.” (T. C. 29). can be given. The representation of the teacher candidate on the visual is as follows:

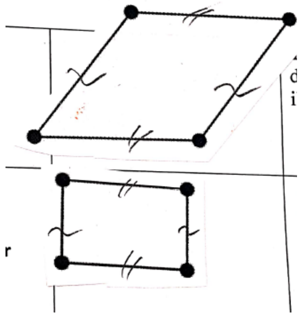

Figure 3. Representation of TC 29

In the sub-theme of focusing on a single shape, TC 55's answer regarding square can be given as an example. The candidate associated the square with a different square in size justified it by explaining the properties of a quadrilateral, such as "All the side lengths are equal, each angle is 90 o. The diagonals also seem to intersect each other at right angles."

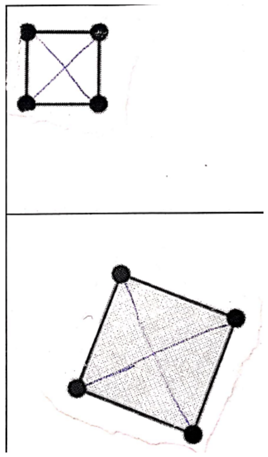

Figure 4. Representation of TC 55.

Teacher candidates did not generalize by checking the features they reached. While reasoning with relationships, one of the geometric habits of mind was used extensively, five teacher candidates used the theme of examining invariants using dynamic thinking. In this context, the candidates established a relationship by applying transformation geometry to the quadrilaterals or splitting them into parts. For example, in one of the answers given to the parallelogram question, which is among the questions, the parallelogram is associated with the rectangle. "I associated the parallelogram with the rectangle. Because a parallelogram is a rectangle. When I draw a rectangle from its two corners, it becomes a parallelogram..." (TC 53), "In a parallelogram, when we cut the shape from the perpendicular sides, we get a square and two triangles. A rectangle is formed when triangles are joined, and squares are added." (TC 5.) expressions exemplify that different quadrilaterals are formed by smashing the quadrilaterals or moving the quadrilaterals.

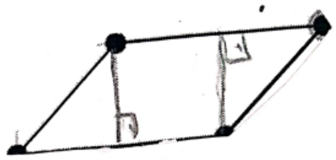

Figure 5. Representation of TC 5

The following answer was received in associating square and rhombus: "Both shapes have four sides. The side lengths are equal. If we turn it into a rhombus and look from its corner, it becomes a square, and if we turn the square around one of its corners, it becomes a rhombus" (TC 10).

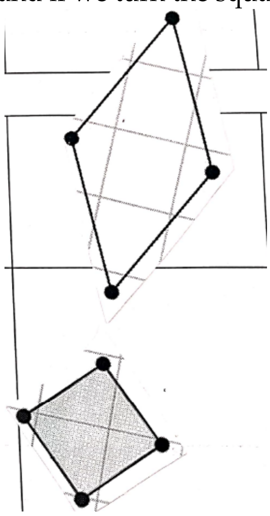

Figure 6. Representation of TC 10

4. Conclusion and Discussion

When the definitions made by the primary school teacher candidates about the quadrilaterals were examined; It was observed that they could not define the deltoid, and most misidentification was made about the trapezoid. In addition, it was noticed that the inclusive definitions of quadrilaterals were not adequately addressed in the answers of the teacher candidates, so there was a lack of knowledge in the hierarchical classification between quadrilaterals. The findings are supported by the findings in the studies of Akuysal (2007), Tümer and Filiz (2006), and Horzum (2018). As a result of their studies, Tümer and Filiz (2006) determined that teacher candidates do not know the exceptional cases of trapezoid, deltoid, rectangle, parallelogram, and rhombus. In the study of Horzum (2018), it was concluded that the teacher candidates based their drawings and explanations on the shape and used the visual properties and static positions of the geometric shapes. He interpreted this situation as insufficient or incorrect for the teacher candidates' conceptual knowledge. When the findings obtained from this research and the literature are analyzed together, it can be interpreted that the teacher candidates' conceptual knowledge about quadrilaterals is incomplete, and they recognize shapes with their prototype properties. In the study in which the prospective elementary mathematics teachers' knowledge of polygons was examined, it was concluded that the teacher candidates' knowledge of geometry was incomplete. They were more successful in questions requiring mathematical definitions (Kartal & Çınar, 2017). Ay and Başbay (2017) found in their research with seventh-grade students that they could not establish a relationship between square and rhombus and lacked knowledge about these concepts. This study determined that although the teacher candidates made the academic definition of the rhombus, they made an incomplete definition in the definition of the square and that the pre-service teachers could not make a hierarchical classification between these two concepts could not define the concepts comprehensively. Therefore, it can be stated based on the findings that pre-service teachers generally have a lack of knowledge about quadrilaterals. The reasons for the findings are considered to be due to the insufficient use of concrete materials in the teaching of the subject of quadrilaterals in the field of geometry learning in schools, the insufficient use of transformation and translation geometry in the teaching of quadrilaterals, and the use of uniform representations in general, the failure to allow students to reason on the properties of quadrilaterals and thus not emphasizing the hierarchical classification between them.

When the definitions made by the teacher candidates were examined according to the properties of the quadrilaterals, it was seen that the definitions were made according to the edge properties of the quadrilaterals, and only one teacher candidate made use of the diagonal property while defining the rhombus. This shows that pre-service teachers benefit from the shape prototype in their definitions of quadrilaterals. In his research, Tümer (2017) concluded that prospective mathematics teachers define the concept of diagonal incompletely and incorrectly. The lack of knowledge of the pre-service teachers about the concept of diagonal explains their inability to define quadrilaterals according to their diagonal features. In the study conducted by Horzum (2018), the teacher candidates' lack of knowledge about the properties of quadrilaterals such as sides, angles, and diagonals, and the fact that they predominantly use edge features in their definitions with some misconceptions support the finding of this study.

In the last aim of the study, in which the geometric thinking of the minds of the teacher candidates about the relations between the quadrilaterals was examined, it was concluded that the candidates concentrated on the theme of reasoning with relations, and they could not make any ideas on the themes of balancing generalization with exploration and reflection. When the results are evaluated in general, it shows that teacher candidates are not successful in finding relationships between quadrilaterals, generalizing the relationships they find by using strategies, associating the characteristic valid for a quadrilateral with quadrilaterals that may have that feature, in other words, with the whole, and reasoning. Yavuzsoy-Köse and Tanışlı (2014) concluded that teacher candidates' mental habits in geometry are not at a sufficient level, as a result of the study in which they investigated the ways of geometric thinking in perimeter and area problems. In this study conducted with teacher candidates, the failure of the candidates to achieve the expected level is similar to the study conducted by Yavuzsoy-Köse and Tanışlı (2014). This may be the content of the courses related to mathematics taken during the undergraduate years. Again, in this study, teacher candidates' excessive use of reasoning with relationships is in line with the longitudinal study of Bülbül and Güven (2019). Candidates exhibited more association habits, and the candidates' change in the process was attributed to the courses conducted by the researchers at universities. In the study conducted by Koç and Bozkurt (2012), approximately half of the teacher candidates could not correctly compare the cylinder volumes. In addition, it was observed that one-fourth of the candidates could not write any name of the geometric habit of the mind (GHM), and most of them could not explain why and how they used this GHM by simply writing their GHM names. The results reached by Koç and Bozkurt (2012) support this study. Korkmaz, Dündar, and Yaman (2016) evaluated mathematics teachers' mathematical habits of mind in the problem-solving process. In the study, it was observed that as the professional seniority of the teachers increased, there was a difference in the generalization habits of their thoughts about the habits of mind, and the preferences of the teachers to generalize increased depending on the seniority.

The effect of the seniority year can be interpreted as explained related to the subject area. Therefore, it can be interpreted that the teacher candidates' inability to make geometric generalizations in this study may be due to the lack of mathematics teachers' similar course work in the subject in general and quadrilaterals in particular. In this study, it can be said that five pre-service teachers exhibit dynamic thinking by applying the actions of transforming into a quadrilateral, applying translations, or dissecting. On the other hand, a few teachers have the habit of geometric thinking in researching invariants, which can be attributed to the mathematics and geometry courses taken during the undergraduate years. Yavuzsoy Köse and Tanışlı (2014) encountered a similar situation in their study. They stated that dynamic geometry programs could develop dynamic thinking most effectively. However, because these programs have only recently been used in Turkey, teacher candidates have not been successful in dynamic thinking.

4.1. Suggestions

In order to improve the geometric thinking levels of teacher candidates who will work as teachers in the future, the geometric habits of mind model can be included in the undergraduate courses of education faculties.

This survey has attempted to reveal the level of geometric habits of mind of classroom teacher candidates. In future research on the subject, experimental studies can be designed to examine the effects of the activities that are thought to develop the geometric habits of the mind and research that can obtain in-depth information.

Dynamic geometry software can be used in teaching geometry subjects in mathematics lessons. Group works can be encouraged by creating a classroom environment where students can discuss their ideas and demonstrate their reasoning and argumentation skills.

5. Declarations

Statement of Research and Publication Ethics Researchers have adhered to all ethical principles and rules in collecting, analyzing, and reporting data. Therefore, the ethics committee approved the study of Süleyman Demirel University with the number E-87432956-050.99-125461

Consent to participate The participant has given his voluntary consent to participate in the study and permission to use the data in a pseudonymized form in publications.

Declarations Conflict of interest. The authors declare no competing interests.

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