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GUP-Corrected ΛCDM Cosmology

Salih KİBAROĞLU*[1]

Abstract

In this study, we investigate the effect of the generalized uncertainty principle on the ΛCDM cosmological model. Using quantum corrected Unruh effect and Verlinde's entropic gravity idea, we find Planck-scale corrected Friedmann equations with a cosmological constant. These results modify the ΛCDM cosmology.

Keywords: Generalized gravity theories, cosmology, uncertainty principle.

1. INTRODUCTION

In general, the dynamics of our Universe is explained by cosmological models. But there is no cosmological model that explains all phases of the evolution of the Universe. According to the Friedmann-Lemaitre-Robertson-Walker model, the Universe is assumed that it has a homogeneous and isotropic characteristic on a large scale. Furthermore, we know that this is an approximate theory and we should find more general cosmological models to explain our Universe more accurate.

In recent years, the Λ-cold dark matter (ΛCDM) cosmological model is known as the standard model of cosmology. According to this model, Einstein's theory of general relativity is a correct theory and the Universe has a characteristic like an Einstein-de Sitter space-time for the late time. This model provides well explanations from today to the radiation-dominated era such as the accelerated expansion of the Universe, cosmic microwave background. On the other hand, this theory is not successful to explain the early time period of the Universe such as the inflation era. For this reason, it is reasonable to extend/modify this theory to a more general form.

At this point, the extended theories of gravity help us to find generalized cosmological models. For instance, in 1998 it was discovered that the expansion rate of our Universe is accelerating [1-4]. To explain this acceleration, the main candidates are the cosmological constant and dark energy which can be obtained by adding extra source terms to the Einstein field equations [5, 6].

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There is another alternative approach for constructing an extended theory of gravity put forwarded by Verlinde [7, 8]. According to his theory, gravity is not a fundamental force but emerges as an entropic force. He formulated a thermodynamical description of gravity by considering the holographic principle and Bekenstein-Hawking entropy [9-11]. In the light of this idea, if one has modified/deformed thermodynamical quantity this leads to finding modified gravitational equations. This idea provides a very useful theoretical background to derive generalized versions of gravity theories [12-20] and cosmological models [21-30].

The quantum gravity studies show that Heisenberg's uncertainty principle should be modified [31]. In this context, one of the main studies is the Generalized Uncertainty Principle (GUP) [32-39] which provide a quantum gravitational correction to the Heisenberg relation as,

$$\Delta x \Delta p \geq \frac{\hbar}{2}\left[1+\beta l_p^2 (\Delta p)^2\right], \tag{1}$$

where l_p is the Planck length, β is a constant of order unity and dimensionless and $c=1$ is assumed. Furthermore, another study is the Extended Uncertainty Principle (EUP) have a position-uncertainty correction to the ordinary uncertainty relation,

$$\Delta x \Delta p \geq \frac{\hbar}{2}\left[1+\frac{\alpha}{l_H^2} (\Delta x)^2\right], \tag{2}$$

where α is taken to be of order unity and l_H is a large distance scale for instance the (anti)-de Sitter radius. Thus, EUP provides a way to introduce quantum effects on large scales [40-43].

In this paper, we investigate the possible effects of the GUP model on the ΛCDM cosmology. For this purpose, we use the entropic force approach and the GUP model that used in [39].

In Sec. 2, we summarize the GUP-corrected gravity model based on [39]. Then in Sec. 3, we give a brief description of the ΛCDM cosmology. In Sec. 4, we find a Planck-scale-corrected Friedmann equations based on the selected GUP model. We also note that this paper uses the natural units where the speed of light c and Boltzmann's constant k_B equal to one. The last section concludes the paper.

2. QUANTUM CORRECTED GRAVITY MODEL

One can derive the Unruh effect with the help of Heisenberg's uncertainty principle when we assume that a photon has crossed the Rindler event horizon [33, 38]. If we consider the Planck scale where the gravitational effect is neglected, we know that there are some modifications of HUP named as generalized uncertainty principle and extended uncertainty principle. So, these modified models affect the structure of thermodynamical quantities such as the Unruh effect. According to the paper [39] a GUP model is given as follows,

$$\Delta x \Delta p \geq \frac{\hbar}{2}\left[1+\beta \left(\frac{\Delta p}{m_p}\right)^2\right], \tag{3}$$

and

$$[\hat{x}, \hat{p}] = i\hbar\left[1+\beta \left(\frac{\hat{p}}{m_p}\right)^2\right], \tag{4}$$

where the constant β is used as a dimensionless deformation parameter. By the help of this modified background, we can derive the modified Unruh temperature as follows,

$$T \cong T_U f(a). \tag{5}$$

Here $T_U = \hbar a/2\pi$ represents the ordinary Unruh temperature and the function $f(a)$ is defined as

$$f(a) = \left[1 + \frac{\beta}{2}\left(\frac{l_p a}{\pi}\right)^2\right]. \tag{6}$$

Here, a is the acceleration of the reference frame. According to this background and using the Verlinde's entropic gravity idea, a GUP-corrected gravitational field equation can be found as follows [30],

$$R_{ab} - \frac{1}{2} g_{ab} R + \frac{\Lambda}{f(a)} g_{ab} = \frac{8\pi G}{f(a)} \{T_{ab} + T_{ab}^{GUP}\}, \tag{7}$$

where Λ represents the cosmological constant, T_{ab} is originated from ordinary matter and T_{ab}^{GUP} comes from GUP modification and its form,

$$T_{ab}^{GUP} = \frac{1}{8\pi G}\left(\nabla_a \nabla_b f - \frac{1}{2} g_{ab} \nabla^2 f\right). \tag{8}$$

Moreover, in the concept of this GUP model, Newton's second law of gravity takes the following form,

$$F = maf(a), \tag{9}$$

where $f(a)$ is given by Eq.(6).

3. REVIEW OF ΛCDM COSMOLOGY

In this section, we present a brief review of the ΛCDM cosmological model which is also known as the standard model of cosmology [44]. According to this model, the Friedmann equation is given the following form,

$$H(t)^2 = \frac{8\pi G}{3}\rho(t) - \frac{k}{a^2} + \frac{\Lambda}{3}, \tag{10}$$

and the acceleration equation is,

$$\frac{\ddot{a}(t)}{a(t)} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}, \tag{11}$$

here $a(t)$ is a dimensionless arbitrary function of time, known as the scale factor, which is related to the expansion of the universe, $\rho(t)$ and $p(t)$ represents total energy density and the pressure of cosmological fluids, $H = \dot{a}(t)/a(t)$ is the Hubble parameter which describes the expansion rate of the universe and the dot represents the time derivative of the corresponding component. Moreover, the driving term $\Lambda/3$ is responsible for the acceleration. Besides, the continuity equation can be given as follows,

$$\dot{\rho} + 3H(\rho + p) = 0. \tag{12}$$

Using these equations, the Raychaudhuri equation can be written as follows,

$$\dot{H} = -4\pi G(\rho + p) + \frac{k}{a^2}. \tag{13}$$

4. GUP-CORRECTED FRIEDMANN EQUATIONS

We start with the Friedmann–Robertson–Walker universe with the metric

$$ds^2 = dt^2 - a(t)^2(dr^2 + r^2 d\Omega^2), \tag{14}$$

where $a(t)$ is the scale factor, and Ω denotes the line element of a unit sphere. According to Verlinde's paper, we have a spherical holographic screen with a spatial region which has the following physical radius,

$$\tilde{r} = a(t)r. \tag{15}$$

By assuming that our universe has homogeneity and isotropic form in a large scale, the matter content of the universe forms a perfect fluid with the following stress-energy tensor,

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu}, \tag{16}$$

and if we take the trace of this tensor we get,

$$T = T_\mu^\mu = \rho - 3p. \tag{17}$$

Here $u^\mu = (1,0,0,0)$ is the four-velocity and satisfies the relation $g_{\mu\nu}u^\mu u^\nu = 1$. Also $\rho(t)$ and $p(t)$ represents energy density and the pressure of cosmological fluids, respectively. By using the conservation of the energy–stress tensor, the continuity equation takes the following form,

$$\dot{\rho} + 3H(\rho + p) = 0. \tag{18}$$

The number of bits on the screen is defined as

$$N = \frac{A}{G\hbar}, \tag{19}$$

where $A = 4\pi\tilde{r}^2$ represents the area of the screen. According to the equipartition law of energy, the total energy on the screen can be written as follows,

$$E = \frac{1}{2}Nk_B T. \tag{20}$$

On the other hand, the energy can be represented as

$$E = M, \tag{21}$$

where the mass M represents the mass in the spatial region V. Using Eq.(16) and Eq.(21), the total mass in the spatial region can also be defined as

$$M = \int_V dV\left(T_{\mu\nu}u^\mu u^\nu\right), \tag{22}$$

where $T_{\mu\nu}u^\mu u^\nu$ is the energy density of the system. Moreover, the acceleration of the radial observer at r can be written,

$$a_r = -\frac{d^2\tilde{r}}{dt^2} = -\ddot{a}r. \tag{23}$$

By using this result the modified Unruh temperature should be the following form,

$$T = \frac{\hbar a_r}{2\pi} f(a_r) = T_U f(a_r). \tag{24}$$

From this background and by assuming the area as $A = 4\pi\tilde{r}^2$ and the volume as $V = \frac{4}{3}\pi\tilde{r}^3$, we get

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3f}\rho, \tag{25}$$

This equation represents the dynamical equations for the Newtonian cosmology for the modified case. To obtain the Friedmann equations for the GUP-deformed general relativity, we use active gravitational mass (Tolman-Komar mass) $\mathcal{M}$ rather than total mass M in the spatial region V and it is defined as,

$$\mathcal{M} = 2\int_V dV\left(T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu} - \frac{\Lambda}{8\pi G}g_{\mu\nu}\right)u^\mu u^\nu, \tag{26}$$

After taking this integral, we get the active gravitational mass as

$$\mathcal{M} = \left(\rho + 3p - \frac{\Lambda}{4\pi G}\right) V. \tag{27}$$

Then using this new mass definition, we find the following equation,

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3f}(\rho + 3p) + \frac{\Lambda}{3f}. \tag{28}$$

So, we find the acceleration equation including the cosmological term for the dynamical evolution of the FRW universe. Multiplying $\dot{a}a$ on both sides of the last equation and using the continuity equation in Eq.(16), we get

$$H^2 = \frac{8\pi G}{3f}\rho - \frac{k}{a^2} + \frac{\Lambda}{3f}, \tag{29}$$

Thus, we get the second Friedmann equation that is responsible for the time evolution of the FRW universe. Here, k is the integration constant and it can be interpreted as the spatial curvature of the region V in Einstein's theory of gravity. Here, $k = 1, 0, -1$ correspond to a closed, flat, and open FRW universe, respectively. The Eq.(29) can also be written as follows,

$$H^2 = \frac{8\pi G}{3f}(\rho - \rho_\Lambda) - \frac{k}{a^2}, \tag{30}$$

where an energy density corresponding to the cosmological constant is defined as,

$$\rho_\Lambda := -\frac{\Lambda}{8\pi G}. \tag{31}$$

In other words, it can be interpreted as an additional energy density to the universe. Moreover, time derivation of the Hubble parameter can be written as follows,

$$\dot{H} = \frac{\ddot{a}}{a} - H^2. \tag{32}$$

Using this equation together with Eqs.(28) and (29), we get,

$$\dot{H} = -\frac{4\pi G}{f}(\rho + p) + \frac{k}{a^2}. \tag{33}$$

Thus, we derive the GUP-corrected Raychaudhuri equation. With the help of Eq.(29), one can also write the matter density as follows,

$$\rho(t) = \rho_c + \rho_k + \rho_\Lambda, \tag{34}$$

where the critical density ρ_c and the energy density corresponding to the integration constant ρ_k are defined as follows,

$$\rho_c := \frac{3H^2}{8\pi G} f, \quad \rho_k := \frac{3k}{8\pi G a^2} f, \tag{35}$$

On the other hand, the density parameter is defined as $\Omega(t) = \rho/\rho_c$ and using this definition one can write,

$$\Omega(t) + \Omega_k(t) + \Omega_\Lambda(t) = 1, \tag{36}$$

where,

$$\Omega_k(t) = -\frac{k}{H^2 a^2}, \quad \Omega_\Lambda(t) = \frac{\Lambda}{3H^2 f}. \tag{37}$$

These expressions represent density parameters with respect to the integral constant and the cosmological constant, respectively. The density parameter contains information about the shape of our universe. If we suppose $\Omega(t) = 1$, this model describes a flat universe. The other conditions $\Omega(t) < 1$ and $\Omega(t) > 1$ corresponds to an open universe closed universe. Today it is known that the value of the density parameter is close to one. From here, we can write the following expression,

$$\Lambda \cong \frac{3k}{a^2}\left[1+\frac{\beta}{2}\left(\frac{l_p a_r}{\pi}\right)^2\right]. \tag{38}$$

If we set the deformation parameter as $\beta = 0$ the modified equations go to their standard forms. Furthermore, some possible values of the deformation parameter β, both gravitational and non-gravitational cases, are summarized in [45]. According to this, for gravitational cases, the values of β change between $\beta < 10^{21}$ and $\beta < 10^{78}$. On the other hand, in some cases, β could get negative values [46-49]. This idea may lead to interesting results because a negative deformation parameter can change the sign of the cosmological constant in a special condition. If we consider very high acceleration values and at appropriate β values, the sign of Eq.(38) would be changed.

5. CONCLUSIONS

In this paper, we aimed to find the possible contribution of the generalized uncertainty principle on a cosmological model. We know that Verlinde's entropic gravity idea provides a useful theoretical background to extend/modify cosmological models.

From this background, we used GUP-corrected gravitational model in Eq.(7) (see also [30]) to derive modified Friedmann equations. Eventually, we derived quantum corrected Friedman equations Eq.(28) and Eq.(29). Also, these equations can be seen as quantum corrected ΛCDM model. In a certain condition, if the deformation parameter β in the function $f(a)$ in Eq.(6) goes to zero, the model reduces its conventional form.

According to the function $f(a)$ in Eq.(6) and modified Friedmann equations, the contribution that comes from GUP is very little because the function contains a square of the Planck length. In this condition, the corrections can be ignored when the Universe becomes large. But there may be a considerable contribution for the early universe such as the inflation phase. This is very important because the ΛCDM model is insufficient for the explanation the early Universe period.

In addition, $f(a)$ could have a negative sign when we chose negative deformation parameter and extreme acceleration conditions. This may lead to change some signs of the resulting Friedmann equations and the cosmological constant in Eq.(38). Thus, we can say that this kind of deformation could affect the evolution of the universe.

Consequently, one can say that modifications of the entropic gravity lead to derive extended cosmological models and these studies may provide a deeper understanding of our Universe.

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No conflict of interest or common interest has been declared by the authors.

The Declaration of Ethics Committee Approval

This study does not require ethics committee permission or any special permission.

The Declaration of Research and Publication Ethics

The authors of the paper declare that they comply with the scientific, ethical and quotation rules of SAUJS in all processes of the paper and that they do not make any falsification on the data collected. In addition, they declare that Sakarya University Journal of Science and its editorial board have no responsibility for any ethical violations that may be encountered, and that this study has not been evaluated in any academic publication environment other than Sakarya University Journal of Science.

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