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A modified Mann algorithm for the general split problem of demicontractive operators

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Abstract

This work proposes a novel method for solving the general split common fixed point problem of demicontractive operators in the framework of real Hilbert spaces. Our proposed technique is principally based on the Mann algorithm. The proof of the weak convergence theorem is additionally established under some particular conditions. We subsequently verify the convergence of our algorithm via numerical examples.

Keywords: general split common fixed point problem demicontractive operator Mann algorithm
2020 MSC: 47J25, 47H10, 65K10.

1. Introduction

The Split Feasibility Problem (SFP) was first proposed by Censor and Elfving [5] in 1994. In this problem, we assume that $\mathcal{H}_1$ and $\mathcal{H}_2$ are real Hilbert spaces with inner product $\langle \cdot, \cdot \rangle$ and norm $\| \cdot \|$. By letting C and Q be nonempty closed and convex subsets of $\mathcal{H}_1$ and $\mathcal{H}_2$ respectively, an arduous endeavor is to find a point $x \in \mathcal{H}_1$ that satisfies the following condition:

$$x \in C \text{ such that } Ax \in Q, \tag{1}$$

where $A : \mathcal{H}_1 \to \mathcal{H}_2$ is a given bounded linear operator. Several algorithms for solving SFP (1) were proposed in various ways in both finite and infinite-dimensional spaces with the requirement of the existence of the inverse of A. A classical way to solve the SFP (1) is to employ the CQ algorithm which was introduced by Byrne [3], which is defined in the following manner: for any $x_0 \in \mathcal{H}_1$,

$$x_{n+1} = P_C(x_n - \gamma A^*(I - P_Q)Ax_n), \ \ \forall n \geq 0,$$

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where the operator A^* is adjoint of A, the step size γ is in an open interval $\left(0, \frac{2}{\|A\|^2}\right)$, P_C and P_Q are the orthogonal projections on to C and Q, respectively. Many researchers have developed novel methods that do not require P_C and P_Q calculations. For example, Kesornprom et al. [9] proposed two gradient-CQ algorithms in 2020 and demonstrated their weak and strong convergence under specific situations. Motivated by the aforementioned problem, Censor and Segal [4] accordingly introduced the Split Common Fixed Point Problem (SCFP) in a purpose of searching a point

$$x \in Fix(U) \quad \text{such that} \quad Ax \in Fix(T),$$

where $U : \mathcal{H}_1 \to \mathcal{H}_1$ and $T : \mathcal{H}_2 \to \mathcal{H}_2$ are both nonlinear operators. The notations $Fix(U)$ and $Fix(T)$ indicate the sets of fixed points of U and T respectively. It is worth noting that the SCFP is a generalization of the SFP. Moudafi [11] later provided an algorithm for solving the SCFP in the case of demicontractive operators where the weak convergence theorem of such algorithm was also acquired. Zheng et al. [16] set forth the iterative Algorithm 1.1 that is given below for the SCFP of a pair of demicontractive operators U and T along with the proof of the weak convergence theorem.

Algorithm 1.1 Initialization: Let $x_0 \in \mathcal{H}_1$ be arbitrary and x_{n+1} be recursively defined by

$$x_{n+1} = (1-\alpha)x_n + \alpha[Ux_n - \tau A^*(I-T)Ax_n], \; n \geq 0,$$

where $\alpha \in \left(0, \frac{1-k_1}{2}\right)$ and $\tau \in \left(0, \frac{1-k_2}{2\alpha\|A\|^2}\right)$ with constants $k_1 \in [0,1)$ and $k_2 \in [0,1)$. Some interesting studies of the SCFP and the associated problems can be found in [6, 13, 14, 15]. Kangtunyakarn [8] recently presented the General Split Feasibility Problem (GSFP) in which its aim is to determine a point, x, satisfying

$$x \in C \quad \text{such that} \quad Ax, Bx \in Q,$$

where $B : \mathcal{H}_1 \to \mathcal{H}_2$ is another bounded linear operator.

Inspired by both SCFP and GSFP, we put forward the General Split Common Fixed Point Problem (GSCFP) in which we attempt to seek a point

$$x \in Fix(U) \quad \text{such that} \quad Ax, Bx \in Fix(T).$$

In this work, an algorithm based on the Mann algorithm for the GSCFP of demicontractive operators is presented. This proposed algorithm defines a sequence that weakly converges to a solution to the problem under some additional conditions. Some results of the associated problems of the prolbem have been reported, see

The structure of this paper is as follows. Section 2 contains the underlying backgrounds including the technical lemma (Lemma 2.4) which is a key to our main result. Whilst the novel algorithm for the GSCFP as well as its proof for the weak convergence of a sequence (Theorem 3.2) are described in Section 3. Last but not least, Section 4 provides some numerical examples of the convergence result of the algorithm.

2. Preliminaries

Let C be a nonempty, closed and convex subset of a real Hilbert space $\mathcal{H}$. For $x \in \mathcal{H}$, we define the metric projection P_C from $\mathcal{H}$ onto C by

$$P_C x := \arg\min_{y \in C} \|x - y\|^2.$$

It is somewhat noteworthy the following equality

$$2\langle x, y\rangle = \|x\|^2 + \|y\|^2 - \|x-y\|^2 \tag{2}$$

for all $x, y \in \mathcal{H}$.

Definition 2.1. [7, 10, Section 2] *A self-operator T on C is said to be demicontractive (or k-demicontractive) if there exists a constant $k \in [0, 1)$ such that*

$$\|Tx - x^*\|^2 \leq \|x - x^*\|^2 + k\|x - Tx\|^2,$$

or equivalently,

$$\langle x - Tx, x - x^* \rangle \geq \frac{1-k}{2}\|x - Tx\|^2,$$

for all $x \in C$ and $x^* \in Fix(T)$.

Definition 2.2. [2, Definition 5.1] *Let D be a nonempty subset of $\mathcal{H}$ and let $\{x_n\}$ be a sequence in $\mathcal{H}$. Then $\{x_n\}$ is Fejér monotone with respect to D if for every $x \in D$*

$$\|x_{n+1} - x\| \leq \|x_n - x\| \text{ for all } n \geq 0.$$

Throughout this work, the notation $\rightharpoonup$ will be used for the weak convergence theorem, whereas the notation $\to$ for the strong convergence theorem and the notation $\omega_w(x_n)$ is the weak ω-limit set of the sequence $\{x_n\}$.

Definition 2.3. [12, Section 1] *Let $T : C \to \mathcal{H}$ be an operator. Then T is said to be demiclosed at $y \in \mathcal{H}$ if $Tx_n \to y$ implies $Tx = y$ for any sequence $\{x_n\}$ in C such that $x_n \rightharpoonup x \in C$.*

Now, let us state some essential facts which will be used to present our main result.

Lemma 2.4. [1, Theorem 2.16] *If the sequence $\{x_n\}$ is Fejér monotone with respect to C, we consequently have the following conclusions:*
(i) $x_n \rightharpoonup x^* \in C$ if and only if $\omega_w(x_n) \subset C$;
(ii) the sequence $\{P_C(x_n)\}$ converges strongly;
(iii) if $x_n \rightharpoonup x^* \in C$, then $x^* = \lim_{n\to\infty} P_C(x_n)$.

3. Main Results

Before embarking on the main results, some essential assumptions are primarily assumed to be held:
- $\mathcal{H}_1$ and $\mathcal{H}_2$ are real Hilbert spaces;
- $A, B : \mathcal{H}_1 \to \mathcal{H}_2$ are bounded linear operators;
- $U : \mathcal{H}_1 \to \mathcal{H}_1$ is a k_1-demicontractive operator with $k_1 \in [0, 1)$;
- $T : \mathcal{H}_2 \to \mathcal{H}_2$ is a k_2-demicontractive operator with $k_2 \in [0, 1)$;
- $I - U$, $I - T$ are demiclosed at zero;
- $\Omega := \{r : r \in Fix(U) \text{ and } Ar, Br \in Fix(T)\}$ is nonempty.

The succeeding lemma plays a crucial role in solving the GSCFP.

Lemma 3.1. *$r \in \Omega$ if and only if $r \in Fix\left(U - \tau\left(A^*(I-T)A + B^*(I-T)B\right)\right)$ for any $\tau > 0$.*

Proof. Suppose $r \in \Omega$, we accordingly have $r = Ur$ and $(I - T)Ar = (I - T)Br = 0$. This also leads to $r \in Fix\left(U - \tau\left(A^*(I-T)A + B^*(I-T)B\right)\right)$ for any $\tau > 0$. Conversely, when we assume $r \in Fix\left(U - \tau\left(A^*(I-T)A + B^*(I-T)B\right)\right)$ for any $\tau > 0$. By taking $z \in \Omega$ and exploiting the equivalence of demicontractive operators, we obtain

$$\begin{aligned}
0 &= \langle r - Ur + \tau A^*(I-T)Ar + \tau B^*(I-T)Br, r - z\rangle \\
&= \langle r - Ur, r - z\rangle + \tau\langle A^*(I-T)Ar, r - z\rangle + \tau\langle B^*(I-T)Br, r - z\rangle \\
&= \langle r - Ur, r - z\rangle + \tau\langle (I-T)Ar, Ar - Az\rangle + \tau\langle (I-T)Br, Br - Bz\rangle \\
&\geq \frac{1-k_1}{2}\|r - Ur\|^2 + \frac{1-k_2}{2}\tau\|(I-T)Ar\|^2 + \frac{1-k_2}{2}\tau\|(I-T)Br\|^2.
\end{aligned}$$

This implies that $r \in Fix(U)$ and $Ar, Br \in Fix(T)$, that is, $z \in \Omega$. This completes the proof. □

The iterative scheme based on the Mann algorithm in which it converges weakly to the solution of the GSCFP for demicontractive operators can now be described.

Algorithm 3.1: By choosing an initial guess $x_0 \in \mathcal{H}_1$ arbitrarily, x_{n+1} can be recursively computed through the formula

$$x_{n+1} = (1-\alpha_n)x_n + \alpha_n \left[Ux_n - \tau\left(A^*(I-T)Ax_n + B^*(I-T)Bx_n\right)\right], \ n \geq 0 \tag{3}$$

where $\alpha_n \in [a,b] \subset \left(0, \frac{1-k_1}{2}\right)$. We see that the proposed method is easy to compute as it has only one step and it can be applied to solve the SCFP, GSFP, SFP and signal recovery problem.

Theorem 3.2. *Assume that $\tau \in \left(0, \frac{1-k_2}{4b\max\{\|A\|^2,\|B\|^2\}}\right)$. Then the sequence $\{x_n\}$ generated by Algorithm 3.1 converges weakly to a point $p \in \Omega$, that is $\lim\limits_{n\to\infty} P_\Omega(x_n)$.*

Proof. We start the proof by showing the sequence $\{x_n\}$ is bounded. This is simply done by taking $z \in \Omega$ and utilizing the equivalence of demicontractive operators, we therefore have

$$\begin{aligned}
&\langle x_n - Ux_n + \tau A^*(I-T)Ax_n + \tau B^*(I-T)Bx_n, x_n - z\rangle \\
&\quad = \langle x_n - Ux_n, x_n - z\rangle + \tau\langle A^*(I-T)Ax_n, x_n - z\rangle + \langle B^*(I-T)Bx_n, x_n - z\rangle \\
&\quad = \langle x_n - Ux_n, x_n - z\rangle + \tau\langle (I-T)Ax_n, Ax_n - Az\rangle + \langle (I-T)Bx_n, Bx_n - Bz\rangle \\
&\quad \geq \frac{1-k_1}{2}\|x_n - Ux_n\|^2 + \frac{1-k_2}{2}\tau\|(I-T)Ax_n\|^2 + \frac{1-k_2}{2}\tau\|(I-T)Bx_n\|^2.
\end{aligned} \tag{4}$$

This henceforth results in

$$\begin{aligned}
&\alpha_n^2\|x_n - Ux_n + \tau A^*(I-T)Ax_n + B^*(I-T)Bx_n\|^2 \\
&\quad \leq \alpha_n^2\left[\|x_n - Ux_n\| + \tau\|A^*(I-T)Ax_n + B^*(I-T)Bx_n\|\right]^2 \\
&\quad \leq 2\alpha_n^2\left[\|x_n - Ux_n\|^2 + \tau^2\|A^*(I-T)Ax_n + B^*(I-T)Bx_n\|^2\right] \\
&\quad \leq 2\alpha_n^2\left[\|x_n - Ux_n\|^2 + 2\tau^2\|A\|^2\|(I-T)Ax_n\|^2 + 2\tau^2\|B\|^2\|(I-T)Bx_n\|^2\right] \\
&\quad \leq 2\alpha_n^2\left[\|x_n - Ux_n\|^2 + 2\tau^2\max\left\{\|A\|^2,\|B\|^2\right\}\left(\|(I-T)Ax_n\|^2 + \|(I-T)Bx_n\|^2\right)\right].
\end{aligned} \tag{5}$$

Define $R_1 = a(1-k_1-2b)$ and $R_2 = a\tau\left(1-k_2-4b\tau\max\left\{\|A\|^2,\|B\|^2\right\}\right)$. By applying (2) and (3)-(5), we obtain

$$\begin{aligned}
&\|x_{n+1} - z\|^2 \\
&\quad = \|(1-\alpha_n)x_n + \alpha_n\left[Ux_n - \tau A^*(I-T)Ax_n - \tau B^*(I-T)Bx_n\right] - z\|^2 \\
&\quad = \|x_n - z - \alpha_n\left[x_n - Ux_n + \tau A^*(I-T)Ax_n + \tau B^*(I-T)Bx_n\right]\|^2 \\
&\quad = \|x_n - z\|^2 - 2\alpha_n\langle x_n - Ux_n + \tau A^*(I-T)Ax_n + \tau B^*(I-T)Bx_n, x_n - z\rangle \\
&\qquad + \alpha_n^2\|x_n - Ux_n + \tau A^*(I-T)Ax_n + B^*(I-T)Bx_n\|^2 \\
&\quad \leq \|x_n - z\|^2 - \alpha_n\left[(1-k_1)\|x_n - Ux_n\|^2 + \tau(1-k_2)\left(\|(I-T)Ax_n\|^2 + \|(I-T)Bx_n\|^2\right)\right] \\
&\qquad + 2\alpha_n^2\left[\|x_n - Ux_n\|^2 + 2\tau^2\max\left\{\|A\|^2,\|B\|^2\right\}\left(\|(I-T)Ax_n\|^2 + \|(I-T)Bx_n\|^2\right)\right] \\
&\quad = \|x_n - z\|^2 - \alpha_n(1-k_1-2\alpha_n)\|x_n - Ux_n\|^2 \\
&\qquad - \alpha_n\tau\left(1-k_2-4\alpha_n\tau\max\left\{\|A\|^2,\|B\|^2\right\}\right)\left(\|(I-T)Ax_n\|^2 + \|(I-T)Bx_n\|^2\right). \\
&\quad \leq \|x_n - z\|^2 - R_1\|x_n - Ux_n\|^2 - R_2\left(\|(I-T)Ax_n\|^2 + \|(I-T)Bx_n\|^2\right).
\end{aligned} \tag{6}$$

The sequence $\{x_n\}$ is therefore Fejér monotone with respect to Ω. This leads to the conclusion that $\{x_n\}$ is bounded. It is then required to create a particular restraint in which Lemma 2.4 can be applied. In other words, the condition $\omega_w(x_n) \subset \Omega$ must be proven. By using (6), it leads to

$$R_1\|x_n - Ux_n\|^2 + R_2\|(I-T)Ax_n\|^2 + R_2\|(I-T)Bx_n\|^2 \leq \|x_n - z\|^2 - \|x_{n+1} - z\|^2.$$

By the induction hypothesis, it is rather straightforward to justify

$$\begin{aligned} R_1 \sum_{j=1}^{n} \|x_j - Ux_j\|^2 + R_2 \sum_{j=1}^{n} \|(I-T)Ax_j\|^2 + R_2 \sum_{j=1}^{n} \|(I-T)Bx_j\|^2 \\ \leq \|x_0 - z\|^2 - \|x_{n+1} - z\|^2 \leq \|x_0 - z\|^2. \end{aligned}$$

This thus gives

$$R_1 \sum_{j=1}^{\infty} \|x_j - Ux_j\|^2 + R_2 \sum_{j=1}^{\infty} \|(I-T)Ax_j\|^2 + R_2 \sum_{j=1}^{\infty} \|(I-T)Bx_j\|^2 < \infty,$$

which implies that

$$\lim_{n\to\infty} \|x_n - Ux_n\| = \lim_{n\to\infty} \|(I-T)Ax_n\| = \lim_{n\to\infty} \|(I-T)Bx_n\| = 0.$$

By the demiclosedness property of $I-U$ and $I-T$ (at zero), we can deduce that $\omega_w(x_n) \subset \Omega$. Finally, by Lemma 2.4, we arrive at the conclusion that $x_n \rightharpoonup p = \lim_{n\to\infty} P_\Omega(x_n)$. The proof is now complete. $\square$

By using Theorem 3.2, we can solve the SCFP, GSFP, and SFP where $C \subset \mathcal{H}_1$ and $Q \subset \mathcal{H}_2$ are two nonempty closed and convex sets.

Algorithm 3.2: Let $x_0 \in \mathcal{H}_1$ be an arbitrarily initial guess. Then x_{n+1} can be evaluated recursively via the formula

$$x_{n+1} = (1-\alpha_n)x_n + \alpha_n \left[Ux_n - \tau_A A^*(I-T)Ax_n\right],\ n \geq 0$$

where $\alpha_n \in [a,b] \subset \left(0, \frac{1-k_1}{2}\right)$.

Corollary 3.3. *Assume that $\tau_A \in \left(0, \frac{1-k_2}{2b\|A\|^2}\right)$ and $\Omega_A \neq \emptyset$, where $\Omega_A := \{r : r \in Fix(U) \text{ and } Ar \in Fix(T)\}$. The sequence $\{x_n\}$ derived from Algorithm 3.2 converges weakly to a point $p \in \Omega_A$, precisely $\lim_{n\to\infty} P_{\Omega_A}(x_n)$.*

Algorithm 3.3: Let $x_0 \in \mathcal{H}_1$ be an arbitrarily initial guess. Then x_{n+1} can be calculated recursively by means of

$$x_{n+1} = (1-\alpha_n)x_n + \alpha_n \left[P_C x_n - \tau \left(A^*(I-P_Q)Ax_n + B^*(I-P_Q)Bx_n\right)\right],\ n \geq 0$$

where $\alpha_n \in [a,b] \subset \left(0, \frac{1}{2}\right)$.

Corollary 3.4. *Assume that $\tau \in \left(0, \frac{1}{4b\max\{\|A\|^2, \|B\|^2\}}\right)$ and $\Gamma \neq \emptyset$, where $\Gamma := \{r : r \in C \text{ and } Ar, Br \in Q\}$. The sequence $\{x_n\}$ generated by Algorithm 3.3 converges weakly to a point $p \in \Gamma$ which means $\lim_{n\to\infty} P_\Gamma(x_n)$.*

Algorithm 3.4: Let $x_0 \in \mathcal{H}_1$ be an arbitrarily initial guess. Then x_{n+1} can be computed recursively using the following formula

$$x_{n+1} = (1-\alpha_n)x_n + \alpha_n \left[P_C x_n - \tau_A A^*(I-P_Q)Ax_n\right],\ n \geq 0$$

where $\alpha_n \in [a,b] \subset \left(0, \frac{1}{2}\right)$.

Corollary 3.5. *Assume that $\tau_A \in \left(0, \frac{1}{2b\|A\|^2}\right)$ and $\Gamma_A \neq \emptyset$, where $\Gamma_A := \{r : r \in C \text{ and } Ar \in Q\}$. The sequence $\{x_n\}$ gained from Algorithm 3.4 converges weakly to a point $p \in \Gamma_A$ that is $\lim_{n\to\infty} P_{\Gamma_A}(x_n)$.*

4. Numerical experiments

In this section, numerical examples are provided to demonstrate the performance of our algorithms.

Example 4.1. *Let $\mathcal{H}_1 = \mathcal{H}_2 = L^2([0,1])$ with norm $\|x\| := (\int_0^1 |x(t)|^2 dt)^{\frac{1}{2}}$ and inner product $\langle x, y\rangle := \int_0^1 x(t)y(t)dt,\ x, y \in L^2([0,1])$. Define*

$$(Ux)(t) = \left(\frac{t+1}{4}\right) x(t),\ (Tx)(t) = \frac{x(t)}{2},\ (Ax)(t) = 3x(t),\ \text{and } (Bx)(t) = 5x(t),$$

where $x \in L^2([0,1])$. Then $\alpha_n \in [a,b] \subset \left(0, \frac{1}{2}\right)$ and $\tau \in \left(0, \frac{1}{100b}\right)$, and we set $\tau = \frac{1}{120b}$.

To investigate the sensitivity of the initial guess x_0, three different choices are undertaken; $x_0 = t$ (Choice 1), $x_0 = \frac{t^2}{2}$ (Choice 2) and $x_0 = \frac{\cos(t)}{5}$ (Choice 3). We additionally examine the impact of the sequence $\{x_n\}$ on each iterative scheme by classifying α_n into three cases; $\alpha_n = \frac{1}{6}$ (Case 1), $\alpha_n = \frac{1}{4}$ (Case 2) and $\alpha_n = \frac{e}{6}$ (Case 3).

We compute x_n by Algorithm 3.1 until the stopping criterion is satisfied, i.e. $E_n := \|x_{n+1} - x_n\| < 5 \times 10^{-3}$. The results are presented next.

		Case 1	Case 2	Case 3
Choice 1	No. of Iter.	14	12	9
	Elapsed Time (s)	8.6537	6.0198	4.2984
Choice 2	No. of Iter.	10	9	7
	Elapsed Time (s)	4.5562	4.7548	2.9943
Choice 3	No. of Iter.	9	8	6
	Elapsed Time (s)	3.8632	3.6014	2.4522

Table 1: Numerical experiments of Example 4.1.

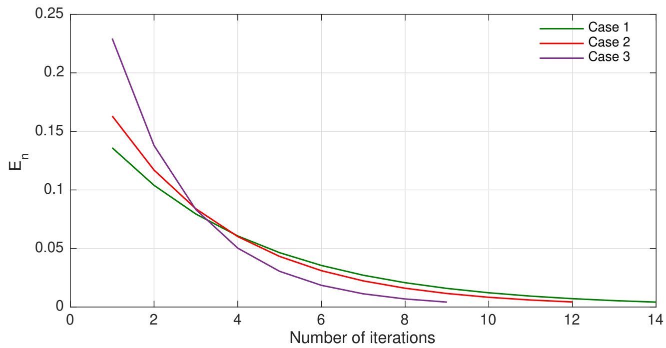

Figure 1: E_n versus number of iterations of Choice 1.

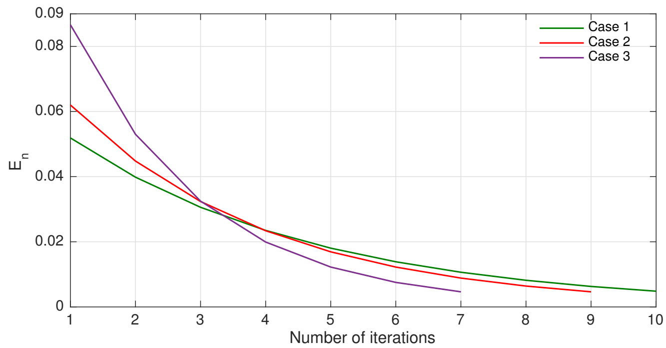

Figure 2: E_n versus number of iterations of Choice 2.

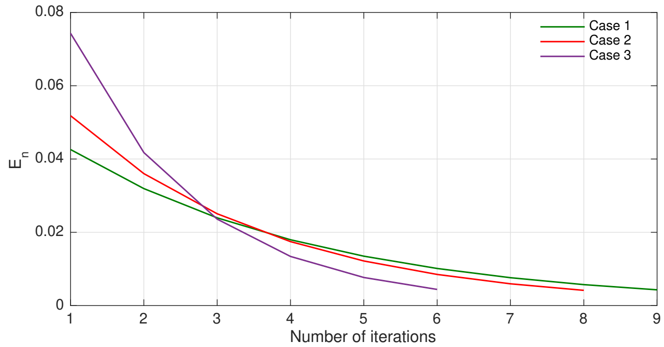

Figure 3: E_n versus number of iterations of Choice 3.

The number of iterative processes as well as the total elapsed time required for different x_0 and α_n values are provided in Table 1. Whilst Figures 1-3 display the errors obtained from 3 distinct choices of x_0 with different values of α_n. According to the results shown in the Table and Figures, in the case when step size α_n approaching $\frac{1}{2}$, it is apparent that the sequence $\{x_n\}$ generated by Algorithm 3.1 converges faster than the other cases.

Example 4.2. *We apply Algorithm 3.4 to solve the problem of recovering the original signal from compressive measurements in this example. Let $\bar{x} \in \mathbb{R}^N$ and $y \in \mathbb{R}^M$ be the original signal and the observed data with noise $\varepsilon \in \mathbb{R}^M$, respectively. Consider*

$$y = A\bar{x} + \varepsilon, \tag{7}$$

where $A \in \mathbb{R}^{M\times N}$ $(M < N)$. The compressive sensing signal reconstruction described in the preceding equation is what we want to solve. However, it is well known that solving (7) is identical to the LASSO problem:

$$\min_{x\in\mathbb{R}^N} \frac{1}{2}\|Ax - y\|_2^2 \ \ \textit{subject to} \ \ \|x\|_1 \le \zeta,$$

where $\zeta > 0$. This problem can be seen as the SFP through the following settings: $\mathcal{H}_1 = \mathbb{R}^N$, $\mathcal{H}_2 = \mathbb{R}^M$, $C = \{x \in \mathbb{R}^N : \|x\|_1 \le \zeta\}$, and $Q = \{y\}$. Suppose that the signal size to be $N = 1024$ and $M = 512$, and the original signal $\bar{x}$ is generated by the uniform distribution in $[-2, 2]$ with k nonzero elements. Let A be the Gaussian matrix generated by the MATLAB routine randn(M, N), the observation y be generated by white Gaussian noise with signal-to-noise ratio SNR $= 40$ and $\zeta = k$. For any $n \ge 0$, let $\alpha_n = \frac{e}{6}$. Select $\gamma = \frac{1}{5\|A\|_2^2}, \tau_A = \frac{10}{3e\|A\|_2^2}$ and $x_0 = A^t y$ as the initial point. Then, we compare the accuracy between the recovered signals with the mean-squared error: $MSE_n = \frac{1}{N}\|x_n - \bar{x}\|_2^2 < 5 \times 10^{-4}$. The results are presented next.

		k Nonzero Elements			
		$k=25$	$k=50$	$k=75$	$k=100$
CQ algorithm	No. of Iter.	250	351	1255	1923
	Elapsed Time (s)	0.0855	0.1303	0.4026	0.5788
Algorithm 3.4	No. of Iter.	111	169	610	840
	Elapsed Time (s)	0.0384	0.0542	0.1587	0.2430

Table 2: Numerical experiments of Example 4.2.

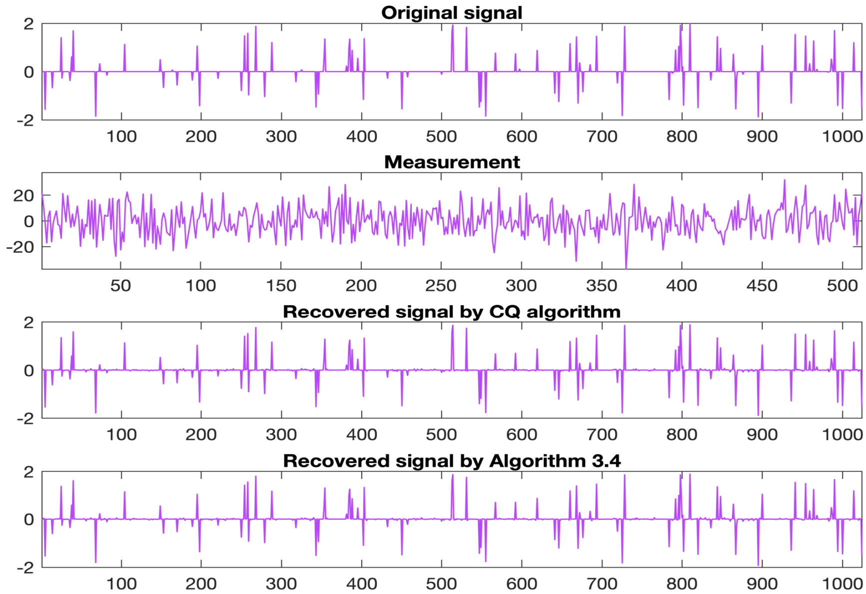

Figure 4: From top to bottom: the original signal, the measurement, and the reconstructed signals by CQ algorithm and Algorithm 3.4 in Table 2 for $k = 100$.

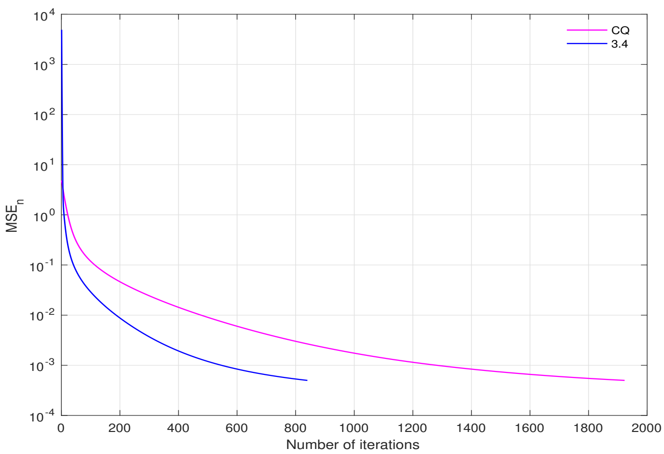

Figure 5: Plots of MSE$_n$ over number of iterations when $k = 100$.

The numerical tests in Table 2 were done with different numbers of nonzero elements: $k = 25, 50, 75, 100$. For these four cases, the CPU times and the number of iterations for CQ algorithm and Algorithm 3.4

are reported. In Figure 4, we also show the recovery signals for $k = 100$. We compute the errors of each reconstructed signal displayed in Figure 5 to detect the differences between these findings. To summarize, Algorithm 3.4 requires fewer iterations and takes less time than CQ algorithm.

5. Conclusion

This work presents the unprecedented approach in order to solve the GSCFP. Under some straightforward conditions, the algorithm provides a sequence that converges weakly to a solution of the problem. The convergence of our main theorem is furthermore confirmed by the numerical experiments.

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