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Erratum to "A New Pre-Order Relation for Set Optimization using ℓ-difference" [Communications in Advanced Mathematical Sciences, 4(3) (2021), 163-170]

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Abstract
In this work, an erratum for a proposition in the paper "A New Pre-Order Relation for Set Optimization using ℓ-difference" is outlined. It was pointed out by Stefan Rocktäschel and Ernest Quintana that the proof of Proposition 3.11 is wrong in [1]. A small detail in the proof of Proposition 3.11 has been overlooked. A new proposition, which is closely related to Proposition 3.11 in [1], is presented. The main results of the paper are not affected by this erratum.

Keywords: Order relation, pre-order relation, set optimization

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1. A new order relation for set approach

There is a subtle error in the proof of Proposition 3.11 in [1]. So, Proposition 3.11 in [1] needs to be restate. The main results of the paper are not effected from this erratum. The following example can be given as a counter example for Proposition 3.11 (i) in [1]:

Example 1.1. *Let $Y = \mathbb{R}^2$, $K = \{(x,y) \in \mathbb{R}^2 \mid y = x \text{ and } x \geq 0\}$, $A = \{(x,y) \in \mathbb{R}^2 \mid y = x\}$ and $B = \{0_{\mathbb{R}^2}\}$. Then, we can find $a \in A$ and $b \in B$ such that $b \leq_K a$. But, $A \preceq^{\ell_1} B$ is not satisfied.*

I want to put the following proposition instead of Proposition 3.11 in [1]:

Proposition 1.2. *Let $A, B \in \mathscr{P}(Y)$. If $b \leq_K a$ for all $a \in A$ and all $b \in B$, then $A \preceq^{\ell_1} B$.*

Proof. Assume that $b \leq_K a$ for all $a \in A$ and all $b \in B$. By contradiction, suppose that $A \npreceq^{\ell_1} B$. Then, $(B \ominus_\ell A) \cap K = \emptyset$, and we have $k + A \not\subset B + K$ for all $k \in K$. By setting $k = 0_Y \in K$, we have $A \not\subset B + K$. Hence, there exists $a \in A$ with $a \notin B + K$. Consequently, it holds $a \notin b + K$ for all $b \in B$ and therefore, $b \nleq_K a$ for all $b \in B$, which is contradict. □

Besides of all them, we can easily show that the order relation $\preceq^3$ implies the order relation $\preceq^{\ell_1}$, where the order of sets should be changed. That is, if $B \preceq^3 A$ (or $A \subset B + K$) for any $A, B \in \mathscr{P}(Y)$, then $A \preceq^{\ell_1} B$. But, the inverse inclusion may not be true. For example, let $Y = \mathbb{R}^2$, $K = \mathbb{R}^2_+$, $A = (-1,0)$ and $B = (0,0)$, where $\mathbb{R}^2_+$ is nonnegative orthant of the space. Although $A \preceq^{\ell_1} B$, we have $B \npreceq^3 A$.

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Competing interests

The authors declare that they have no competing interests.

Author's contributions

All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

References

[1] E. Karaman, *A new pre-order relation for set optimization using l-difference*, Comm. Adv. Math. Sci., **4**(3) (2021), 163-170.