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Interval Valued q- Rung Orthopair Hesitant Fuzzy Choquet Aggregating Operators in Multi-Criteria Decision Making Problems

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Abstract

In this paper, we introduce Interval valued q- Rung Orthopair Hesitant fuzzy sets (IVq-ROHFS) with motivation of Interval valued pythagorean Hesitant fuzzy sets [44] as a new concept. Then, we give some basic operations as complement, union, intersection, addition, scalar multiplication, scalar power. Also, we combine to (IVq-ROHFS) and choquet integral together with aggregating operators and develop to Interval valued q- rung orthopair hesitant fuzzy Choquet averaging operator (IVq-ROHCA) and Interval valued q- rung orthopair hesitant fuzzy Choquet geometric operator (IVq-ROHCG). Then, we offer to indicate soft approach of proposed IVq-ROHCA and IVq-ROHCG an example adopted from Interval-valued intuitionistic hesitant fuzzy Choquet integral based TOPSIS (IVIHCI) [43]. The obtained results are agreement with IVIHCI but presented IVq-ROHCA and IVq-ROHCG have more advantages than existing structures as Interval-valued intuitionistic hesitant fuzzy sets (IVIHFS), interval-valued Pythagorean Hesitant fuzzy sets (IVPHFS) owing to reasons changing according to need, requirement, prefer of decision makers and moreover because of IVIHFS and IVPHFS are special cases of IVq-ROHFS. It is open from comparative analysis that while some of offered approaches like IVIHFS, IVPHFS etc. are giving no solution for some values, our operators present to needed results.

1. INTRODUCTION

Decision making is almost a subject to be used for each area of life. Therefore, this subject has been worked by several scholars but these problems can include some complexity information, limitations, undefined statements. To handle with these various cases, more approaches have been defined like fuzzy sets [1], Intuitionistic fuzzy sets, Interval valued intuitionistic fuzzy sets [2, 6], Pythagorean fuzzy sets [3], Hesitant fuzzy sets [4, 5]. One of the most important structures is Intuitionistic fuzzy sets [2] which is known as generalizing of fuzzy sets. In here, the basic idea is that sum of the membership degree and the nonmembership degree is equal or less than 1 and this concept has been applied over novel constructions as following; Chen [7] offered distance measures based on the Hausdorff metric, Chen and Chang [8] presented similarity measure based on transformation techniques and applied MCDM, Guo and Song [9] used entropy measures over Atanassov' s intuitionistic fuzzy sets, Xu and Yager [10] put forward some geometric aggregation operators with helping to Intuitionistic fuzzy sets, Xu [11] discussed Intuitionistic fuzzy aggregation operators in 2007, Liu et al. [12] defined the intuitionistic fuzzy linguistic cosine similarity measure and tested its application in pattern recognition and also Park et al. [13] presented a decision making algorithm based on TOPSİS under Interval valued intuitionistic fuzzy sets environment, Tan and Zhang [14] offered DCM by using Interval valued intuitionistic fuzzy sets. In next time, this approach has been updated owing to shortcomings of intuitionistic set like $\langle 0.3,0.9\rangle$ and $0.3+0.9>1$ and Yager [3] proposed pythagorean fuzzy sets which is defined as square of sum of the membership degree and the nonmembership degree is equal or less than 1. Then, several authors extended over different concepts to this cluster. Some of these papers can be ordered as follow; Yager and coauthor, Yager [15, 16] worked Pythagorean membership grade and made an application in DCM and weighted average operator and weighted power average operator, respectively, Peng and yang [17] revealed some results for Pythagorean fuzzy sets in 2015, Garg [18, 19] gave correlation coefficient based on Pythagorean fuzzy sets and also However, hesitation degree of information can face with different statements especially if novel

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decision makers who have different opinions about an element want to have comment, in this statement, the existing clusters do not overcome. Therefore, Torra and, Torra and Narukawa [4, 5] have presented to Hesitant fuzzy sets. Then, this concept has been converted to different clusters, Dual hesitant fuzzy sets [22], Generalized hesitant fuzzy sets [23], Triangular hesitant fuzzy set [24], multi-hesitant fuzzy sets [25], then Pythagorean Hesitant Fuzzy Set [26] was defined by combining hesitant fuzzy sets and Pythagorean fuzzy sets. Moreover, Zhang and others [44] put forward Interval Valued Pythagorean Hesitant Fuzzy Set and gave its application to MCDM, Wei and Lu, [27] Tang and Wei [28] proposed Dual hesitant Pythagorean fuzzy Hamacher aggregation operators and Dual hesitant Pythagorean fuzzy information in a decision making method, Khan et al. [29] aggregated Pythagorean hesitant fuzzy information. The Choquet integral is very useful tool to eliminate hesitation degree and Choquet integral operator has been widely used with Pythagorean fuzzy sets, hesitant fuzzy sets, Pythagorean hesitant fuzzy sets, intuitionistic fuzzy sets see [30, 31, 32, 33, 34].

Sometimes, the experts can encounter with situations which not to be expressed with helping to above existing clusters for Decision making problems in real life as $\langle 0.8,0.7\rangle$ and $0.8^2+0.7^2>1$ but this problem can be solved by defining $0.8^q + 0.7^q < 1$ for $q\geq 3$. To eliminate this limitation, Yager [35] defined q-rung orthopair fuzzy sets. It can be said that this concept is generalization of IFS and PFS and wider area is scanned than IFS and PFS. Also, in short time this cluster has been started to be worked by several scholars. Liu and Wang [36] developed some aggregation operators over q-rung orthopair fuzzy sets, Liu and coworker [37] presented Bonferroni mean operators based on q-ROF, Hamy mean operators over q-rung orthopair fuzzy sets were tested by Wang et al. [38]. Moreover, Wei et al. [39] proposed Maclaurin symmetric mean operators and surveyed applications over potential evaluation of emerging technology. In addition to, some works for q-ROF can be seen by surveying [40, 41, 42]. Although the above defined approaches, the restricted information is not completely eliminated. Interval Valued Pythagorean Hesitant Fuzzy Set was worked by Zhang et al [44] in 2020. In here, Zhang et al. defined with intervals by dividing membership value and non-membership value as $\langle[\mu^-,\mu^+],[\nu^-,\nu^+]\rangle$ with condition $(\mu^+)^2+(\nu^+)^2\leq 1$ but this definition includes some vagueness statements. When $\mu^+=0.8$, $\nu^+=0.7$ are determined, this cluster encounters with non- solution cases. With this motivation, in this paper, we define interval valued q- rung orthopair hesitant fuzzy set (IVq-ROHFS) having soft, changeable structure. The above defined problem is relaxable solved with IVq-ROHFS for q=3, $0.8^3+0.7^3\leq 1$. Then, some basic definitions, theorems are produced for this new concept as union, intersection, addition, multiplication, scalar multiplication, scalar power. Moreover, score function is investigated over IVq-ROHFS. In addition to, we combine IVq-ROHFS and choquet integral together with aggregating operators. Then, the obtained concept is applied for an example over Interval-valued intuitionistic hesitant fuzzy Choquet integral (IVIHFC) based on TOPSIS [43]. In here, the example is solved for a lot of values of q and the results are almost agreement but it should be noted that our proposed operators have more advantages than existing used IVIHFC because of soft construction of q-ROF. Finally, we offer comparative analysis and change some values of decision making matrix in previous example, while compared papers do not give result, our proposed concept achieves this. This structure can be utilized in several areas as medical diagnosis, engineering, MCDM environment etc..

The remaining of paper is organized as follow; second section includes the basic definitions and propositions of PFS, HFS, q-ROF; in third section, we present interval valued q-rung orthopair hesitant fuzzy sets and put forward the basic operations; section 4 introduces Interval valued q- rung orthopair hesitant fuzzy Choquet averaging operator and Interval valued q- rung orthopair hesitant fuzzy Choquet geometric operator and some properties; section 5 includes a numerical example; section 6 proposes a comparative analysis; the end section offers conclusion and some notes.

2. PRELIMINARY

In this section, we recall some basic notions of hesitant fuzzy sets and the q- rung orthopair fuzzy set.

Definition 2.1 *[1] Let E be a universe. A fuzzy set X over E is a mapping defined as follows:*
$X=\{(\mu_X(x)/x): x\in E\}$
where $\mu_X: E\to[0,1]$.
Here, μ_X called membership function of X, and the value $\mu_X(x)$ is called the grade of membership of $x\in E$. The value represents the degree of x belonging to the fuzzy set X.
Then, Fuzzy sets have been extended as HFSs in [5] .

Definition 2.2 *[5] Let X be a non-empty set. Then, a hesitant fuzzy set (shortly HFS) in X is in terms of a function that when applied to X return a subset of $[0,1]$. We express the HFS by*
$A=\{(x,\xi_A(x)): x\in X\}$,
where $\xi_A(x)$ is a set of some values in $[0,1]$, denoting the possible membership degrees of the element $x\in X$ to the set A, $\xi=\xi_A(x)$ is called a hesitant fuzzy element (HFE) and $\mathcal{H}(X)$ denotes the set of all HFEs on X.
While this structure is not meeting enough needs, *DHPFSs* have been defined.

Definition 2.3 *[27] Let X be a non-empty set. Then, a dual hesitant pythagorean fuzzy set (shortly DHPFS) in X is defined in terms of two functions that when applied to X return a subset of different values of $[0,1]$. We express the DHPFS by*
$A=\{(x,\mu_A(x),\nu_A(x)): x\in X\}$,
where $\mu_A(x),\nu_A(x)$ are two sets of some values in $[0,1]$, denoting the membership degrees and non-membership degrees of the element $x\in X$ to the set A and $\mathcal{D}(X)$ denotes the set of all DHPFEs on X and it holds that
$0\leq\gamma_+^2+\eta_+^2\leq 1$
where for all $x\in X$, $\mu_A(x)=\bigcup_{\gamma\in\mu_A(x)}\{\gamma\}$ $\nu_A(x)=\bigcup_{\eta\in\nu_A(x)}\{\eta\}$ and γ^+,η^+ are maximum elements in $\mu_A(x),\nu_A(x)$.
Then, interval valued pythagorean hesitant fuzzy set was defined with intervals by dividing membership value and non-membership value as $\langle[\mu^-,\mu^+],[\upsilon^-,\upsilon^+]\rangle$ with condition $(\mu^+)^2+(\upsilon^+)^2\leq 1$.

Definition 2.4 *[44] Let X be a reference set. A interval valued pythagorean hesitant fuzzy set P is defined as follows:*
$P=\langle(x,h_P(x)): x\in X\rangle$,
for
$h_P(x)=\{\langle\mu_P(x),\nu_P(x)\rangle: \mu_P(x)=[\mu_P^-(x),\mu_P^+(x)]\in[0,1], \nu_P(x)=[\nu_P^-(x),\nu_P^+(x)]\in[0,1], (\mu_P^+(x))^2+(\nu_P^+(x))^2\leq 1\}$
and
$\pi_P(x)=\{[\pi_P^-(x),\pi_P^+(x)]: \pi_P^-(x)=(1-(\mu_P^-(x))^2-(\nu_P^-(x))^2)^{\frac{1}{2}}, \pi_P^+(x)=(1-(\mu_P)^+(x)^2-(\nu_P)^+(x)^2)^{\frac{1}{2}}, \langle[\mu_P^-(x),\mu_P^+(x)],[\nu_P^-(x),\nu_P^+(x)]\rangle\}=\langle\mu_P(x),\nu_P(x)\rangle$
.

Definition 2.5 *[35] Let X be a non-empty set. Then, a q- rung orthopair fuzzy set (shortly q-ROFS) in X is defined in terms of two functions that when applied to X return a subset of different values of $[0,1]$. We*

express the q-ROFS by
$A = \{(x, \mu_A(x), \nu_A(x)) : x \in X\}$,
where $\mu_A(x), \nu_A(x)$ are two elements in $[0,1]$, denoting the membership degrees and non-membership degrees of the element $x \in X$ and it holds that $0 \le \gamma^q + \eta^q \le 1$ where for all $x \in X$.

Definition 2.6 *[44] Let* $h_P = \{(\mu, \nu) : \mu = [\mu^-, \mu^+], \nu = [\nu^-, \nu^+]\}, h_{P_1} = \{(\mu_1, \nu_1) : \mu = [\mu_1^-, \mu_1^+], \nu = [\nu_1^- = [\nu_1^-, \nu_1^+]\}$ *and* $h_{P_2} = \{(\mu_2, \nu_2) : \mu = [\mu_2^-, \mu_2^+], \nu = [\nu_2^-, \nu_2^+]\}$ *be three* $IVPHFS$ *sets over* X*. The basic concepts of IVPHFS are defined as follows:*

- $h_{P_1} \cup h_{P_2} = \{\langle [max\{\mu_1^-, \mu_2^-\}, max\{\mu_1^+, \mu_2^+\}], [min\{\nu_1^-, \nu_2^-\}, min\{\nu_1^+, \nu_2^+\}] \rangle : \langle \mu_i, \nu_i \rangle \in h_{P_i}, i = 1,2\}$,
- $h_{P_1} \cap h_{P_2} = \{\langle [min\{\mu_1^-, \mu_2^-\}, min\{\mu_1^+, \mu_2^+\}], [max\{\nu_1^-, \nu_2^-\}, max\{\nu_1^+, \nu_2^+\}] \rangle : \langle \mu_i, \nu_i \rangle \in h_{P_i}, i = 1,2\}$,
- $h_P^c = \{\langle [\nu^-, \nu^+], [\mu^-, \mu^+] \rangle : \langle \mu, \nu \rangle \in h_P\}$,
- $h_P^\lambda = \{\langle [(\mu^-)^\lambda, (\mu^+)^\lambda], [(1 - (1 - (\nu^-)^2)^\lambda)^{\frac{1}{2}}, (1 - (1 - (\nu^+)^2)^\lambda)^{\frac{1}{2}}] \rangle : \langle \mu, \nu \rangle \in h_P\}$,
- $\lambda h_P = \{\langle [(1 - (1 - (\mu^-)^2)^\lambda)^{\frac{1}{2}}, (1 - (1 - (\mu^+)^2)^\lambda)^{\frac{1}{2}}], [(\nu^-)^\lambda, (\nu^+)^\lambda] \rangle : \langle \mu, \nu \rangle \in h_P\}$,
-

$h_{P_1} \oplus h_{P_2} = \{\langle [((\mu_1^-)^2 + (\mu_2^-)^2 - (\mu_1^-)^2(\mu_2^-)^2)^{\frac{1}{2}}, ((\mu_1^+)^2 + (\mu_2^+)^2 - (\mu_1^+)^2(\mu_2^+)^2)^{\frac{1}{2}}], [\nu_1^- \nu_2^-, \nu_1^+ \nu_2^+] : \langle \mu_i, \nu_i \rangle \in h_{P_i}, i = 1,2 \rangle$
,
-

$h_{P_1} \otimes h_{P_2} = \{\langle [\mu_1^- \mu_2^-, \mu_1^+ \mu_2^+], [((\nu_1^-)^2 + (\nu_2^-)^2 - (\nu_1^-)^2(\nu_2^-)^2)^{\frac{1}{2}}, ((\nu_1^+)^2 + (\nu_2^+)^2 - (\nu_1^+)^2(\nu_2^+)^2)^{\frac{1}{2}}] : \langle \mu_i, \nu_i \rangle \in h_{P_i}, i = 1,2 \rangle$

3. INTERVAL VALUED Q- RUNG ORTHOPAIR HESITANT FUZZY SETS

The concept of q- Rung Orthopair fuzzy sets (q-ROFS) was defined by Yager [35] in 2016. In this section, interval valued q- rung orthopair hesitant fuzzy set (IVq-ROHFS) is defined and some basic properties are given.

Definition 3.1 *Let* X *be a reference set. A interval valued q- rung orthopair hesitant fuzzy set* $\mathfrak{R}$ *is defined as follows:*

$$\mathfrak{R} = \langle (x, h_{\mathfrak{R}}(x)) : x \in X \rangle,$$

For
$h_{\mathfrak{R}}(x) = \{\langle \mu_{\mathfrak{R}}(x), \nu_{\mathfrak{R}}(x) \rangle : \mu_{\mathfrak{R}}(x) = [\mu_{\mathfrak{R}}^-(x), \mu_{\mathfrak{R}}^+(x)] \in [0,1], \nu_{\mathfrak{R}}(x) = [\nu_{\mathfrak{R}}^-(x), \nu_{\mathfrak{R}}^+(x)] \in [0,1], (\mu_{\mathfrak{R}}^+(x))^q + (\nu_{\mathfrak{R}}^+(x))^q \le 1\}$
and
$\pi_{\mathfrak{R}}(x) = \{[\pi_{\mathfrak{R}}^-(x), \pi_{\mathfrak{R}}^+(x)] : \pi_{\mathfrak{R}}^-(x) = (1 - (\mu_{\mathfrak{R}}^-(x))^q - (\nu_{\mathfrak{R}}^-(x))^q)^{\frac{1}{q}}, \pi_{\mathfrak{R}}^+(x) = (1 - (\mu_{\mathfrak{R}})^+(x)^q - (\nu_{\mathfrak{R}})^+(x)^q)^{\frac{1}{q}}, \langle [\mu_{\mathfrak{R}}^-(x), \mu_{\mathfrak{R}}^+(x)], [\nu_{\mathfrak{R}}^-(x), \nu_{\mathfrak{R}}^+(x)] \rangle\} = \langle \mu_{\mathfrak{R}}(x), \nu_{\mathfrak{R}}(x) \rangle$

in here we call cluster of pairs $h_{\mathfrak{R}} = h_{\mathfrak{R}}(x)$ as interval valued q- rung orthopair hesitant fuzzy set (IVq-ROHFS) and is indicated $h_{\mathfrak{R}} = \{(\mu, \nu): \mu = [\mu^-, \mu^+], \nu = [\nu^-, \nu^+], (\mu^+)^q + (\nu^+)^q \leq 1\}$.

Definition3.2 *Let* $h_{\mathfrak{R}} = \{(\mu, \nu): \mu = [\mu^-, \mu^+], \nu = [\nu^-, \nu^+]\}, h_{\mathfrak{R}_1} = \{(\mu_1, \nu_1): \mu = [\mu_1^-, \mu_1^+], \nu = [\nu_1^-, \nu_1^+]\}$ *and* $h_{\mathfrak{R}_2} = \{(\mu_2, \nu_2): \mu = [\mu_2^-, \mu_2^+], \nu = [\nu_2^-, \nu_2^+]\}$ *be three IVq-ROHFS sets over* X*. The basic concepts of IVq-ROHFS are defined as follows:*

- $h_{\mathfrak{R}_1} \cup h_{\mathfrak{R}_2} = \{\langle [max\{\mu_1^-, \mu_2^-\}, max\{\mu_1^+, \mu_2^+\}], [min\{\nu_1^-, \nu_2^-\}, min\{\nu_1^+, \nu_2^+\}]\rangle : \langle \mu_i, \nu_i \rangle \in h_{\mathfrak{R}_i}, i = 1,2\}$,
- $h_{\mathfrak{R}_1} \cap h_{\mathfrak{R}_2} = \{\langle [\min\{\mu_1^-, \mu_2^-\}, \min\{\mu_1^+, \mu_2^+\}], [\max\{\nu_1^-, \nu_2^-\}, max\{\nu_1^+, \nu_2^+\}]\rangle : \langle \mu_i, \nu_i \rangle \in h_{\mathfrak{R}_i}, i = 1,2\}$
- $h_{\mathfrak{R}}^c = \{\langle [\nu^-, \nu^+], [\mu^-, \mu^+]\rangle : \langle \mu, \nu \rangle \in h_{\mathfrak{R}}\}$,
- $h_{\mathfrak{R}}^\lambda = \{\langle [(\mu^-)^\lambda, (\mu^+)^\lambda], [(1-(1-(\nu^-)^q)^\lambda)^{\frac{1}{q}}, (1-(1-(\nu^+)^q)^\lambda)^{\frac{1}{q}}]\rangle : \langle \mu, \nu \rangle \in h_{\mathfrak{R}}\}$,
- $\lambda h_{\mathfrak{R}} = \{\langle [(1-(1-(\mu^-)^q)^\lambda)^{\frac{1}{q}}, (1-(1-(\mu^+)^q)^\lambda)^{\frac{1}{q}}], [(\nu^-)^\lambda, (\nu^+)^\lambda]\rangle : \langle \mu, \nu \rangle \in h_{\mathfrak{R}}\}$,
- $h_{\mathfrak{R}_1} \oplus h_{\mathfrak{R}_2} = \{\langle [((\mu_1^-)^q + (\mu_2^-)^q - (\mu_1^-)^q(\mu_2^-)^q)^{\frac{1}{q}}, ((\mu_1^+)^q + (\mu_2^+)^q - (\mu_1^+)^q(\mu_2^+)^q)^{\frac{1}{q}}], [\nu_1^-\nu_2^-, \nu_1^+\nu_2^+] : \langle \mu_i, \nu_i \rangle \in h_{\mathfrak{R}_i}, i = 1,2\rangle$
- $h_{\mathfrak{R}_1} \otimes h_{\mathfrak{R}_2} = \{\langle [\mu_1^-\mu_2^-, \mu_1^+\mu_2^+], [((\nu_1^-)^q + (\nu_2^-)^q - (\nu_1^-)^q(\nu_2^-)^q)^{\frac{1}{q}}, ((\nu_1^+)^q + (\nu_2^+)^q - (\nu_1^+)^q(\nu_2^+)^q)^{\frac{1}{q}}] : \langle \mu_i, \nu_i \rangle \in h_{\mathfrak{R}_i}, i = 1,2\rangle$

,

Definition 3.3 *Let* $h_{\mathfrak{R}_1} = \{(\mu_1, \nu_1): \mu = [\mu_1^-, \mu_1^+], \nu = [\nu_1^-, \nu_1^+]\}$ *be IVq-ROHFe. In this statement score function of* $h_{\mathfrak{R}}$ *is defined as follow;*

$$S(h_{\mathfrak{R}}) = \sum_{\mathfrak{f} \in h_{\mathfrak{R}}} = \frac{1}{2l(h_{\mathfrak{R}})}[((\mu_1^-)^q - (\nu_1^-)^q) + ((\mu_1^+)^q - (\nu_1^+)^q)]$$

where $l(h_{\mathfrak{R}})$ indicates number of elements in IVq-ROHFS and also for two IVq-ROHFe which is showed as $h_{\mathfrak{R}_1}$ and $h_{\mathfrak{R}_2}$;

1. if $S(h_{\mathfrak{R}_1}) > S(h_{\mathfrak{R}_2}), h_{\mathfrak{R}_1} > h_{\mathfrak{R}_2}$,
2. if $S(h_{\mathfrak{R}_1}) < S(h_{\mathfrak{R}_2}), h_{\mathfrak{R}_1} < h_{\mathfrak{R}_2}$,
3. if $S(h_{\mathfrak{R}_1}) = S(h_{\mathfrak{R}_2}), h_{\mathfrak{R}_1} = h_{\mathfrak{R}_2}$.

3.1 Fuzzy measure and Choquet integral operator

The fuzzy measure introduced by Sugeno in 1974 [45] was successfully utilized for MCDM problems. Then, choquet integral [46] was used as a powerful notation to aggregating operators. In here, we give to fuzzy measure, choquet integral, discrete choquet integral.

Definition 3.4 *A fuzzy measure over X is a function $\Re: P(X) \longrightarrow [0,1]$ satisfying following conditions;*

1. $\Re(\emptyset) = 0, \Re(X) = 1$,

2. If M and $N \in P(X)$ and $M \subseteq N$ then $\Re(M) \leq \Re(N)$.

But Sugeno [45] proposed some special conditions of fuzzy measure due to reasons like complexity into calculations as follow;

$\Re(M \cup N) = \Re(M) + \Re(N) + \lambda\Re(M)\Re(N)$ and $\lambda \in (-1, \infty)$ for all $M, N \in P(X)$ and $M \cap N = \emptyset$. If $\lambda = 0$, $\lambda -$ fuzzy measure is induced additive measure as follow;

1. $\Re(M \cup N) = \Re(M) + \Re(N)$

If all elements in X are finite, then;

$$\bullet\ \Re(X) = \begin{cases} \frac{1}{\lambda}\prod_{i=1}^{n} [1 + \lambda\Re(x_i)] - 1, & \lambda \neq 0 \\ \sum_{x_i \in M} \Re(x_i), & \lambda = 0. \end{cases}$$

Definition 3.5 *[46] Let f be positive reel valued function on X and $\Re$ be fuzzy measure on X. The discrete Choquet integral of f with respect to $\Re$ is proposed as follow;*

$$C_{\Re}(f) = \sum_{i=1}^{n} f_{\sigma(i)}[\Re(A_{\sigma(i)}) - \Re(A_{\sigma(i-1)})],$$

where $\sigma(i)$ indicates a permutation on X such that $f_{\sigma(1)} \geq f_{\sigma(2)} \geq \ldots \geq f_{\sigma(n)}$ and $A_{\sigma(i)} = 1,2,\ldots,i$, $A_{\sigma(0)} = \emptyset$.

4. INTERVAL VALUED Q- RUNG ORTHOPAIR HESITANT FUZZY CHOQUET AGGREGATING OPERATORS

In this section, we define averaging operators, geometric operators based on choquet integral and their some properties.

Definition 4.1 *Let determine collection of IVq-ROHFSs that $\Re_i = \{(\mu_i, \nu_i): \mu_i = [\mu_i^-, \mu_i^+], \nu_i = [\nu_i^-, \nu_i^+]\}$ where $(i = 1,2,\ldots,n)$ and $\sigma(i)$ indicates to a permutation of $1,2,\ldots,n$ such that $\Re_{\sigma(1)} \geq \Re_{\sigma(2)} \geq \ldots \geq \Re_{\sigma(n)}$ and $\Re_{\sigma(0)} = \emptyset$ in here, Interval valued q- rung orthopair hesitant fuzzy Choquet integral averaging operator (IVq-ROHFCA) is defined as follow;*

$$IV - qROHFCA(\Re_1, \Re_2, \ldots, \Re_n) = \{\langle[(1 - \prod_{i=1}^{n} (1 - (\mu_{\sigma(i)}^-)^q)^{(\mu(A_{\sigma(i)}) - \mu(A_{\sigma(i-1)}))})^{\frac{1}{q}}, (1 - \prod_{i=1}^{n} (1 - (\mu_{\sigma(i)}^+)^q)^{(\mu(A_{\sigma(i)}) - \mu(A_{\sigma(i-1)}))})^{\frac{1}{q}}], [\prod_{i=1}^{n} (\nu_{\sigma(i)}^-)^{(\mu(A_{\sigma(i)}) - \mu(A_{\sigma(i-1)}))}, \prod_{i=1}^{n} (\nu_{\sigma(i)}^+)^{(\mu(A_{\sigma(i)}) - \mu(A_{\sigma(i-1)}))}]\rangle\}$$

Theorem 4.2 *Let accept collection of $\Re_i = \{(\mu_i, \nu_i): \mu_i = [\mu_i^-, \mu_i^+], \nu_i = [\nu_i^-, \nu_i^+]\}$ and $\sigma(i)$ indicates to a permutation of $1,2,\ldots,n$ such that $\Re_{\sigma(1)} \geq \Re_{\sigma(2)} \geq \ldots \geq \Re_{\sigma(n)}$ and $\Re_{\sigma(0)} = \emptyset$. $\Re_{\sigma(i)}$ is the ith largest element of $\Re_i$ and their aggregated value is still IVq-ROHF.*

Proof. It is trivial to show for $n = 1$, we proof for $n = 2$ and can write as follow;

$$(\mu(A_{\sigma(1)})\Re_1 = \{\langle[(1-(1-(\mu^-_{\sigma(1)})^q)^{(\mu(A_{\sigma(1)})})^{\frac{1}{q}},(1-(1-(\mu^+_{\sigma(1)})^q)^{(\mu(A_{\sigma(1)}))})^{\frac{1}{q}}],[(\nu^-_{\sigma(1)})^{(\mu(A_{\sigma(1)}))},(\nu^+_{\sigma(1)})^{(\mu(A_{\sigma(1)}))},]\rangle\}$$

and

$$(\mu(A_{\sigma(2)})-\mu(A_{\sigma(1)}))\Re_2 = \{\langle[(1-(1-(\mu^-_{\sigma(2)})^q)^{\left(\mu(A_{\sigma(2)})-\mu(A_{\sigma(1)})\right)})^{\frac{1}{q}},(1-(1-(\mu^+_{\sigma(2)})^q)^{\left(\mu(A_{\sigma(2)})-\mu(A_{\sigma(1)})\right)})^{\frac{1}{q}}],[(\nu^-_{\sigma(2)})^{\left(\mu(A_{\sigma(2)})-\mu(A_{\sigma(1)})\right)},(\nu^+_{\sigma(2)})^{(\mu(A_{\sigma(2)})-\mu(A_{\sigma(1)}))}]\rangle\}$$

from here

$$(\mu(A_{\sigma(1)})\Re_1 \oplus (\mu(A_{\sigma(2)})-\mu(A_{\sigma(1)}))\Re_2 = \{\langle[((1-(1-(\mu^-_{\sigma(1)})^q)^{\left(\mu(A_{\sigma(1)})\right)})+(1-(1-(\mu^-_{\sigma(2)})^q)^{\left(\mu(A_{\sigma(2)})-\mu(A_{\sigma(1)})\right)})-(1-(1-(\mu^-_{\sigma(1)})^q)^{\left(\mu(A_{\sigma(1)})\right)})(1-(1-(\mu^-_{\sigma(2)})^q)^{\left(\mu(A_{\sigma(2)})-\mu(A_{\sigma(1)})\right)}))^{\frac{1}{q}},((1-(1-(\mu^+_{\sigma(1)})^q)^{\left(\mu(A_{\sigma(1)})\right)})+(1-(1-(\mu^+_{\sigma(2)})^q)^{\left(\mu(A_{\sigma(2)})-\mu(A_{\sigma(1)})\right)})-(1-(1-(\mu^+_{\sigma(1)})^q)^{\left(\mu(A_{\sigma(1)})\right)})(1-(1-(\mu^+_{\sigma(2)})^q)^{\left(\mu(A_{\sigma(2)})-\mu(A_{\sigma(1)})\right)}))^{\frac{1}{q}}],[((\nu^-_{\sigma(1)})^{\left(\mu(A_{\sigma(1)})\right)})((\nu^-_{\sigma(2)})^{\left(\mu(A_{\sigma(2)})-\mu(A_{\sigma(1)})\right)}),((\nu^+_{\sigma(1)})^{(\mu(A_{\sigma(1)}))})((\nu^+_{\sigma(2)})^{(\mu(A_{\sigma(2)})-\mu(A_{\sigma(1)}))})]\rangle\}$$

and thus for $n = 2$, it holds, we look for $n = k$;

$$IVq-ROHFCA(\Re_1,\Re_2,\dots,\Re_k) = \{\langle[(1-\prod_{i=1}^{k}(1-(\mu^-_{\sigma(i)})^q)^{\left(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)})\right)})^{\frac{1}{q}},(1-\prod_{i=1}^{k}(1-(\mu^+_{\sigma(i)})^q)^{\left(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)})\right)})^{\frac{1}{q}}],[\prod_{i=1}^{k}(\nu^-_{\sigma(i)})^{\left(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)})\right)},\prod_{i=1}^{k}(\nu^+_{\sigma(i)})^{(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))}]\rangle\}$$

and from here we can write for $n = k + 1$ as follow;

$$IVq-ROHFCA(\Re_1,\Re_2,\dots,\Re_{k+1}) = \{\langle[(1-\prod_{i=1}^{k}(1-(\mu^-_{\sigma(i)})^q)^{\left(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)})\right)})^{\frac{1}{q}},(-\prod_{i=1}^{k}(-(\mu^+_{\sigma(i)})^q)^{\left(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)})\right)})^{\frac{1}{q}}],$$

$$[\prod_{i=1}^{k}(\nu^-_{\sigma(i)})^{\left(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)})\right)},$$

$$\prod_{i=1}^{k} (\nu_{\sigma(i)}^{+})^{\left(\mu\left(A_{\sigma(i)}\right)-\mu\left(A_{\sigma(i-1)}\right)\right)}])\} \oplus \{\langle[(1-(1-(\mu_{\sigma(k+1)}^{-})^{q})^{\left(\mu\left(A_{\sigma(k+1)}\right)-\mu\left(A_{\sigma(k)}\right)\right)})^{\frac{1}{q}},(1-(1-(\mu_{\sigma(k+1)}^{+})^{q})^{\left(\mu\left(A_{\sigma(k+1)}\right)-\mu\left(A_{\sigma(k)}\right)\right)})^{\frac{1}{q}}],$$

$$[(\nu_{\sigma(k+1)}^{-})^{(\mu(A_{\sigma(k+1)})-\mu(A_{\sigma(k)}))},(\nu_{\sigma(k+1)}^{+})^{(\mu(A_{\sigma(k+1)})-\mu(A_{\sigma(k)}))}])\}$$

and from here

$$\{\langle[((1-\prod_{i=1}^{k}(1-(\mu_{\sigma(i)}^{-})^{q})^{\left(\mu\left(A_{\sigma(i)}\right)-\mu\left(A_{\sigma(i-1)}\right)\right)})+(1-(1-(\mu_{\sigma(k+1)}^{-})^{q})^{\left(\mu\left(A_{\sigma(k+1)}\right)-\mu\left(A_{\sigma(k)}\right)\right)})-(1-\prod_{i=1}^{k}(1-(\mu_{\sigma(i)}^{-})^{q})^{\left(\mu\left(A_{\sigma(i)}\right)-\mu\left(A_{\sigma(i-1)}\right)\right)})(1-(1-(\mu_{\sigma(k+1)}^{-})^{q})^{\left(\mu\left(A_{\sigma(k+1)}\right)-\mu\left(A_{\sigma(k)}\right)\right)}))^{\frac{1}{q}},((1-\prod_{i=1}^{k}(1-(\mu_{\sigma(i)}^{+})^{q})^{\left(\mu\left(A_{\sigma(i)}\right)-\mu\left(A_{\sigma(i-1)}\right)\right)})+(1-(1-(\mu_{\sigma(k+1)}^{+})^{q})^{\left(\mu\left(A_{\sigma(k+1)}\right)-\mu\left(A_{\sigma(k)}\right)\right)})-(1-\prod_{i=1}^{k}(1-(\mu_{\sigma(i)}^{+})^{q})^{\left(\mu\left(A_{\sigma(i)}\right)-\mu\left(A_{\sigma(i-1)}\right)\right)})(1-(1-(\mu_{\sigma(k+1)}^{+})^{q})^{\left(\mu\left(A_{\sigma(k+1)}\right)-\mu\left(A_{\sigma(k)}\right)\right)}))^{\frac{1}{q}}],$$

$$[(\prod_{i=1}^{k}(\nu_{\sigma(i)}^{-})^{\left(\mu\left(A_{\sigma(i)}\right)-\mu\left(A_{\sigma(i-1)}\right)\right)})$$

$$((\nu_{\sigma(k+1)}^{-})^{\left(\mu\left(A_{\sigma(k+1)}\right)-\mu\left(A_{\sigma(k)}\right)\right)}),$$

$$(\prod_{i=1}^{k}(\nu_{\sigma(i)}^{+})^{\left(\mu\left(A_{\sigma(i)}\right)-\mu\left(A_{\sigma(i-1)}\right)\right)})$$

$$((\nu_{\sigma(k+1)}^{+})^{(\mu(A_{\sigma(k+1)})-\mu(A_{\sigma(k)}))})])\}$$

and if the basic operations are made, it holds for $n = k + 1$ as follow;

$$IVq-ROHFCA(\Re_1,\Re_2,\ldots,\Re_{k+1}) = \{\langle[(1-\prod_{i=1}^{k+1}(1-(\mu_{\sigma(i)}^{-})^{q})^{(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))})^{\frac{1}{q}},(1-\prod_{i=1}^{k+1}(1-(\mu_{\sigma(i)}^{+})^{q})^{(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))})^{\frac{1}{q}}],[\prod_{i=1}^{k+1}(\nu_{\sigma(i)}^{-})^{(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))},\prod_{i=1}^{k+1}(\nu_{\sigma(i)}^{+})^{(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))}]\rangle\}$$

the proof is completed.

Theorem 4.3 *(idempotency) Let accept collection of* $\Re_i = \{(\mu_i, \nu_i): \mu = [\mu_i^-, \mu_i^+], \nu = [\nu_i^-, \nu_i^+]\}$ *and* $\Re_i = \Re$ *for* $(i = 1,2,\ldots,n)$. *Thus,* $IVq-ROHFCA(\Re_1, \Re_2, \ldots \Re_n) = \Re$.

Proof. From the theorem 4.2, can be written as follow;

$$= \{\langle[(1-\prod_{i=1}^{n}(1-(\mu_{\sigma(i)}^{-})^{q})^{(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))})^{\frac{1}{q}},(1-\prod_{i=1}^{n}(1-(\mu_{\sigma(i)}^{+})^{q})^{(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))})^{\frac{1}{q}}],[\prod_{i=1}^{n}(\nu_{\sigma(i)}^{-})^{(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))},\prod_{i=1}^{n}(\nu_{\sigma(i)}^{+})^{(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))}]\rangle\}$$

from here

$$= \{\langle[(1-(1-(\mu_{\sigma(i)}^{-})^q)^{\sum_{i=1}^{n}(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))})^{\frac{1}{q}},(1-(1-(\mu_{\sigma(i)}^{+})^q)^{(\sum_{i=1}^{n}\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))})^{\frac{1}{q}}],[(\nu_{\sigma(i)}^{-})^{\sum_{i=1}^{n}(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))},(\nu_{\sigma(i)}^{+})^{\sum_{i=1}^{n}(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))}]\rangle\}$$

since $\sum_{i=1}^{n}\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)})=1$

$$IVq-ROHFCA(\Re_1,\Re_2,\ldots,\Re_n)=\{\langle[(1-(1-(\mu_{\sigma(i)}^{-})^q))^{\frac{1}{q}},(1-(1-(\mu_{\sigma(i)}^{+})^q))^{\frac{1}{q}}],[(\nu_{\sigma(i)}^{-}),(\nu_{\sigma(i)}^{+})]\rangle\}$$
$$=\Re$$

Theorem 4.4 *(Monotonicity) If* $\Re_i \le \Re_i^*$,

$IVq-ROHFCA(\Re_1,\Re_2,\ldots,\Re_n) \le IVq-ROHFCA(\Re_1^*,\Re_2^*,\ldots,\Re_n^*)$.

Proof. Since $\Re_i \le \Re_i^*$ for all i, then we can write;

- if $\mu_{\sigma(i)}^{-} \le \mu_{\sigma(i)}^{-*}$;

$$(\mu_{\sigma(i)}^{-})^q \le (\mu_{\sigma(i)}^{-*})^q$$
$$1-(\mu_{\sigma(i)}^{-})^q \ge 1-(\mu_{\sigma(i)}^{-*})^q$$
$$(1-(\mu_{\sigma(i)}^{-})^q)^{(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))} \ge (1-(\mu_{\sigma(i)}^{-*})^q)^{(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))}$$
$$\prod_{i=1}^{n}(1-(\mu_{\sigma(i)}^{-})^q)^{(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))} \ge \prod_{i=1}^{n}(1-(\mu_{\sigma(i)}^{-*})^q)^{(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))}$$
$$(1-\prod_{i=1}^{n}(1-(\mu_{\sigma(i)}^{-})^q)^{(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))})^{\frac{1}{q}} \le$$
$$(1-\prod_{i=1}^{n}(1-(\mu_{\sigma(i)}^{-*})^q)^{(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))}$$

- if $\mu_{\sigma(i)}^{+} \le \mu_{\sigma(i)}^{+*}$;

$$(\mu_{\sigma(i)}^{+})^q \le (\mu_{\sigma(i)}^{+*})^q$$
$$1-(\mu_{\sigma(i)}^{+})^q \ge 1-(\mu_{\sigma(i)}^{+*})^q$$
$$(1-(\mu_{\sigma(i)}^{+})^q)^{(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))} \ge (1-(\mu_{\sigma(i)}^{+*})^q)^{(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))}$$
$$\prod_{i=1}^{n}(1-(\mu_{\sigma(i)}^{+})^q)^{(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))} \ge \prod_{i=1}^{n}(1-(\mu_{\sigma(i)}^{+*})^q)^{(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))}$$
$$(1-\prod_{i=1}^{n}(1-(\mu_{\sigma(i)}^{+})^q)^{(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))})^{\frac{1}{q}} \le$$
$$(1-\prod_{i=1}^{n}(1-(\mu_{\sigma(i)}^{+*})^q)^{(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))})^{\frac{1}{q}}$$

- if $\nu^-_{\sigma(i)} \geq \nu^-_{\sigma(i)}{}^*$;

$$(\nu^-_{\sigma(i)})^{(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))} \geq (\nu^-_{\sigma(i)}{}^*)^{(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))}$$
$$\prod_{i=1}^n (\nu^-_{\sigma(i)})^{(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))} \geq \prod_{i=1}^n (\nu^-_{\sigma(i)}{}^*)^{(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))}$$

- if $\nu^+_{\sigma(i)} \geq \nu^+_{\sigma(i)}{}^*$;

$$(\nu^+_{\sigma(i)})^{(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))} \geq (\nu^+_{\sigma(i)}{}^*)^{(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))}$$
$$\prod_{i=1}^n (\nu^+_{\sigma(i)})^{(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))} \geq \prod_{i=1}^n (\nu^+_{\sigma(i)}{}^*)^{(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))}$$

the proof is completed.

Theorem 4.5 *(Boundedness) Let accept collection of* $\Re_i = \{(\mu_i, \nu_i): \mu_i = [\mu_i^-, \mu_i^+], \nu_i = [\nu_i^-, \nu_i^+]\}$ *, in this statement,*

$$\min\{\Re_1, \Re_2, \dots, \Re_n\} \leq IVq-ROHFCA(\Re_1, \Re_2, \dots, \Re_n) \leq \max\{\Re_1, \Re_2, \dots, \Re_n\}.$$

Proof. Firstly, we know that

$$= (1 - \prod_{i=1}^{n} (1 - min(\mu^-_{\sigma(i)})^q)^{(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))})^{\frac{1}{q}}$$
$$\leq (1 - \prod_{i=1}^{n} (1 - (\mu^-_{\sigma(i)})^q)^{(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))})^{\frac{1}{q}}$$
$$\leq (1 - \prod_{i=1}^{n} (1 - max(\mu^-_{\sigma(i)})^q)^{(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))})^{\frac{1}{q}}$$

and from here;

$$= (1 - (1 - min(\mu^-_{\sigma(i)})^q)^{\sum_{i=1}^n(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))})^{\frac{1}{q}}$$
$$\leq (1 - \prod_{i=1}^{n} (1 - (\mu^-_{\sigma(i)})^q)^{(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))})^{\frac{1}{q}}$$
$$\leq (1 - (1 - max(\mu^-_{\sigma(i)})^q)^{\sum_{i=1}^n(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))})^{\frac{1}{q}}$$

thus, $min(\mu^-_{\sigma(i)}) \leq (1 - \prod_{i=1}^{n} (1 - (\mu^-_{\sigma(i)})^q)^{(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))})^{\frac{1}{q}} \leq max(\mu^-_{\sigma(i)})$.

Similarly,

$$= (1 - \prod_{i=1}^{n} (1 - min(\mu_{\sigma(i)}^{+})^{q})^{(\mu(A_{\sigma(i)}) - \mu(A_{\sigma(i-1)}))})^{\frac{1}{q}}$$
$$\leq (1 - \prod_{i=1}^{n} (1 - (\mu_{\sigma(i)}^{+})^{q})^{(\mu(A_{\sigma(i)}) - \mu(A_{\sigma(i-1)}))})^{\frac{1}{q}}$$
$$\leq (1 - \prod_{i=1}^{n} (1 - max(\mu_{\sigma(i)}^{+})^{q})^{(\mu(A_{\sigma(i)}) - \mu(A_{\sigma(i-1)}))})^{\frac{1}{q}}$$

and from here

$$= (1 - (1 - min(\mu_{\sigma(i)}^{+})^{q})^{\sum_{i=1}^{n}(\mu(A_{\sigma(i)}) - \mu(A_{\sigma(i-1)}))})^{\frac{1}{q}}$$
$$\leq (1 - \prod_{i=1}^{n} (1 - (\mu_{\sigma(i)}^{+})^{q})^{(\mu(A_{\sigma(i)}) - \mu(A_{\sigma(i-1)}))})^{\frac{1}{q}}$$
$$\leq (1 - (1 - max(\mu_{\sigma(i)}^{+})^{q})^{\sum_{i=1}^{n}(\mu(A_{\sigma(i)}) - \mu(A_{\sigma(i-1)}))})^{\frac{1}{q}}$$

thus, $min(\mu_{\sigma(i)}^{+}) \leq (1 - \prod_{i=1}^{n} (1 - (\mu_{\sigma(i)}^{+})^{q})^{(\mu(A_{\sigma(i)}) - \mu(A_{\sigma(i-1)}))})^{\frac{1}{q}} \leq max(\mu_{\sigma(i)}^{+})$.

From other hand,

$$\prod_{i=1}^{n} \min(v_{\sigma(i)}^{-})^{\left(\mu(A_{\sigma(i)}) - \mu(A_{\sigma(i-1)})\right)} \leq$$

$$\prod_{i=1}^{n} (v_{\sigma(i)}^{-})^{\left(\mu(A_{\sigma(i)}) - \mu(A_{\sigma(i-1)})\right)} \leq$$

$$\prod_{i=1}^{n} max(v_{\sigma(i)}^{-})^{(\mu(A_{\sigma(i)}) - \mu(A_{\sigma(i-1)}))}$$

and from here

$$min(v_{\sigma(i)}^{-})^{\sum_{i=1}^{n}(\mu(A_{\sigma(i)}) - \mu(A_{\sigma(i-1)}))} \leq$$

$$\prod_{i=1}^{n} (v_{\sigma(i)}^{-})^{(\mu(A_{\sigma(i)}) - \mu(A_{\sigma(i-1)}))} \leq$$

$$max(v_{\sigma(i)}^{-})^{\sum_{i=1}^{n}(\mu(A_{\sigma(i)}) - \mu(A_{\sigma(i-1)}))}$$

thus, $min(v_{\sigma(i)}^{-}) \leq \prod_{i=1}^{n} (v_{\sigma(i)}^{-})^{(\mu(A_{\sigma(i)}) - \mu(A_{\sigma(i-1)}))} \leq max(v_{\sigma(i)}^{-})$. Similarly,

thus, $min(v_{\sigma(i)}^{+}) \leq \prod_{i=1}^{n} (v_{\sigma(i)}^{+})^{(\mu(A_{\sigma(i)}) - \mu(A_{\sigma(i-1)}))} \leq max(v_{\sigma(i)}^{+})$

and IVq-ROHFCA carries Boundedness property.

Now, we discuss some special cases of IVq-ROHFCA as follow.

- If $q = 1$, IVq-ROHFCA is reduced to Interval valued intuitionistic hesitant choquet integral averaging operator (IVIHCA).

- If $q = 2$, IVq-ROHFCA is reduced to Interval valued Pythagorean hesitant choquet integral averaging operator (IVPHCA).

- If $\mu(\{x_{\sigma(i)}\}) = \mu(A_\sigma(i)) - \mu(A_\sigma(i-1))$ for $(i = 1,2,\ldots,n)$, IVq-ROHFCA is reduced to Interval valued q-rung orthopair hesitant weighted averaging operator (IVq-ROHWA).

- If $\mu(A) = \sum_{i=1}^{|A|} w_i$ for all $A \in X$ where $|A|$ indicates number of elements in A and $w_i = \mu(A_{\sigma(i)}) - \mu(A_{\sigma(i-1)})$ for $(i = 1,2,\ldots,n)$ also $w = (w_1, w_2,\ldots,w_n)^T$ and $\sum_{i=1}^{n} w_i = 1$, in this statement IVq-ROHFCA is reduced to Interval valued q-rung orthopair hesitant ordered weighted averaging operator (IVq-ROHOWA).

Definition 4.6 *Let determine collection of IVq-ROHFSs that $\Re_i = \{(\mu_i, \nu_i) : \mu_i = [\mu_i^-, \mu_i^+], \nu_i = [\nu_i^-, \nu_i^+]\}$ where $(i = 1,2,\ldots,n)$ and $\sigma(i)$ indicates to a permutation of $1,2,\ldots,n$ such that $\Re_{\sigma(1)} \geq \Re_{\sigma(2)} \geq \ldots \geq \Re_{\sigma(n)}$ and $\Re_{\sigma(0)} = \emptyset$ in here, Interval valued q- runk orthopair hesitant fuzzy Choquet integral geometric operator (IVq-ROHFCG) is defined as follow;*

$$IVq-ROHFCG(\Re_1, \Re_2, \ldots, \Re_n) = \{\langle [\prod_{i=1}^{n} (\mu_{\sigma(i)}^-)^{(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))}, \prod_{i=1}^{n} (\mu_{\sigma(i)}^+)^{(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))}],$$

$$[(1-\prod_{i=1}^{n} (1-(\nu_{\sigma(i)}^-)^q)^{(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))})^{\frac{1}{q}}, (1-\prod_{i=1}^{n} (1-(\nu_{\sigma(i)}^+)^q)^{(\mu(A_{\sigma(i)})-\mu(A_{\sigma(i-1)}))})^{\frac{1}{q}}]\rangle\}$$

Theorem 4.7 *Let accept collection of $\Re_i = \{(\mu_i, \nu_i) : \mu = [\mu_i^-, \mu_i^+], \nu = [\nu_i^-, \nu_i^+]\}$ and $\sigma(i)$ indicates to a permutation of $1,2,\ldots,n$ such that $\Re_{\sigma(1)} \geq \Re_{\sigma(2)} \geq \ldots \geq \Re_{\sigma(n)}$ and $\Re_{\sigma(0)} = \emptyset$. $\Re_{\sigma(i)}$ is the ith largest element of $\Re_i$ and their aggregated value is still IVq-ROHF.*

Proof. It can be made as similarity to Theorem 4.2. IVq-ROHFCG provides following properties.

Theorem 4.8 *(idempotency) Let accept collection of $\Re_i = \{(\mu_i, \nu_i) : \mu_i = [\mu_i^-, \mu_i^+], \nu_i = [\nu_i^-, \nu_i^+]\}$ and $\Re_i = \Re$ for $(i = 1,2,\ldots,n)$. Thus, $IVq-ROHFCG(\Re_1, \Re_2, \ldots \Re_n) = \Re$.*

Theorem 4.9 *(Monotonicity) If $\Re_i > \Re_i^*$,*

$IVq-ROHFCG(\Re_1, \Re_2, \ldots, \Re_n) \leq IVq-ROHFCG(\Re_1^*, \Re_2^*, \ldots, \Re_n^*)$.

Theorem 4.10 *(Boundedness) Let accept collection of $\Re_i = \{(\mu_i, \nu_i) : \mu_i = [\mu_i^-, \mu_i^+], \nu_i = [\nu_i^-, \nu_i^+]\}$, in this statement,*

$\min\{\Re_1, \Re_2, \ldots, \Re_n\} \leq IVq-ROHFCG(\Re_1, \Re_2, \ldots, \Re_n) \leq \max\{\Re_1, \Re_2, \ldots, \Re_n\}$.

Now, we discuss some special cases of IVq-ROHFCG of follow.

- If $q = 1$, IVq-ROHFCG is reduced to Interval valued intuitionistic hesitant choquet integral geometric operator (IVIHCG).

- If $q = 2$, IVq-ROHFCG is reduced to Interval valued Pythagorean hesitant choquet integral geometric operator (IVPHCG).

- If $\mu(\{x_{\sigma(i)}\}) = \mu(A_\sigma(i)) - \mu(A_\sigma(i-1))$ for $(i = 1,2,\ldots,n)$, IVq-ROHFCG is reduced to Interval valued q-rung orthopair hesitant weighted geometric operator (IVq-ROHWG).

- If $\mu(A) = \sum_{i=1}^{|A|} w_i$ for all $A \in X$ where $|A|$ indicates number of elements in A and $w_i = \mu(A_{\sigma(i)}) - \mu(A_{\sigma(i-1)})$ for $(i = 1,2,\ldots,n)$ also $w = (w_1, w_2, \ldots, w_n)^T$ and $\sum_{i=1}^{n} w_i = 1$, in this statement IVq-ROHFCG is reduced to Interval valued q-rung orthopair hesitant ordered weighted geometric operator (IVq-ROHOWG).

5. AN APPLICATION OF MULTI-ATTRIBUTE DECISION-MAKING METHOD UNDER IVQ-ROHFS

In this section, we apply the presented Interval Valued q- Rung Orthopair Hesitant fuzzy averaging, geometric operators into an algorithm and test over a MCDM problem with n alternatives and m criteria to indicate effective of aggregating operators over IVq-ROHFS. Let $A = \{A_1, A_2, \ldots, A_m\}$ be a set of alternatives, $C = \{C_1, C_2, \ldots, C_n\}$ be a set of criterions. Then, the following steps have been defined for algorithm. Firstly, experts consist of a matrix according to their own ideas. Then, fuzzy measures are calculated by using Def. 3.4. and obtained values are converted through of $IVq - ROHFCA$. Then, score values are determined and ranked.

1. Define Decision making matrix as $(\mathfrak{R}_{ij})_{m\times n} = \{(\mu,\nu): \mu = [\mu_{ij}^-, \mu_{ij}^+], \nu = [\nu_{ij}^-, \nu_{ij}^+]\}$ for $i = 1,2,\ldots,m$

   and $j = 1,2,\ldots,n$,

2. Confirm Fuzzy measures for m attributes,

3. Calculate Interval Valued q- Rung Orthopair Hesitant fuzzy elements by utilizin $\mathfrak{R}_i = IVq - ROHFCA(\mathfrak{R}_{i1}, \mathfrak{R}_{i2}, \ldots, \mathfrak{R}_{in})$g or $\mathfrak{R}_i = IVq - ROHFCG(\mathfrak{R}_{i1}, \mathfrak{R}_{i2}, \ldots, \mathfrak{R}_{in})$ for $i = 1,2,\ldots,m$,

4. Consist of score values of IVq-ROHF elements,

5. Determine alternatives rankings in descending order.

Numerical example

In this section, we adopt an example over Interval valued intuitionistic hesitant fuzzy sets [43] and solve by utilizing the above defined averaging operator over IVq-ROHFS. In here, a committee of experts think to take a project manager and the committee determines three criterions $C_i(i = 1,2,3)$ as follow;

1. knowledge; Does he/she have previous experience or backround about this job?

2. reliability; The reliability is important for business because of nobody wants to work with a noreliability person.

3. demanding; The demanding is essential for businees. If not demanding into mood of worker, the businees losts its future

and four candidates $A_i(i = 1,2,3,4)$ apply for this position;

Decision makers determine decision making matrix according to their experiences, ideas as follow in Table 1.

Table 1*. Evaluations of alternatives made by decision makers*

	C_1	C_2
A_1	$\{\langle[0.7,0.9],[0.1,0.1]\rangle,\langle[0.6,0,8],[0.1,0.2]\rangle,$ $\langle[0.3,0,4],[0.6,0.6]\rangle\}$	$\{\langle[0.5,0.6],[0.2,0.3]\rangle,\langle[0.1,0,$ $[0.8,0.9]\rangle\}$
A_2	$\{\langle[0.2,0.3],[0.5,0.6]\rangle\}$	$\{\langle[0.1,0.3],[0.6,0.6]\rangle\}$
A_3	$\{\langle[0.3,0.5],[0.4,0.5]\rangle\}$	$\{\langle[0.8,0,9],[0.1,0.1]\rangle,\langle[0.5,0,$ $[0.1,0.2]\rangle\}$
A_4	$\{\langle[0.2,0,4],[0.3,0.5]\rangle,\langle[0.5,0,7],[0.1,0.2]\rangle\}$	$\{\langle[0.1,0,1],[0.7,0.9]\rangle\}$
	C_3	
	$\{\langle[0.8,0.9],[0.1,0.1]\rangle\}$	
	$\{\langle[0.3,0.4],[0.4,0.6]\rangle,\langle[0.5,0,6],[0.4,0.5]\rangle\}$	
	$\{\langle[0.2,0.3],[0.5,0.6]\rangle,\langle[0.1,0,3],[0.6,0.6]\rangle\}$	
	$\{\langle[0.1,0.3],[0.6,0.7]\rangle,\langle[0.2,0,2],[0.7,0.8]\rangle,$ $\langle[0.3,0,4],[0.6,0.6]\rangle\}$	

Determine fuzzy measures of n attributes. Suppose fuzzy measures as follow; $\mu(C_1)=0.4$, $\mu(C_2)=0.3$ and $\mu(C_3)=0.4$ in this statement $\lambda=0.5$ is obtained by Definition 3.5 and thus $\mu(C_1,C_2)=0.76$, $\mu(C_1,C_3)=0.88$, $\mu(C_2,C_3)=0.76$ and $\mu(C_1,C_2,C_3)=1$.

Calculate a new permutation with helping to score function for $q=1$ and reorder elements of decision making matrix, and aggregate based on IVq-ROHFCA as below;

$$\mathfrak{R}_1=IVq-ROHFCA(\mathfrak{R}_{11},\mathfrak{R}_{12},\mathfrak{R}_{13})$$
$$=\{\langle[0.7504,0.8941],[0.0895,0.0987]\rangle,\langle[0.7134,0.8524],[0.0895,0.1377]\rangle,\langle[0.6251,0.7499],$$
$$[0.2117,0.2333]\rangle,\langle[0.7126,0.8714],[0.1249,0.1285]\rangle,\langle[0.6700,0.8207],[0.1249,0.1792]\rangle,$$
$$\langle[0.5684,0.6962],[0.2952,0.3037]\rangle\}$$

$$\mathfrak{R}_2=IVq-ROHFCA(\mathfrak{R}_{21},\mathfrak{R}_{22},\mathfrak{R}_{23})$$
$$=\{\langle[0.2404,0.3694],[0.4396,0.5643]\rangle,\langle[0.3361,0.4638],[0.4396,0.5246]\rangle,\langle[0.1988,0.3293],$$

$[0.4806, 0.5643])\}$

$\Re_3 = IVq - ROHFCA(\Re_{31}, \Re_{32}, \Re_{33})$
$= \{\langle [0.5804, 0.7379], [0.2171, 0.2525] \rangle, \langle [0.3946, 0.5933], [0.2171, 0.3331] \rangle, \langle [0.5684, 0.7379],$
$[0.2268, 0.2525] \rangle, \langle [0.3773, 0.5933], [0.2268, 0.3331] \rangle, \langle [0.5684, 0.7379], [0.2268, 0.2620] \rangle,$
$\langle [0.3773, 0.5933], [0.2268, 0.3457] \rangle\}$

<
$\Re_4 = IVq - ROHFCA(\Re_{41}, \Re_{42}, \Re_{43})$
$= \{\langle [0.1522, 0.3302], [0.4437, 0.6226] \rangle, \langle [0.2975, 0.1783], [0.2859, 0.4315] \rangle, \langle [0.1988, 0.2858],$
$[0.4778, 0.6638] \rangle, \langle [0.3361, 0.4588], [0.3079, 0.4601] \rangle, \langle [0.2485, 0.3779], [0.4437, 0.5782] \rangle,$
$\langle [0.3773, 0.5286], [0.2859, 0.4008] \rangle\}$

If we utilize score function over IVq-ROHFS to aggregate attribute values, then the scores of alternatives are found;

$S(\Re_1) = 0.5756461$, $S(\Re_2) = -0.179201534$, $S(\Re_3) = 0.311660828$ and $S(\Re_4) = -0.136024305$. Thus, this means that ordering of candidates $A_1 \succ A_3 \succ A_4 \succ A_2$.

In here, we only give for $q = 1$ and Table 2 is obtained by using Def 3.3. score values and ranking alternatives for $q = 2,3,5,7,10,15$ as follow;

***Table 2**. Ranking alternatives of Score Function Values under IVq-ROHFCA*

different parameters	Score function	Ranking Alternatives
$q = 2$	$S(\Re_1) = 0.53925, S(\Re_2) = -0.15598,$ $S(\Re_3) = 0.28806, S(\Re_4) = -0.07662$	$A_1 \succ A_3 \succ A_4 \succ A_2$
$q = 3$	$S(\Re_1) = 0.44818, S(\Re_2) = -0.08682,$ $S(\Re_3) = 0.22676, S(\Re_4) = -0.04145$	$A_1 \succ A_3 \succ A_4 \succ A_2$
$q = 5$	$S(\Re_1) = 0.30643, S(\Re_2) = -0.02606,$ $S(\Re_3) = 0.13797, S(\Re_4) = -0.01008$	$A_1 \succ A_3 \succ A_4 \succ A_2$
$q = 7$	$S(\Re_1) = 0.21751, S(\Re_2) = -0.00731,$ $S(\Re_3) = 0{,}09052, S(\Re_4) = -0{,}00182$	$A_1 \succ A_3 \succ A_4 \succ A_2$
$q = 10$	$S(\Re_1) = 0.00968, S(\Re_2) = 0.00178,$ $S(\Re_3) = 0.01124, S(\Re_4) = 0.01192$	$A_4 \succ A_3 \succ A_1 \succ A_2$

If the table is surveyed, the best alternative is A_1 and the worst alternative is A_2 for all values of q out $q = 10$ under IVq-ROHFCA. For $q = 10$, the best alternative is A_4, if the worst alternative, it is agreement with the other results. This statement indicates that our proposed cluster and method are agreement, reality, flexible approach according to IVIHFCI because while IVIHFCI is being surveyed for $q = 1$, our proposed cluster proposes an approach changing according to need, prefer, requirement of decision makers by including different values.

6. COMPARATIVE AND DISCUSSION

In this section, we discuss the highlighting of IVq-ROHFCA in decision making. To do so, the proposed aggregating operator is compared with correlation coeeficient of IVPHFSs [47], IVPHFWA [44], IVPHFWG [44], GIVPHFWA [44] and GIVPHFWG [44]. Firstly, we change some values of above decision making matrix and redesign as follow;

If the above values are carefully surveyed, it is open that $0.8^2 + 0.7^2 > 1$ and thus the compared operators do not propose solution for the defined decision making matrix;

***Table 4**: Ranking alternatives of Score Function Values under IVq-ROHFCA*

Different methods	Score value	Ranking Alternatives
Zheng et al. [47]	Null*	not ordering**
IVPHFWA [44]	Null*	not ordering**
IVPHFWG [44]	Null*	not ordering**
GIVPHFWA [44]	Null*	not ordering**
GIVPHFWG [44]	Null*	not ordering**
IVq-ROHFCA ($q = 3$)	$S(\Re_1) = 0.46612$, $S(\Re_2) = -0.06397$, $S(\Re_3) = 0.23579$, $S(\Re_4) = -0{,}00737$	$A_1 \succ A_3 \succ A_4 \succ A_2$
IVq-ROHFCG ($q = 3$)	$S(\Re_1) = -0.04283$, $S(\Re_2) = -0.20775$, $S(\Re_3) = -0.05056$, $S(\Re_4) = -0.31569$	$A_1 \succ A_3 \succ A_2 \succ A_4$

7. CONCLUSION

The changeable constructions according to need, prefer, requirement have presented a possibility to give more objective decisions for decision makers. Therefore, q- Rung Orthopair fuzzy sets (q-ROFs) defined as generalized intuitionistic fuzzy sets have been produced by Yager [35]. In this paper, we propose interval valued q- rung orthopair hesitant fuzzy set (IVq-ROHFS) by combining interval hesitant fuzzy sets and q-ROFs with motivation of Interval valued pythagorean Hesitant fuzzy sets (IVPHFS). This cluster is essential with respect to including more data, flexible construction, presenting of several clusters into its own construction especially for MCDM problems. Then, we present some basic concepts of IVq-ROHFS like complement, union, intersection, addition, multiply, scalar multiplication, scalar power and a score function to determine large and small relationship between two IVq-ROHFes. After then, Interval valued q- rung orthopair hesitant fuzzy Choquet integral has been developed by combining the soft construction of IVq-ROHFS and choquet integral and this concept has been integrated with aggregation operators from here obtained some structures as called Interval valued q- rung orthopair hesitant fuzzy Choquet averaging operator (IVq-ROHCA) and Interval valued q- rung orthopair hesitant fuzzy Choquet geometric operator (IVq-ROHCG). In addition to, an example has been given adopted from Interval-valued intuitionistic hesitant fuzzy sets (IVIHFS) to indicate effective, realistic, flexible of method. Our results are agreement when compared with this method based on TOPSIS but have more advantages. The calculated values for $q = 1$ show that our approach is IVIHFS [43], for $q = 2$, it is IVPHFS [44], for $q = 3$, it is interval valued cubic hesitant fuzzy sets (IVCHFS). This statement eliminates to error margin because of comparative analysis in its own. Flexible structure changing according to the prefer, requirement, need of experts will provide to eliminate non-objective comments. Finally, we offer a comparative analysis and change some values in decision making matrix and compare with some papers over IVPHFS. Besides, this manuscript can be utilized to construct several basic constructions like Hamacher aggregation operators, Prioritized Aggregation Operators, Geometric hybrid operators, Power aggregation operator so on and also some the basic measures like Hamming, Housdorff, Euclidean, correlation coefficient, similarity measures.

Declaration of Ethical Code

In this study, we undertake that all the rules required to be followed within the scope of the "Higher Education Institutions Scientific Research and Publication Ethics Directive" are complied with, and that none of the actions stated under the heading "Actions Against Scientific Research and Publication Ethics" are not carried out.

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