Maltepe Journal of Mathematics
ISSN:2667-7660, URL:http://dergipark.org.tr/tr/pub/mjm
Volume IV Issue 1 (2022), Pages 1-8. Doi: https://doi.org/ 10.47087/mjm.1037337

SOME RESULTS FOR (s,m)-CONVEX FUNCTION IN THE SECOND SENSE

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Abstract. In this paper, it is given some properties for an (s,m)-convex function defined on $[0,d]$, $d>0$ in the first sense and the second sense with $m\in(0,1)$. Also, some integral inequalities are examined for any non positive (s,m)-convex function in the second sense with any measure space.

1. Introduction

Convex functions, like differentiable functions, have a important role in many fields of pure and applied mathematics. It connects concepts from topology, algebra, geometry and analysis, and is an important tool in optimization, mathematical programming and game theory [3].

In recent years, after Miheşan [14] defined (s,m)-convex functions in the first sense, several investigations have emerged resulting in applications in mathematics, as it can be seen in [1, 2, 12, 4, 5, 10, 7, 6, 8, 9, 13].

Definition 1.1. *A function $f:[0,d]\to\mathbb{R}$ is called an (s,m)-convex function in the first sense, where $(s,m)\in[0,1]$ and $d>0$, if for all $x,y\in[0,d]$ and $t\in[0,1]$*

$$f\left(tx+m\left(1-t\right)y\right)\le t^{s}f(x)+m(1-t^{s})f(y).$$

Moreover, Eftekhari [15] introduced (s,m)-convex functions in the second sense in 2014 as follows:

Definition 1.2. *$f:[0,d]\to\mathbb{R}$, $d>0$ is called to be an (s,m)- convex in the second sense function for some $(s,m)\in(0,1]^2$ if*

$$f\left(tx+m\left(1-t\right)y\right)\le t^{s}f\left(x\right)+m(1-t)^{s}f\left(y\right)$$

for any $x,y\in[0,d]$.

2020 *Mathematics Subject Classification.* Primary:26D15; Secondaries: 26D10; 39B62 .
Key words and phrases. (s,m)-convex function; Integral inequalities; Measure space.

Submitted on December 16th, 2021. Published on March 18th, 2022
Communicated by Hacer SENGUL KANDEMIR, and Canan Hazar GULEC.

Example 1.1. *Let* $s, m \in (0,1]$, $p \in [1,+\infty)$ *and* $f : [0,+\infty) \to \mathbb{R}$ *defined by* $f(x) = x^p + c$, $c \leq 0$, *then* f *is an* (s,m)*-convex function in the second sense. Indeed, for all* $x, y \in [0,+\infty)$, $t \in [0,1]$ *and* $(s,m) \in [0,1]$ *we have*

$$\begin{aligned} f(tx+m(1-t)y) &= (tx+m(1-t)y))^p + c \leq ty^p + m^p(1-t)y^p + c \\ &\leq t^s x^p + m(1-t)^s y^p + (t^s + m(1-t)^s)c \\ &\leq t^s f(x) + m(1-t)^s f(y). \end{aligned}$$

We note that if a nonnegative function is convex and starshaped, then it is an (s,m)-convex function in the second sense function for all $(s,m) \in (0,1]^2$. This function class is an extension of s-convex functions in the second sense that are $(s,1)$- convex functions in the second sense [12]. Dragomir and Fitzpatrick proved that a s-convex functions in the second sense f is Riemann integrable if $f(c) = 0$ for any point c in domain of the function f in [17]. Also, when f is Lebesgue integrable on $[a,b]$ they give the Hermite-Hadamard type inequality for a s-convex functions in the second sense f on $[a,b]$ as the following inequality

$$2^{s-1} f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a} \int_a^b f(x)dx \leq \frac{f(a)+f(b)}{s+1}.$$

However, there is not any result for integrability of (s,m)- convex functions in the second sense with $m \in (0,1)$, and so researchers like [18, 19, 20] have to stipulate integrability.

In this paper, we deal with some properties and some inequalities for (s,m)-convex functions in the second sense with $m \in (0,1)$.

2. **Some Properties**

Let's first recall the well known H. Lebesgue Theorem ([21] p.257).

Theorem 2.1 (H. Lebesgue)**.** *Let* f *be a real-valued increasing function on* $[a,b]$. *Then the derivative* f' *exists and is nonnegative in* $(a,b)\backslash E$ *where* E *is a null set in* $(\mathbb{R}, \mathfrak{M}_L, \mu_L)$ *contained in* (a,b). *Further more* f' *is* $\mathfrak{M}_L$ *measurable and* μ_L*-integrable on* $(a,b)\backslash E$ *with*

$$\int_{[a,b]} f' d\mu_L \leq f(b) - f(a).$$

Theorem 2.2. *If* $f : [0,d] \to \mathbb{R}$, $d > 0$ *is an* (s,m)*- convex in the first sense or the second sense function for* $m \in (0,1)$, *then the derivative* f' *exists and*

$$sf(x) \leq xf'(x)$$

is hold for all $x \in (a,b)\backslash E$, $[a,b] \subset (0,d]$ *where* E *is a null set in* $(\mathbb{R}, \mathfrak{M}_L, \mu_L)$ *contained in* (a,b).

Proof. Let $f : [0,d] \to \mathbb{R}$, $d > 0$ be an (s,m)- convex in the first or the second sense function for $m \in (0,1)$. In this case,

$$f(0) \leq mf(0),$$

and so it is obtained $f(0) \le 0$. Also for all $0 < x \le y$ we have

$$f(x) = f\left(\frac{x}{y}y + m\left(1-\frac{x}{y}\right)0\right) \le \left(\frac{x}{y}\right)^s f(y) + m\left(1-\left(\frac{x}{y}\right)^s\right)f(0) \le \left(\frac{x}{y}\right)^s f(y)$$

or

$$f(x) = f\left(\frac{x}{y}y + m\left(1-\frac{x}{y}\right)0\right) \le \left(\frac{x}{y}\right)^s f(y) + m\left(1-\frac{x}{y}\right)^s f(0) \le \left(\frac{x}{y}\right)^s f(y)$$

i.e., $\frac{f(x)}{x^s} \le \frac{f(y)}{y^s}$, $0 < x \le y \le d$. This means that the function $g(x) = \frac{f(x)}{x^s}$ is monotone increasing function on $[a,d]$, $a > 0$. Since the functions $h(x) = x^s$ and $g(x) = \frac{f(x)}{x^s}$ are differentiable, according to H. Lebesgue Theorem we gain that the derivative f' exists and

$$sf(x) \le xf'(x)$$

is satisfied for all $x \in (a,b)\backslash E$, $(a,b] \subset (0,d]$ where E is a null set in $(\mathbb{R}, \mathfrak{M}_L, \mu_L)$ contained in (a,b). □

Corollary 2.3. *If $f : [0,d] \to \mathbb{R}$, $d > 0$ is an (s,m)- convex in the first sense or the second sense function for $m \in (0,1)$, f is Riemann integrable on $[a,d]$, $a > 0$.*

Corollary 2.4. *If $f : [0,d] \to \mathbb{R}$, $d > 0$ is a nonnegative (s,m)- convex in the first sense or second sense function for $m \in (0,1)$, then f is continuous at the zero, $f(0) = 0$ and monotone increasing, and so Riemann integrable on $[0,d]$.*

Corollary 2.5. *If $f : [0,d] \to \mathbb{R}$, $d > 0$ is a nonnegative (s,m)- convex in the first sense or second sense function for $m \in (0,1)$ and is the derivative of a function on $(0,d)$, then f is continuous on $[0,d)$.*

Proof. This result is taken from the fact that the derivative function has points of discontinuity only if it has points of the second type discontinuity. □

Theorem 2.6. *Let $f : [0,d] \to \mathbb{R}$, $d > 0$ be a nonnegative (s,m)- convex in the first sense or second sense function for $m \in (0,1)$ and continuous on any subinterval $[0,c]$, $c \le d$. Then, the limit $\lim_{x\to 0} \frac{f(x)}{x^s}$ exists.*

Proof. Suppose that $f : [0,d] \to \mathbb{R}$, $d > 0$ be a nonnegative (s,m)- convex in the first sense or second sense function for $m \in (0,1)$ and continuous on any subinterval $[0,c]$, $0 < c \le d$. Therefore $g : [0,c] \to \mathbb{R}$ defined as $g(x) = x^{1-s} f(x)$ is continuous on $[0,c]$ and for all $n \in \mathbb{N}$ and all $x \in [0,c]$

$$g(\frac{1}{n}x) = \left(\frac{1}{n}x\right)^{1-s} f\left(\frac{1}{n}x\right) \le x^{1-s}\left(\frac{1}{n}\right)^{1-s}\left(\frac{1}{n}x\right)^s f(x) = \frac{1}{n}g(x)$$

is satisfied. According to Theorem 6 in [24], $g(x)$ is differentiable at $x = 0$. This means that the limit $\lim_{x\to 0} \frac{f(x)}{x^s}$ exists. □

Theorem 2.7. *If $f : [0,d] \to \mathbb{R}$, $d > 0$ is a negative valued (s,m)- convex in the first sense or the second sense function for $m \in (0,1)$, f is a starshaped function on $[0,d]$.*

Proof. Under the assumption of theorem, for all $x \in [0,d]$, $f(x) < 0$. Now, we suppose that the function is not starshaped. From here, there exist two point $x_0 \in [0,d]$ and $t_0 \in (0,1)$

$$t_0 f(x_0) < f(t_0 x_0).$$

Because f is an (s,m)- convex in the first sense and second sense function for $m \in (0,1)$,

$$t_0 f(x_0) < f(t_0 x_0) \leq t_0^s f(x_0)$$

is hold. However, since f is a negative valued function, for $t_0 \in (0,1)$

$$t_0^s < t_0$$

is obtained. This is a contradiction. Therefore, it is gained that the function f is starshaped on $[0,d]$. □

Corollary 2.8. *If $f:[0,d] \to \mathbb{R}$, $d>0$ is a negative valued (s,m)- convex in the first sense and second sense function for $m \in (0,1)$, there exists a point $c \in [0,d]$ such that $f\chi_{[0,c]}$ is a non positive starshaped function on $[0,c]$ and $f\chi_{[c,d]}$ is a nonnegative monotone increasing function on $[c,d]$, where χ_A is the characteristic function of the subset A of $\mathbb{R}$.*

3. Some Inequalities

Theorem 3.1. *Let $f:[0,d] \to \mathbb{R}$, $d>0$ be an (s,m)-convex function in the second sense and Riemann integrable on $[a,b]$, $0 \leq a \leq mb \leq b \leq d$. Then*

$$2^{s-1} f\left(m\frac{a+b}{2}\right) \leq \frac{m}{b-a}\int_a^b f(x)\,dx \leq m\frac{(b-ma)\,f(b)+(mb-a)\,f(a)}{(s+1)(b-a)}.$$

Proof. Because f is an (s,m)-convex function in the second sense, for all $x,y \in [a,b]$ we have

$$f\left(m\frac{x+y}{2}\right) \leq m\frac{f(x)+f(y)}{2^s}.$$

If $x = ta+(1-t)b$ and $y = tb+(1-t)a$ are chosen, then we get

$$f\left(m\frac{a+b}{2}\right) \leq \frac{m}{2^s}\left(f(ta+(1-t)b)+f(tb+(1-t)a)\right).$$

We obtain by integrating the last inequality

$$f\left(m\frac{a+b}{2}\right) \leq \frac{m}{2^{(s-1)}}\frac{1}{b-a}\int_a^b f(x)\,dx.$$

Since $a \leq mb$, and $f:[0,d] \to \mathbb{R}$, $d>0$ is an (s,m)-convex function in the second sense

$$\begin{aligned}
&\int_a^b f(y)\,dy = \int_a^{mb} f(x)\,dx + \int_{mb}^b f(x)\,dx \\
&= (mb-a)\int_0^1 f(ta+m(1-t)b)\,dt + (b-mb)\int_0^1 f(tb+m(1-t)b)\,dt \\
&\leq (mb-a)\int_0^1 \left(t^s f(a)+m(1-t)^s f(b)\right)dt + (b-mb)\int_0^1 \left(t^s+m(1-t)^s\right) f(b)\,dt \\
&= \tfrac{(b-ma)f(b)+(mb-a)f(a)}{s+1}
\end{aligned}$$

we have

$$f\left(m\frac{a+b}{2}\right) \leq m\frac{2^{1-s}}{b-a}\int_a^b f(x)\,dx \leq m2^{1-s}\frac{(b-ma)\,f(b)+(mb-a)\,f(a)}{(s+1)(b-a)}.$$

$\square$

Remark. *If we take $m=1$ and $s=1$ in Theorem 3.1, then*

$$f\left(\frac{a+b}{2}\right)\leq\frac{1}{b-a}\int_a^b f(x)\,dx\leq\frac{f(b)+f(a)}{2}$$

is famous Hermite-Hadamard inequality.

Corollary 3.2. *If $f:[0,d]\to\mathbb{R}$, $d>0$ is an (s,m)-convex function in the second sense and the derivative function f' is Riemann integrable on $[a,b]$, $0\leq a\leq mb\leq b\leq d$, then*

$$\int_a^b f(x)\,dx\leq\min\left\{\frac{bf(b)-af(a)}{s+1},\frac{(b-ma)\,f(b)+(mb-a)\,f(a)}{s+1}\right\}.$$

Theorem 3.3. *Let $f:[0,d]\to\mathbb{R}$ be a differentiable on $[0,d]$ and $|f'|$ is an (s,m)-convex function in the second sense in $[0,d]$ for $m\in(0,1)$, then for all $x\in[a,b]$, $[a,b]\subset[0,d]$*

$$\left|f(mx)-\frac{1}{b-a}\int_a^b f(y)\,dy\right|\leq\frac{|f'(b)|}{b-a}\left[\frac{(m(s+1)+1)\left((mx-a)^2+(b-mx)^2\right)}{(s+1)(s+2)}\right].$$

Proof. In this case, we use the equality given by Cerone and Dragomir in [22], and so

$$\begin{aligned}
&\left|f(mx)-\frac{1}{b-a}\int_a^b f(y)dy\right|\\
=&\left|\frac{(mx-a)^2}{b-a}\int_0^1 tf'(tmx+(1-t)\,a)dt-\frac{(b-mx)^2}{b-a}\int_0^1 tf'(tmx+(1-t)\,b)dt\right|\\
\leq&\frac{(mx-a)^2}{b-a}\int_0^1 t|f'(tmx+(1-t)\,a)|dt+\frac{(b-mx)^2}{b-a}\int_0^1 t|f'(tmx+(1-t)\,b)|dt\\
\leq&\frac{(mx-a)^2}{b-a}\int_0^1 t\,(mt^s+(1-t)^s)|f'(b)|dt+\frac{(b-mx)^2}{b-a}\int_0^1 t\,(mt^s+(1-t)^s)|f'(b)|dt\\
=&\frac{|f'(b)|}{b-a}\left[\frac{(m(s+1)+1)\left((mx-a)^2+(b-mx)^2\right)}{(s+1)(s+2)}\right]
\end{aligned}$$

is obtained. $\square$

Remark. *If it is chosen as $m=1$ in Theorem 3.3, it is obtained the inequality given Alomari et. al. in* [23].

Theorem 3.4. *If $f:[0,d]\to\mathbb{R}$, $d>0$ is an (s,m)-convex function in the second sense for any $m\in(0,1)$ then the following inequality is hold*

$$f\left(m\sum_{k=1}^{n}t_kx_k\right)\leqslant m\sum_{k=1}^{n}t_k^sf\left(x_k\right) \tag{3.1}$$

where $\sum_{k=1}^{n}t_k\leqslant 1,\ t_k\in[0,1]$ *and* $x_k\in[a,b]$.

Proof. It can be proved by using the mathematical induction method as in [25]. First of all, since $f:[0,d]\to\mathbb{R},\ d>0$ is an (s,m)-convex function in the second sense with $m\in(0,1)$ for $n=1$, $t\in[0,1]$ and $x\in[a,b]$

$$f(mtx)=f(mtx+(1-t)0)\leq(1-t)^sf(0)+mt^sf(x)\leq mt^sf(x).$$

Now, for the next step of induction we consider that the equation 3.1 is true for $n-1$. In this case, if $\sum_{k=1}^{n}t_k\leqslant 1$, then $\frac{t_k}{1-t_n}\leq 1$, $1\leq k\leq n-1$ is hold and

$$\begin{aligned}
f\left(m\sum_{k=1}^{n}t_kx_k\right) &= f\left(m(1-t_n)\sum_{k=1}^{n-1}\frac{t_k}{1-t_n}x_k+mt_nx_n\right)\\
&\leq (1-t_n)^sf\left(m\sum_{k=1}^{n-1}\frac{t_k}{1-t_n}x_k\right)+mt_n^sf(x_n)\\
&\leq m\sum_{k=1}^{n-1}t_k^sf(x_k)+mt_n^sf(x_n)=m\sum_{k=1}^{n}t_k^sf\left(x_k\right).
\end{aligned}$$

This conclusion completes the proof of the theorem. □

Theorem 3.5. *Suppose that (X,Σ,μ) is a finite measure space and $h:X\to[0,+\infty)$ is a μ-integrable function such that $h\left(x\right)\leqslant\frac{1}{\mu(X)}$ a.e. . If $f:[0,d]\to\mathbb{R}$, $d>0$ is a non positive continuous (s,m)-convex function in the second senses for any $m\in(0,1)$ and $g:X\to[0,d]$ is a μ-integrable function, then we have*

$$f\left(m\int_E h\left(x\right)g\left(x\right)d\mu\left(x\right)\right)\leqslant m\int_E h\left(x\right)f\left(g\left(x\right)\right)d\mu\left(x\right)$$

for any $E\in\Sigma$.

Proof. Let $I=\bigcup_{k=1}^{n}I_{n_k}$ be any partition of disjoint intervals I_{n_k} for $n\in\mathbb{N}$. Because g is an μ-integrable function, the set $E_{n_k}:=g^{-1}\left(I_{n_k}\right)\cap E$ is in Σ for any set $E\in\Sigma$ and $E=\bigcup_{k=1}^{n}E_{n_k}$. Choosing any point x_{n_k} in each set E_{n_k}. Since h is a positive valued function and $\int_X h\left(x\right)d\mu\left(x\right)\leqslant 1$, the linear combination

$$\sum_{k=1}^{n}\mu\left(E_{n_k}\right)h\left(x_{n_k}\right)g\left(x_{n_k}\right)$$

is in $[0,d]$ for large enough $n\in\mathbb{N}$. Because f is a non positive (s,m)-convex function for any $m\in(0,1)$ on $[0,d]$, the following inequality is satisfied by using the previous theorem

$$\begin{aligned} f\left(m\sum_{k=1}^{n}\mu\left(E_{n_k}\right)h\left(x_{n_k}\right)g\left(x_{n_k}\right)\right) &\leq m\sum_{k=1}^{n}\mu^s\left(E_{n_k}\right)h^s\left(x_{n_k}\right)f\left(g\left(x_{n_k}\right)\right) \\ &\leq m\sum_{k=1}^{n}\mu\left(E_{n_k}\right)h\left(x_{n_k}\right)f\left(g\left(x_{n_k}\right)\right) \end{aligned}$$

The proof of the theorem is completed under the continuity assumption of the function f. □

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