

MACHINING OF HYPOCYCLOIDAL SURFACES BY ADDING ROTATIONS AROUND PARALLEL AXES, PART II: GEOMETRY OF THE BODY OF THE TOOL

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Abstract: This article presents the geometry of the tool body, which implements a method of machining hypocycloidal internal and external polyhedral surfaces by adding rotations around parallel axes. Mathematical models of the major cutting angles when machining hypocycloidal surfaces by adding rotations around parallel axes have been developed. The models obtained allow the design of a serviceable cutting tool, securing the mechanical diagram of the method.

Key words: Cutting; cutting tools; hypocycloidal surfaces; machining methods; polygon shaft joints; tool angles.

Paralel eksenlere dönme ekleyerek hypocycloid yüzeylerin talaşlı işlenmesi: Bölüm 2: metodun kinematiği e mümkün olan uygulama alanları

Özet: Bu makalede paralel eksenlere dönme ilave ederek hypocycloid iç ve polihedral dış yüzeylerin talaşlı işlenmesinde kullanılan takım geometrisi sunuldu. Bu metod torna, freze ve matkap tezgahları ve dikey işleme merkezlerinde kullanılarak bu tezgahların imalat kapasitesini genişletir. Metodun kinematiği takım tasarımcılarının bakışı ve hypocycloid yüzeylerin paralel eksenlere dönme ilave edilerek talaşlı işlenmesinde temel kesme açılarının geliştirilmiş modelinin geliştirilmesi bakışı ile tanımlandı. Bu metodun uygulanabilir alanları belirlendi.

Anahtar kelimeler: Kesme, Kesme takımları, Hypocycloid yüzeyler, talaşlı işleme yöntemleri, poligon şaft bağlantıları, kesme açıları

Introduction

To perform effective machining by adding rotations around parallel axes and to provide size accuracy of hypocycloidal surfaces, it is necessary to employ a serviceable cutting tool securing the mechanical diagram of machining. The matching of these conditions establishes certain requirements to the tool design and first and foremost to its body geometry.

The geometric parameters determine the sharpness of the cutting wedges of the body and their position with respect to the vector of the relative cutting speed and predetermine the character of the current machining process. In this aspect and taking into account the kinematics of the method, when designing the tool, it is necessary to be able to determine the current values of the body geometric parameters, which appear to be variable along the cutting edges length in the operating system. The analytic determining of the geometric parameters in the operating system allows the determining of their critical values maintaining the machining under incessantly changing conditions and making correct decisions about the sizes of the static angles during design.

The objective of this study is to develop a mathematical model of the cutting tool angles for implementing the method of machining external and internal hypocycloidal surfaces by adding rotations around parallel axes.

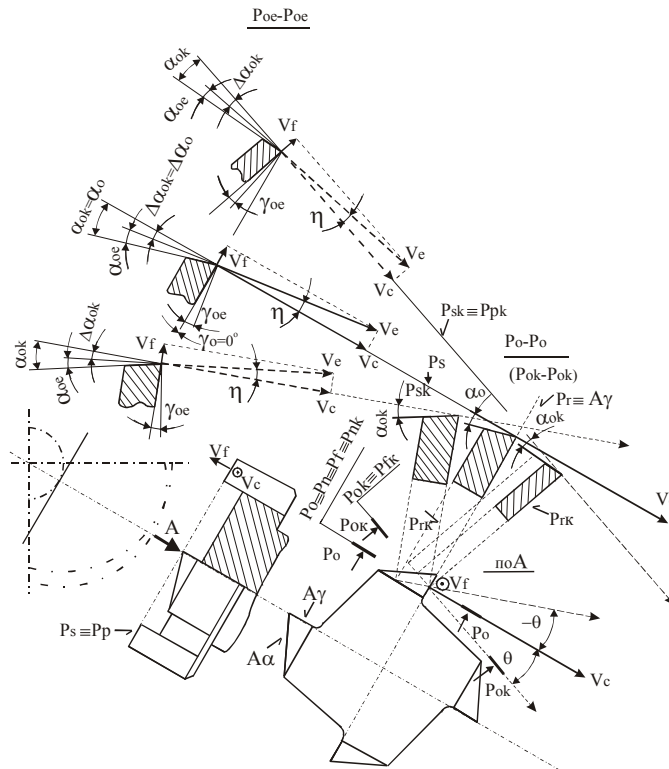


Figure 1. A diagram for determination of the major cutting angles of the front teeth

Tool body geometry

The geometry in the static and in the operating systems has been expounded for a cutting tool for internal hypocycloidal n -hedron with a simplified design diagram, shown in Fig. 1. It has $(n-1)$ cutting edges, situated on the vertices of a regular linear $(n-1)$ -gon. Rectilinear face (A_γ) is accepted, one and the same for the teeth on the front face and cylindrical surface and also rectilinear flanks (A_α) on these teeth, respectively the teeth are milled.

The position of the vectors of the primary (v_c) and feed (v_f) motions and the static coordinate planes of the front teeth $(P_r, P_s, P_0, P_f, P_p, P_n)$ are shown in the tool diagram (Fig. 1). The static major angles (γ_0) and (α_0) of the front teeth are shown in the section of the principal plane (P_0) .

In the cutting process, for a considered point from the cutting edge, the cutting speed vector changes its position (illustrated by a dotted line) depending on the function of the kinematic angle θ (Maximov and Hristov, 2004). In this aspect, it is accepted that the major angles formed as a result of the change in θ to be treated as “kinematic” with respective symbols $\gamma_{0,k}$ and $\alpha_{0,k}$. The latter are shown in the kinematic principal plane $P_{0,k}$ (in Fig. 1 for two boundary values of θ).

On the basis of the sections $P_0 - P_0$ and $P_{0,k} - P_{0,k}$ from the calculating diagram the current values of the designated as kinematic clearance angles are defined in a common case:

$$\alpha_{0,k} = \arctg(\tg \alpha_0 \cos \theta) \quad (1)$$

and as kinematic rakes

$$\gamma_{0,k} = \arctg(\operatorname{tg} \gamma_0 / \cos \theta) \quad (2)$$

where (Maximov and Hristov, 2004):

$$\theta = \frac{\pi}{2} - \frac{\varphi_e}{n-1} - \arctg \frac{-k \cos \varphi_e + \frac{1-2k}{2(n-1)} \cos \frac{\varphi_e}{n-1}}{k \sin \varphi_e + \frac{1-2k}{2(n-1)} \sin \frac{\varphi_e}{n-1}}$$

φ_e is the angle of the tool axis rotation around the hole axis; $k = e/D_n$; e is the distance between the tool and hole axes; D_n is the diameter of the circumference circumscribed around the hypocycloidal n -gon.

For the accepted case, because of design considerations, of one and the same face (A_γ) for the teeth on the front face and on the cylindrical surface, the static rake γ_0 on both surfaces is $\gamma_0 = 0$, hence:

$$\gamma_{0,k} = \gamma_0 = 0 \quad (2a)$$

The current values of the performance major rakes and clearance angles are determined in the sections of the performance principal plane $P_{0,e}$. Its position versus the performance coordinate planes $P_{r,e}$ and $P_{s,e}$ depends on the relative position of the latter ($P_{r,e} \perp P_{s,e}$), which is determined by the current directions of the cutting speed vector ($P_{r,e} \perp v_e$):

$$\vec{v}_e = \vec{v}_c + \vec{v}_f$$

where (Maximov and Hristov, 2004):

$$v_c = 2\pi n_e D_n F(\varphi_e) \quad (3)$$

is the current magnitude of the cutting speed in the primary motion of the tool,

$$F(\varphi_e) = \sqrt{\left[k \sin \varphi_e + \frac{1-2k}{2(n-1)} \sin \frac{\varphi_e}{n-1} \right]^2 + \left[-k \cos \varphi_e + \frac{1-2k}{2(n-1)} \cos \frac{\varphi_e}{n-1} \right]^2}$$

n_e in tr/min is the rotation frequency of the tool axis around the hole axis, D_n is in m;

$$v_f = 10^{-3} f n_e \quad (4)$$

is the feed motion speed, f is the feed per revolution of the tool axis around the hole axis, mm/tr.

The positions of the basic performance plane $P_{r,e}$ as regards the static P_r are determined by the current values of the kinematic angle η :

$$\eta = \arctg(v_f / v_c) \quad (5)$$

Due to the relative perpendicularity of the planes $P_{r,k}$ and $P_{s,k}$ (respectively $P_{r,e}$ and $P_{s,e}$), the current values of η account for the changes in the current values of the kinematic major angles:

$$\eta = |\Delta \gamma_{0,k}| = |\Delta \alpha_{0,k}| = \arctg(v_f / v_c) \quad (6)$$

The current values of the effective major clearance angles and rakes are determined by the formulae:

$$\alpha_{0,e} = \alpha_{0,k} - \Delta \alpha_{0,k} \quad (7)$$

$$\gamma_{0,e} = \gamma_{0,k} + \Delta \gamma_{0,k} \quad (8)$$

From (7), (8), (1) and (6):

$$\alpha_{0,e} = \operatorname{arctg} \frac{\operatorname{tg}\alpha_0 \cos\theta - \frac{v_f}{v_c}}{1 + \frac{v_f}{v_c} \operatorname{tg}\alpha_0 \cos\theta} \quad (9)$$

$$\gamma_{0,e} = \operatorname{arctg} \frac{\operatorname{tg}\gamma_0 \cos\theta + \frac{v_f}{v_c}}{1 - \frac{v_f}{v_c} \operatorname{tg}\gamma_0 \cos\theta} \quad (10)$$

For the cutting process, it is of paramount importance, that the performance major clearance angles to be positive. It follows from (9) that the condition for working with a positive clearance angle $\alpha_{0,e}$ is:

$$0 \leq \theta < \arccos \frac{v_f}{v_c \operatorname{tg}\alpha_0} \quad (11)$$

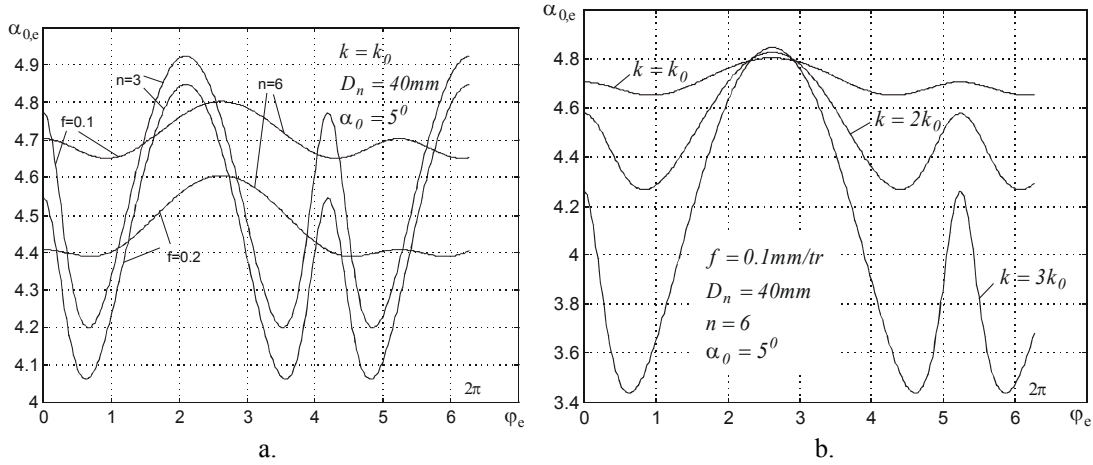


Figure 2. Dependence of $\alpha_{0,e}$ on φ_e : a. when $k = \text{const}$; b. when $n = \text{const}$

It is obvious from the speed v_c that the functions of the major angles with an argument φ_e (the angle of tool axis rotation around the hole axis) will be periodic functions with a period $\frac{2(n-1)\pi}{n}$. Figure 2 shows the dependence of $\alpha_{0,e}$ on φ_e . The amplitude of the functions decreases when the number of sides n of the hypocycloidal profile increases and when k decreases as well.

Similarly, making use of the calculating diagram from Fig. 3, the major angles for the teeth on the cylindrical surface are determined. From sections $P_{0,k} - P_{0,k}$ kinematic clearance angles are determined as a common case

$$\alpha_{0,k} = \alpha_0 + \Delta\alpha_{0,k} \quad (12)$$

and kinematic rakes

$$\gamma_{0,k} = \gamma_0 - \Delta\gamma_{0,k} \quad (13)$$

where:

$$|\Delta\gamma_{0,k}| = |\Delta\alpha_{0,k}| = |\theta|$$

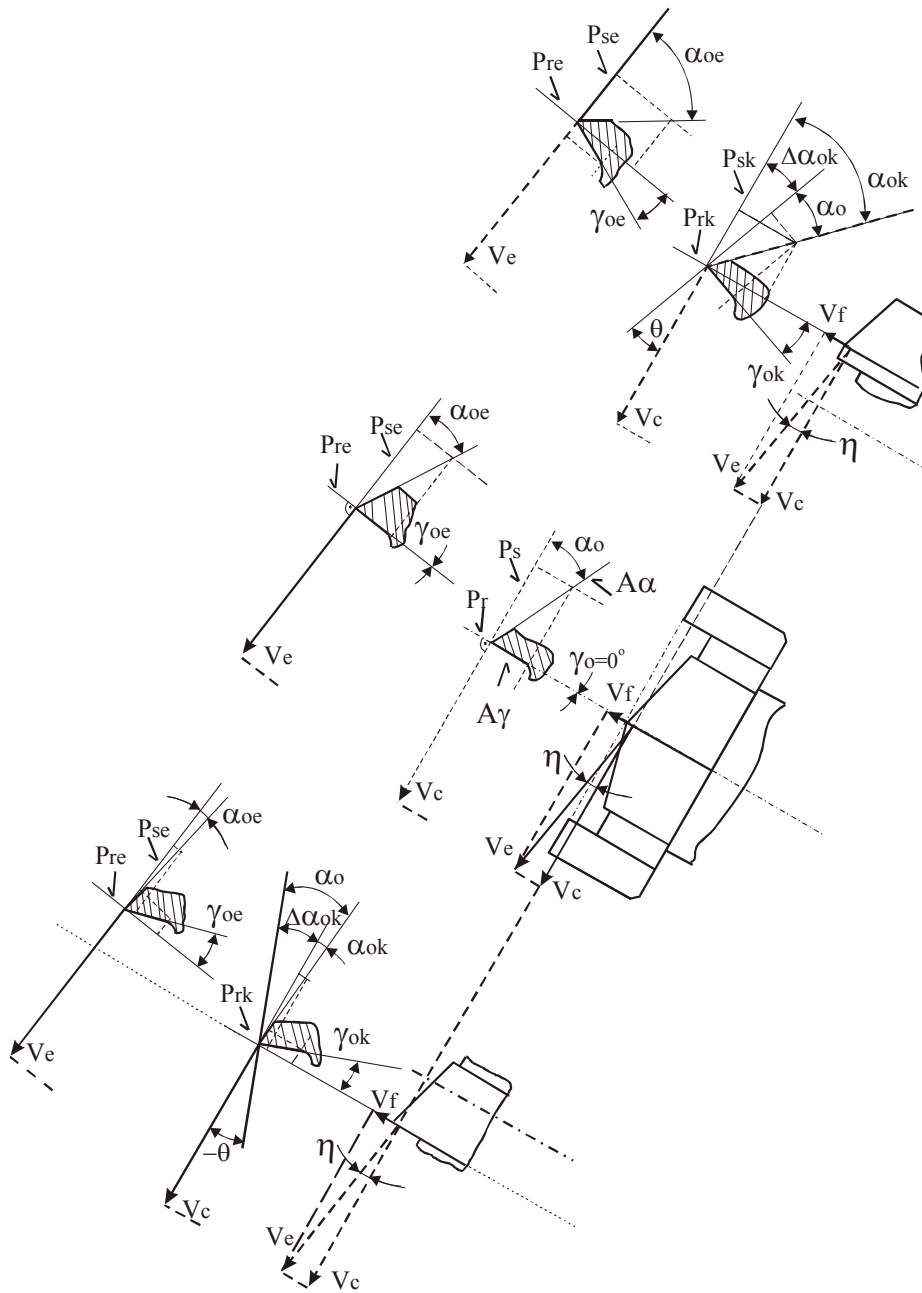


Figure 3. A diagram for determination of the major cutting angles of the teeth on the cylindrical surface

On the basis of sections $P_{0,e} - P_{0,e}$, the current values of the effective clearance angles and rakes are determined by the formulae:

$$\alpha_{0,e} = \arctg \left\{ \operatorname{tg} \alpha_{0,k} / \cos [\arctg (v_f / v_c)] \right\} \quad (14)$$

and

$$\gamma_{0,e} = \arctg \left\{ \operatorname{tg} \gamma_{0,k} \cos [\arctg (v_f / v_c)] \right\} \quad (15)$$

From (12)-(15) for the current values of the effective major clearance angles and rakes respectively it is obtained:

$$\alpha_{0,e} = \arctg \left\{ \frac{\operatorname{tg}(\alpha_0 + \theta)}{\cos[\arctg(v_f / v_c)]} \right\} \quad (16)$$

$$\gamma_{0,e} = \arctg \left\{ \frac{\operatorname{tg}(\gamma_0 - \theta)}{\cos[\arctg(v_f / v_c)]} \right\} \quad (17)$$

For this case, taking (2a) into account:

$$\gamma_{0,e} = \arctg \left\{ -\operatorname{tg}\theta \cos[\arctg(v_f / v_c)] \right\} \quad (17a)$$

It follows from (16) that the condition for working with positive major clearance angles is:

$$\alpha_0 > |\theta| \quad (18)$$

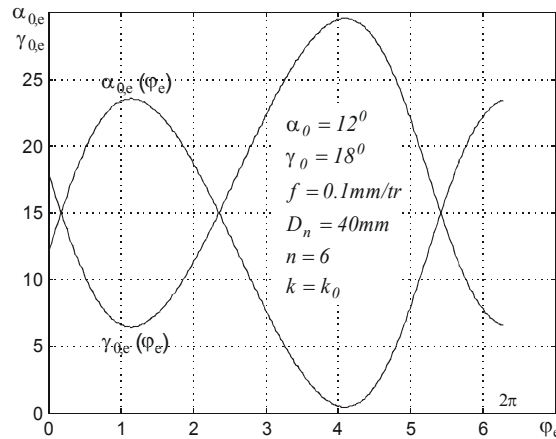


Figure 5. Dependence of $\alpha_{0,e}$ and $\gamma_{0,e}$ on φ_e of the teeth on the cylindrical surface

The impact of each parameter on $\alpha_{0,e}$ is as in the case of front teeth (relationship (9)). Figure 4 shows the dependence of $\alpha_{0,e}$ and $\gamma_{0,e}$ on φ_e of the teeth on the cylindrical surface. The major performance angles $\alpha_{0,e}$

and $\gamma_{0,e}$ are periodic functions with a period $\frac{2(n-1)\pi}{n}$.

Conclusions

Mathematical models of the major cutting angles when machining hypocycloidal surfaces by adding rotations around parallel axes have been developed. The models obtained allow the design of a serviceable cutting tool, securing the mechanical diagram of the method.

Since the clearance angles are especially significant of the cutting process, when setting minimally admissible values of the effective major angles, from (9) and (16) are determined the values of the static major clearance angles for the teeth on the front face and on the cylindrical surface required for making the tool.

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