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Araştırma Makalesi / Research Article

RESPONSE SPECTRA OF SEMI-RIGID SUPPORTED SINGLE STOREY FRAMES MODELED AS CONTINUOUS SYSTEM

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Abstract: The assumptions of frame supports being fully rigid and distributed masses and deformations of the columns being neglected are frequently taken into consideration in dynamic analysis of plane frames. In practice, however, column bases of frames may usually rotate and translate a little due to seismic effects. In this case, support behavior can be modeled using elastic springs at the column bases of frames. Besides, columns, in fact, are not massless, and have distributed mass and stiffness. In this study, the support of the frame modeled as Timoshenko column is modeled by elastic springs against rotation and translation. The generalized equation of motion is obtained by Lagrange equation using energy relations of the system, the dynaimc response is computed for different elastic spring coefficients using incremental Newmark-β method, and response spectra are obtained using ground accelerations of 1999 Izmit earthquake.
Key words: *response spectrum, single-storey frame, semi-rigid support, timoshenko model*

Sürekli Sistem Olarak Modellenen Yarı-Rijit Mesnetli Tek Katlı Çerçevelerin Tepki Spektrumları

Özet: Çerçevelerin dinamik analizinde, ankastre bağlı oldukları ve kolonlarının kütle ve deformasyonunun ihmal edildiği kabulleri sıkça yapılır. Ancak gerçekte, çerçeve kolonlarının temele bağlı noktaları sismik etkiler sebebiyle ötelenme ve dönme gösterebilmektedir. Bu durumda, mesnet davranışı elastik yaylarıyla modellenebilir. Ayrıca, gerçekte kolonlar yayılı kütleye ve rijitliğe sahiptir. Bu çalışmada, Timoshenko kolonu olarak modellenen çerçevenin mesnedi ötelenmeye ve dönmeye karşı elastik yaylar ile modellenmiştir. Genelleştirilmiş hareket denklemi sistemin enerji ifadeleri kullanılarak Lagrange denklemi ile elde edilmiş, Newmark-β metodu kullanılarak sistemin dinamik tepkisi hesaplanmış ve 1999 İzmit depremi yer ivmeleri kullanılarak tepki spektrumları elde edilmiştir.
Anahtar kelimeler: *tepki spektrumu, timoshenko modeli, tek katlı çerçeve, yarı-rijit mesnet*

Introduction

In dynamic analysis of single-storey frames, it is generally assumed that distributed mass of column is negligible and supports are fully rigid. These assumptions make dynamic analysis of mathematical calculation model easy.

Michaltsos and Ermopoulos (2001) studied free and forced vibration of the model in this study neglecting shear deformation and rotatory inertia. Glabisz (1999) studied vibration and stability of elastically supported continuous bars subjected to static loading. Güler (1996) searched the effects of soil flexibility on free vibration of tower-like structures using Euler model.

Dynamic analysis of semi-rigid supported framed systems modeled as discrete parameter in which deformations and distributed mass of the columns are neglected is also frequently studied by many researchers (Rodriguez & Montes, 2000; Bhattachaya & Dutta, 2004; Hjelmstad, 1998).

Behavior of the column bases of frames is more appropriate to semi-rigid support model. Dynamic mathematical model of semi-rigid supported single storey frame is presented in Fig. 1. Floor mass of the frame with its rotational inertia is concentrated at the top of elastic bar in the model, and base of the bar is supported by elastic springs against rotation and translation modeling the semi-rigid support behavior. The model is assumed to be subjected to Izmit earthquake. Generalized equation of motion is derived using energy relations of the model in Lagrange's equation, and dynamic response of the model is evaluated by incremental (non-iterative) Newmark-β method.

Theory

Differential equation of motion for Timoshenko column of the mathematical model in Fig. 1 for free vibration is as in follows

$$\frac{\partial^4 u}{\partial x^4}+\frac{N}{EI}\frac{\partial^2 u}{\partial x^2}-\left[\frac{mk}{AG}+\frac{mI}{EIA}\right]\frac{\partial^4 u}{\partial x^2 \partial t^2}+\frac{m}{EI}\frac{\partial^2 u}{\partial t^2}+\frac{m^2 Ik}{EIAAG}\frac{\partial^4 u}{\partial t^4}=0 \qquad (1)$$

where m, A, I, EI, AG and k are respectively, distributed mass, cross section area, moment of inertia, flexural stiffness, shear stiffness and constant of shear area of the column, N is axial compressive force. Method of separation of variables is applied using transformation given in Eq. (2) to solve differential Eq. (1).

$$u(x,t)=X(x)\cdot T(t)=X(x)\cdot \sin(\omega t) \qquad (2)$$

where ω is natural frequency, X(x) is shape function and T(t) is harmonic time function chosen to solve the equation of motion for free vibration and to obtain X(x) substituted into energy relations that will give the equation of motion for seismic vibration.

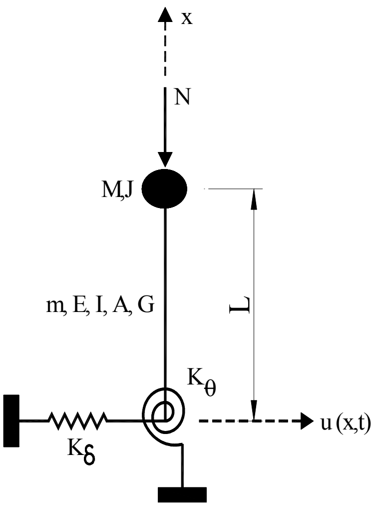

Figure 1. Dynamic mathematical model of semi-rigid supported single-storey frame.

Differentiating successively of Eq. (2) with respect to x and t and substituting in Eq. (1) gives

$$X^{IV}+\left[\frac{N}{EI}+\frac{mk\omega^2}{AG}+\frac{mI\omega^2}{EIA}\right]X^{II}-\left[\frac{m\omega^2}{EI}\right]X=0 \qquad (3)$$

Solving differential equation (3) gives the shape function X(x) of the continuous model used for the frame in Fig. 1 and substituting X(x) in Eq. (2) gives the displacement function of the model as

$$u(x,t)=\left[C_1 \sinh(m_1 x)+C_2 \cosh(m_1 x)+C_3 \sin(m_2 x)+C_4 \cos(m_2 x)\right]\sin(\omega t) \qquad (4)$$

where $m_1=\sqrt{n_1}$; $m_2=\sqrt{|n_2|}$; $n_{1,2}=\frac{-\alpha_5 \mp \sqrt{\Delta}}{2}$; $\Delta=\alpha_5^2+4\alpha_4$; $\alpha_4=\frac{m\omega^2}{EI}$;

$\alpha_5=\alpha_1+\alpha_2+\alpha_3$; $\alpha_3=\frac{mI\omega^2}{EI\cdot A}$; $\alpha_2=\frac{mk\omega^2}{AG}$; $\alpha_1=\frac{N}{EI}$; C_i are integration constants.

Shape function for the considered mode frequency obtained from the solution of Eq. (3) in terms of the integration constants C_1...C_4 is substituted into boundary conditions given in Eq. (5) to evaluate the integration constants for C_4=1, because the three integration constants are always calculated in terms of the fourth one (Chopra, 1995).

$$M(0)=-K_\theta \theta(0),\ V(0)=K_\delta u(0),\ M(L)=J\frac{\partial^2 \theta(L)}{\partial t^2},\ V(L)=-M\frac{\partial^2 u(L)}{\partial t^2} \qquad (5)$$

where M(0), M(L), V(0), V(L) $\theta(0)$, $\theta(L)$ are moment, shear and slope functions at x=0 and x=L respectively; K_θ and

K_δ are spring coefficients of rotation and translation, respectively; M and J are concentrated mass and its rotatory inertia, respectively.

Lagrange's equation of the undamped system is written as follows (Biggs, 1964).

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{Z}_n}\right)-\frac{\partial E_k}{\partial Z_n}+\frac{\partial E_p}{\partial Z_n}=\frac{\partial E_g}{\partial Z_n} \tag{6}$$

where Z(t) is the generalized coordinate of the nth mode; E_k, E_p and E_g are kinetic, potential and external load energies, respectively. Total kinetic and potential energies of the model in Fig. 1 are obtained respectively as (Karami et al, 2003)

$$E_k=\frac{1}{2}m\int_0^L\left(\frac{\partial u(x,t)}{\partial t}\right)^2dx+\frac{1}{2}m\frac{I}{A}\int_0^L\left(\frac{\partial \theta(x,t)}{\partial t}\right)^2dx+\frac{1}{2}M\left(\frac{\partial u(L,t)}{\partial t}\right)^2+\frac{1}{2}J\left(\frac{\partial \theta(L,t)}{\partial t}\right)^2 \tag{7}$$

$$E_p=\frac{1}{2}EI\int_0^L\left(\frac{\partial \theta(x,t)}{\partial x}\right)^2dx+\frac{1}{2}\frac{AG}{k}\int_0^L\left(\frac{\partial u(x,t)}{\partial x}-\theta(x,t)\right)^2dx-\frac{1}{2}N\int_0^L\left(\frac{\partial u(x,t)}{\partial x}\right)^2dx$$
$$+\frac{1}{2}K_\delta[u(0,t)]^2+\frac{1}{2}K_\theta[\theta(0,t)]^2 \tag{8}$$

where total external seismic load energy as

$$E_g=-m\ddot{u}_g(t)\int_0^L u(x,t)dx \tag{9}$$

Displacement function of a continuous system modeled as generalized single-degree-of-freedom system can be written as

$$u(x,t)=X(x)Z(t) \tag{10}$$

Using the transformation of Eq. (10) in the energy equations (7), (8), (9) and substituting them into Lagrange's equation (6) gives the equation of motion for the model in Fig. 1 as in the following

$$M_n^*\ddot{Z}(t)+K_n^*Z(t)=-F_n^*\ddot{u}_g(t) \tag{11}$$

where $M_n^*=m\int_0^L X^2dx+m\frac{I}{A}\int_0^L X^{I^2}dx+M[X(L)]^2+J\left[\alpha_6X^{III}(L)+\alpha_7X^{I}(L)\right]^2$;

$$K_n^*=EI\int_0^L\left(\alpha_6X^{IV}+\alpha_7X^{II}\right)^2dx+\frac{AG}{k}\int_0^L\left[(1-\alpha_7)X^{I}-\alpha_6X^{III}\right]^2dx$$
$$-N\int_0^L\left(X^{I}\right)^2dx+K_\delta[X(0)]^2+K_\theta\left[\alpha_6X^{III}(0)+\alpha_7X^{I}(0)\right]^2\ ;$$

$F_n^*=m\int_0^L Xdx$; $\alpha_6=\frac{EI\cdot k}{AG}$; $\alpha_7=\alpha_6\alpha_5+1$.

Since Eq. (11) is similar to the equation of motion of a discrete parameter sdof system, the following relation can be obtained if both sides are divided by M_n^*.

$$\ddot{Z}(t)+\omega_n^2Z(t)=-\Gamma_n\ddot{u}_g(t) \tag{12}$$

where $\omega_n^2=\frac{K_n^*}{M_n^*}$ (frequency); $\Gamma_n=\frac{F_n^*}{M_n^*}$.

Eq. (12) is evaluated for incremental displacement ΔZ_i by substituting Eq. (13) given for incremental acceleration into incremental equation (14) of motion, as given in Newmark-β method (Chopra,1995).

$$\Delta\ddot{Z}_i = \frac{1}{\beta(\Delta t)^2}\Delta Z_i - \frac{1}{\beta\Delta t}\dot{Z}_i - \frac{1}{2\beta}\ddot{Z}_i \tag{13}$$

$$\Delta\ddot{Z}_i + \omega^2\Delta Z_i = -\Gamma\Delta\ddot{u}_g \tag{14}$$

where β=1/6, Δt is incremental time. Once ΔZ_i is computed, incremental velocity $\Delta\dot{Z}_i$ and acceleration $\Delta\ddot{Z}_i$ are obtained from Eqs. (15) and (13) respectively, and Z_{i+1}, $\dot{Z}_{i+1}$, $\ddot{Z}_{i+1}$ from Eq. (16).

$$\Delta\dot{Z}_i = \frac{\gamma}{\beta\Delta t}\Delta Z_i - \frac{\gamma}{\beta}\dot{Z}_i + \Delta t\left(1-\frac{\gamma}{2\beta}\right)\ddot{Z}_i \tag{15}$$

$$Z_{i+1} = Z_i + \Delta Z_i \qquad \dot{Z}_{i+1} = \dot{Z}_i + \Delta\dot{Z}_i \qquad \ddot{Z}_{i+1} = \ddot{Z}_i + \Delta\ddot{Z}_i \tag{16}$$

Numerical Analysis

Response spectrum analysis of semi-rigid supported timoshenko column having a concentrated mass representing floor at the top are performed. Rotational inertia of the concentrated mass is also included in the dynamic analysis. Numerical data for the model of single-storey semi-rigid supported frame in Fig. 1 are given as follows:

Distributed mass of bar: m=1.7982 ts^2/m^2; Cross-section area of bar: A=2.45 m^2; Cross-sectional moment of inertia of bar: I=0.0625 m^4; Flexural stiffness of bar: EI=198750 tm^2; Shear stiffness of bar: AG=3116400 t; Shear area constant for rectangular cross-section: k=1.2; Length of bar: L=3 m; Concentrated mass: M=53.946 ts^2/m; Rotational inertia of concentrated mass: J=485.514 ts^2m/rad; Axial compression force: N=100 t; Coefficient of translational spring: K_δ=10000-100000-1000000 (t/m); Coefficient of rotational spring: K_θ=10000-100000-1000000 (tm/rad)

Acceleration-time history of 1999 Izmit earthquake is presented in Fig. 2.

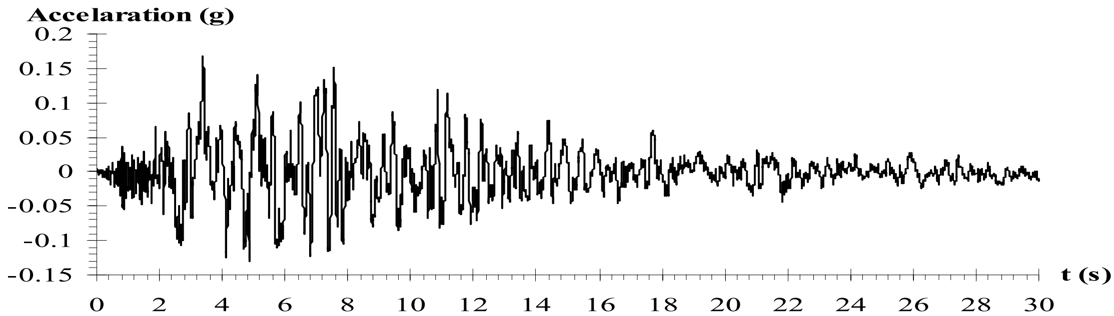

Figure 2. Acceleration-time history of 1999 Izmit earthquake

Response spectrum analysis of semi-rigid supported single-storey frames subjected to Izmit earthquake is made solving Eq. (12) by Newmark-β method for period values of 0-3 seconds using a computer program developed by the author. Displacement, velocity and acceleration response spectrum graphs are presented in respectively Figs. 3, 4 and 5 for translational and rotational spring coefficients of 10000 (t/m; tm/rad); in respectively Figs. 6, 7 and 8 for spring coefficients of 100000 (t/m; tm/rad); respectively Figs. 9, 10 and 11 for t spring coefficients of 1000000 (t/m; tm/rad).

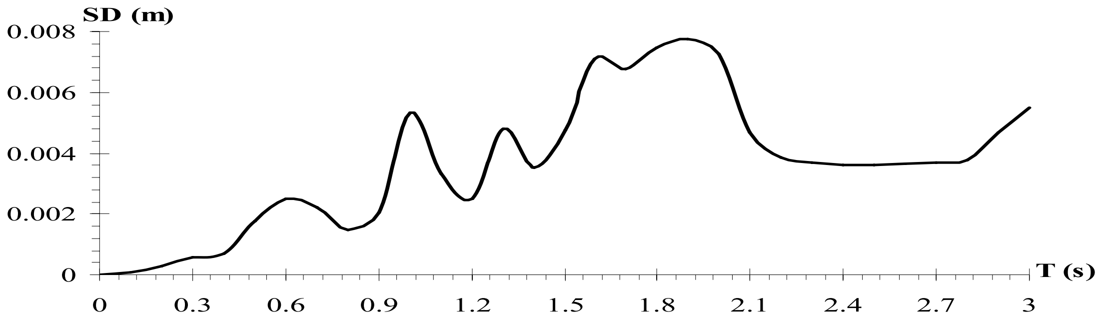

Figure 3. Displacement response spectrum for K_δ=10000 t/m, K_θ=10000 tm/radian

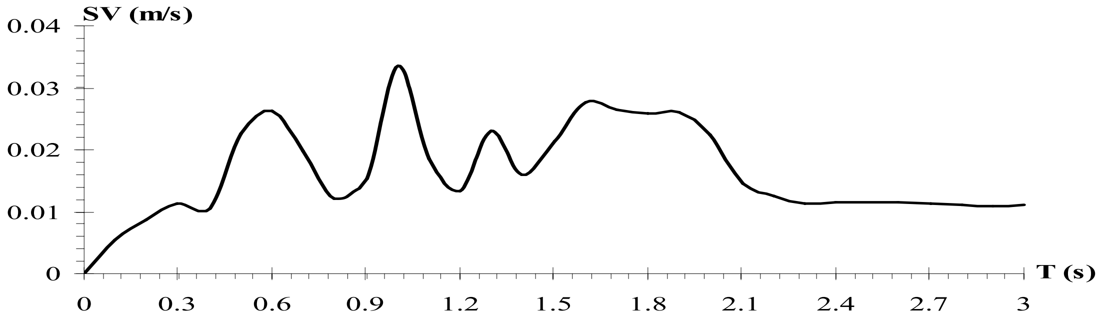

Figure 4. Velocity response spectrum for K_δ=10000 t/m, K_θ=10000 tm/radian

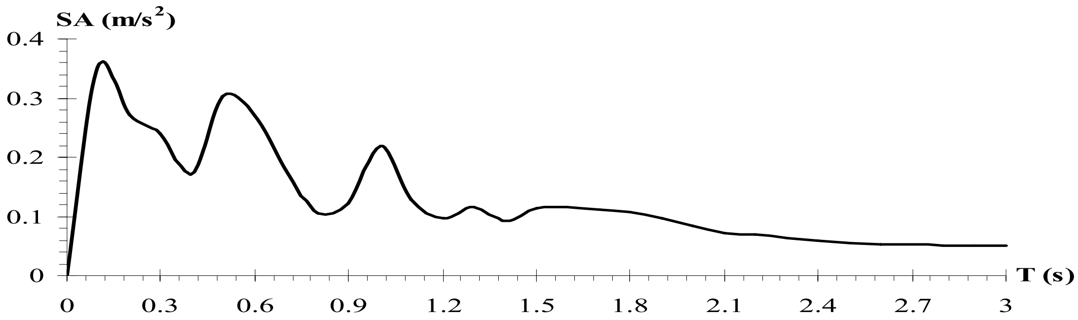

Figure 5. Acceleration response spectrum for K_δ=10000 t/m, K_θ=10000 tm/radian

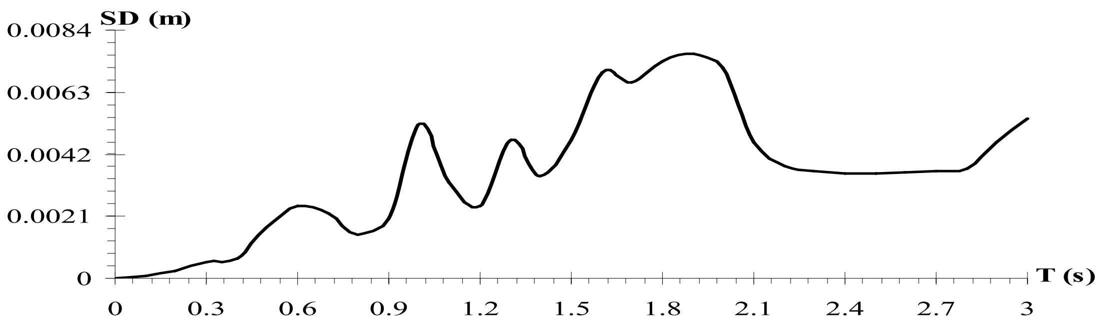

Figure 6. Displacement response spectrum for K_δ=100000 t/m, K_θ=100000 tm/radyan

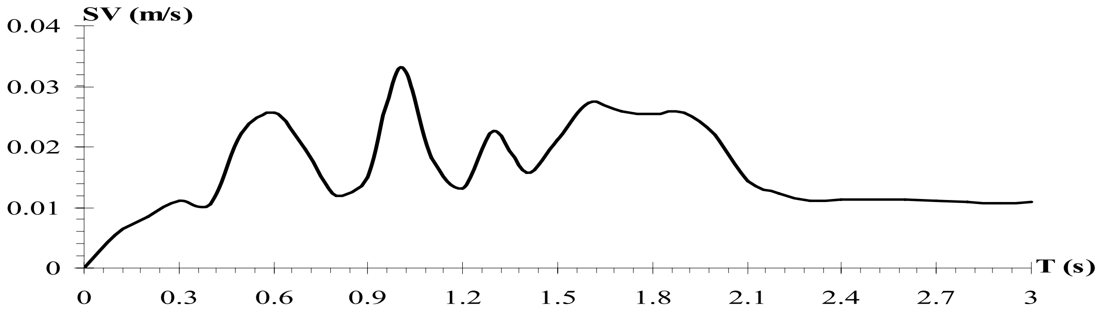

Figure 7. Velocity response spectrum for K_δ=100000 t/m, K_θ=100000 tm/radian

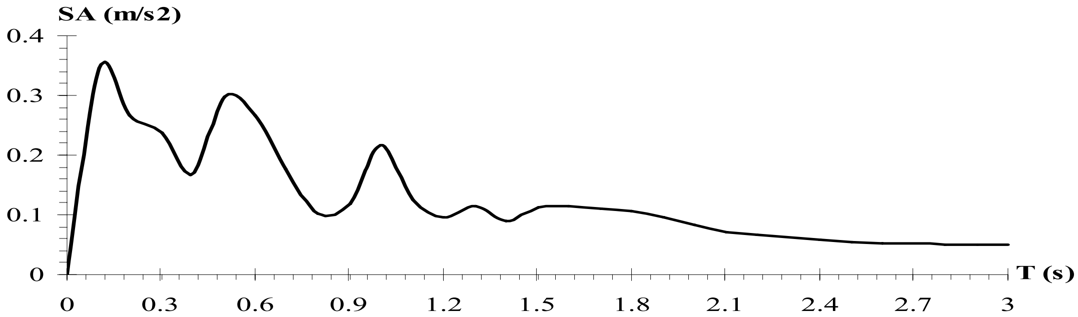

Figure 8. Acceleration response spectrum for K_δ=100000 t/m, K_θ=100000 tm/radian

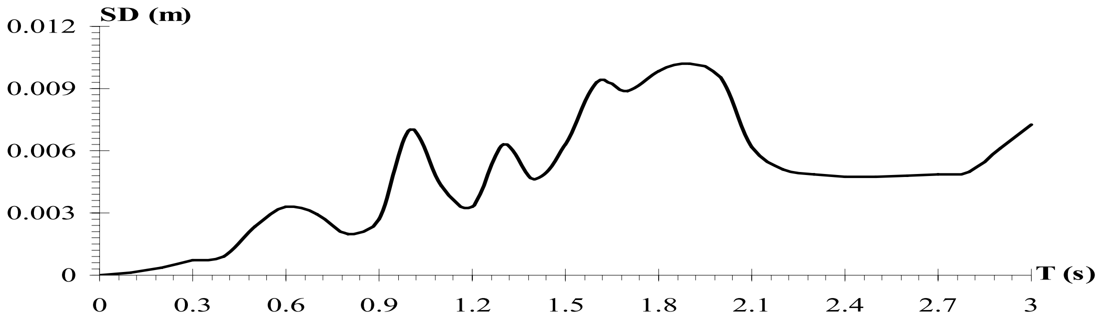

Figure 9. Displacement response spectrum for K_δ=1000000 t/m, K_θ=1000000 tm/radian

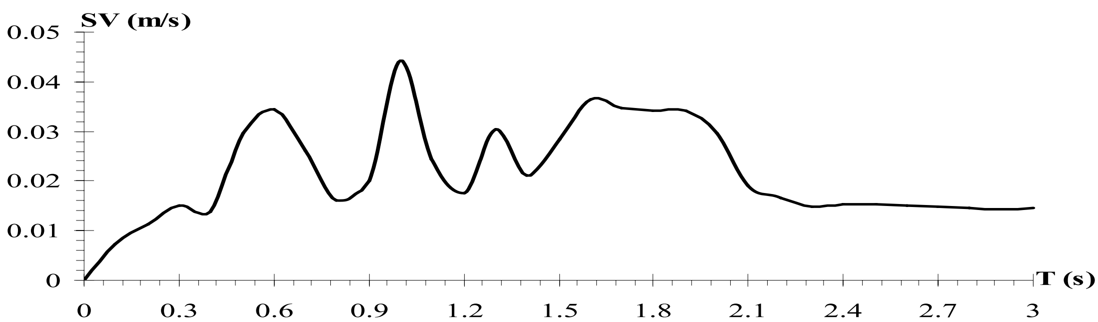

Figure 10. Velocity response spectrum for K_δ=1000000 t/m, K_θ=1000000 tm/radyan

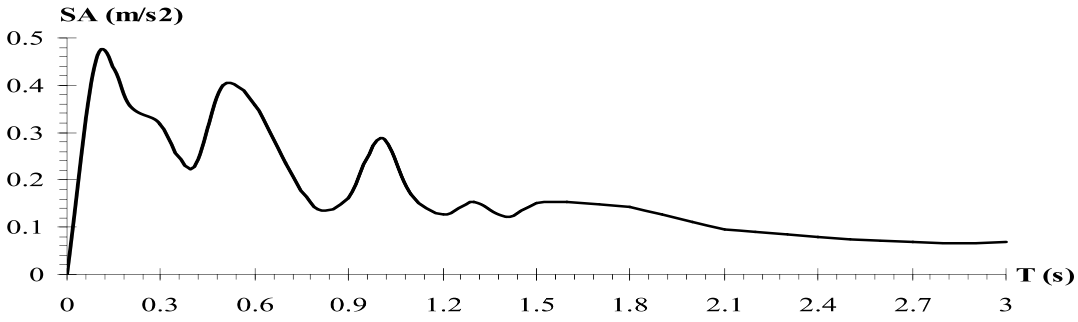

Figure 11. Acceleration response spectrum for K_{δ}=1000000 t/m, K_{θ}=1000000 tm/radyan

Discussion

The generalized mass and generalized seismic force values show a decrease if the spring coefficients are increased, whereas the generalized stiffness increases. It is observed in the response spectrum analysis of semi-rigid supported single storey frames having a period of 0-3 seconds performed for North-South component of 1999 Izmit earthquake ground accelerations that the displacement, velocity and acceleration response spectrum values show a decrease for translational and rotational spring coefficients of 10000 and 100000 (t/m; tm/rad) when the spring coefficients are increased.

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