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Correlation Coefficients of Fermatean Fuzzy Sets with a Medical Application

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Abstract

The FFS is an influential extension of the available IFS and PFS, whose benefit is to better exhaustively characterize ambiguous information. For FFSs, the correlation between them is usually evaluated by the correlation coefficient. To reflect the perspective of professionals, in this paper, a new correlation coefficient of FFSs is proposed and investigated. The correlation coefficient is very important and frequently used in every field from engineering to economics, from technology to science. In this paper, we propose a new correlation coefficient and weighted correlation coefficient formularization to evaluate the affair between two FFSs. A numerical example of diagnosis has been gotten to represent the efficiency of the presented approximation. Outcomes calculated by the presented approximation are compared with the available indices.

1. Introduction

In the studies in the literature, there are different applications such as aggregation operators and information measures to solve decision-making (DM) problems. Except that these solution methods, another method for choosing the best alternative is the correlation coefficients (KK), which are used to measure the level of dependency between two sets. KKs are used to measure how strong a relationship is between two variables. A KK is a bivariate statistic when it summarizes the relationship between two variables, and its a multivariate statistic when you have more than two variables. Therefore, there is a very wide field of study, from science to economics, from engineering to medicine. Although existing probabilistic methods have successful results, they also have limitations. For example, probabilistic techniques are in accordance with a mass collection of data, which is random, to acquire the necessary confidence level. However, on a great scale, the complex system has massive fuzzy ambiguity owing to which it is tough to acquire the complete possibility of the events. Hence, outcomes according to probability theory do not all the time ensure beneficial information to the professionals owing to the limitation of being able to operate only quantitative information. Furthermore, occasionally there is inadequate data to accurately operate the statistics of parameters, in real-world practices. As a natural consequence of these limitations, the outcomes in accordance with probability theory do not all the time ensure beneficial information to the professionals and then the probabilistic approximation is insufficient to account for such built-in uncertainties in the data. There are many possibilities to overcome these difficulties. One of the most successful results of these possibilities for handling uncertainties and impreciseness in DM is methods based on fuzzy set (FS) theory.

In [2], the correlation for fuzzy information according to the classic statistics is served and ensured a formula for KK of FSs. In [3] The KK of fuzzy information by utilization of a mathematical programming approximation according to the standard definition of KKs has been investigated. Based on the results were obtained through FS theory, more comprehensive and more accurate results were obtained with intuitionistic fuzzy set IFS [1]. The theory of IFS considers non-membership degree (ND) together with membership degree (MD) and requires that their sum be 1 or less than 1. The KKs derived based on IFS have been operated in numerous different fields as DM, cluster analysis, image processing, pattern recognition, etc [4]-[8]. Many DM problems related to PF information have entered the literature thanks to PFS [9, 10], [11]-[16], which was introduced to overcome the limitation in IFS.

Medical DM is a complicated condition that largely depends on the knowledge, experience, and judgment of the physician. For medical DM, it is not enough for the physician to follow the current disease process and current treatment alternatives. It also needs to be aware of

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other variables and use this information for medical DM. Such a situation for the physician makes it necessary to consider complex causal models that can be defined with uncertain, imprecise, and incomplete information. For such cases, decision-making mechanisms derived from theories such as FS, IFS, PFS, or Neutrosophic Space(NS) [17] appear as structures that offer powerful and comprehensive solutions [9, 10], [18]-[29].KKs derived according to the FS, IFS, and PFS theories are available in the literature [26]–[42].

The Fermatean fuzzy set (FFS)was initiated by Senepati and Yager [43]. In the FFS, the MD and ND accomplish the property $0 \leq m_A^3 + n_A^3 \leq 1$. The FFS, which is included in the literature as a new concept, gives better results than the IFS and PFS in defining uncertainties. For example $0.9+0.6>1$, $0.9^2+0.6^2>1$ and $0.9^3+0.6^3<1$. In [43], some properties, score and accuracy functions of FFSs are served. Further, the TOPSIS method, which is frequently used in Multi Criteria Decision Making(MCDM) problems, has been applied to FFS. In addition, Senepathy and Yager [43], the TOPSIS technique, which is continually utilized in MCDM problems, has been applied to FFS. As a continuation of this work, Senapati ve Yager [44] investigated several new operations, subtraction, division, and Fermatean arithmetic mean operations over FFSs and employed Fermatean fuzzy weighted product model to solve MCDM problems. In [45], new aggregation operators belonging to FFS have been defined, and properties related to these operators have been examined. In study of Donghai and et al [46], the notion of Fermatean fuzzy linguistic term sets is offered. Operations, score, and accuracy functions belonging to these sets were given. In [47], a new similarity measure related to Fermatean fuzzy linguistic term sets is constructed. The new measurement is a combination of Euclidean distance measure and cosine similarity measure. Kirisci [48] defined fermatean fuzzy soft sets and gave the measure of entropy based on fermatean fuzzy soft sets. In [49], a new hesitant fuzzy set which is called the fermatean hesitant fuzzy set has been given and investigated some properties. In [50], the ELECTRE I method is defined with Fermatean fuzzy sets according to the group DM process in which more than one individual interacts at the same time. In [27], a decision support algorithm in accordance with the Fermatean fuzzy soft set concept is presented to maximize the effectiveness of anti-virus masks. In [51], vairous fermatean fuzzy reference relations (consistent, incomplete, consistent incomplete, acceptable incomplete) have been defined. An additive consistency based on a priority vector has been given. In addition, a model is presented to obtain missing decisions in incomplete fermatean fuzzy preference relations.

It is the aim of this study to give the KK and weighted KK formulation to measure the relationship between two FFS. The effectiveness of the proposed technique will be demonstrated by giving a numerical example of a medical diagnosis. The results of this technique will be compared with previously known techniques.

2. Preliminaries

It will be regarded as the $\mathfrak{X} = \{x_1, x_2, \cdots, x_n\}$ initial universe throughout the work.

The Intuitionistic fuzzy set(IFS) in $\mathfrak{X}$ is defined:

$$N = \{(x, \zeta_N(x), \eta_N(x)) | x \in \mathfrak{X}\}.$$

In this definition, $\zeta_N(x), \eta_N(x) : \mathfrak{X} \to [0,1]$ is said to be MD and ND, with $\zeta_N(x) + \eta_N(x) \leq 1$.

The Pythagorean fuzzy set(PFS) is characterized as,

$$N = \{(x, \zeta_N(x), \eta_N(x)) | x \in \mathfrak{X}\},$$

if $\zeta_N(x), \eta_N(x) : \mathfrak{X} \to [0,1]$ are MD and ND of element of the $x \in \mathfrak{X}$, with $\zeta_N^2(x) + \eta_N^2(x) \leq 1$.

Fermatean fuzzy set (FFS) is given as

$$N = \{(x, \zeta_N(x), \eta_N(x)) | x \in \mathfrak{X}\},$$

if $\zeta_F(x), \eta_F(x) : \mathfrak{X} \to [0,1]$ are MD and ND of element of the $x \in \mathfrak{X}$, with $\zeta_N^3(x) + \eta_N^3(x) \leq 1$.

Principle of recognition is defined as: In discourse universe $\mathfrak{X}$, let it be assumed that there are m patterns defined by FFS $\mathfrak{N}_k$ $(k = 1, 2, \cdots, m)$. Again, let's suppose that there is an model to be identified with FFS $\mathfrak{P}$ in $\mathfrak{X}$.
The relationship index degree between FFSs $\mathfrak{N}_k$ and $\mathfrak{P}$ is described as

$$R(\mathfrak{N}_{k0}, \mathfrak{P}) = \max_{1 \leq k \leq m} \{R(\mathfrak{N}_k, \mathfrak{P})\}.$$

In this case, it is decided that sample $\mathfrak{P}$ belongs to $\mathfrak{N}_{k0}$.

The set

$$IE(N) = \sum_{i=1}^{n} \left[\zeta_N^2(x_i) + \eta_N^2(x_i)\right]$$

is called informational intuitionistic energy of two IFS N. Hence, the correlation and KK of IFSs can be given as

$$
\begin{aligned}
C_I(N,M) &= \sum_{i=1}^{n}\left[\zeta_N(x_i).\zeta_M(x_i)+\eta_N(x_i).\eta_M(x_i)\right]\\
\mathfrak{C}_I(N,M) &= \frac{C_I(N,m)}{\sqrt{IE(N).IE(M)}}\\
&= \frac{\sum_{i=1}^{n}\left[\zeta_N(x_i).\zeta_M(x_i)+\eta_N(x_i).\eta_M(x_i)\right]}{\sqrt{\sum_{i=1}^{n}\left[\zeta_N^2(x_i)+\eta_N^2(x_i)\right]}.\sqrt{\sum_{i=1}^{n}\left[\zeta_M^2(x_i)+\eta_M^2(x_i)\right]}}\\
\mathfrak{D}_I(N,M) &= \frac{C_I(N,m)}{\max\{IE(N).IE(M)\}}\\
&= \frac{\sum_{i=1}^{n}\left[\zeta_N(x_i).\zeta_M(x_i)+\eta_N(x_i).\eta_M(x_i)\right]}{\max\left[\sum_{i=1}^{n}\left[\zeta_N^2(x_i)+\eta_N^2(x_i)\right].\sum_{i=1}^{n}\left[\zeta_M^2(x_i)+\eta_M^2(x_i)\right]\right]}
\end{aligned}
$$

The set

$$
IE_P(N)=\sum_{i=1}^{n}\left[\zeta_N^4(x_i)+\eta_N^4(x_i)+\theta_N^4(x_i)\right]
$$

is called informational energies of PFS N [37].

For FFSs N and M, correlation and KK are defined as:

$$
\begin{aligned}
C_P(N,M) &= \sum_{i=1}^{n}\left[\zeta_N^2(x_i).\zeta_M^2(x_i)+\eta_N^2(x_i).\eta_M^2(x_i)+\theta_N^2(x_i).\theta_M^2(x_i)\right]\\
\mathfrak{C}_P(N,M) &= \frac{C_I(N,M)}{\sqrt{IE(N).IE(M)}}\\
&= \frac{\sum_{i=1}^{n}\left[\zeta_N^2(x_i).\zeta_M^2(x_i)+\eta_N^2(x_i).\eta_M^2(x_i)+\theta_N^2(x_i).\theta_M^2(x_i)\right]}{\sqrt{\sum_{i=1}^{n}\left[\zeta_N^4(x_i)+\eta_N^4(x_i)+\theta_N^4(x_i)\right]}.\sqrt{\sum_{i=1}^{n}\left[\zeta_M^4(x_i)+\eta_M^4(x_i)+\theta_M^4(x_i)\right]}}\\
\mathfrak{D}_P(N,M) &= \frac{C_I(N,M)}{\max\{IE(N).IE(M)\}}\\
&= \frac{\sum_{i=1}^{n}\left[\zeta_N(x_i).\zeta_M(x_i)+\eta_N(x_i).\eta_M(x_i)+\theta_N^2(x_i).\theta_M^2(x_i)\right]}{\max\left[\sum_{i=1}^{n}\left[\zeta_N^4(x_i)+\eta_N^4(x_i)+\theta_N^4(x_i)\right].\sum_{i=1}^{n}\left[\zeta_{MB}^4(x_i)+\eta_M^4(x_i)+\theta_M^4(x_i)\right]\right]}.
\end{aligned}
$$

3. New Correlation Coefficients

Let $\mathfrak{N}=\{x_i,\zeta_{\mathfrak{N}}(x_i,\eta_{\mathfrak{N}}(x_i)|x_i\in\mathfrak{X}\}$ be a FFS, where $\zeta_{\mathfrak{N}}(x_i),\eta_{\mathfrak{N}}(x_i)\in[0,1]$ and $\zeta_{\mathfrak{N}}^3(x_i+\eta_{\mathfrak{N}}^3(x_i)|x_i\leq 1$ for each $x_i\in\mathfrak{X}$, we define the informational energy of the FFS $\mathfrak{N}$ as

$$
IE(\mathfrak{N})=\sum_{i=1}^{n}\left(\zeta_{\mathfrak{N}}^6(x_i)+\eta_{\mathfrak{N}}^6(x_i)+\theta_{\mathfrak{N}}^6(x_i)\right). \tag{3.1}
$$

Suppose that two FFV's $\mathfrak{N}=\{x_i,\zeta_{\mathfrak{N}}(x_i,\eta_{\mathfrak{N}}(x_i)|x_i\in\mathfrak{X}\}$ and $\mathfrak{M}=\{x_i,\zeta_{\mathfrak{M}}(x_i,\eta_{\mathfrak{M}}(x_i)|x_i\in\mathfrak{X}\}$ in X, where $\zeta_{\mathfrak{N}}(x_i,\eta_{\mathfrak{N}}(x_i),\zeta_{\mathfrak{N}}(x_i,\eta_{\mathfrak{N}}(x_i)\in[0,1]$ for each $x_i\in\mathfrak{X}$. Hence, the correlation of the FFVs $\mathfrak{N}$, $\mathfrak{M}$ is defined:

$$
C(\mathfrak{N},\mathfrak{M})=\sum_{i=1}^{n}\left(\zeta_{\mathfrak{N}}^3(x_i)\zeta_{\mathfrak{M}}^3(x_i)+\eta_{\mathfrak{N}}^3(x_i)\eta_{\mathfrak{M}}^3(x_i)+\theta_{\mathfrak{N}}^3(x_i)\theta_{\mathfrak{M}}^3(x_i)\right).
$$

For the correlation of FFSs, the conditions
(1) $C(\mathfrak{N},\mathfrak{N})=IE(\mathfrak{N})$
(2) $C(\mathfrak{N},\mathfrak{M})=C(\mathfrak{M},\mathfrak{N})$
are hold.

Definition 3.1. *Choose two FFSs $\mathfrak{N}$ and $\mathfrak{M}$ on X. Then the KK between $\mathfrak{N},\mathfrak{M}$ is defined by*

$$
\begin{aligned}
\mathfrak{C}(\mathfrak{N},\mathfrak{M}) &= \frac{C(\mathfrak{N},\mathfrak{M})}{[IE(\mathfrak{N}).IE(\mathfrak{M})]^{1/2}} \qquad (3.2)\\
&= \frac{\sum_{i=1}^{n}\left(\zeta_{\mathfrak{N}}^3(x_i)\zeta_{\mathfrak{M}}^3(x_i)+\eta_{\mathfrak{N}}^3(x_i)\eta_{\mathfrak{M}}^3(x_i)+\theta_{\mathfrak{N}}^3(x_i)\theta_{\mathfrak{M}}^3(x_i)\right)}{\sqrt{\sum_{i=1}^{n}\left(\zeta_{\mathfrak{N}}^6(x_i)+\eta_{\mathfrak{N}}^6(x_i)+\theta_{\mathfrak{N}}^6(x_i)\right)}.\sqrt{\sum_{i=1}^{n}\left(\zeta_{\mathfrak{M}}^6(x_i)+\eta_{\mathfrak{M}}^6(x_i)+\theta_{\mathfrak{M}}^6(x_i)\right)}}
\end{aligned}
$$

Theorem 3.2. *For any two FFSs $\mathfrak{N},\mathfrak{M}$ in X, the KK of FFSs satisfies the following conditions:*

(P1) $\mathfrak{C}(\mathfrak{N},\mathfrak{M})=\mathfrak{C}(\mathfrak{M},\mathfrak{N})$,
(P2) If $\mathfrak{N}=\mathfrak{M}$*, then* $\mathfrak{C}(\mathfrak{N},\mathfrak{M})=1$,

(P3) $0 \leq \mathfrak{C}(\mathfrak{N},\mathfrak{M}) \leq 1$.

Proof. We only proved condition (P2). Obviously, $\mathfrak{C}(\mathfrak{N},\mathfrak{M}) \geq 0$.

$$\begin{aligned} C(\mathfrak{N},\mathfrak{M}) &= \sum_{i=1}^{n}\left(\zeta_{\mathfrak{N}}^{3}(x_i)\zeta_{\mathfrak{M}}^{3}(x_i)+\eta_{\mathfrak{N}}^{3}(x_i)\eta_{\mathfrak{M}}^{3}(x_i)+\theta_{\mathfrak{N}}^{3}(x_i)\theta_{\mathfrak{M}}^{3}(x_i)\right)\\ &= \left(\zeta_{\mathfrak{N}}^{3}(x_1)\zeta_{\mathfrak{M}}^{3}(x_1)+\eta_{\mathfrak{N}}^{3}(x_1)\eta_{\mathfrak{M}}^{3}(x_1)+\theta_{\mathfrak{N}}^{3}(x_1)\theta_{\mathfrak{M}}^{3}(x_1)\right)\\ &+ \left(\zeta_{\mathfrak{N}}^{3}(x_2)\zeta_{\mathfrak{M}}^{3}(x_2)+\eta_{\mathfrak{N}}^{3}(x_2)\eta_{\mathfrak{M}}^{3}(x_2)+\theta_{\mathfrak{N}}^{3}(x_2)\theta_{\mathfrak{M}}^{3}(x_2)\right)\\ &+ \cdots+\left(\zeta_{\mathfrak{N}}^{3}(x_n)\zeta_{\mathfrak{M}}^{3}(x_n)+\eta_{\mathfrak{N}}^{3}(x_n)\eta_{\mathfrak{M}}^{3}(x_n)+\theta_{\mathfrak{N}}^{3}(x_n)\theta_{\mathfrak{M}}^{3}(x_n)\right). \end{aligned}$$

Using the Cauchyâ€“Schwarz inequality, for $(\zeta_1+\cdots+\zeta_n)\in R^n$ and $(\eta_1+\cdots+\eta_n)\in R^n$,

$$(\zeta_1\eta_1+\zeta_2\eta_2+\cdots+\zeta_n\eta_n)^2 \leq \left(\zeta_1^2+\cdots+\zeta_n^2\right).\left(\eta_1^2+\cdots+\eta_n^2\right).$$

Then,

$$\begin{aligned} \Big[C(\mathfrak{N},\mathfrak{M})\Big]^2 &\leq \Big[(\zeta_{\mathfrak{N}}^{6}(x_1)+\eta_{\mathfrak{N}}^{6}(x_1)+\theta_{\mathfrak{N}}^{6}(x_1))+(\zeta_{\mathfrak{N}}^{6}(x_2)+\eta_{\mathfrak{N}}^{6}(x_2)+\theta_{\mathfrak{N}}^{6}(x_2))\\ &+ \cdots+(\zeta_{\mathfrak{N}}^{6}(x_n)+\eta_{\mathfrak{N}}^{6}(x_n)+\theta_{\mathfrak{N}}^{6}(x_n))\Big]\times\Big[(\zeta_{\mathfrak{M}}^{6}(x_1)+\eta_{\mathfrak{M}}^{6}(x_1)+\theta_{\mathfrak{M}}^{6}(x_1))+(\zeta_{\mathfrak{M}}^{6}(x_2)+\eta_{\mathfrak{M}}^{6}(x_2)+\theta_{\mathfrak{M}}^{6}(x_2))\\ &+ \cdots+(\zeta_{\mathfrak{M}}^{6}(x_n)+\eta_{\mathfrak{M}}^{6}(x_n)+\theta_{\mathfrak{M}}^{6}(x_n))\Big]\\ &= \sum_{i=1}^{n}(\zeta_{\mathfrak{N}}^{6}(x_i)+\eta_{\mathfrak{N}}^{6}(x_i)+\theta_{\mathfrak{N}}^{6}(x_i))\times\sum_{i=1}^{n}(\zeta_{\mathfrak{M}}^{6}(x_i)+\eta_{\mathfrak{M}}^{6}(x_i)+\theta_{\mathfrak{M}}^{6}(x_i))\\ &= IE(\mathfrak{N}).IE(\mathfrak{M}). \end{aligned}$$

Therefore, $[C(\mathfrak{N},\mathfrak{M})]^3 \leq IE(\mathfrak{N}).IE(\mathfrak{M})$ and $\mathfrak{C}(\mathfrak{N},\mathfrak{M}) \leq 1$. □

Example 3.3. *Take two FFSs* $\mathfrak{N}=\{(x_1,0.8,0.6),(x_2,0.5,0.9),(x_3,0.6,0.6)\}$ *and* $\mathfrak{M}=\{(x_1,0.6,0.7),(x_2,0.8,0.6),(x_3,0.7,0.5)\}$ *in* $\mathfrak{X}$. *By Equation 3.1, the informational energies of* $\mathfrak{N}$ *and* $\mathfrak{M}$*:*

$$\begin{aligned} IE(\mathfrak{N}) &= \sum_{i=1}^{n}\left(\zeta_{\mathfrak{N}}^{6}(x_i)+\eta_{\mathfrak{N}}^{6}(x_i)+\theta_{\mathfrak{N}}^{6}(x_i)\right)\\ &= (0.8^6+0.6^6+0.64^2)+(0.5^6+0.9^6+0.527^2)+(0.6^6+0.6^6+0.828^2)\\ &= 2.3221 \end{aligned}$$

$$\begin{aligned} IE(\mathfrak{M}) &= \sum_{i=1}^{n}\left(\zeta_{\mathfrak{N}}^{6}(x_i)+\eta_{\mathfrak{N}}^{6}(x_i)+\theta_{\mathfrak{N}}^{6}(x_i)\right)\\ &= (0.6^6+0.7^6+0.441^2)+(0.8^6+0.6^6+0.2722)+(0.7^6+0.5^6+0.532^2)\\ &= 1.1579 \end{aligned}$$

By using the Equation 3.2, the correlation between the FFSs $\mathfrak{N},\mathfrak{M}$ is written as

$$\begin{aligned} C(\mathfrak{N},\mathfrak{M}) &= \sum_{i=1}^{n}\left(\zeta_{\mathfrak{N}}^{3}(x_i)\zeta_{\mathfrak{M}}^{3}(x_i)+\eta_{\mathfrak{N}}^{3}(x_i)\eta_{\mathfrak{M}}^{3}(x_i)+\theta_{\mathfrak{N}}^{3}(x_i)\theta_{\mathfrak{M}}^{3}(x_i)\right)\\ &= 0.8^3 0.6^3+0.6^3 0.7^3+(0.272)(0.441)+0.5^3 0.8^3+0.9^3 0.6^3+(0.146)(0.272)+0.6^3 0.7^3+0.6^3 0.5^3+(0.568)(0.532)\\ &= 0.96148 \end{aligned}$$

Hence,the KK between the FFSs $\mathfrak{N},\mathfrak{M}$ is given by

$$\begin{aligned} \mathfrak{C}(\mathfrak{N},\mathfrak{M}) &= \frac{C(\mathfrak{N},\mathfrak{M})}{[IE(\mathfrak{N}).IE(\mathfrak{M})]^{1/2}}\\ &= \frac{0.96148}{[(2.3221).(1.1579)]^{1/2}} = 0.58627 \end{aligned}$$

Definition 3.4. *For* $\mathfrak{N}$ *and* $\mathfrak{M}$*, the definition of KK as:*

$$\begin{aligned} \mathfrak{D}(\mathfrak{N},\mathfrak{M}) &= \frac{C(\mathfrak{N},\mathfrak{M})}{\max\,[IE(\mathfrak{N}).IE(\mathfrak{M})]}\\ &= \frac{\sum_{i=1}^{n}\left(\zeta_{\mathfrak{N}}^{3}(x_i)\zeta_{\mathfrak{M}}^{3}(x_i)+\eta_{\mathfrak{N}}^{3}(x_i)\eta_{\mathfrak{M}}^{3}(x_i)+\theta_{\mathfrak{N}}^{3}(x_i)\theta_{\mathfrak{M}}^{3}(x_i)\right)}{\max\left[\sum_{i=1}^{n}\left(\zeta_{\mathfrak{N}}^{6}(x_i)+\eta_{\mathfrak{N}}^{6}(x_i)+\theta_{\mathfrak{N}}^{6}(x_i)\right),\sum_{i=1}^{n}\left(\zeta_{\mathfrak{M}}^{6}(x_i)+\eta_{\mathfrak{M}}^{6}(x_i)+\theta_{\mathfrak{M}}^{6}(x_i)\right)\right]} \end{aligned}$$

Theorem 3.5. *For any two FFSs $\mathfrak{N},\mathfrak{M}$ $\mathfrak{D}(\mathfrak{N},\mathfrak{M})$ satisfies the following conditions:*

(P1) $\mathfrak{D}(\mathfrak{N},\mathfrak{M})=\mathfrak{D}(\mathfrak{M},\mathfrak{N})$,
(P2) $\mathfrak{N}=\mathfrak{M}$ *iff* $\mathfrak{D}(\mathfrak{N},\mathfrak{M})=1$,
(P3) $0\leq\mathfrak{D}(\mathfrak{N},\mathfrak{M})\leq 1$.

Proof. We only proved condition (P2). It is clear that $\mathfrak{D}(\mathfrak{N},\mathfrak{M})\geq 0$. Since from Theorem 3.2, $\left[C(\mathfrak{N},\mathfrak{M})\right]^3\leq IE(\mathfrak{N}).IE(\mathfrak{M})$. Therefore, $C(\mathfrak{N},\mathfrak{M})\leq\max[IE(\mathfrak{N}).IE(\mathfrak{M})]$, thus $\mathfrak{D}(\mathfrak{N},\mathfrak{M})\leq 1$. □

It is possible to define these KKs in a different way. Weights will be used for these new definitions. Because, in numerous real-life practices, the distinct sets can have got diverse weights. Therefore, weight ω_i of every element $x_i\in\mathfrak{X}$ must be considered in new definitions. The KKs to be defined by weights will be called weighted KKs. FFor these definitions, choose the weight vector as ω. Further, Consider that satisfy the $\sum_{i=1}^{n}\omega_i=1$ condition for $\omega_i\geq 1$. Therefore, $C_\omega(\mathfrak{N},\mathfrak{M})$, $\mathfrak{C}_\omega(\mathfrak{N},\mathfrak{M})$ are defined as follows:

$$\begin{aligned}\mathfrak{C}_\omega(\mathfrak{N},\mathfrak{M}) &= \frac{C_\omega(\mathfrak{N},\mathfrak{M})}{[IE_\omega(\mathfrak{N}).IE_\omega(\mathfrak{M})]^{1/3}} \qquad (3.3)\\ &= \frac{\sum_{i=1}^{n}\omega_i\left(\zeta_{\mathfrak{N}}^3(x_i)\zeta_{\mathfrak{M}}^3(x_i)+\eta_{\mathfrak{N}}^3(x_i)\eta_{\mathfrak{M}}^3(x_i)+\theta_{\mathfrak{N}}^3(x_i)\theta_{\mathfrak{M}}^3(x_i)\right)}{\sqrt[3]{\sum_{i=1}^{n}\omega_i\left(\zeta_{\mathfrak{N}}^6(x_i)+\eta_{\mathfrak{N}}^6(x_i)+\theta_{\mathfrak{N}}^6(x_i)\right)}.\sqrt[3]{\sum_{i=1}^{n}\omega_i\left(\zeta_{\mathfrak{M}}^6(x_i)+\eta_{\mathfrak{M}}^6(x_i)+\theta_{\mathfrak{M}}^6(x_i)\right)}}\\ \mathfrak{D}_\omega(\mathfrak{N},\mathfrak{M}) &= \frac{C\omega(\mathfrak{N},\mathfrak{M})}{\max\left[IE\omega(\mathfrak{N}).IE\omega(\mathfrak{M})\right]} \qquad (3.4)\\ &= \frac{\sum_{i=1}^{n}\omega_i\left(\zeta_{\mathfrak{N}}^3(x_i)\zeta_{\mathfrak{M}}^3(x_i)+\eta_{\mathfrak{N}}^3(x_i)\eta_{\mathfrak{M}}^3(x_i)+\theta_{\mathfrak{N}}^3(x_i)\theta_{\mathfrak{M}}^3(x_i)\right)}{\max\left[\sum_{i=1}^{n}\omega_i\left(\zeta_{\mathfrak{N}}^6(x_i)+\eta_{\mathfrak{N}}^6(x_i)+\theta_{\mathfrak{N}}^6(x_i)\right),\sum_{i=1}^{n}\omega_i\left(\zeta_{\mathfrak{M}}^6(x_i)+\eta_{\mathfrak{M}}^6(x_i)+\theta_{\mathfrak{M}}^6(x_i)\right)\right]}\end{aligned}$$

The following theorems are proved as similar to Theorem 3.2 and Theorem 3.5:

Theorem 3.6. *Take a weight vector of x_i as ω and it is considered to satisfy the $\sum_{i=1}^{n}\omega_i=1$ condition for $\omega_i\geq 1$. The weighted KK between the FFSs $\mathfrak{N},\mathfrak{M}$ defined by Equation 3.3 satisfies:*

(P1) $\mathfrak{C}_\omega(\mathfrak{N},\mathfrak{M})=\mathfrak{C}_\omega(\mathfrak{M},\mathfrak{N})$,
(P2) $0\leq\mathfrak{C}_\omega(\mathfrak{N},\mathfrak{M})\leq 1$;
(P3) $\mathfrak{C}_\omega(\mathfrak{N},\mathfrak{M})=1$ *iff* $\mathfrak{N}=\mathfrak{M}$

Theorem 3.7. *The weighted KK between the FFSs $\mathfrak{N},\mathfrak{M}$ defined by Equation 3.4 satisfies:*

(P1) $\mathfrak{D}_\omega(\mathfrak{N},\mathfrak{M})=\mathfrak{D}_\omega(\mathfrak{M},\mathfrak{N})$,
(P2) $0\leq\mathfrak{D}_\omega(\mathfrak{N},\mathfrak{M})\leq 1$;
(P3) $\mathfrak{D}_\omega(\mathfrak{N},\mathfrak{M})=1$ *iff* $\mathfrak{N}=\mathfrak{M}$

4. Application

Consider the infectious diseases as influenza A(H1N1), Crimean-Congo Hemorrhagic Fever(CCHF), Hepatitis C, norovirus, sandfly fever and denote the set $D=\{h_1,h_2,h_3,h_4,h_5\}$. Let's consider the basic symptoms of these diseases as vomiting, headache, anorexia, temperature, nausea and and show the set of symptoms as $S=\{b_1,b_2,b_3,b_4,b_5\}$.

In order to describe a patient by symptoms, the FFS can be given as follows:

$$\mathfrak{N} = \{(b_1,0.8,0.5),(b_2,0.6,0.4),(b_3,0.3,0.9),(b_4,0.5,0.6),(b_5,0.2,0.8)\}$$

and the relationship between symptoms and diseases are given in the following sets as FFSs:

$$\begin{aligned} h_1 &= \{(b_1,0.6,0.1),(b_2,0.2,0.7),(b_3,0.1,0.8),(b_4,0.5,0.2),(b_5,0.1,0.8)\}\\ h_2 &= \{(b_1,0.7,0.2),(b_2,0.3,0.7),(b_3,0.1,0.9),(b_4,0.7,0.2),(b_5,0.1,0.7)\}\\ h_3 &= \{(b_1,0.3,0.3),(b_2,0.7,0.2),(b_3,0.2,0.7),(b_4,0.3,0.8),(b_5,0.1,0.9)\}\\ h_4 &= \{(b_1,0.1,0.7),(b_2,0.2,0.5),(b_3,0.7,0.2),(b_4,0.4,0.8),(b_5,0.3,0.8)\}\\ h_5 &= \{(b_1,0.1,0.8),(b_2,0.1,0.7),(b_3,0.2,0.9),(b_4,0.3,0.7),(b_5,0.7,0.2)\}\end{aligned}$$

The developed KK $\mathfrak{C}$ and KK $\mathfrak{D}$ were used and they were calculated as follows:

$$\mathfrak{C}(\mathfrak{N},h_1)=0.2308,\quad \mathfrak{C}(\mathfrak{N},h_2)=0.24653,\quad \mathfrak{C}(\mathfrak{N},h_3)=0.2007,$$
$$\mathfrak{C}(\mathfrak{N},h_4)=0.191,\quad \mathfrak{C}(\mathfrak{N},h_5)=0.211$$

and

$$\mathfrak{D}(\mathfrak{N},h_1)=0.7417,\quad \mathfrak{D}(\mathfrak{N},h_2)=0.767,\quad \mathfrak{D}(\mathfrak{N},h_3)=0.6456,$$
$$\mathfrak{D}(\mathfrak{N},h_4)=0.6133,\quad \mathfrak{D}(\mathfrak{N},h_5)=0.6721.$$

Choose $\omega = \{0.20, 0.15, 0.13, 0.27, 0.25\}$. Then,

$$\mathfrak{C}_\omega(\mathfrak{N}, h_1) = 0.6136, \quad \mathfrak{C}_\omega(\mathfrak{N}, h_2) = 0.652, \quad \mathfrak{C}_\omega(\mathfrak{N}, h_3) = 0.52115$$
$$\mathfrak{C}_\omega(\mathfrak{N}, h_4) = 0.585, \quad \mathfrak{C}_\omega(\mathfrak{N}, h_5) = 0.6243$$

and

$$\mathfrak{D}_\omega(\mathfrak{N}, h_1) = 0.7424, \quad \mathfrak{D}_\omega(\mathfrak{N}, h_2) = 0.8, \quad \mathfrak{D}_\omega(\mathfrak{N}, h_3) = 0.656$$
$$\mathfrak{D}_\omega(\mathfrak{N}, h_4) = 0.614, \quad \mathfrak{D}_\omega(\mathfrak{N}, h_5) = 0.6721$$

The recognition principle showed us that the h_2 pattern is the most desired pattern. When the results of all KK indices were compared, it was seen that each result was the same.

4.1. Comparison

The KK based on the IFS defined by Xu [8] is given as follows for the two IFSs M and N:

$$\mathfrak{C}_{Xu}(N, M) = \frac{\sum_{i=1}^{n}\left[\zeta_N(x_i)\zeta_M(x_i) + \eta_N(x_i)\eta_M(x_i) + \theta_N(x_i)\theta_M(x_i)\right]}{\max\left[(\sum_{i=1}^{n}(\zeta_N^2(x_i) + \eta_N^2(x_i) + \theta_N(x_i)))^{1/2}, (\sum_{i=1}^{n}(\zeta_M^2(x_i) + \eta_M^2(x_i) + \theta_M(x_i)))^{1/2}\right]}.$$

For $\{(b_1, 0.1, 0.1), (b_2, q, 0), (b_3, 0, 1)\}$, choose the three patients h_1, h_2, h_3. These patients shown in form the IFS as follows:

$$\begin{aligned} h_1 &= \{(b_1, 0.35, 0.55), (b_2, 0.65, 0.15), (b_3, 0.3, 0.3)\} \\ h_2 &= \{(b_1, 0.55, 0.35), (b_2, 0.65, 0.2), (b_3, 0.4, 0.3)\} \\ h_3 &= \{(b_1, 0.55, 0.35), (b_2, 0.65, 0.2), (b_3, 0.4, 0.3)\}. \end{aligned}$$

Then $\mathfrak{C}_{Xu}(h_1, N) = \mathfrak{C}_{Xu}(h_2, N) = \mathfrak{C}_{Xu}(h_3, N) = 0.4102$. These results showed that Xu's formula cannot classify h_1, h_2, h_3 with N. When the suggested method was used with the same values, $\mathfrak{C}(h_1, N) = 0.4735$ $\mathfrak{C}(h_2, N) = 0.3976$, $\mathfrak{C}(h_3, N) = 0.3933$ were found.

Now, consider the KK of PFS defined by Garg [37]. Taking the values from the illustrative example used by Garg, the resulting values are as follows: $\mathfrak{C}_{Garg}(h_1, N) = 0.6726$, $\mathfrak{C}_{Garg}(h_2, N) = 0.332$, $\mathfrak{C}_{Garg}(h_3, N) = 0.8943$.When the suggested method was used with the same values, $\mathfrak{C}(h_1, N) = 0.987$ $\mathfrak{C}(h_2, N) = 0.237$, $\mathfrak{C}(h_3, N) = 0.285$ were found.

5. Discussion

First of all, let's explain the advantages of the presented technique and the differences with other techniques. As is known, FFSs can investigate the problems with imprecise and incomplete information more effectively than of IFSs. Since the sets of Pythagorean and intuitionistic MDs are not as extensive as the sets of fermatean MDs [43], it is clear that FFSs will have many comprehensive possibilities for identifying and resolving uncertainties than IFS and PFS.

IFS is a successful generalization of FS theory in dealing with uncertainty and uncertainty, which is characterized by $MD + ND \leq 1$. However, there are cases where the sum of the MDs and NDs will be greater than 1. In this case, the IFS technique will be insufficient to solve this problem. To solve this inadequacy, PFS which is initiated by Yager has emerged. PFS is a natural generalization of FS theory, with successful results. However, the sum of the squares of MD and ND of DMR of a particular attribute may also be greater than 1, in which case it will not be an appropriate solution method in PFS.

There are KKs obtained with IFSs and PFSs in the literature, and there are algorithms defined using these KKs. As mentioned earlier, some cases cannot be symbolized by IFSs and PFSs, hence appropriate results may not be obtained from their corresponding algorithms. The KKs obtained with IFSs and PFSs are a specific situation of the KK of FFSs. Then, the suggested KK is more generalized than existing ones and is appropriate for solving real-life problems more accurately.

6. Conclusion

This study is dedicated to defining a KK for FFS. This study has extended the constraint conditions of $MD + ND \leq 1$ for IFS and the $MD^2 + ND^2 \leq 1$ for PFS to the FFS KK theory. The numeric example has been served that represents that The offered KK can easily operate the conditions where the present KKs in the IFS and PFS frameworks fail. The offered KK in FFS has been improved by taking the MD, ND, and their hesitation degree between MD and ND. Furthermore, weighted KKs have been described to handle cases where elements in a set are correlative. The medical diagnosis model is shown that the correlation coefficient given in the study is easy to use and optimum results can be obtained. From the illustrative example study, it has been accomplished that the offered KK in the FFS framework can conveniently operate the real-life DM problem with their objectives.

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