WOMEN MATHEMATICIANS

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ABSTRACT: In any book on the history of mathematics, mention about women mathematicians is hardly found. One wonders if there were any and, if any, why so few? The lives of some women mathematicians from different countries, the first who was born in 370 AD and the last who died in 1935. If there were only a few women mathematicians before the 20th Century, why have there been more in the 21th century what has been done, and what is being done, to increase their number?

There is no enough space in this paper to examine the details of the mathematical research being conducted by these eight women. Instead the main interest will be the difficulties involved and the struggles they underwent while fulfilling their desire to become mathematicians, the first one of them who had a tragic end.

Keywords: *Women mathematicians.*

ÖZET: Herhangi bir matematik tarihi kitabında kadın matematikçilere az rastlanılır. İnsan bunun neden bu kadar az olduğunu merak eder. Bu soruya yanıt verebilmek için bilinenlerin bazılarının yaşam öykülerinin bilinmesi gerekir. Bu yazının ilk kısmı farklı ülkelerin kadın matematikçilerine ayrılmıştır. Bunlardan ilki 370 yılında doğmuş ve sonuncusu da 1935 yılında ölmüştür. Bunlarla ilgili bazı sorular vardır. Yirminci yüzyıldan önce çok az kadın matematikçi varsa yirminci yüzyılda daha fazla olması için ne yapılabilirdi? 2001 yılları için ne tahmin yapılabilir?

Kadın matematikçilerin matematikte neler yaptıklarını burada anlatacak kadar yer yoktur. Çok güçlükler çeken bu kadın matematikçilerin ilki Hypatia'dır.

Anahtar sözcükler: Kadın matematikçiler.

HYPATIA (370 - 415)

Hypatia was a typical representative of Alexandrian Neopolitanism. She was the Greek philosopher of the famous 5th Century AD, and perhaps the most famous woman philosopher in history. She was the daughter of Theon of Alexandria, a mathematics writer from whom she seems to have derived and become interested in the subject.

Theon of Alexandria flourished in about 370 AD. He was not actually a mathematician of any important of special note, but mathematical science is indebted to him for an edition of Euclid's Elements and a commentary on the Almagest. This book, translated with comments by M. Halma and published in Paris in 1821, submits a great deal of miscellaneous information about the numerical methods used by the GREEKS.

Hypatia was more distinguished than her father and was the last Alexandrian mathematician of any general reputation. She wrote a commentary on the Conics of Apollonius and possibly other works, but none of her works have survived.

The fate of Hypatia may serve to remind that the Eastern Christians, as soon as they became the dominant party in the state, showed themselves bitterly hostile to all forms of learning. That very singleness of the purpose, which had at first so materially aided their progress developed into a one-sidedness, which refused to see any good outside their own body; and all who did not actively assist them were ultimately persecuted. She was the first woman who took any noteworthy position in mathematics, and perhaps because of her martyrdom she has occupied an unduly exalted place in history. But was this also the end of Alexandrian mathematics?

She was a student of her father, and as such were her attainments that she was called upon, so the tradition says, to preside over the neoplatonic School at Alexandria. Much that passes for history in her case seems to be fiction, as the statement of Suidas (10th century AD) that she married Isidorus of Gaza, the Neoplatonist. It seems certain, however, that she was slain in one of the city brawls between followers of rival sects. Suidas says that she wrote a commentary on an astronomical table of a certain Diophantus, possibly the algebraist, and one on the Conics of Apollonius. Her works, however, are all lost today. For the romantic side of her life, J. Toland wrote, in 1720, " ...the history of a most beautiful, most virtuous, most learned... lady ".

Renowned for beauty, modesty, learning and eloquence, she became probably the most important figure in the Neoplatonic School of Alexandria, where she was said to support Orestes, the pagan prefect of Egypt, in his political opposition to St. Cyril, patriarch of Alexandria. She is said to have occupied the Chair of Platonic Philosophy and to have lectured on Plato, Aristotle and other philosophers. Her most notable student was Synesius of Cyrene, who became Bishop of Plotemais in 411 AD and whose affectionate and admiring letters to her are the main source of information about her personality. This close friendship between the pagan Neoplatonist teacher and the Christian Neoplatonist student illustrates on aspect of the complex relationship between the Christians and pagan at Alexandria under the Christian Roman Empire, the close and fruitful contact, which was so important for later development of Christianity with Pagan philosophy.

Scandalous stories about her friendship with Orestes, as well as disapproval of her non-Christian beliefs eventually caused her tragic end. One day, in March 415 AD, a Christian mob in Alexandria, incited by fanatical clergy, stopped her carriage, dragged her into the church of the Caesarium, flayed her skin by using seashells, tore her limbs apart and burned her broken body in the street.

The impact of her dramatic death in Alexandria has caused that year to be taken by some to mark the end of ancient mathematics, let alone Alexandrian mathematics. As the death of Hypatia had marked the closure of Alexandria as a mathematical center, the final establishment of Christianity in the East marks the end of the Greek scientific schools.

GABRIELLE EMLIE LE TOLELLIER DE BRETEUIL MARQUISE DU CHATELET (1706 – 1749)

The world does not often connect the name of Voltaire (1694 – 1778) with mathematics, and when it connects that of the Marquise de Chatelet with the science, it is largely by courtesy. Each, however, did something to make the Newtonian theory, and each absorbed enough mathematics to make the labor fairly serious.

Francois Marie Arouet, known to the world as Voltaire who was born in Paris, November 21, 1694; died in Paris, May 30, 1778, and as the foremost leader of the 18th century in the contest for human liberty, was interested in mathematics chiefly because he was interested in all things English, was interested in Newton, was interested in getting out a work on Newton's philosophy which was "Éléments de la philosophie de Newton, Amsterdam, 1738", and was interested in Emilie, Marquise du Chatelet.

Marquise du Chatelet, French mathematician, physicist, philosopher, and translator who did much to free French thought from subservience to Cartesianism, was born in Paris on December 17, 1706, the daughter of Luis Nicholas le Tonnelier, baron de Breteuil. An accomplished linguist and musician, she became prominent in the social life of the time. She was married at nineteen to the general marquise Florent du Chatelet-Lemont in 1725, and became a marquise. Then she was a favorite woman and the first woman of the court of the Queen (1725). They had three children; but before and after her marriage she had alliances with other men, the most important being that with Voltaire, dating from 1733. She left her husband her place in the palace and went to Voltaire's habitation (1733) with Voltaire whom she was engaged because of her ambition for scientific matters and philosophy. She became Voltaire's mistress, and provided him with the protection he need when his

"Letters philosophiques" published in 1734, incurred the wrath of the authorities. She continued her scientific studies during this 15-year engagement with Voltaire. She gave him a reliable and a continuous support.

Daughter of the Baron de Breteuil, Marquise du Chatelet, turned her brilliant mind to Euclid, to Newton, to the literary classics of Greece and Rome, to Locke, and to Voltaire. She had had studies about mathematics under Maupertuis and Koenig, read Newton, and understood him, at least in part, and in due time translated the last principia, completing it a few days before her death. It was published posthumously at Paris in 1759. There is a bibliography of her works in A. Rebiere, Les femmes dans la science, 2nd ed., p. 65, Paris 1897. Her most important work was her translation of Newton's Principia, which appeared first in 1756. Influenced by Samuel Koenig, she wrote "Institutions de physique" (1740), a work pervaded by the views of Leibniz. But she was only an amateur in science. Voltaire, in one his many epigrams about her, wrote:

"Her spirit is very philosophical,
But her heart loves pompons."

In French:

"Son esprit est trés philosophe,
Mais son coeur aime les pompons."

In his work on Newton he addresses a poem to her, beginning:

"Tu m'appelles à toi, vaste and puissant Génie,
Minerve de la France, immartal Emilie,
Disciple de Newton, and de la Vérite."

In English:

"You call me to you, great and powerful genius,
Minerve of France, immortal Emilie,
Disciple of Newton and of Truth."

The long so-journs at the château of the Du Chatelets in Champagne provided a haven for writing, as well as refuge from the Paris police whenever it became necessary for her to extricate the intemperate Voltaire from personal and political difficulties.

Marquise de Chatelet and Voltaire in 1738 competed independently for a prize offered by the Academy of Sciences in Paris. Although it failed to win the prize, the academy published it in 1744. A very famous mathematician Leonhard Euler won the prize. Her work was "Dissertation sur la nature et la propagation du feu".

Voltaire and Madame de Chatelet (or du Chatelet-Lomont) continued to live together even after she had transferred her affections to the poet Jean Francois de Saint-Lambert; and when, on September 10, 1749, she died in childbirth in Luneville, at the palace of King Stanislas of Poland, in the presence of her husband and Voltaire and the poet Jean François de Saint-Lambert, who was the father of her child. These men and her husband were with her.

She had worked until the end on the translation of the Principia, and this was published, with a preface by Voltaire and under the direction of A.C. Clairaunt, in 1756. Frederick, the Great, who loved an epigram far more than he loved the courtesies of life, suggested this epitaph: " Here lies one who lost her life in given birth to an unfortunate child and to a treatise on philosophy."

The many hundreds of letters that passed between Madame du Chatelet and Voltaire are assumed to have been destroyed; but others were included in Voltaire's "Correspondence" edited by Theondore Besterman, 24 Volumes, 1953-1957.

MARIA GAETANAAGNESI (1718-1799)

Among the women of Italy who have added to the store of the world's knowledge of mathematics the most erudite one of this period was Marie Gaetana Agnesi who was Born at Milan, March 16, 1718; died at Milan, January 9, 1799. She lived at a time when it was acceptable for women in Italy to be educated, contrary to the customs in other European countries. This point is very important, because at that time women and their educations were problems in European countries because in Europe, which was the center of civilization, women excluded the sense of education. It was more suitable for women to sew, to etc. in short to be housewife. If we pay attention we conclude that her family was different from other European families in positive way because her father was a mathematics professor at the University of Bologna.

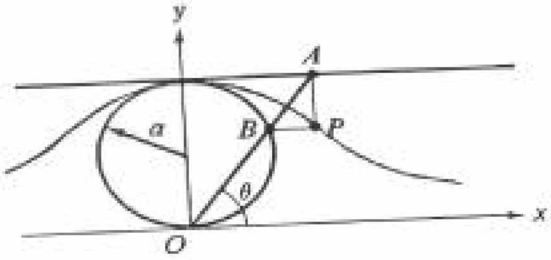
Agnesi was the eldest of 21 children born to a literate family. She was a very precocious child, and her mother and father, latter a professor of mathematics, encouraged her. Marie Agnesi was lucky at this point to born in Italy. She took the advantage of their supply. At the age of nine, at one of the gatherings of intellectuals in her home, she delivered in Latin a discourse defending higher education for women.

Her best-known work, Treatise on Analysis for the Use of Italian Youth, dedicated to the Empress Maria Theresa, was published in two volumes in 1748. Two volumes have four books and they were the first comprehensive calculus texts. In four books, the texts treated algebra and geometry, differential calculus, integral calculus, and differential equations. The text was translated into French and English. There is an English translation, two volumes, London, 1801, with a brief biography. Her work on analytic geometry is well known. There is a mistranslation that is responsible for our calling Agnesi's bell-shaped curve "the witch" today. This name, in fact, is founded only in texts written in English. Agnesi's own name for the curve was "versiera", from the Latin verb vertere, to turn. The translator, a Cambridge scholar who had learned Italian expressly for the purpose of translating Agnesi's text, probably confused the Latin versiera with the Italian avversiera, "wife of the devil", carefully translating the latter as "the witch".

The witch of Maria Agnesi is a bell-shaped curve that may be constructed as follows: Let C be a circle of radius a having its center at $(0,a)$ on the y-axis. The variable line through the origin $\overline{OA}$ intersects the line $y = 2a$ in the point A and intersects the circle in the point B. A point P on the witch is now located by taking the intersection of lines through A and B parallel to y-and x-axes, respectively.

a. Find parametric equations of the witch for the set of all points P determined as shown in the following figure. (Hint: Show $\underline{OB}=2a \sin \theta$). Sketch the curve.
b. Also find a cartesian equation for the witch of Maria Agnesi.

It was in the Treatise that she discussed the cubic curve, $(x^2 + a^2)y = a^3$ which became know as the Witch of Agnesi due to a misunderstanding. Today many students who study analytical geometry know at least the name Agnesi.
She had been tutoring her younger brothers and sisters, and this work was written



for them, but others soon discovered it and it was translated into many languages and used as a textbook. This is remarkable that she was doing her best while tutoring. She took her job, her responsibilities seriously. She had collected much of the then known work on plane curves, the calculus, and differential equations for these volumes. Because she had a mastery of many languages, including Latin, Greek and Hebrew. She had been able to read the papers in which research on these subjects was reported.

Her work was sufficiently well known in France that a committee of the 1 French Academy of Sciences was appointed to assess it. One member wrote to her, as quoted by Mary R. Beard On Understanding Woman:

I do not know of any work of this kind that is clearer, more methodical or more comprehensive...There is none in mathematical sciences. I admire particularly the art with which you bring under uniform methods the diverse conclusions scattered among the works of geometers reached by methods entirely different.

Although members of the French Academy thought highly of her work, had they wanted to invite her join the Academy they could not have done so, for membership in the Academy was denied to women. There is a question among historians as to whether Agnesi ever held a chair at the University of Bologna, but it is clear that after her father's death in 1752 she devoted herself completely to religion and to aiding the poor. May be this was a way of escaping from the real world which could not understand the value of her. May be she tried to find the happiness in the world of the poor. If she could have continued to work mathematics we would have more knowledge of course.

SOPHIE GERMAIN (1776-1831)

While Marie Agnesi was still alive, another woman, Sophie Germain (1776-1831) destined also to be a mathematician, was born in Paris. She was not fortunate as Agnesi in her choice of parents and her country, because of the education. Her parents were prosperous, and could have allowed her to study whatever field she wished. They did indeed encourage her in intellectual pursuits until she chose mathematics. That was too much for a girl to choose this subject. For them, girls were like flowers. When they discover that she was secretly studying mathematics in her own room at night, they took away her candles, her fire, and her clothes, leaving her only her bedcovers. They wanted to take away their kid from this disease, because they were egoist. They did not care of her honor and could not think the result. They insulted her and were full of hatred in the deep part of her heart. She wanted to prove herself. She was not only their daughter but she was Sophie Germain. She managed to secrete candles in her room and, after the other members of the family were asleep, she continued her study of mathematics. In the meantime she taught herself Latin so that she could read some of the mathematics books she had obtained. After finding her cold and asleep at her desk many times, her family gave in and allowed her to study mathematics. Now the world was hers and she was very happy. She was working very hard.

It was just at this time that the École Polytechnique was founded, and Germain looked forward to studying mathematics there only to learn that women were excluded from attending the Polytechnique. However, lecture notes were available to all who asked, and Germain obtained these. Students were also allowed to submit written observations, which Germain did under an assumed name, M. LeBlanc. In this way communication between her and the well-known mathematicians of the day was started. In fact, J.L. Lagrange, who was at the École Polytechnique, was so impressed with "his" work that he insisted on meeting "him". When Lagrange discovered that M. LeBlanc was a woman his respect for her work continued. Through Lagrange, Germain got to know all the French scientists of the day, and her home soon became a center for meetings of some of the most distinguished of the group.

In the years 1804 and 1805, Gauss had some correspondence about number-theoretic problems with a then unknown French mathematician. We quote twice from the correspondence with Olbers, First from a letter of December 7, 1804:

"Recently I had the pleasure to receive a letter from LeBlanc, a young geometer in Paris, who made himself enthusiastically familiar with higher mathematics and showed how deeply he penetrated into my Disquisitiones Aritmeticae..."

The second quotation is more than two years later, March 24, 1807:

"Recently, I was greatly surprised on account of my Disquisitiones Aritmeticae. Did I not repeatedly write you of a correspondence in Paris, one M. LeBlanc, who had perfectly understood all my investigations? This LeBlanc recently explained himself to me. You will certainly be as surprised as I was when you hear that LeBlanc is the assumed name of a young woman, Sophie Germain".

One of the later proofs of the law of quadratic reciprocity is connected to an idea of Sophie Germain; her name is remembered in number theory because she was the first to find the solution for certain special cases of Fermat's last theorem. That a solution to the equation $x^n + y^n = z^n$ is impossible in positive integers for any integer n greater than 2. L.E. Dickson used her work on this problem in 1908 to prove Fermat's last theorem for every odd prime n less than 1700. In 1910, E. Dubois named a special type of prime number as a Sophien, thus ensuring that Sophie Germain's name lives on in the theory of numbers, too. By now she was known under her own name and had begun to publish the results of her research, which were in many different fields. She also worked on curvature of surfaces, and was even a philosopher; after her death, a nephew collected her writings in philosophy and publish them under the title "Considerations generale sur l'etat des sciences et des letters aux differentes epoques de leur culture".

Having disposed of quadratic second-degree reciprocity, it was natural for Gauss to consider the general question of binomial congruences of any degree. If m is a given integer not divisible by the prime p, and if n is a given positive integer, and if further an integer x can be founded such that $x^n \equiv m \pmod{p}$, m is called an n-ic residue of p; when n=4, m is a biquatratic residue of p.

The case of quadratic binomial congruences (n=2) suggests but little to do when exceeds 2. One of the matters Gauss was to have included in the discarded eighth section, (as he told Sophie Germain, in the projected but unachieved second volume) of the Disquisitiones Aritmeticae was a discussion of these higher congruences and a search for the corresponding laws of reciprocity, namely the interconnections (as to solvability or non-solvability) of the pair $x^n \equiv p \pmod{q}$, $x^n \equiv q \pmod{p}$, where p, q are rational primes. In particular cases n=3, n=4 were to have been investigated.

That Gauss was not merely being polite to a young woman admirer is shown by a letter of July 21, 1807 to his friend Olbers. "... Lagrange is warmly interested in astronomy and the higher arithmetic; the two test-theorems (for what primes 2 is a cubic or a biquadratic residue), which I also communicated to him some time ago, he consider among the most beautiful things and the most difficult to prove. But Sophie Germain has sent me the proofs of these; I have not yet been able to go through them, but I believe they are good; at least she had attacked the matter from the right side, only somewhat more diffusely than would be necessary..." The theorems to which Gauss refers are those stating for what odd primes p each of the congruences $x^3 \equiv 2 \pmod{p}$, $x^4 \equiv 2 \pmod{p}$ is solvable.

The lady in question was Mademoiselle Sophie Germain just a year than Gauss. She and Gauss never met. Gauss recommended to the faculty of the University of Göttingen that she be awarded an honorary doctor's degree, but unfortunately, she died of cancer, in 1831, before it could be conferred. By a curious coincidence we shall see the most celebrated woman mathematician of the nineteenth century, another Sophie, getting her degree from the same liberal University many later after Berlin had refused her on account of her sex.

In 1809, Napoleon ordered the Academy of Sciences to after a prize for a solution to the problem of finding a mathematical theory for elastic surface and comparing it with experimental data. Twice, anonymously, Germain submitted solutions. The first time her entry was the only one submitted. Each time a mistake was found and no prize was awarded. The third time she submitted an entry using her real name and won, despite a lock of rigor due to lack of formal training. Her equation for elastic laminae is still the fundamental equation of the theory and now known as Germain's equation. She had won prestigious prize without having published a single paper. Sophie Germain's scientific interests embraced acoustics, the mathematical theory of elasticity, and the higher arithmetic, in all of which she did notable work.

Entranced by the Disquisitiones Arithmeticae, Sophie wrote to Gauss some of her own arithmetical observations. Fearing that Gauss might be prejudiced against a women mathematician, she assumed a man's name. Gauss formed a high opinion of the talented correspondent whom he addressed in excellent French as "Mr. LeBlanc ".

Germain never held a post at academic institution. The story of her correspondence with the famous mathematician C.F. Gauss is well known. A mathematician friend recommended to her Gauss's Disquisitiones Arithmeticae (published in 1801), although at that time not many mathematicians had been able to penetrate deeply into the work. Germain was greatly interested in it and in 1804 wrote her first letter to Gauss, including some problems in number theory on which she was working at the time. She again used the assumed name of M. LeBlanc; later by accident, Gauss learned that LeBlanc was a woman, but, to his credit, their correspondence continued. Over a four-year period they exchanged many letters concerning mathematical questions.

LeBlanc dropped her-or his- disguise when she was forced to divulge her true name to Gauss on the occasion of her having done him a good turn with the French infesting Hanover. Writing on April 30, 1807, Gauss thanks his correspondent for her intervention on his behalf with the French General Pernety and deplores the war. Continuing, he pays her a high compliment and expresses something of his own love for the theory of numbers. As the latter is particularly of interest we shall quote from this letter, which shows Gauss in one of his cordially human moods.

"But how describe to you my admiration and astonishment at seeing my esteemed correspondent Mr. LeBlanc metamorphose himself into this illustrious personage (Sophie Germain) who gives such a brilliant example of what I would find it difficult to believe. A taste for the abstract sciences in general and above all the mysteries of numbers is excessively rare: one is not astonished at it; the enchanting charms of this sublime science reveal themselves only to those who have the courage to go deeply into it. But when a person of the sex, which according to our customs and prejudices, must encounter infinitely more difficulties than men to familiarize herself with these thorny researches, succeeds nevertheless in surmounting these obstacles and penetrating the most obscure parts of them, then without doubt she must have the noblest courage, quite extraordinary talents and a superior genius. Indeed nothing could prove to me in so flattering and less equivocal manner that the attractions of this science, which has enriched my life with so many joys, are not chimerical, as the predilection with which you have honored it." He then goes on to discuss mathematics with her. A delightful touch is the date at the end of the letter: "Bronsvic ce 30 Avril 1807 jour de ma naissance-Brunswick, this 30^{th} of April 1807, my birthday."

A completely different experience was connected with a war tax levied by the French government in 1808. Gauss, who was subjected to it as a member of the university, was asked to pay ffrs 2000, a very considerable sum for a man who had just joined the university and not yet received his first salary. Without being asked, Lagrange in Paris and Olbers in Bremen offered their help, but Gauss did not want to accept any money from them. In the end, the contribution was paid by an anonymous donor who, somewhat surprisingly, turned out to be Count Dahlberg, formerly the arch-chancellor of the Roman Empire and then Lord Bishop of Frankfurt. There were other signs of his growing fame. In 1810, only two years later, Gauss won a medal from the Institut de France. He refused the money that accompanied it, but accepted the astronomical clock that was purchase for him by Sophie Germain.

If Gauss was somewhat cool in his printed expressions of appreciation he was cordial enough in his correspondence and in his scientific relations with those who sought him out in a spirit of disinterested inquiry. One of his scientific friendships is of more than mathematical interest as it shows the liberality of Gauss' views regarding women scientific workers. His broadmindedness in this respect would have been remarkable for any man of his generation; for a German it was almost without precedent.

What a pity that she had no choice to continue for living.

MARY FAIRFAX SOMERVILLE (1780-1872)

Four years later the birth of Sophie Germain in France, Mary Fairfax Somerville (1780-1872) was born in Scotland. She grew up in a small seacoast village and received no formal education until after she was ten years old, when her father sent her to a fashionable girls' school. She hated the school and returned home after a year. She was not interested in elementary subjects, around, friends, teacher and books. They were very dull for her. It was her first school. At school she seems to have inspired a curious mixture of fear and anger in minds of her teachers and fellow students. Her teachers were good men and patient, but it was merely a strong word to describe the heinous inability of a mathematical genius of the first rank to squander her intellect on the futilities of rhetoric as expounded. Some madness dominates this girl. She lost for one year there. But at least she could now read, and read she did, despite the complaints of relatives who saw that she was reading instead of sewing.

It was by accident that she saw a problem in magazine that was solved by algebra, and she wondered what this algebra. She thought days and days about what algebra was! This question that was seen by chance by her would make her to penetrate the world of mathematics. At the end she wanted to buy a book on algebra, but it was not acceptable at that time for a young girl to go into a bookstore and by such a book instead of buying earrings, rings, and skirts. Why? Perhaps, if they entered the world of mathematics, they would have learned to use their sense and they would never have let men to use of them in any case.

One day she heard of Euclid's Elements of Geometry when her painting teacher recommended it to male student, saying it would help him with perspective. Once more she could not obtain a copy because of her sex. Which was important, sexuality or brain? Again, by accident, she was sewing in the room where a brother was being tutored in mathematics and, when he could not answer a question asked by the tutor, she prompted him. The tutor became sympathetic to her desire to learn mathematics, obtained a copy of Euclid for her, and helped her as much as he could. Her mother was horrified at her daughter's desire to study mathematics and, naturally, instructed the servants to take away her candles so that she could not study at night. Her father predicted that she would soon be in a straitjacket. Today, we must thank this tutor who helped her. Again thanks coincidences, which made her entering the world of mathematics. But she continued to study mathematics by herself.

It was only after second marriage to a cousin, William Somerville, a surgeon, that she found someone sympathetic to her desire for knowledge. There were now sufficient funds to allow her to follow her interests, but there were also four children. She managed somehow to find time to work. When the family moved to London, her husband's work brought them into contact with intellectuals. In this way she met the scientists of the day, among them the Herschels (Caroline, Sir William, Sir John), Sir Edward Parry, Lord Brougham and P.S. de LaPlace. Soon she was one of them.

Soon after she presented her first paper, "The Magnetic Properties of the Violet Rays of the Solar Spectrum," to the Royal Society in 1826. In 1827 she was invited (by letter to her husband, asking him to persuade her) by Society for the Diffusion of Useful Knowledge to write a popularization of LaPlace's Mécanique Céleste. She accepted the invitation and translated the text, adding her own extensive notes. She called the work The Mechanisms of the Heavens. It was her most popular work, but she also wrote The Connection of the Physical Sciences, 1858; Molecular and

Microscopic Sciences, 1869; Physical Geography, 1870, as well as many articles and monographs including "On Curves and Surfaces of Higher Order". The question of Euclid's fifth postulate relating to parallel lines, has occupied the attention of geometers ever since Elements was written. The first scientific investigation of this part of the foundation of geometry was made by Girolamo Sacchery (1733) that Somerville has read this of course.

Somerville took the view that mathematical truths existed in the mind of the Deity and that humans could only discover them, not create them. However, she was not so theological in her outlook that she escaped the criticism of the Church. Dean Cockburn of York Cathedral denounced her by name from his pulpit, for her support of science.

All her life Somerville was an advocate of education for women and in her later years she wrote "Age has not abated my zeal for the emancipation of my sex from the unreasonable prejudice to prevalent in Great Britain against a literary and scientific education for women".

In 1879, seven years after her death, Somerville College was founded at Oxford University as a women's college, which it still is today. In the last few years when three of the five women's colleges at Oxford have become coeducational, the Somerville College faculty has voted to keep the faculty, as well as the student body, all female.

CHRISTINE LADD FRANKLIN (1848-1930)

One might have hoped that opportunities for women mathematicians were better in the New World, but unfortunately they were not. Christine Ladd Franklin (1848-1930), an 1869 graduate of Vassar College, wanted to be a physicist. She later learned that women were not allowed to work in laboratories and changed her interest to mathematics. When Johns Hopkins University founded in 1876, announced a fellowship program in mathematics, her application submitted under the name C. Ladd, was one of the first to arrive. Her credentials proved to be so outstanding that she was awarded a fellowship, sight unseen.

When the Board of Trustees discover that she was a woman, they accused her of using trickery in order to gain admission and her fellowship was revoked. Fortunately this occurred while the world-famous English mathematician, James J. Sylvester was at Hopkins. He had read some of her papers in English mathematical journals and insisted that the gifted young woman be admitted. The Trustees gave in, and Ladd entered Johns Hopkins in 1878 on a three-year fellowship. However, the Trustees forbade that her name appear in print in any list of fellowship holders at Johns Hopkins. In 1882 she submitted her dissertation, "The Algebra of Logic", which her adviser, Charles Sanders Peirce, said was brilliant. That was not good enough for the Trustees however, and they ruled that no Ph.D. should be granted to her on the grounds that a precedent might be set.

Soon after this the Franklins left Johns Hopkins and went to Göttingen to study. There they found that women were not allowed to attend lectures. However, a member of the Göttingen faculty was so impressed with Ladd Franklin's abilities that he gave his lectures to her privately and let her work in his laboratory. Out of this period came the beginning of her work on color vision, now known as the Ladd Franklin theory. In 1904 she and her husband returned to Baltimore, he as editor of a Baltimore newspaper and she as a lecturer in logic and psychology at Johns Hopkins, the only woman member of the faculty. (Apparently, lecturers could be appointed without permission of the Trustees). Still the Trustees refused to grant her the Ph.D. degree. In 1909 when Columbia University invited her to join its psychology faculty, she accepted.

It was while she was at Columbia that a member of the Psychology Department at Harvard University invited her to give a lecture there. She had accepted, and he had made all arrangements for her talk, which was to be followed by a dinner in her honor, when Harvard's President heard of it. He immediately wrote to the psychologist who had invited her, saying that no woman was to speak at Harvard and that he must withdraw the invitation. She replied that, unless she heard directly from the President of Harvard, she was coming anyway. It seems that the President did not write, and Ladd Franklin came to Harvard. Plans for her talk and visit went off as originally scheduled.

Finally, 44 years after Christine Ladd Franklin had completed all the work for the Ph.D., and when she was 78 years old, Johns Hopkins awarded her the degree.

SONJA (1850-1891)

In Russia in the 1800's encouragement for women to become mathematicians was not any greater than it had been in France in the 1700's. Sonja Corvin-Kurkovsky Kovalevsky was born at Moscow, Russia, on January 15, 1850, and died at Stockholm, Sweden, on February 10, 1891, six years before the death of Weierstrass. She was the daughter of an artillery officer in the Russian army. In her teens she developed intellectual interests and was particularly fond of Algebra. Her father's concept of a woman did not include her being an intellectual. When her father learned that she liked algebra, he immediately threw away her algebra book. However, she was able to obtain another and continued her study secretly. Both she and her sister wanted university educations, but at that time in Russia women were not admitted to the universities. They determined to go abroad, but this was not possible as long as they needed the permission of their parents.

At fifteen Sonja began the study of mathematics. By eighteen she had made such rapid progress the she was ready for advanced work and was enamored of the subject. As she came of an aristocratic and prosperous family, she was enabled to gratify her ambition for foreign study and matriculated at the University of Heidelberg. With the aid of intellectual friends of their own age in St. Petersburg, they arranged a fictitious marriage for Sonja, when she was 18, with Vladimir

Kovalevsky. The three sets out for Heidelberg, the couple acting as chaperone for the other sister.

However, in Heidelberg the story was the same; women were not admitted to the University. This highly gifted girl became not only the leading woman mathematician of modern times, but also made a reputation as a leader in the movement for the emancipation of women, particularly as regarded their age-old disabilities in the field of higher education. After much effort on her part she was allowed to audit lectures. It was at this time that she decided that she definitely wanted to become a mathematician.

In addition to all this she was a brilliant writer. As a young girl she hesitated long between mathematics and literature as a career. After the composition of her most important mathematical work (the prize memoir noted later), she turned to literature as a relaxation and wrote the reminiscences of her childhood in Russia in the form of a novel (publish first in Swedish and Danish). Of this work it is reported "the literary critics of Russia and Scandinavia were unanimous in declaring that Sonja Kovalevsky had equaled the best writers of Russian literature in style and thought." Unfortunately this promising start was blocked by her premature death, and only fragments of other literary works survive. Her one novel was translated into many languages.

We must first tell how Sonja and Weierstrass met. Weierstrass used to enjoy his summer vacation in a thoroughly human manner. The Franco-Prussian war caused him to forego his usual summer trip in 1870, and he stayed in Berlin, lecturing on elliptic functions. Owing to the war his class had dwindled to only twenty instead of the fifty who heard the lectures two years before.

Since the autumn of 1869 Sonja Kowalewski; then a dazzling young woman of nineteen, had been studying elliptic functions under Leo Königsberger (born 1837) at the University of Heidelberg, where she had also followed the lectures on physics by Kirchhoff and Helmholtz and had met Bunsen, the famous chemist under rather amusing circumstance to be related presently. Königsberger, one of Weierstrass first pupils, was a first-rate publicity agent for his master. Sonja caught her teacher's enthusiasm and resolved to go directly to the master himself for inspiration and enlightenment She wanted to work with Karl W.T. Weierstrass at the University of Berlin. So she went to Berlin, only to learn that opportunities there were even worse for women than in Heidelberg; women were not even allowed to audit classes. She wrote to Weierstrass and asked him to let her study with him as a private pupil. He sent her some problems to solve and was so impressed with her solutions that he consented to let her study with him.

The years (1864-97) of Weierstrass' career at Berlin as Professor of Mathematics were full of scientific and human interests for the man who was acknowledged as the leading analyst in the world. One phase of these interests demands more than the passing reference that might suffice in a purely scientific biography of Weierstrass:

his friendship with his favorite pupil, Sonja Kowalewski. Although Weierstrass never married but he was no panicky bachelor who took to his heels every time he saw a pretty woman coming. Sonja, according to competent judges who knew her, was extremely good looking, charming and a very beautiful girl.

The status of unmarried women students in the 1870's was somewhat anomalous. To forestall gossip, Sonja at the age of eighteen contracted what was to have been a nominal marriage, left her husband in Russia, and set out for Germany. Her one indiscretion in her dealings with Weierstrass was her neglect to inform him at the beginning that she was married.

Having decided to learn from the master himself, Sonja took her courage in her hands and called on Weierstrass in Berlin. She was twenty, very earnest, very eager, and very determined; he was fifty five, vividly grateful for the lift Gudermann had given him toward becoming a mathematician by taking him on as a pupil, and sympathetically understanding of the ambitions of young people. To hide her trepidation Sonja wore a large and floppy hat, "so that Weierstrass saw nothing of those marvelous eyes whose eloquence, when she wished it none could resist".

Sonja's evident earnestness on her first visit impressed Weierstrass favorably and he wrote to Königsberger inquiring about her mathematical aptitudes. He asked also whether "the lady's personality offers the necessary guarantees". On receiving an enthusiastic reply, Weierstrass tried to get the university senate to admit Sonja to his mathematical lectures. Being brusquely refused he took care of her himself on his own time. Every Sunday afternoon was devoted to teaching Sonja at his house, and once a week Weierstrass returned her visit. After the first few lessons Sonja lost her hat and Weierstrass saw her beautiful eyes immediately. The lessons began in autumn of 1870 and continued with slight interruptions due to vacations or illnesses till the autumn of 1874. He soon came to consider her to be one of his most promising students. For four years she worked very hard with his teacher.

In 1874 she finished three articles, anyone of which, according to Weierstrass, was suitable for a Ph.D. dissertation. The question obviously was where she might receive a Ph.D. degree since it would be hard to receive a degree from the University of Berlin at which she had been denied admission. Weierstrass approached Göttingen, which had in the past awarded degrees in absentia. Finally, after many objections by Göttingen and efforts by Weierstrass, Göttingen awarded her the degree. In the one of her three articles, which was used as a dissertation, she solved a problem in partial differential equations posed by Cauchy. Her solution is now known as the Cauchy-Kovalevsky Theorem.

After taking her degree in absentia from Göttingen in 1874, Sonja returned to Russia for a rest as she was worn, out by excitement and overwork. Her fame had preceded her and she "rested" by plunging into the hectic utilities of a crowded social season in St. Petersburg.

There followed a six-year period in which she did no mathematics; for reasons that are unknown, she returned to Russia where it was impossible for her to teach. She and her husband turned their fictitious marriage into a real one; she developed and followed other interest; she did not answer Weierstrass' letters. There were financial problems; her marriage deteriorated. When Weierstrass came back in Berlin, pulled wires all over Europe trying to get his favorite pupil a position worthy of her talents. His fruitless efforts disgusted him with the narrowness of the orthodox academic mind.

In October 1875, Weierstrass received from Sonja the news that her father had died. In August 1878, he writes to ask whether she ever received a letter he had written her so long before that he has forgotten its date. Sonja now demonstrated what a woman could do in that line when she puts her mind to it. She did not answer her old friend's letter for two years although she knew he had been unhappy and in poor health.

The answer when it did come was rather a letdown. Sonja's sex had got the better of her ambitions and she had been living happily with her husband. In October 1878, Sonja's daughter "Foufie" was born. The forced quiet after Foufie's arrival roused the mother's dormant mathematical interests once more, and she wrote to Weierstrass for technical advice. He replied that he must look up the relevant literature before venturing an opinion.

After three years later she decided to return to mathematics and asked Weierstrass for advice. She soon tired of domesticity, and when P.S. Tchebyscheff invited her to give a paper at the Sixth Congress of Natural Scientists in 1880, she accepted, giving one of the three papers she had written in Berlin. Gösta Mittag-Leffler heard her talk at this Congress, and offered to try to get her a position at the University of Helsinki where he was on the faculty. She decided against this, but did go abroad and began to work on mathematics again, sometimes in Paris, sometimes in Berlin and sometimes in Moscow. She left her daughter with a friend in Berlin.

Sonja's domestic difficulties presently resolved themselves through the sudden death of her husband in March 1883 by suicide, Mittap-Leffler wrote Weierstrass that he had convinced the administration at Stockholm University, where he now was a professor, to allow Kovalevsky to lecture there, unpaid, of course. Finally, in 1884, the University offered her a five-year professorship.

She began working on a problem for which the French Academy of Sciences had offered a prize of 3000 Francs, the Bordin Prize. The problem was that of determining the path of rotation of a solid body around a fixed point, where the path is contained in an ellipsoidal shell. The solutions were judged without knowledge of the identity of the authors. She received the prize, her solution having being so worthy that the prize money was increased to 5000 francs.

In the meantime she became emotionally involved with a Russian philosopher who was living in France. He proposed marriage on the condition that she gives up her work. She refused. In 1889, at the end of her five-year professorship at Stockholm University, she was made professor for life; in today's parlance, she received tenure. On return from one of her visits to her philosopher friend, she contracted influenza and died in Stockholm at the age of 41 and at the height of her mathematical powers. Shortly before, she had been elected to the St. Petersburg Academy of Sciences.

EMMY NOETHER (1882-1935)

In the year in which Ladd Franklin should have received her Ph.D., Emmy Noether (1882-1935) was born in Germany. Her father was the mathematician Max Noether a professor at the University of Erlangen. Until she was 18 she seems to have followed the usual pattern for daughters of the bourgeoisie in Erlangen; she attended the state Girls' School, learned English and French, and took the Bavarian state examinations for certification as a teacher of those languages, should she ever need to earn her livelihood. She had first mathematics education from her father. It was soon after this that she decided she wanted to attend the University and to study mathematics. There are apparently no records left which might show why this young woman suddenly wanted to change the routine of her life and become a mathematician. At that time women were allowed to audit courses at the universities, providing they received permission from the professors. On this basis Noether audited courses at the University of Erlangen from 1900 to 1902. However, she wanted to enroll at the University as a regular student, which was, in general, not allowed. In fact, at that time women could not attend the Gymnasium, which prepared male students for admission to the universities. But there was a loophole. Students could take a matriculation examination for admission to the university without having attended the Gymnasium. Noether took and passed this examination. Finally, she was permitted to enroll at Erlangen and in July 1908 she was awarded her degree.

Emmy Noether had little in common with the legendary "female mathematician" Sonja Kowalewski, who had bewitched even Weierstrass with her young charms as well as her mind. Noether was not even feminine in her appearance or manner. This is the first thing even today, that the men who knew her recall. "She had a loud and disagreeable voice, she looked like an energetic and very nearsighted washerwoman. Her clothes were always baggy". And they still quote with delight the gentle remark of Hemann Weyl that "the groces did not preside at her cradle." But she was to be much more important to mathematics than the bewitching Sonja. Even at this time, she had an impressive knowledge of certain subjects which Hilbert and Klein needed for their work on relativity theory, and they were both determined that she must stay in Göttingen. But in spite of the fact that Göttingen had been the first university in Germany to grand a doctoral degree to a woman, it was still not an easy matter to obtain habilitation for one. The entire Philosophical Faculty, which included philosophers, philologists and historians as well as natural scientists and mathematicians, had to vote on the acceptance of the habilitation thesis. Particular opposition came from the non-mathematical members of the Faculty.

They argued formally: "How can it be allowed that a woman become a privatdozent? Having become a privatdozent, she can then become a professor and a member of the University Senate. Is it permitted that a woman enter the Senate?" They argued informally, "What will our soldiers think when they return to the University and find that they are expected to learn at the feet of a woman?"

Hilbert had heard what to him were similarly irrelevant arguments when he had been attempting to have Gromme's dissertation approved by the same faculty members. "If students without the gymnasium diploma will always write such dissertations as Grommer's," he had told them, "it will be necessary to make a law forbidding the taking of the examination for the diploma." Now he answered their formal argument against habilitating Emmy Noether with equal directness:

"Meine Herren, I do not see that the sex of the candidate is an argument against her admission as a Privatdozent. After all, the Senate is not a bath-house".

When, in spite of this rejoinder, he still could not obtain her habilitation, he solved the problem of keeping her at Göttingen in his own way. Lectures would be announced under the name of Professor Hilbert, but delivered by Fräulein Noether. She was delivering the lectures in Hilbert's name and with his support because, since she was a woman, she had not been permitted to become a Privatdozent.

Noether's Ph.D. dissertation, "On complete systems of invariants for ternary biquadratic forms," was written under the direction of Paul Gordan. From 1908 until 1915 she stayed in Erlangen, without a position, doing research in mathematics, giving invited lectures, publishing papers and toward the end of his life, substituting for her father at the University when he was ill. During this time she was beginning to be recognized as a mathematician of the rank. In 1915 Felix Klein and David Hilbert invited her to come to Göttingen, which she did, staying until 1933 when she was forced to leave by the Nazis. Klein and Hilbert were particularly interested in her work on invariant theory for its usefulness in general relativity theory, on which they were working at the time. Yet however liberal Göttingen may have been in 1915 when compared to other German universities, it was still not ready for a woman faculty member. Despite the efforts of Hilbert and Klein, she was not granted a position which carried any stipend until 1923 when she was given a Lectureship in Algebra with a minimal stipend.

In the years between 1915 and 1923, she lectured at Göttingen, not under her own name, but under Hilbert's name. In her first year at Göttingen her research continued in invariant theory, resulting in two papers on differential invariants, which are still used today. Noether's axiomatic approach to the study of abstract rings and ideal theory, for which she is best known, first appeared in a paper, written jointly with W. Schmeidler, in 1920. P.S. Alexandroff, the Russian mathematician, in his address to the Moscow Mathematical Society in 1935 at the time of her death said: Emmy Noether entered upon her wholly individual path of mathematical work in 1919-1920… This work with W. Schmeidler serves as a prologue to her general theory of ideals, opening with the classical memoir of 1921, "Idealtheorie in Ringbereichen." I think that of all that Emmy Noether did, the bases of the general theory of ideals and all the work related to them have exerted, and will continue to exert, the greatest influence on mathematics as a whole... She taught us just to think in simple, and thus general, terms-homomorphic representation, the group or ring with operators, the ideal- and not in complicated algebraic calculations, and she therefore opened a path to the discovery of algebraic regularities where before these regularities had been obscured by complicated specific conditions.

In her work Noether stressed the use of chain conditions. It is because of her work that rings with the ascending chain condition on ideals are now known as Noetherian rings.

To quote Alexandroff again: From 1927 the influence of the ideas of Emmy Noether on contemporary mathematics continually grew, and along with it grew scientific praise for the author of those ideas. The direction of her work at this time moved more and more into the region of noncommutative algebra, the theory of representation and of the general arithmetic of hypercomplex areas...

Emmy Noether at last received recognition for her ideas. If in the years 1923-1925 she had to demonstrate the importance of the theories that she had developed, in 1932, at the International Mathematical Congress in Zürich. She was crowned with the laurel of her success. A summary of her work read by her at this gathering was the triumph of the direction she represented...

It is interesting to note that both Noether and Germain did their most significant work relatively late for a mathematician.

According to Hermann Weyl, who became a member of the Göttingen faculty in 1930, Noether was "the strongest center of mathematics activity" at Göttingen from 1930 to 1933, "considering both the fertility of her scientific research program and her influence upon a large circle of pupils." Many of her younger colleagues, among them Weyl, recognized the injustice done to her in her lack of a suitable position, and tried to get a better position for her. In 1935 Weyl wrote: When I was called permanently to Göttingen in 1930, I earnestly tried to obtain from the Ministerium a better position for her, because I was ashamed to occupy such a preferred position beside her whom I knew to be my superior as a mathematician in many respects. I did not succeed. Tradition, prejudice, external considerations, weighted the balance against her scientific merits and scientific greatness, by that time denied by no one.

Emmy Noether was not a full professor, but she contributed importantly to the mathematical atmosphere of Göttingen during this period. She and her students, few in number and many of them foreign, represented the trend toward abstraction and generalization which was to become more and more dominant in mathematics during the coming years.

She was a very poor lecturer, writing on the board and wiping almost immediately what she had written. She spoke quickly and sometimes condensed many syllables into one or two. To Friedrichs it seemed that her speaking never quite caught up with her thinking. "I have no doubt she had a very clear understanding of what she was saying," Hans Lewy said, "but she did not have a clear idea of what she was going to say." She was devoted to her students, who come to her with a11 their problems, personal as well as mathematical. She was especially popular with the Russian visitors; and when they began to go around Göttingen in their shirtsleeves-a startling departure from proper dress for students- the style was christened the Noether- guard uniform.

The leading woman mathematician of our own times, Emmy Noether also came from Göttingen. The word "came from" is right. When the sagacious Nazis expelled Fräulein Noether from Germany because she was a Jewess, Bryn Mawr College, Pennsylvania, took her in. She was the most creative abstract algebraist in the world. In less than a week of the new German enlightenment, Göttingen lost the liberality which Gauss cherished and which he strove all his life to maintain.

One of the most fertile circles of research in post-war Göttingen revolved around Emmy Noether. The desired position of Privatdozent had at last been obtained for her in 1919. This was still the lowest possible rank on the university scale, not a job but a privilege. But Emmy Noether was delighted with the appointment. In the thirteen years that had passed, she had had to defend her doctoral dissertation before Gordan; she had come a long way. Already she had achieved important results in differential invariants, which the Soviet mathematician Paul Alexandroff was to consider sufficient to secure her a reputation as a first-rate mathematician, "hardly less a contribution to mathematical science than the notable researches of Sonja." She herself was always to dismiss these works as standing to the side of her main scientific path, on which at last she was now at the age of 39, taking her first step, the building up on an axiomatic basis of a completely general theory of ideals. This work would have it source in the early algebraic work of Hilbert, but in her hands the axiomatic method would become no longer "merely a method for logical clarification and deepening of the foundations as it was with Hilbert, but a powerful weapon of concrete mathematical research."

In 1922 she became an associate professor. There were no obligations connected with this now title and no salary, such an extraordinary professor being consider more than usual inferior to an ordinary professor. Only a Göttingen saying to the effect "an extraordinary professor knows nothing ordinary and an ordinary professor knows nothing extraordinary" could explain the title.

She and her work were not on the whole much admired in her native land. She was never even elected to the Göttingen Scientific Society. "It is time that we begin to elect some people of real stature to this society," Hilbert once remarked at a meeting "Now, how many people of stature have we indeed elected in the past few years?" He looked thoughtfully around at the members. "Only zero," he said at last. "Only zero!"

I said she was not a good lecturer and her classes usually numbered no more than five or ten. Once though, she arrived at the appointed hour to find more than a hundred students waiting for her. "You must have the wrong class," she told them. But they began the traditional noisy shuffling of the feet, which, in lieu of clapping, preceded and ended each university class. So she went ahead and delivered her lecture to this unusually large number of students. When she finished, a note was passed up to her by one of her regular students who were in the group. "The visitors," it read, "have understood the lecture just as well as any of the regular students."

It was true she had no pedagogical talents. Her mind was open only to those who were in sympathy with it. Her teaching approach, like her thinking, was wholly conceptual. But of all the new generation in Göttingen, Emmy Noether was to have the greatest influence on the course of mathematics.

1933 with the Nazis in power in Germany, Noether, a Jew, was dismissed from her post at Göttingen. Alexandroff attempt to get her an appointment at the University of Moscow, but Russian red tape was slow and, when Bryn Mawr offered her a one-year visiting appointment she accepted it. From Bryn Mawr it was an easy trip to the Institute for Advanced Study and she soon began to give weekly lectures. At the age of 51 this remarkable mathematician held her first position at what might be called a "normal" salary. It is to the great credit of Anna Pell Wheeler, Chairman of the Mathematics Department at Bryn Mawr College at the time, and to the College itself, that Noether was invited to this country. In the second year at Bryn Mawr she entered the hospital for a routine operation to remove a tumor. She seemed to be recovering well, when suddenly she died, still at the bight of her mathematical powers.

REFERENCES

1. Encyclopedia Britannica, (1988), Encyclopedia Britannica Inc., 22 Vols, New York.

2. KATZ, W. J. A (1998), History of Mathematics, Addison-Wesley Publishers Inc., New York.

3. Lexicon Universal Encyclopedia (1989), Lexicon Publications, Inc., 20 Vols, New York.