



## OPTIMUM DESIGN OF GEODESIC STEEL DOMES UNDER CODE PROVISIONS USING METAHEURISTIC TECHNIQUES

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### Abstract

*Metaheuristic search techniques strongly employ randomized decisions while searching for solutions to structural optimization problems. These techniques play an increasingly important role for practically solving hard combinatorial problems from various domains. Over the past few years there has been considerable success in developing metaheuristic search algorithms as well as randomized systematic search methods for obtaining solutions to discrete programming problems. This paper examines minimum weight design of pin-jointed geodesic steel domes using seven metaheuristic search techniques; namely, simulated annealing, genetic algorithms, evolution strategies, particle swarm optimizer, tabu search, ant colony optimization and harmony search methods. The optimum design problem of geodesic steel domes is formulated according to design limitations stipulated by ASD-AISC (Allowable Stress Design Code of American Institute of Steel Institution). The minimum design loads and combined load effects are established as specified by ASCE 7-98 (Minimum Design Loads for Buildings and Other Structures, American Society of Civil Engineers). A numerical example is presented, where seven metaheuristic methods are implemented to achieve minimum weight design of a 130-member geodesic steel dome subjected to a total of eight combined load cases of dead, live, snow and temperature loads.*

**Keywords:** structural optimization, discrete optimization, metaheuristic search techniques, minimum weight design, geodesic steel domes.

### 1. Introduction

The field of structural optimization is a relatively new area undergoing rapid changes in methods and focus. Until recently there was a severe imbalance between enormous amount of literature on the subject and paucity of applications to practical design problems. There is still no shortage of new publications, but there are also exciting applications of the methods of structural optimizations in the civil engineering, machine design and other engineering fields. As a result of the growing pace of applications, research into structural optimization methods is increasingly driven by real-life problems.

Structural optimization when first emerged has attracted a widespread attention among designers. It has provided a systematic solution to age-old structural design problems which were handled by using trial-error methods or engineering intuition or both. Structural optimization provides tools for structural designers to determine the optimum topology or the optimum geometry and/or optimum cross sectional dimensions for the members of a structure. Application of mathematical programming methods to structural design problems has paved the way in developing a design procedure which was capable of producing structures with cross-sectional dimensions [1]. In the last four decades vast amount of research work has been conducted in structural optimization which covers the field from

optimum design of individual elements to rigid frames and finite element structures. However, due to the fact that mathematical programming techniques deal with continuous design variables, the algorithms developed has provided to designer cross-sectional dimensions that were neither standard nor practical [2-4]. The reality of the practice is that there are certain steel sections produced by steel mills that are available for a designer to choose from in the case of steel structures and there are practically accepted dimensions for the beams and columns among which the selection can be carried out in a reinforced concrete structure due to architectural reasons. Hence, the structural designer finds himself/herself in a restricted area where only discrete values are available when it comes to make a decision what sections he/she has to select for the members of a steel or reinforced concrete frame. Consequently, the discrete structural optimization algorithms developed thus far utilizing the mathematical programming methods has not found widespread applications in practical design of structures. Naturally this has led the researches to seek better algorithms for the solution of discrete optimum design problems.

The recent novel and innovative metaheuristic search techniques emerged make use of ideas inspired from the nature and they do not suffer the discrepancies of mathematical programming based optimum design methods. The basic idea behind these techniques is to simulate the natural phenomena, such as survival of the fittest, immune system, swarm intelligence and the cooling process of molten metals through annealing into a numerical algorithm [5-13]. These methods are non-traditional stochastic search and optimization methods, and they are very suitable and effective in finding the solution of combinatorial optimization problems. They do not require the gradient information of the objective function and constraints and they use probabilistic transition rules not deterministic ones. The optimum structural design algorithms that are based on these techniques are robust and quite effective in finding the solution of discrete programming problems. There are large numbers of such metaheuristic techniques available in the literature nowadays. A detailed review of these algorithms as well as their applications in the optimum structural design is carried out in Saka [14].

In the present study, the optimum design of pin-jointed geodesic steel dome structures is investigated using seven metaheuristic search techniques; namely simulated annealing [15], evolution strategies [16], particle swarm optimizer [17], tabu search method [18], ant colony optimization [19], harmony search method [20] and simple genetic algorithm [21]. Domes are arched shaped space structures that may be built in different patterns (such as geodesic, lamella, schwedler, grid, ribbed, etc.) using one or more layers of elements. These systems offer very economical and viable solutions for covering large spaces, especially when no internal supports (such as column, stud or cable) are preferable in the structural design. In the paper the optimum design problem of geodesic steel domes is formulated, where design limitations including strength and serviceability requirements are imposed according to ASD-AISC [22] specification. The design loads and combined load effects for these systems are computed according to the minimum load requirements as specified by ASCE 7-98 [23]. A single numerical example is demonstrated, where a 130-member geodesic steel dome with eight member groups (design variables) is sized for minimum weight using standard pipe sections by conducting three independent runs with each of the seven optimization techniques abovementioned. The convergence rates and reliabilities of the techniques in attaining the optimum design of the structure are compared, and the results are discussed extensively.

## **2. Optimum Design Problem Of Geodesic Steel Domes**

The design of steel dome structures requires the selection of members from a standard steel pipe section table such that the dome satisfies the strength and serviceability requirements

specified by a chosen code of practice, while the economy is observed in the overall or material cost of the dome. For a pin-jointed geodesic steel dome which consists of  $N_m$  members grouped into  $N_d$  design variables, this problem can be formulated as follows.

## 2.1. Objective Function

Find a vector of integer values  $\mathbf{I}$  (Eqn. 1) representing the sequence numbers of standard sections in a given section table

$$\mathbf{I}^T = [I_1, I_2, \dots, I_{N_d}] \quad (1)$$

to generate a vector of cross-sectional areas  $\mathbf{A}$  for  $N_m$  members of the dome (Eqn. 2)

$$\mathbf{A}^T = [A_1, A_2, \dots, A_{N_m}] \quad (2)$$

such that  $\mathbf{A}$  minimizes the objective function

$$W = \sum_{m=1}^{N_m} r_m L_m A_m \quad (3)$$

where  $W$  refers to the weight of the dome;  $A_m$ ,  $L_m$ ,  $r_m$  are cross-sectional area, length and unit weight of the  $m$ -th dome member, respectively.

## 2.2. Design Constraints

The structural behavioral and performance limitations of pin-jointed geodesic steel domes can be formulated as follows:

$$g_m = \frac{S_m}{(S_m)_{all}} - 1 \leq 0, \quad m = 1, \dots, N_m \quad (4)$$

$$s_m = \frac{I_m}{(I_m)_{all}} - 1 \leq 0, \quad m = 1, \dots, N_m \quad (5)$$

$$d_{j,k} = \frac{d_{j,k}}{(d_{j,k})_{all}} - 1 \leq 0, \quad j = 1, \dots, N_j \quad (6)$$

In Eqns. (4-6), the functions  $g_m$ ,  $s_m$  and  $d_{j,k}$  are referred to as constraints being bounds on stresses, slenderness ratios and displacements, respectively;  $S_m$  and  $(S_m)_{all}$  are the computed and allowable axial stresses for the  $m$ -th member, respectively;  $I_m$  and  $(I_m)_{all}$  are the slenderness ratio and its upper limit for  $m$ -th member, respectively;  $N_j$  is the total number of joints; and finally  $d_{j,k}$  and  $(d_{j,k})_{all}$  are the displacements computed in the  $k$ -th direction of the  $j$ -th joint and its permissible value, respectively. In the present study, these limitations are implemented according to ASD-AISC [22] code provisions.

Accordingly, the maximum slenderness ratio is limited to 300 for tension members, and it is taken as 200 for compression members. Hence, the slenderness related design constraints are formulated as follows:

$$I_m = \frac{K_m L_m}{r_m} \leq 300 \text{ (for tension members)}$$

$$I_m = \frac{K_m L_m}{r_m} \leq 200 \text{ (for compression members)}$$
(7)

where,  $K_m$  is the effective length factor of  $m$ -th member ( $K_m = 1$  for all members), and  $r_m$  is its minimum radii of gyration.

The allowable tensile stresses for tension members are calculated as in Eqn. (8):

$$(s_t)_{all} = 0.60F_y$$

$$(s_t)_{all} = 0.50F_u$$
(8)

where  $F_y$  and  $F_u$  stand for the yield and ultimate tensile strengths, and the smaller of the two formulas is considered to be the upper level of axial stress for a tension member.

The allowable stress limits for compression members are calculated depending on two possible failure modes of the members known as elastic and inelastic buckling, Eqns. (9-11).

$$C_c = \sqrt{\frac{2p^2 E}{F_y}}$$
(9)

$$(s_c)_{all} = \frac{\left[1 - \frac{(K_m L_m / r_m)^2}{2C_c^2}\right] F_y}{\frac{5}{3} + \frac{3(K_m L_m / r_m)}{8C_c} - \frac{(K_m L_m / r_m)^3}{8C_c^3}}, \quad I_m < C_c \text{ (inelastic buckling)}$$
(10)

$$(s_c)_{all} = \frac{12p^2 E}{23(K_m L_m / r_m)^2}, \quad I_m \geq C_c \text{ (elastic buckling)}$$
(11)

In Eqns. (9-11),  $E$  is the modulus of elasticity, and  $C_c$  is referred to as the critical slenderness ratio parameter. For a member with  $I_m < C_c$ , it is assumed that the member buckles inelastically, and its allowable compression stress is computed according to Eqn. (10). Otherwise ( $I_m \geq C_c$ ), elastic buckling of the member takes place, in which case the allowable compression stress is computed as to Eqn. (11).

### 3. Design Loads and Combinations

For the design of structural systems, it is assumed that the structures are exposed to various gravity (e.g., dead, snow) and lateral (e.g., wind, earthquake) loads during their service life.

The structures must be proportioned to safely accommodate these loads or their combined effects without any failure, cracking or excessive deformation. Amongst a variety of different loadings, the most critical load cases that must be strictly considered in the design of dome structures appear to be dead, snow, wind as well as temperature induced loads. In the study, the minimum values for these loads are computed according to the provisions of ASCE 7-98 [23], which is explained in the following subsections.

### 3.1. Dead Load

The dead load includes the weight of the members, joints, cladding and other components of domes acting with gravity on the foundations below.

### 3.2. Snow Load

In ASCE 7-98 [23], snow loads are categorized into three groups as ground snow loads, flat-roof snow loads and sloped-roof snow loads. Because of the arched shape geometry of a dome structure, the sloped-roof snow load values are adopted here and the design snow load  $p_f$  is computed using the following equation in ASCE 7-98 [23]:

$$p_f = 0.7C_s C_e C_t I p_g \quad (12)$$

where  $C_s$  is the roof slope factor,  $C_e$  is the exposure coefficient,  $C_t$  is the temperature factor,  $I$  is the importance factor, and  $p_g$  is the ground snow load.

### 3.3. Wind Load

ASCE 7-98 [23] recommends three different approaches for calculation of wind loads referred to as (i) simplified procedure, (ii) analytical procedure and (iii) wind tunnel procedure. In the present study, wind loads acting on dome structures are computed in accordance with the analytical procedure. In this approach, the velocity pressure is first computed using the following equation in the specification

$$q_h = 0.613K_z K_{zt} K_d V^2 I \quad (13)$$

where  $q_h$  (in  $\text{N/m}^2$ ) is the velocity pressure evaluated at mean roof height,  $K_z$  is the velocity exposure coefficient,  $K_{zt}$  is the topographic factor,  $K_d$  is the wind directionality factor,  $V$  (in  $\text{m/s}$ ) is the basic wind speed, and  $I$  is the importance factor.

Next, the design wind pressure is computed considering a combined effect of internal and external pressures acting on the roof, as follows:

$$p_w = q_h G C_p - q_h (G C_{pi}) \quad (14)$$

where  $p_w$  is the design wind pressure,  $G$  is the gust effect factor (taken as 0.85),  $C_p$  is the external pressure coefficient, and  $(G C_{pi})$  is the internal pressure coefficient. The first term in Eqn. (15) considers the effect of external pressure, whereas the second term accounts for the effect of internal pressure.

### **3.4. Temperature Changes**

It is known that temperature changes may create significant additional axial stresses in the members. Hence it is essential to take into account the effects of temperature induced loads by applying a positive and negative change in temperature of the entire dome system.

### **3.5. Load Combinations**

For the design purpose, the dome is subjected to a total of eight combined load cases considering various combinations of dead load (DL), snow load (SL), wind load with external pressure (WEP), wind load with positive internal pressure (WIP), wind load with negative internal pressure (WIN), positive temperature change (TP) and negative temperature change (TN). These combined loads cases are listed below. It is important to highlight that some of the combined load effects are reduced by combination factors in compliance with allowable stress design requirements.

- (i) DL + SL
- (ii) DL + TP
- (iii) DL + TN
- (iv) D + WEP + WIP
- (v) D + WEP + WIN
- (vi)  $D + 0.75 (WE + WIP) + 0.75SB$
- (vii)  $D + 0.75 (WE + WIN) + 0.75SB$
- (viii)  $D + 0.75TN + 0.75SLB$

## **4. Metaheuristic Search Techniques: An Overview**

A combinatorial optimization problem requires exhaustive search and effort to determine an optimum solution which is computationally expensive and in some cases may even not be practically possible. Metaheuristic search techniques are established to make this search within computationally acceptable time period. Amongst these techniques are simulated annealing (SA), evolution strategies (ESs), particle swarm optimizer (PSO), tabu search method (TS), ant colony optimization (ACO), harmony search method (HS), genetic algorithms (GAs) and others. All of these techniques implement particular metaheuristic search algorithms that are developed based on simulation of a natural phenomenon into numerical optimization procedure. They have gained a worldwide popularity recently and have proved to be quite robust and effective methods for finding solutions to discrete programming problems in many disciplines of science and engineering, including structural optimization.

Simulated annealing (SA), which is a well-known member of metaheuristic search techniques, searches for minimum energy states using an analogy based upon the physical annealing process. In this process, a solid initially at a high energy level is cooled down gradually to reach its minimum energy and thus to regain proper crystal structure with perfect lattices. The idea that this process can be simulated to solve optimization problems was pioneered independently by Kirkpatrick et al. [15] and Cerny [24], establishing a direct analogy between minimizing the energy level of a physical system and lowering the cost of an objective function. Successful applications of SA in discrete structural optimization problems have been reported in a number of early works in the literature, such as Refs. [25-27]. The

enhancement of the technique is accomplished in some recent publications, such as Refs. [28, 29] for accelerating its search capability in complex design domains.

The most well known stream of evolutionary algorithms is genetic algorithms (GAs), which have been initially pioneered by Holland [30]. These algorithms are based on the evolutionary ideas of natural selection and genetics mechanism. The first application of the technique in optimum structural design is presented by Goldberg and Samtani [31], where the weight minimization of the classical 10-bar truss is accomplished with GAs. Today, many variations and extensions of the technique have been proposed, and successful applications of the technique are accumulated in a vast amount of discrete and continuous optimization literature [32-36]. In the present study, a genetic algorithm with standard components referred to as simple genetic algorithm (SGA) has been implemented due to its generality and wide acceptability.

Evolution strategies (ESs) are another promising representative of evolutionary algorithms. The fundamentals of the technique were originally laid in the pioneering studies of Rechenberg [16]. They were first developed in a rather simple form known as  $(1+1)$ -ES that implements on the basis of two designs; a parent and an offspring individual. Today, the modern variants of ESs are accepted as  $(m+1)$ -ES and  $(m, I)$ -ES, which were developed by Schwefel [37]. Both variants employ design populations consisting of  $m$  parent and  $I$  offspring individuals, and are intended to carry out a self-adaptive search in continuous design spaces. The extensions of these variants to solve discrete optimization problems were put forward in Refs. [38-40]. A literature survey turns up several publications reporting a very successful use of this method in discrete optimum design of structural systems [41, 42].

Particle swarm optimization is a population based metaheuristic search technique inspired by social behavior of bird flocking or fish schooling. This behaviour is concerned with grouping by social forces that depend on both the memory of each individual as well as the knowledge gained by the swarm [17, 43]. The procedure involves a number of particles which represent the swarm being initialized randomly in the search space of an objective function. Each particle in the swarm represents a candidate solution of the optimum design problem. The particles fly through the search space and their positions are updated using the current position, a velocity vector and a time step. The successful applications of this technique have also been reported in the field of structural optimization, especially in size/shape optimum design of skeletal structures. Amongst some recent applications are Perez and Behdinan [44], He et al. [45] and Fourie and Groenwold [46].

Tabu search (TS) is another metaheuristic method, which was first developed by Glover [18]. The method implements a simple yet an efficient iterative based local search strategy for solving combinatorial optimization problems. At each step a number of candidate solutions are sampled in the close vicinity of the current design by perturbing a single design variable called a move. The best candidate is chosen and replaced with the current design even if it offers a non-improving solution, and the move leading to this candidate is recognized as a successful move. To protect the search against cycling within the same subset of solutions, information regarding most recently visited solutions is collected in a list referred to as tabu list. A candidate is allowed to replace the current design provided that its move is not in tabu list; otherwise the search is preceded with the current solution. The method has been mostly employed for weight minimization of structural systems in the literature, such as Bland [47], and Kargahi et al. [48].

Ant colony optimization technique is inspired from the way that ant colonies find the shortest route between the food source and their nest. Ants being completely blind individuals can successfully discover as a colony the shortest path between their nest and the food source. They manage this through their characteristic of employing a volatile substance called pheromones. When finding food, the ants deposit pheromones on the ground while traveling,

which is used by other ants in the colony as a guide to find the food sources. Ant colony optimization was developed by Colorni et al. [49] and Dorigo [50] and used in the solution of traveling salesman problem. The optimum structural design applications of the technique have been presented in Camp et al. [51], Aydoğdu and Saka [52].

Harmony search method is based on natural musical performance processes that occur when a musician searches for a better state of harmony. The resemblance, for example between jazz improvisation that seeks to find musically pleasing harmony and the optimization is that the optimum design process seeks to find the optimum solution as determined by the objective function. The pitch of each musical instrument determines the aesthetic quality just as the objective function is determined by the set of values assigned to each design variable. The recent applications of HS algorithm in structural optimization reveal that it is a very powerful technique for relatively small-to-medium scale discrete optimization problems [53-56]. An enhancement of the technique is proposed in Hasançebi et al. [57] for larger scale problems, where an adaptive change of its parameters is facilitated for establishing the most advantageous search automatically by the algorithm.

It should be underlined that there is no a unique formulation or a standardized algorithm used to implement any of the metaheuristic search techniques mentioned above. Rather, each technique has been devised in various algorithmic forms and has numerous extensions and modifications. In this study the algorithms to implement the techniques are selected on the basis of their generalities and reported performances in the published literature. The implementation specifics and detailed outlines of these algorithms can be found in Hasançebi et al. [58] with complete parameter sets and enhancements proposed to accelerate their performances.

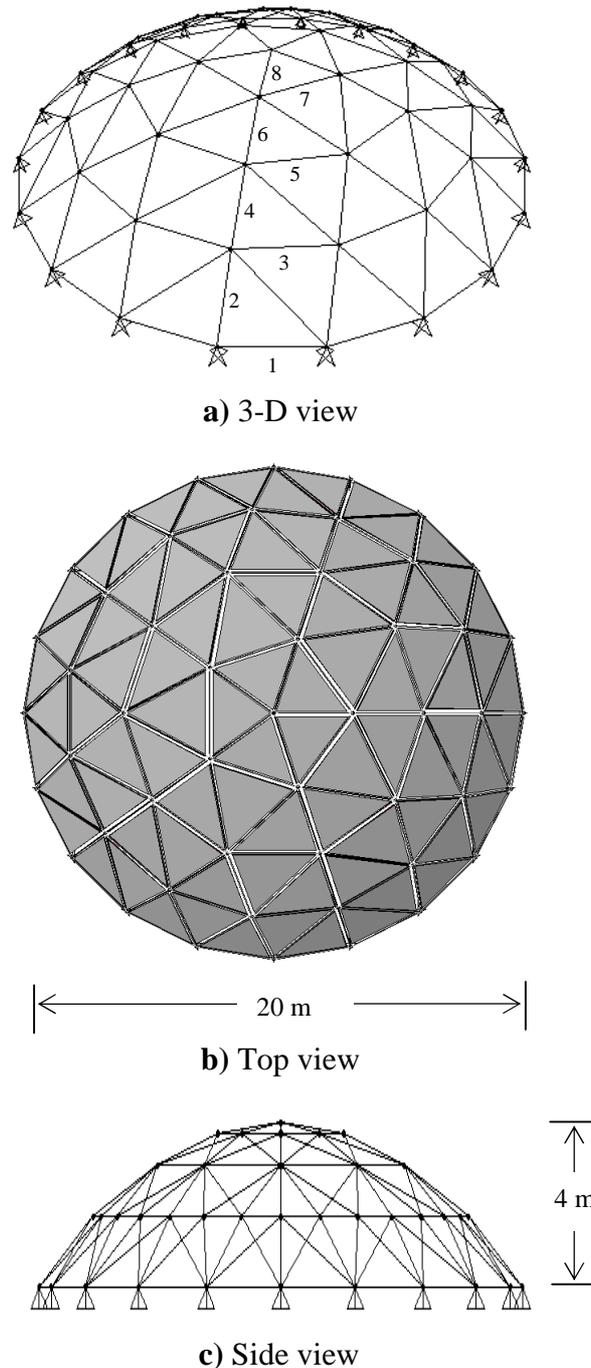
## 5. Numerical Example

Figure 1 shows plan, elevation and 3-D views of a geodesic steel dome with a base diameter of 20 m (65.6 ft) and a total height of 4 m (13.1 ft). The structure consists of 51 joints and 130 members that are grouped into 8 independent size design variables. The size variables are to be selected from a database of 37 pipe (circular hollow) sections issued in ASD-AISC [22] standard section tables. The stress and stability limitations of the members are calculated according to the provisions of ASD-AISC [22], as explained in Section 2. The displacements of all nodes are limited to 5.55 cm (2.18 in) in any direction. The following material properties of the steel are used: modulus of elasticity ( $E$ ) = 29000ksi (203893.6MPa) and yield stress ( $F_y$ ) = 36ksi (253.1MPa).

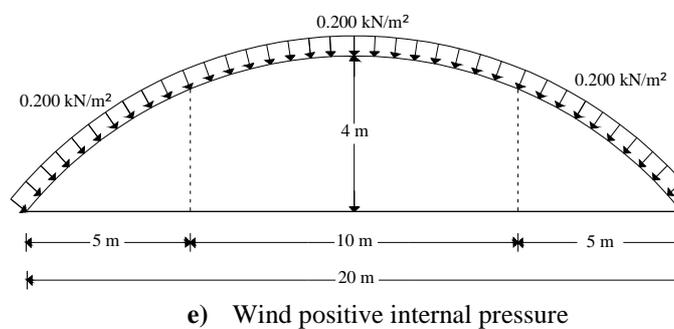
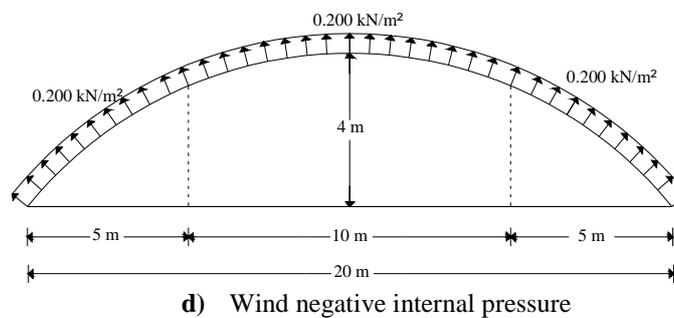
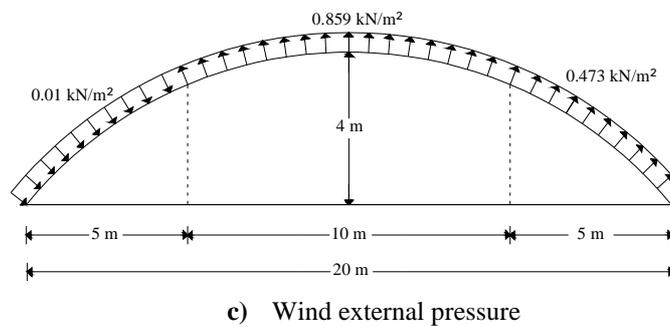
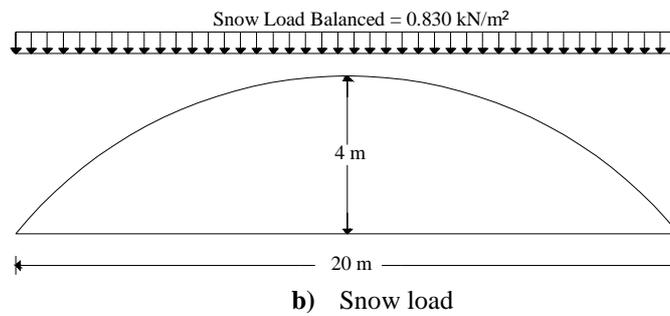
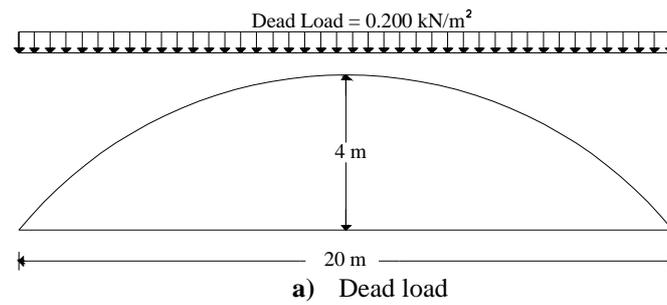
The design loads and combined loads effects are applied on the dome as explicated in Section 3. A sandwich type aluminum cladding material is used to cover the dome surface, resulting in a uniform dead load pressure of 200 N/m<sup>2</sup>, including the frame elements used for the girts (Figure 2a). The design snow load is computed by using the following parameter values in Equation (12):  $C_s = 1.0$ ,  $C_e = 0.9$ ,  $C_t = 1.0$ ,  $I = 1.1$  and  $p_g = 1.1975\text{kN/m}^2$  (25.0lb/ft<sup>2</sup>), which results in a uniform design snow pressure of  $p_f = 830\text{ N/m}^2$  (17.325 lb/ft<sup>2</sup>) as displayed in Figure 2(b).

The design wind load is calculated on the basis of an assumed basic wind speed of  $V = 40\text{ m/s}$  (90mph), and the other quantities in Equation (13) are set to the following parameter values:  $K_z = 1.07$ ,  $K_d = 0.85$ ,  $K_{zt} = 1.087$ ,  $I = 1.15$ , and  $V = 40\text{ m/s}$  (90mph), resulting in a velocity pressure of  $q_h = 1.115\text{ kN/m}^2$  (23.285 lb/ft<sup>2</sup>). To calculate external wind pressure by Equation (14), the dome is divided into three parts; a windward quarter, a

centre half and a leeward quarter as recommended by ASCE 7-98 [23]. The external pressure coefficient  $C_p$  is then calculated for each part considering rise-to-span ratio of the dome, as follows:  $C_p = 0.0105$  for windward quarter,  $C_p = -0.907$  for centre half and  $C_p = -0.5$  for leeward quarter (Figure 2c). On the other hand,  $GC_{pi}$  is taken as  $-0.18$  and  $+0.18$  in the second and third load cases over the entire internal surface to take into account the suction and uplift effects of the internal pressure, respectively (Figures 2d and e). The net pressure acting on different parts of the dome is obtained by combining internal and external wind pressures as per Eqn. (14). Finally, the effects of temperature induced loads are taken into account by applying  $\pm 20$  degree change in the temperature of entire dome system.



**Figure 1.** 130-member geodesic steel dome a) 3D view b) top view c) side view.



**Figure 2.** Loads acting on 130-member geodesic steel dome **a.** dead load, **b.** snow load, **c.** wind external pressure, **d.** wind negative internal pressure and **e.** wind positive internal pressure.

The dead and snow loads represent gravity forces that can easily be applied on the joints of the dome. Nevertheless, accurate representation of wind loads in structural model is somewhat more difficult since these forces must act perpendicular to dome surface, whereas the surface angle vary from point to point due to arched shape of the system. To be able to apply wind forces as accurately as possible, the triangular areas between truss members on dome surface are modeled using weightless shell elements as depicted in Figure 1(b), and the wind forces are acted on these mesh elements as surface pressure loads. During the analysis of the dome, these surface loads are transmitted to joints as point loads and the displacement and force response calculations are obtained accordingly.

The minimum weight design of the dome is sought by conducting three independent runs with each of the seven optimization techniques due to their stochastic natures. The number of structural analyses is taken as 10,000 in each run and for each technique to make sure that all the techniques are given the equal opportunity to grasp the global optimum, and that it is not a restraint for not being able to reach the global optimum. The results are tabulated in Table 1. The second through fourth rows in this table represent the minimum feasible weight designs attained by each technique in its three runs, and the averages of these three runs are displaced in the last row to indicate overall performances of the techniques.

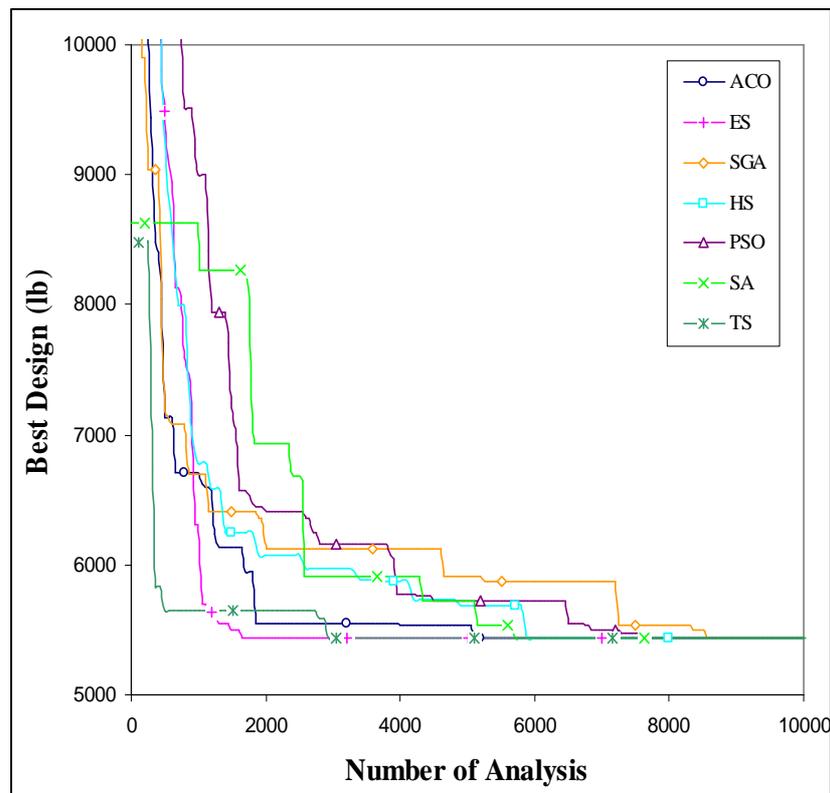
**Table 1.** The minimum weight designs (in lb) obtained for 130-member geodesic steel dome using metaheuristic search techniques.

Designs of three different runs	SA	ESs	PSO	HS	TS	ACO	SGA
First Run	5438.97	5438.97	5438.97	5438.97	5438.97	5608.19	5787.28
Second Run	5438.97	5438.97	5438.97	5475.46	5495.03	5438.97	5438.97
Third Run	5438.97	5438.97	5438.97	5438.97	5438.97	5438.97	5968.03
Average of Three Runs	<b>5438.97</b>	<b>5438.97</b>	<b>5438.97</b>	5451.13	5457.66	5495.38	5731.42

**Table 2.** The optimum design of 130-member geodesic steel dome.

Size Variable	Ready Section	Area, in <sup>2</sup> (cm <sup>2</sup> )
1	P2.5	1.70 (10.97)
2	P2	1.07 (6.90)
3	P2	1.07 (6.90)
4	P2	1.07 (6.90)
5	P2	1.07 (6.90)
6	P2	1.07 (6.90)
7	P2	1.07 (6.90)
8	P1.5	0.799 (5.16)
Weight		5438.97 lb (2467.05 kg)

It is observed from Table 1 that SA, ESs and PSO techniques located the same solution with a design weight of 5438.97 lb (2467.05 kg) in each of their three runs. This solution is given in Table 2 with pipe section designations attained for each size design variable, and is considered to be the optimum solution of the problem reached in the present study. The fact that the same solution is attained in each run by SA, ESs and PSO evinces high convergence reliabilities of these techniques in optimum structural design applications. On the other hand, HS, TS and ACO techniques have identified the optimum solution two times out of three runs, indicating that these techniques are more susceptible to performance variations associated with stochastic behaviour. As compared to other techniques, SGA has exhibited a substandard performance, being able to identify the optimum solution only once and relatively poorer solutions are produced by the algorithm at other two times.



**Figure 3.** The design history graph with metaheuristic search techniques obtained for 130-member geodesic steel dome.

The design history graph is plotted in Figure 3, which demonstrates the improvement of the feasible best design during search process with all the techniques in their best performances. The term “best performance” is used to refer to the best run of the algorithm with the minimum design weight attained or to the run where the fastest convergence of the algorithm is achieved if the optimum solution is located in more than one runs. It is seen from Figure 3 that amongst all the techniques the optimum solution is attained earliest by ESs within the first 2,000 design cycles. TS method also shows a fairly rapid and linear progress towards the optimum, identifying the optimum solution before performing 4,000 design cycles. Another promising performance in terms of convergence rate is exhibited by ACO method, in which case the optimum solution is attained after carrying out approximately

5,500 design cycles. PSO, HS and SA methods display a rather slow and gradual convergence in the course of optimization process, and get to locate the optimum solution between 7,000-8,000 design cycles. Despite an encouraging performance of SGA in the early stages, the convergence rate of the technique has downgraded with increasing number of design cycles and the optimum solution could only be obtained after 9,000 design cycles with SGA.

## **6. Conclusions**

The optimum design of pin-jointed geodesic steel domes is investigated in conjunction with seven metaheuristic search techniques, which have emerged recently as robust and promising tools for successfully handling discrete programming problems encountered in structural optimization. The optimum design process is implemented in conformity with design requirements and specifications prescribed for these systems in actual design practice. Accordingly, the functional and structural requirements of the geodesic dome structures, such as allowable stress levels, acceptable deflections, service loads, etc., are enforced according to chosen codes of practice, namely ASD-AISC [22] and ASCE 7-98 [23].

The performance of the metaheuristic search techniques in finding optimum solutions to the problems of interest is numerically scrutinized in conjunction with a 130-member geodesic steel dome design example. All the techniques could manage to find the optimum solution in at least one of the three independent runs conducted. ESs, SA and PSO can be characterized as methods that have high convergence reliabilities since they found the same solution (the optimum) in all of their three runs. On the other hand, performance variations are observed for TS, ACO and HS methods associated with their stochastic nature. ESs, TS and ACO method come into prominence in terms of their satisfactory convergence rates, whereas a steady convergence attribute has been observed with PSO, HS and SA methods. Finally, SGA exhibited a satisfactory performance neither in terms of convergence reliability nor its convergence rate.

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