



COMPARISON OF EFFLUX TIMES BETWEEN CYLINDRICAL AND SPHERICAL TANK THROUGH AN EXIT PIPE

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Abstract

Mathematical equations for efflux time during gravity draining of a Newtonian liquid (below its bubble point) from large open storage tanks of different geometries (Where the flow in the respective tanks is essentially laminar) through an exit pipe of same length and cross sectional area (the flow in the exit pipe line being turbulent) located at the bottom of the respective tanks are developed. The equations are ultimately simplified and written in dimensionless forms. These equations will be of use in arriving at the minimum time required for draining the contents of respective storage vessels. To drain the same volume of liquid, the efflux time equations developed are compared to find out which of the tanks considered drain faster.

Key Words: Efflux time, Newtonian liquid, open storage tank, Laminar, turbulent, exit pipe

1.Introduction

Process industries use different shapes of storage vessels. The time required to drain these vessels off their liquid contents is known as efflux time and this is of crucial importance in many emergency situations besides productivity considerations as reported by Hart, W. Peter and Sommerfeld.T [1]. Present work considers draining a Newtonian liquid from large storage vessels of cylindrical and spherical tanks through an exit pipe. The diameter and length of the exit pipe for both the geometries is assumed to be same.

2. Literature review

Donald and Barret [2] reported theoretical as well as experimental works for efflux time for draining the contents of cylindrical storage vessels for laminar flow conditions in the exit pipe. The same authors also reported work for draining a Newtonian liquid for turbulent flow conditions. Their model assumes constant friction factor in the exit pipe line. Vandogen and Roche [3] presented experimental work for draining a liquid under turbulent flow conditions in the exit pipe. The Reynolds number considered was more than 40,000-60,000. The effect of pipes and fittings were expressed in terms of equivalent length. Morrison [4] also carried out modelling of draining a liquid through an exit pipe at around a Reynolds number of 6,000. Subbarao et al [5, 6] modelled the efflux time for single exit pipe as well as two exit pipe using macroscopic balances. They simplified the efflux time equation to modified form of Torricelli equation. They reported that polymer solutions influence the efflux time. Efflux time can also be influenced by the geometry of the vessel. The present work considers deriving efflux time for draining a Newtonian liquid through different geometries through an exit pipe of same length and diameter. The geometries considered are

- Cylindrical tank with a flat bottom
- Complete spherical tank

The expressions developed are compared to find out which one of the tanks considered drains faster.

3. Development of mathematical equation for efflux time for different geometries

Suppose an open tank of given geometry (Fig1, 2) provided with an exit pipe is plugged and initially filled with a Newtonian and incompressible liquid. The liquid leaves the station-2 when the exit pipe is unplugged. It is desired to find the time required to drain the contents of the respective storage vessels.

Writing the mass balance equation,

Rate of mass in –Rate of mass out = Rate of mass accumulation

$$W_1 - W_2 = \frac{d}{dt}(V\rho) \quad (1)$$

where W_1 and W_2 are mass flow rate in and out of the system and V is the volume of liquid, ρ is the density of the liquid.

For the present system $w_1 = 0$ and $w_2 = \rho V_2 \frac{\pi}{4} d^2$, V_2 is the average velocity of the liquid from the exit pipe

$$- W_2 = \frac{d}{dt}(V\rho) \quad (2)$$

The mechanical energy balance equation between station-1 and station-2 can be written as

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} + gZ_1 = \frac{P_2}{\rho} + \frac{V_2^2}{2} + gZ_2 + 4f \frac{L}{2d} \frac{V_2^2}{2} \quad (3)$$

Where P_1 and P_2 are pressure at station-1 and station-2, Z_1 & Z_2 are elevations at 1 & 2 and V_1 is the velocity at station-1. For the present system, $P_1 = P_2$ (since the top and bottom are open to atmosphere), at any height h , $Z_1 = Z_2 + h$, f is the turbulent friction factor and d is the diameter of the exit pipe.

Subbarao et al [5,6] used macroscopic balances and accounted for both $4f \frac{L}{d} \frac{V_2^2}{2}$ and

$\frac{V_2^2}{2}$ while simplifying the mathematical equation for efflux time. They have also assumed

friction factor to be constant while simplifying the equation. Their experiments suggested that

$4f \frac{LV_2^2}{2d} \gg \frac{V_2^2}{2}$. Hence neglecting $\frac{V_2^2}{2}$ allows development of an alternative equation for

efflux time. More over, the assumption of constant friction factor can also be eliminated.

With the above, Eq.3 can be written as

$$\frac{V_1^2}{2} + g(h+L) = \frac{4fLV_2^2}{d2} \quad (4)$$

Where L is the length of the exit pipe.

Further, liquid drains very slowly (Since the diameter of tank is very large compared to the hole diameter through which the liquid drains), $V_1 = 0$ & for turbulent flow in the exit pipe,

the friction factor is given by $f = \frac{0.0791}{\text{Re}^{0.25}}$ and $f = \frac{0.0791}{\left(\frac{dV_2\rho}{\mu}\right)^{0.25}}$ Eqn.4 becomes where μ is

the viscosity of liquid,

$$V_2 = \frac{2.87 * g^{4/7} (h+L)^{4/7} (d\rho)^{1/7} d^{4/7}}{\mu^{1/7} L^{4/7}} \quad (5)$$

3.1. Development of Mathematical equation for efflux time for a cylindrical tank

Substituting the value of V_2 from Eq.5 and $V = \frac{\pi}{4} D_1^2 h_1$ (where D_1 is the diameter of the tank) in Eq.2

$$\frac{d}{dt} \left(\frac{\pi}{4} \rho D_1^2 h_1 \right) = - \frac{2.87 * \rho * g^{4/7} (h+L)^{4/7} (d\rho)^{1/7} d^{4/7} d^2}{\mu^{1/7} L^{4/7} 4} \pi \quad (6)$$

For incompressible liquid, ρ is constant and hence Eqn.6 becomes

$$\frac{d}{dt} \left(\frac{1}{4} D_1^2 h_1 \right) = - \frac{2.87 * g^{4/7} (h+L)^{4/7} (d\rho)^{1/7} d^{4/7} d^2}{\mu^{1/7} L^{4/7} 4} \quad (7)$$

Separating the variables and integrating between $h_1 = H_1(t=0)$ and to complete draining $h_1 = 0$ and ($t = t_1$, t_1 is the efflux time for cylindrical tank)

$$t_1 = \frac{7}{3 * 2.87} * L^{3/7} \left[\left(\frac{H_1}{L} + 1 \right)^{3/7} - 1 \right] * \frac{D_1^2}{d^2} \left(\frac{\mu}{d\rho} \right)^{1/7} \left(\frac{L}{d} \right)^{4/7} \frac{1}{g^{4/7}} \quad (8)$$

$$t_1 = 0.813 * \frac{D_1^2}{d^2} \left[\left(\frac{H_1}{L} + 1 \right)^{3/7} - 1 \right] * \left(\frac{\mu d^2}{g^4 \rho} \right)^{1/7} \left(\frac{L}{d} \right) \quad (9)$$

$$\frac{t_1}{\left(\frac{\mu d^2}{\rho g^4} \right)^{1/7}} = 0.813 * \frac{D_1^2}{d^2} \left[\left(\frac{H_1}{L} + 1 \right)^{3/7} - 1 \right] * \frac{L}{d} \quad (10)$$

$$\theta_1 = 0.813 * \frac{D_1^2}{d^2} \left[\left(\frac{H_1}{L} + 1 \right)^{3/7} - 1 \right] * \frac{L}{d} \quad (11)$$

$$\text{Where } \theta_1 = \frac{t_1}{\left(\frac{\mu d^2}{\rho g^4}\right)^{1/7}} \quad (12)$$

Where θ_1 is dimensionless time.

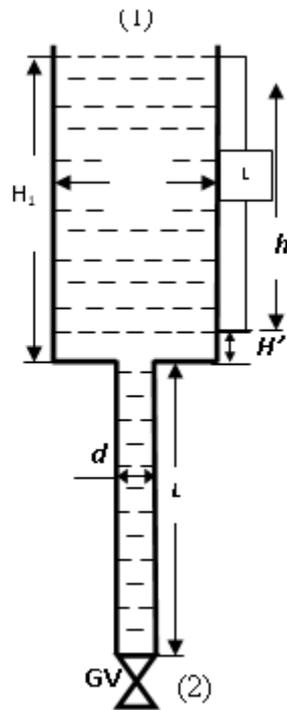


Fig:1 Cylindrical tank with exit pipe

3.2. Development of Mathematical equation for efflux time for a spherical tank

As shown in the Figure (Fig.2), an open spherical tank has to be drained by means of an exit pipe (when the flow in the exit pipe is turbulent). The dimensions are shown in the Figure. The tank is filled with a Newtonian liquid and the liquid is drained from 2. It is desired to find the efflux time required to drain the liquid from the storage vessel.

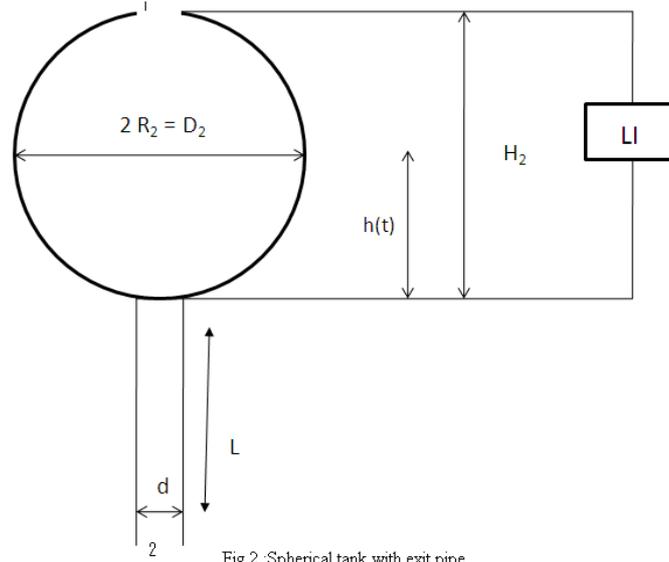


Fig 2 :Spherical tank with exit pipe

Applying mass balance equation for incompressible liquids

$$\frac{d}{dt} \left(\frac{1}{3} \pi R_2^2 h_2^2 \left(1 - \frac{h_2}{3R_2} \right) \right) = - \frac{2.87 * \rho * g^{4/7} (h_2 + L)^{4/7} (d\rho)^{1/7} d^{4/7} d^2 \pi}{\mu^{1/7} L^{4/7} 4} \quad (13)$$

Where h_2 is the height of the liquid at any time t and R_2 is radius at height h_2 . The above equation upon integration between the limits ($h_2=H_2$ at $t=0$ and $h_2=0$ at $t=t_2$) and writing in dimensionless form gives the following equation for efflux time

$$\theta_2 = 1.393 * \left(\frac{L}{d} \right)^3 X_2 \quad (14)$$

$$\text{Where } \theta_2 = \frac{t_2}{\left(\frac{\mu d^2}{\rho g^4} \right)^{1/7}} \quad (15)$$

Where θ_2 is dimensionless time for spherical tank and

Where

$$X_2 = \left\{ \frac{7R_2}{5L} \left[\left(1 + \frac{H_2}{L} \right)^{10/7} - 1 \right] - \frac{14R_2}{3L} \left[\left(1 + \frac{H_2}{L} \right)^{3/7} - 1 \right] - \left[\frac{7}{17} \left(1 + \frac{H_2}{L} \right)^{17/7} - 1 \right] - \left[\frac{7}{3} \left(1 + \frac{H_2}{L} \right)^{3/7} - 1 \right] + \left[\frac{7}{5} \left(1 + \frac{H_2}{L} \right)^{10/7} - 1 \right] \right\} \quad (16)$$

4. Results and discussion

To drain the same volume of liquid from both the tanks, the mathematical equations derived are now compared to find out which of the tanks considered drain faster (i.e, whose efflux time is the lowest).

4.1. Comparison of Efflux time of cylinder with that of a complete sphere

The efflux time equation for spherical tank is compared with that of a cylinder. This is obtained by dividing Eq.11 with Eq.14

$$\frac{\theta_2}{\theta_1} = \frac{1.393 \left(\frac{L}{d}\right)^3 \times 2}{0.813 \frac{D_1^2}{d^2} \frac{L}{d} \left[\left(1 + \frac{H_1}{L}\right)^{3/7} - 1 \right]} \quad (17)$$

$$\text{When same volume of liquid is to be drained, i.e. } \frac{\pi}{4} D_1^2 H_1 = \frac{1}{6} \pi D_2^3 \quad (18)$$

The following two cases can be considered

- a) When the diameter of cylinder is same as Diameter of cone i.e $D_1=D_2$)

Eq23. becomes $H_1 = \frac{2H_2}{3}$ and eq.17 can be written in terms of H_2 and length of exit pipe

L as

$$\frac{\theta_2}{\theta_1} = \frac{1.713 * \left(\frac{L}{H_2}\right)^2 \times 2}{\left(1 + \frac{2H_2}{3L}\right)^{3/7} - 1} \quad (19)$$

The ratio is a function of H_2/L (since for complete spherical tank $R_2=H_2$) and H_2/L is always >0 for all values of H & L, a plot of $\frac{H_2}{L}$ vs $\frac{\theta_2}{\theta_1}$ is shown in fig.3

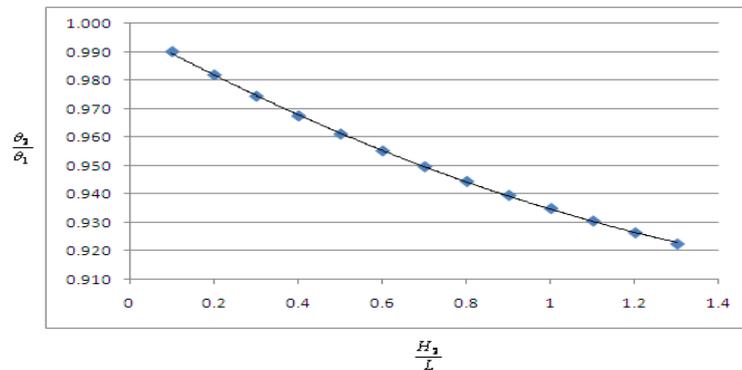


Fig.3. Ratio of efflux time for different values of H_2/L ($D_1=D_2$)

The plot suggests that the ratio is <1 for all values of H_2/L suggesting the efflux time for cylindrical tank is greater than spherical tank. Hence cylindrical tank drains faster than a complete spherical tank when the diameter of sphere is same as that of cylinder.

b) When the height of liquid in both the tanks is same i.e $H_1=H_2$

In this case, Eq.23 becomes $D_2 = 0.866D_1$ (20)

$$\frac{\theta_2}{\theta_1} = \frac{0.321 * \left(\frac{L}{H_2}\right)^2 X_2}{\left(1 + \frac{H_2}{L}\right)^{3/7} - 1} \quad (21)$$

A plot of $\frac{H_2}{L}$ vs $\frac{\theta_2}{\theta_1}$ is shown in the following figure (fig.4)

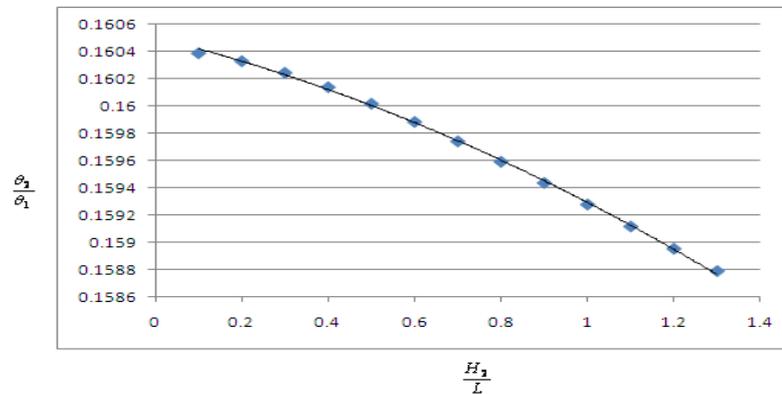


Fig.4. Ratio of efflux time for different values of H_2/L ($H_1=H_2$)

In this case also the ratio is < 1 suggesting draining time for cylinder is greater than that of a sphere under turbulent flow conditions in the exit pipe.

Conclusions: Some of the conclusions of the present work are

1. The order of efflux time is Efflux time for cylinder < efflux time for sphere and is only influenced by ratio of height of liquid to the exit pipe.
2. The influence of equal diameters of sphere and cylinder is more on efflux time ratio than that of equal heights of liquids in the tanks.
3. The theoretical values obtained are to be verified experimentally to find out the exact ratio of efflux times of any two geometries of tanks.

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