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# THE EFFECT OF MEMBER GROUPING ON THE OPTIMUM DESIGN OF GRILLAGES WITH PARTICLE SWARM METHOD

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**Abstract:** *Member grouping of a steel grillage system has an important effect in the minimum weight design of these systems. In the present study, this effect is investigated using an optimum design algorithm which is based on a recently developed particle swarm optimization method (PSO). Particle swarm optimizer is a simulator of social behavior that is used to realize the movement of a birds' flock, which is a population based numerical optimization technique. The optimum design problem of a grillage system is formulated by implementing LRFD-AISC (Load and Resistance Factor Design-American Institute of Steel Construction) limitations. It is decided that W-Sections are to be adapted for the longitudinal and transverse beams of the grillage system. 272 W-Section beams given in LRFD code are collected in a pool and the optimum design algorithm is expected to select the appropriate sections from this pool so that the weight of the grillage is the minimum correspondingly the design limitations implemented from the design code are satisfied. The solution for this discrete programming problem is determined by using the PSO algorithm. In order to demonstrate the effect of member grouping in the optimum design of grillage systems, a design example is presented.*

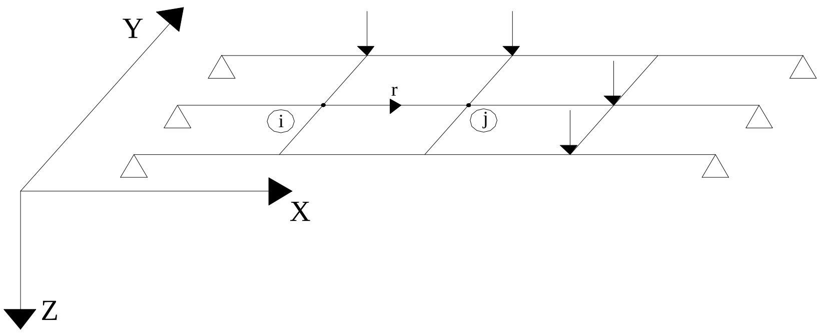
**Keywords:** Grillage optimization, discrete optimum design, member grouping, stochastic search technique, particle swarm algorithm.

## 1. Introduction

Grillage systems are used in structures to cover large spaces such as in bridge decks and in floors. They consist of crosswise longitudinal and transverse beams which constitute an orthogonal system. It is generally up to the designer to select the different member groupings between these beams unless some restrictions are imposed. It is apparent that the selection of varied numbers of member groupings between the longitudinal and transverse beams yields the adaptation of large or small steel sections for these beams. While a single member grouping increases the weight of the system to construct the grillage, an increase in the number of member grouping reduces the weight of the grillage system. Hence, there exist an optimum number of groups in both directions which provides a grillage system with the minimum weight. The number of beams in longitudinal and transverse directions is treated as design variables along with selecting the steel sections for the beams of both directions. The integrated design algorithm determines optimum number of beams in both directions as well as universal beam section designations required for these beams. The technique is based on particle swarm algorithm [1-4] which is a recent addition to stochastic search techniques of combinatorial optimization. Particle swarm approach is inspired by social behavior of bird flocking or fish schooling. This behavior is concerned with grouping by social forces that depend on both the memory of each individual as well as the knowledge gained by the swarm. It can be thought of as a process whereby particles move in n-dimensional space, each particle being a solution and the space being the problem. Particle swarm algorithm defines three main properties, first of which is the velocity that directs movement throughout the solution space, and the rest of which are particle's best and global best which are communicated throughout the swarm. Particle's best represents the fitness of each solution so far and global best represents global fitness of each solution as it passes through the problem space. Particles follow the neighboring optimum particles by adapting these properties in each iteration or generation. From the optimum structural design point of view the objective is to determine the appropriate steel sections for each group of a structure from the available steel sections set such that with these particles set of sections the response of the structure is within the limitations imposed by the design code and the system has the minimum weight. In recent applications particle swarm algorithm is successfully utilized to determine the optimum solutions of different structural design problems [5-7]. In this study, the particle swarm based design algorithm is used to investigate the effect of member grouping in the optimum design of grillage systems.

## 2. Optimum Design Problem to LRFD-AISC

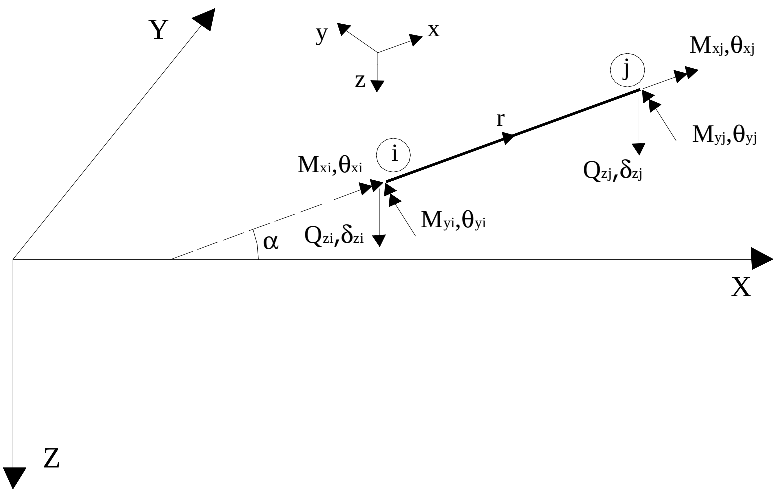
The optimum design problem of a typical grillage system shown in Figure 1 where the behavioral and performance limitations are implemented from LRFD-AISC [8] and the design variables which are selected as the sequence number of W sections given in the W steel profile list of LRFD-AISC can be expressed as follows.




a) Steel grillage system

$$\{D_i\} = \begin{Bmatrix} \theta_{xi} \\ \theta_{yi} \\ \delta_{zi} \end{Bmatrix} , \quad \{P_i\} = \begin{Bmatrix} M_{xi} \\ M_{yi} \\ Q_{zi} \end{Bmatrix}$$

b) Displacements and forces at joint i




c) End forces and end displacements of a grillage member

**Fig. 1.** Typical grillage structure

$$\min \quad W = \sum_{k=1}^{ng} m_k \sum_{i=1}^{n_k} l_i \tag{1}$$

Subject to

$$\delta_j / \delta_{ju} \leq 1 \quad , \ j = 1,2,........, p \tag{2}$$

$$M_{ur} / (\phi_b M_{nr}) \leq 1 \, , r = 1,2,......., nm \tag{3}$$

$$V_{ur} / (\phi_v V_{nr}) \leq 1 \quad , r = 1,2,........, nm \tag{4}$$

Where $m_k$ in Eq. 1 is the unit weight of the W-section selected from the list of LRFD-AISC for the grillage element belonging to group k, $n_k$ is the total number of members in group k, and $n_g$ is the total number of groups in the grillage system. $l_i$ is the length of member *i*. $\delta_j$ in Eq. 2 is the displacement of joint j and $\delta_{ju}$ is its upper bound. The joint displacements are computed using the matrix displacement method for grillage systems. Eq. 3 represents the strength requirement for laterally supported beam in load and resistance factor design according to LRFD-F2. In this inequality $Ø_b$ is the resistance factor for flexure which is given as 0.9, $M_{nr}$ is the nominal moment strength and $M_{ur}$ is the factored service load moment for member r. Eq. 4 represents the shear strength requirement in load and resistance factor design according to LRFD-F2. In this inequality $Ø_v$ represents the resistance factor for shear given as 0.9, $V_{nr}$ is the nominal strength in shear and $V_{ur}$ is the factored service load shear for member r. The details of obtaining nominal moment strength and nominal shear strength of a W-section according to LRFD are given in the following.

## *2.1 Load and Resistance Factor Design for Laterally Supported Rolled Beams*

The computation of the nominal moment strength $M_n$ of a laterally supported beam, it is necessary first to determine whether the beam is compact, non-compact or slender. In compact sections, local buckling of the compression flange and the web does not occur before the plastic hinge develops in the cross section. On the other hand in practically compact sections, the local buckling of compression flange or web may occur after the first yield is reacted at the outer fiber of the flanges. The computation of $M_n$ is given in the following as defined in LRFD-AISC.

a) If $\lambda \le \lambda_p$ for both the compression flange and the web, then the section is compact and

$$M_n = M_p \text{ (Plastic moment capacity)} \tag{5}$$

b) If□ $\lambda_p < \lambda \le \lambda_r$ for the compression flange or web, then the section is partially compact and

$$M_n = M_p - (M_p - M_r)\frac{\lambda - \lambda_r}{\lambda_r - \lambda_p} \tag{6}$$

c) If $\lambda \succ \lambda_r$ for the compression flange or the web, then the section is slender and

$$M_n = M_{cr} = S_x F_{cr} \tag{7}$$

where $\lambda = b_f/(2t_f)$ for I-shaped member flanges and the thickness in which $b_f$ and $t_f$ are the width and the thickness of the flange, and $\lambda = h/t_w$ for beam web, in which $h=d-2k$ plus allowance for undersize inside fillet at compression flange for rolled I-shaped sections. $d$ is the depth of the section and $k$ is the distance from outer face of flange to web toe of fillet. $t_w$ is the web thickness. $h/t_w$ values are readily available in W-section properties table. $\lambda_p$ and $\lambda_r$ are given in table LRFD-B5.1 of the code as

$$\left.\begin{aligned} \lambda_p &= 0.38\sqrt{\frac{E}{F_y}} \\ \lambda_r &= 0.83\sqrt{\frac{E}{F_y - F_r}} \end{aligned}\right\} \text{ for compression flange} \tag{8}$$

$$\left.\begin{aligned} \lambda_p &= 3.76\sqrt{\frac{E}{F_y}} \\ \lambda_r &= 5.70\sqrt{\frac{E}{F_y}} \end{aligned}\right\} \text{ for the web} \tag{9}$$

in which $E$ is the modulus of elasticity and $F_y$ is the yield stress of steel. $F_r$ is the compressive residual stress in flange which is given as 69 $MPa$ for rolled shapes in the code. It is apparent that $M_n$ is computed for the flange and for the web separately by using corresponding $\lambda$ values. The smallest among all is taken as the nominal moment strength of the $W$ section under consideration.

### *2.2 Load and Resistance Factor Design for Shear in Rolled Beams*

Nominal shear strength of a rolled compact and non-compact $W$ section is computed as follows as given in LRFD-AISC-F2.2

$$\text{For } \frac{h}{t_w} \leq 2.45\sqrt{\frac{E}{F_{yw}}}, V_n = 0.6 F_{yw} A_w \tag{10}$$

$$\text{For } 2.45\sqrt{\frac{E}{F_{yw}}} < \frac{h}{t_w} \leq 3.07\sqrt{\frac{E}{F_{yw}}}, V_n = 0.6 F_{yw} A_w \left( \frac{2.45\sqrt{\frac{E}{F_{yw}}}}{\frac{h}{t_w}} \right) \tag{11}$$

$$\text{For } 3.07\sqrt{\frac{E}{F_{yw}}} < \frac{h}{t_w} \leq 260 \ , \quad V_n = A_w \frac{4.52 E t_w{}^2}{h^2} \tag{12}$$

where **E** is the modulus of elasticity and $F_{yw}$ is the yield stress of web steel. $V_n$ is computed from one of the expressions of (10)-(12) depending upon the value of $h/t_w$ of the **W**-section under consideration.

## 3. Particle Swarm Method

Particle swarm optimizer (PSO) is based on the social behavior of animals such as fish schooling, insect swarming and birds flocking. This behavior is concerned with grouping by social forces that depend on both the memory of each individual as well as the knowledge gained by the swarm [1-3]. The procedure involves a number of particles which represent the swarm being initialized randomly in the search space of an objective function. Each particle in the swarm represents a candidate solution of the optimum design problem. The particles fly through the search space and their positions are updated using the current position, a velocity vector and a time step. The steps of the algorithm are outlined in the following as given in [9-11]:

**Step 1.** ***Initializing Particles:*** A swarm consists of a predefined number of particles referred to as swarm size ($\mu$). Each particle ($\mathbf{P}$) incorporates two sets of components; a position (design) vector $\mathbf{I}$ and a velocity vector **V** (Eqn. 13). The position vector $\mathbf{I}$ retains the values (positions) of design variables, while the velocity vector **V** is used to vary these positions during the search. Each particle in the swarm is constructed by a random initialization such that all initial positions $I_i^{(0)}$ and velocities $v_i^{(0)}$ are assigned from Eqns. (14-15):

$$\mathbf{P} = (\mathbf{I}, \mathbf{v}), \quad \mathbf{I} = [I_1, I_2, ..., I_{N_d}] \quad , \quad \mathbf{v} = [v_1, v_2, ..., v_{N_d}] \tag{13}$$

$$I_i^{(0)} = I_{\min} + r\left(I_{\max} - I_{\min}\right), \quad i = 1,.., N_d \tag{14}$$

$$v_i^{(0)} = \frac{I_{\min} + r(I_{\max} - I_{\min})}{\Delta t}, \quad i = 1,.., N_d \tag{15}$$

Where, $r$ is a random number sampled between 0 and 1; $\Delta t$ is the time step; and $I_{\min}$ and $I_{\max}$ are the sequence numbers of the first and last standard steel sections in the profile list, respectively.

**Step 2.** ***Evaluating Particles:*** All the particles are analyzed, and their objective function values are calculated using design space positions.

**Step 3.** ***Updating the Particles' Best and the Global Best:*** A particle's best position (the best design with minimum objective function) thus far is referred to as particle's best and is stored separately for each particle in a vector $\mathbf{B}$. On the other hand, the best feasible position located by any particle since the beginning of the process is called the global best position, and it is stored in a vector $\mathbf{G}$. At the current iteration *k*, both the particles' bests and the global best are updated (15).

$$\mathbf{B}^{(k)} = \left[B_1{}^{(k)},...B_i{}^{(k)}..,B_{N_d}{}^{(k)}\right] \qquad \mathbf{G}^{(k)} = \left[G_1{}^{(k)},...G_i{}^{(k)}..,G_{N_d}{}^{(k)}\right] \tag{16}$$

**Step 4.** ***Updating a Particle's Velocity Vector:*** The velocity vector of each particle is updated considering the particle's current position, the particle's best position and global best position, as follows:

$$v_i^{(k+1)} = wv_i^{(k)} + c_1 r_1 \left(\frac{G_i^{(k)} - I_i^{(k)}}{\Delta t}\right) + c_2 r_2 \left(\frac{B_i^{(k)} - I_i^{(k)}}{\Delta t}\right) \tag{17}$$

Where, $r_1$ and $r_2$ are random numbers between 0 and 1; $w$ is the inertia of the particle which controls the exploration properties of the algorithm; and $c_1$ and $c_2$ are the trust parameters, indicating how much confidence the particle has in itself and in the swarm, respectively.

**Step 5.** ***Updating a Particle's Position Vector:*** Next, the position vector of each particle is updated with the updated velocity vector (Eqn. 18), which is rounded to nearest integer value for discrete variables.

$$I_i^{(k+1)} = I_i^{(k)} + v_i^{(k+1)} \Delta t \tag{18}$$

**Step 6.** ***Termination:*** The steps 2 through 5 are repeated in the same way for a predefined number of iterations $N_{ite}$.

***Constraint handling:*** In this study fly-back mechanism is used for handling the design constraints which is proven to be effective in [12]. Once all particle positions are generated, the objective functions are evaluated for each of these and the constraints in the problem are then computed with these values to find out whether they violate the design constraints. If one or a number of the particle gives infeasible solutions, these are discarded and new ones are re-generated. If a particle is slightly infeasible then such particles are kept in the solution. These particles having one or more constraints slightly infeasible are utilized in the design process that might provide a new particle that may be feasible. This is achieved by using larger error values initially for the acceptability of the new design vectors and then reduce this value gradually during the design cycles and uses finally an error value of 0.001 or whatever necessary value that is required to be selected for the permissible error term towards the end of iterations. This adaptive error strategy is found quite effective in handling the design constraints in large design problems.

## 4. Optimum Design Algorithm

The optimum design algorithm is based on the particle swarm method, steps of which are given previous section. The discrete set from which the design algorithm selects the sectional designations for grillage members is considered to be the complete set of 272 W-sections which start from W100×19.3mm to W1100×499mm as given in LRFD-AISC [8]. The design variables are the sequence numbers of W-sections that are to be selected for member groups in the grillage system. These sequence numbers are integer numbers which can take any value between 1 and 272. Particle swarm method then randomly selects integer number for each member group within the bounds. Once these numbers are decided, then the sectional designation and cross sectional properties of that section becomes available for the algorithm. The grillage system is then analyzed with these sections under the external loads and the response of the system is obtained. If the design constraints given in Eqs. 2-4 are satisfied this set of sections are placed in the solution vector, if not the selection is discarded. This process is continued until the PSO algorithm finds the optimum solution for grillage system.

## 5. Design Example

The optimum design algorithm presented in the previous sections is used to demonstrate the effect of member grouping in the design of grillages. In order to demonstrate this effect, 40-member grillage system shown in Figure 2 is designed several times by considering different member groupings. For this purpose, 12.5m×10m square area is considered. The design problem is to set up a grillage system that is supposed to carry 25.6kN/m$^2$ uniformly distributed load total of which is 3200kN. The total external loading is distributed to the joints as 200kN point load. The grillage system that can be used to cover the area will have 12.5m long longitudinal beams and 10m long transverse beams. The total external load is distributed to joints of the grillage system as a point load value of which is calculated according to beam spacing. A36 mild steel is selected for the design, which has the yield stress of 250MPa, the modulus of elasticity of 205 kN/mm$^2$ and shear modulus of 81 kN/mm$^2$ respectively. The vertical displacements of joints 6, 7, 10 and 11 are restricted to 25 mm. The result of the sensitivity analysis carried out to determine the appropriate value ranges of the particle swarm parameters is given in [13].

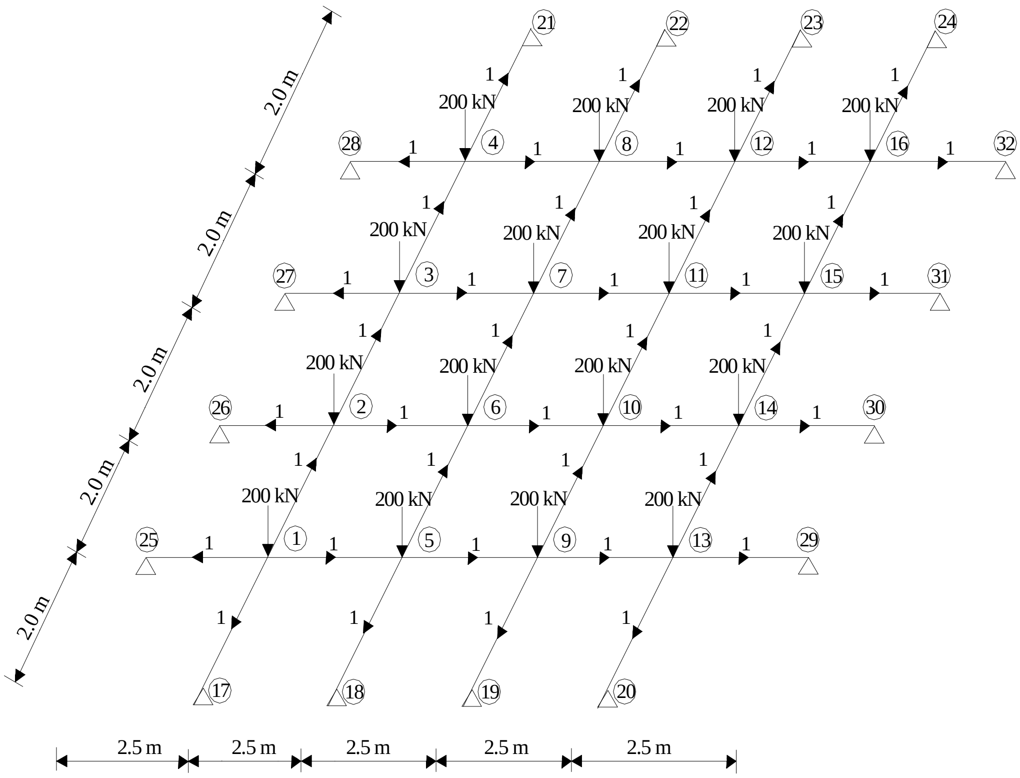


**Fig. 2.** 40-member grillage system with single grouping

It is noticed that particle swarm parameter values of 10 for number of particles ($\mu$), 1.0 for the self-confidence parameter of particles (*c1*) and swarm confidence parameter (*c2*), 0.08 for the inertia weight (*w*) and 2 for maximum velocity of particles ($V_{max}$) and velocity time increment ($\Delta t$) produce the least weight design for this grillage after carrying out several trials in the design of all grillage systems. When the optimum design problem is carried out considering only single group shown in Figure 2, the minimum weight of the system turns out to be 14499.8kg. The optimum design of the grillage system is carried out by the algorithm presented and the optimum results obtained are given in Table 1.

**Table 1.** Optimum design for 40-member grillage system with one group

| **Optimum W-Section Designations** | $\delta_{MAX}$ (mm) | **Maximum Strength Ratio** | **Minimum Weight** (kg) |
|---|---|---|---|
| W760X161 | 24.2 | 0.73 | 14499.8 |

When the longitudinal members are considered as one group and the transverse ones are collected in another member group shown in Figure 3, the minimum weight drops down almost by half to 7729.5kg. Optimum sectional designations of the 40-member grillage system under the external loading, obtained by design method presented, are given in Table 2.

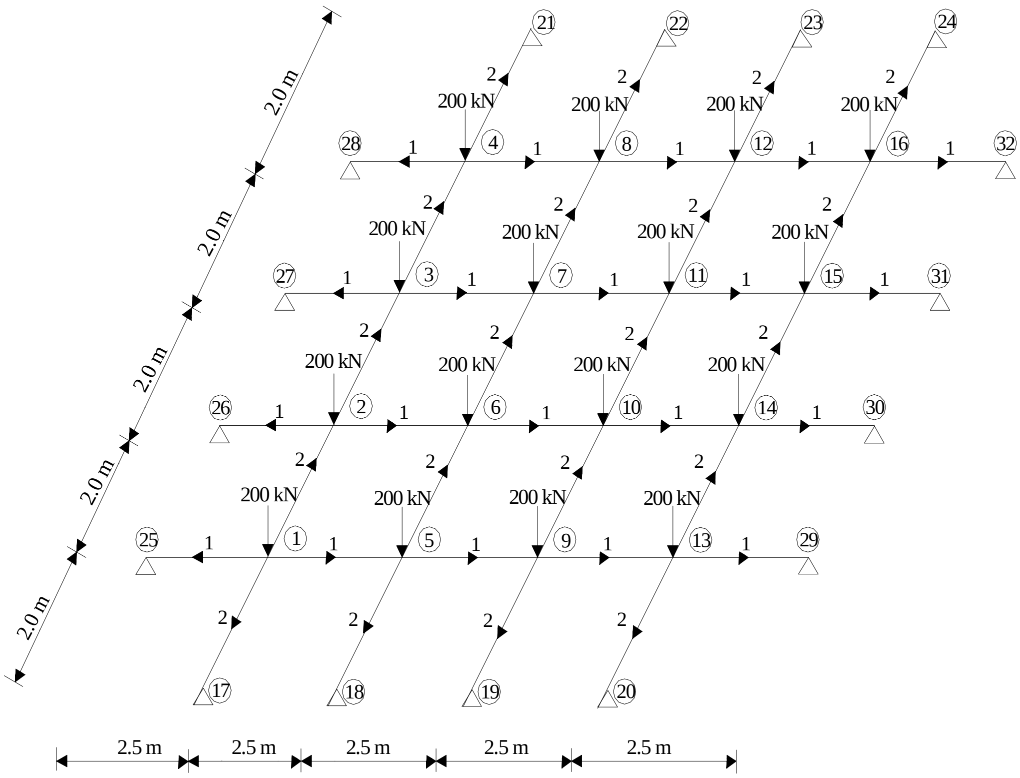



**Fig. 3.** 40-member grillage system with two groups

**Table 2.** Optimum design for 40-member grillage system with two groups

| **Optimum W-Section Designations** | | $\delta_{MAX}$ (mm) | **Maximum Strength Ratio** | **Minimum Weight** (kg) |
|---|---|---|---|---|
| **Group 1** | **Group 2** | | | |
| W150×13.5 | W840×176 | 24.2 | 0.80 | 7729.5 |

Further reduction is possible if longitudinal members are collected in two groups and transverse members are considered as another two groups. It is apparent from Figure 4 that consideration of four member groups represents the optimum grouping for 40-member grillage system. The optimum design of this grillage system with four groups is carried out by the algorithm presented and the optimum results obtained are given in Table 3.

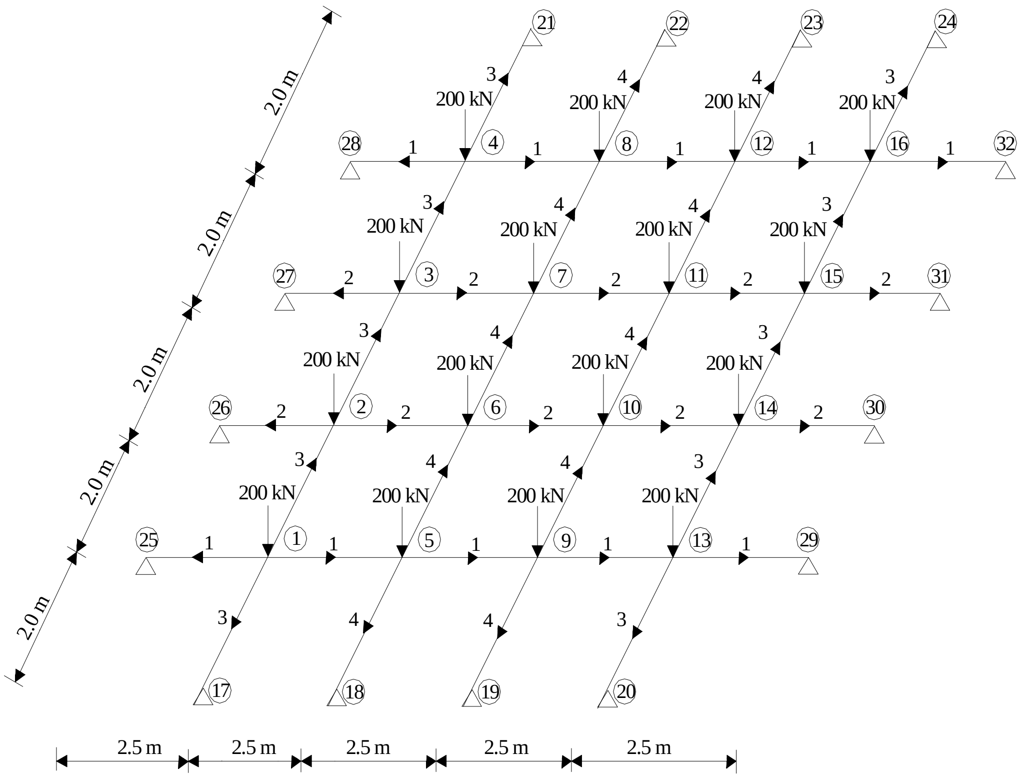



**Fig. 4.** 40-member grillage system with four groups

**Table 3.** Optimum design for 40-member grillage system with four groups

| **Optimum W-Section Designations** | | | | $\delta_{MAX}$ (mm) | **Maximum Strength Ratio** | **Minimum Weight** (kg) |
|---|---|---|---|---|---|---|
| **Group 1** | **Group 2** | **Group 3** | **Group 4** | | | |
| W410×46.1 | W460×52 | W200×15 | W1000×222 | 22.3 | 0.99 | 7198.2 |

Finally, the number of groups is increased from 4 to 8 in both directions. It is interesting to notice that when all the members are allowed to have separate groups, shown in Figure 5, the minimum weight of the grillage system also increases from 7198.2kg to 9403.1kg. The optimum sectional designations obtained for the 40-member grillage system with 8 groups is given in Table 4. Furthermore, it is clear from the same table that for the larger number of groups, the strength constraints becomes dominant in the design problem, while for the cases where less number of groups is considered, the displacement constraints become dominant.

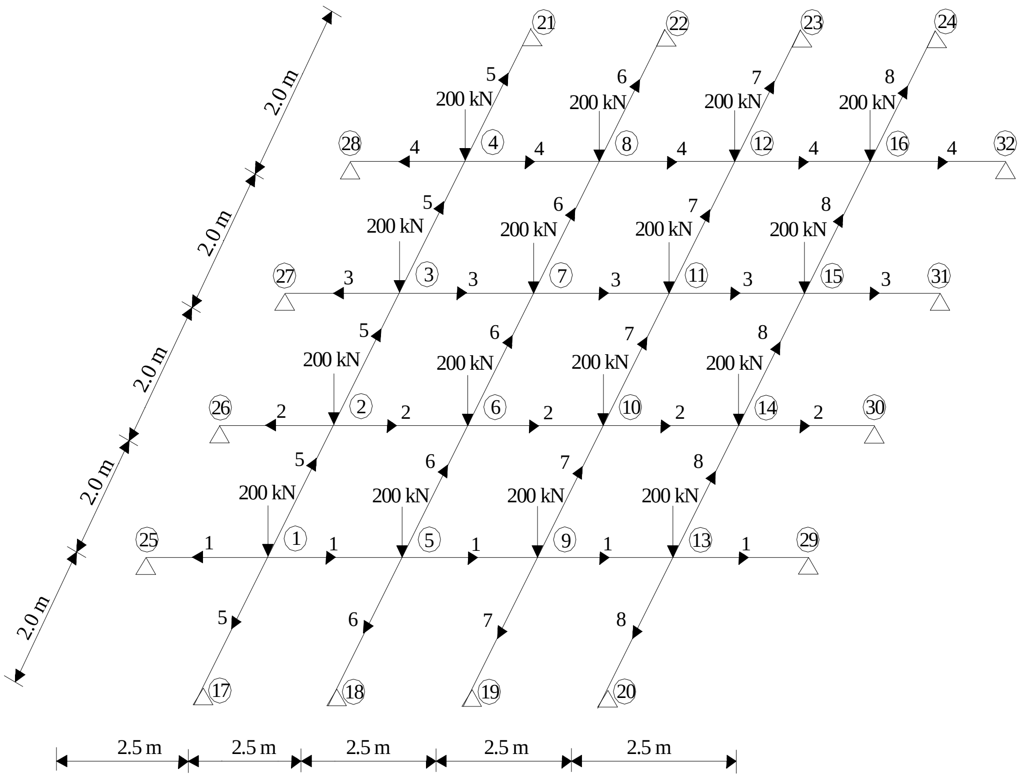


**Fig. 5.** 40-member grillage system with eight groups

**Table 4.** Optimum design for 40-member grillage system with eight groups

| Optimum W-Section Designations | | | | | | | | $\delta_{Max}$ (mm) | Maximum Strength Ratio | Minimum Weight (kg) |
|---|---|---|---|---|---|---|---|---|---|---|
| **Group 1** | **Group 2** | **Group 3** | **Group 4** | **Group 5** | **Group 6** | **Group 7** | **Group 8** | | | |
| W150×13.5 | W760X147 | W150×13.5 | W1000X272 | W410X46.1 | W610X101 | W460X52 | W760X134 | 24.9 | 0.99 | 9403.1 |

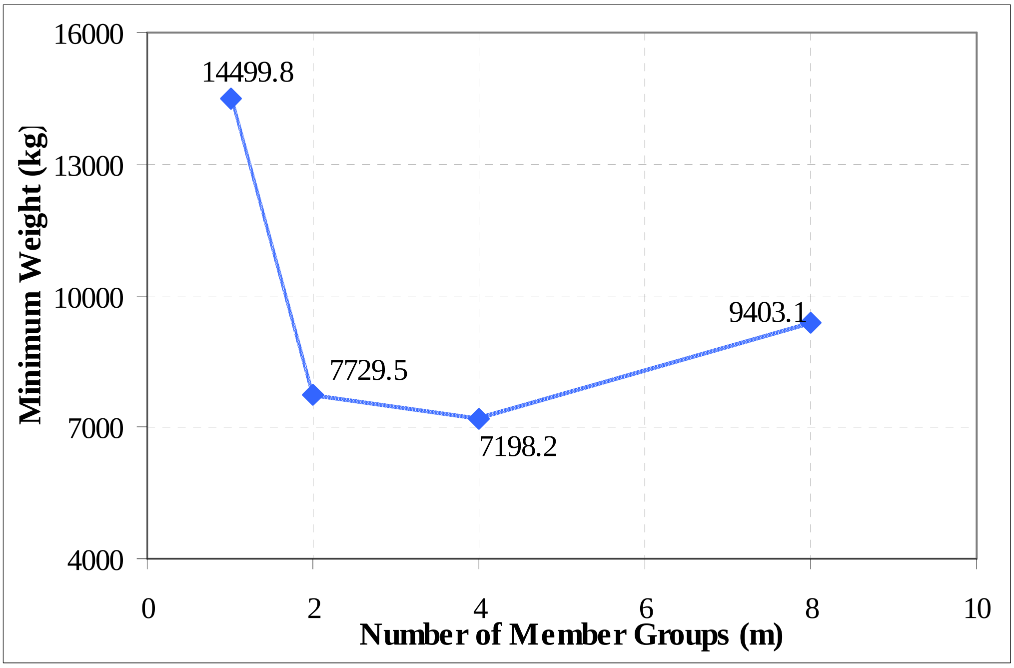



**Fig. 6.** Variation of weight versus member groups

## 6. Conclusions

It is shown that the particle swarm method which is one of the recent additions to metaheuristic algorithms can successfully be used in the optimum design of grillage systems. Particle swarm method has three parameters that are required to be determined prior to its use in determining the optimum solution. These parameters are problem dependent and some trials are necessary to determine their appropriate values for the problem under consideration. It is also shown that member grouping in the optimum design of grillage systems has a considerable effect on the minimum weight and it is more appropriate to consider this parameter as a design variable if a better design is looked for. It is also interesting to notice that while for the larger values of member grouping the optimum design problem is strength dominant, for the smaller values of member grouping the problem becomes displacement dominant.

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