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VIBRATION ANALYSIS OF VISCO-ELASTIC SQUARE PLATE OF VARIABLE THICKNESS WITH THERMAL GRADIENT

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Abstract

In the modern technology, the plates of variable thickness are widely used in engineering applications i.e. nuclear reactor, aeronautical field, naval structure, submarine, earth-quake resistors etc. In this paper thermal gradient effect on vibration of square plate having one -direction thickness variations is studied. The non-homogeneity is assumed to arise due to the variation in the density of the plate material. Rayleigh Ritz method is used to evaluate the fundamental frequencies. Both the modes of the frequency are calculated by the latest computational technique, MATLAB, for the various values of taper parameters and temperature gradient. All the results are presented in the graphs.

Keyword: Visco-elastic, Square plate, Vibration, Thermal gradient, Taper constant.

1. Introduction

With the advancement of technology, the requirement to know the effect of temperature on visco-elastic plates of variable thickness has become vital due to their applications in various engineering branches such as nuclear power plants, engineering, industries etc. Further in mechanical system where certain parts of machine have to operate under elevated temperature, its effect is far from negligible and obviously cause non-homogeneity in the plate material i.e. elastic constants (young modulus etc.) of the materials becomes functions of space variables. Many researchers [1-9] have analysed the free vibration of visco-elastic plates with variable thickness for many years.

The aim of present investigation is to study the one dimensional thermal effect on the vibration of visco-elastic square plate whose thickness varies linearly in one directions. Due to temperature variation, we assume that non homogeneity occurs in Modulus of Elasticity (E)For various numerical values of thermal gradient and taper constants; frequency for the first two modes of vibration are calculated with the help of MATLAB. All the numerical calculations will be carried out using the material constants of alloy 'Duralium'.

2. **Equqtion of Motion And Analysis**

Differential equation of motion for visco-elastic square plate of variable thickness in Cartesian coordinate is given by equation (2.1) respectively [1]:

$$\begin{aligned}&[D_1\left(W_{,xxxx}+2W_{,xxyy}+W_{,yyyy}\right)+2D_{1,x}\left(W_{,xxx}+W_{,xyy}\right)+2D_{1,y}\left(W_{,yyy}+W_{,yxx}\right)+\\&D_{1,xx}(W_{,xx}+\nu W_{,yy})+D_{1,yy}(W_{,yy}+\nu W_{,xx})+2(1-\nu)D_{1,xy}W_{,xy}]-\rho h p^2 W=0\end{aligned} \tag{2.1}$$

which is a differential equation of transverse motion for non-homogeneous plate of variable thickness. Here, D_1 is the flexural rigidity of plate i.e.

$$D_1 = Eh^3 / 12(1 - v^2) \tag{2.2}$$

and corresponding two-term deflection function is taken as [7]

$$W = [(x/a)(y/a)(1-x/a)(1-y/a)]^2 [A_1 + A_2 (x/a)(y/a)(1-x/a)(1-y/a)] \tag{2.3}$$

Assuming that the square plate of engineering material has a steady one dimensional temperature distribution i.e

$$\tau = \tau_0 (1 - x/a) \tag{2.4}$$

where, τ denotes the temperature excess above the reference temperature at any point on the plate and τ_0 denotes the temperature at any point on the boundary of plate and "*a*" is the length of a side of square plate. The temperature dependence of the modulus of elasticity for most of engineering materials can be expressed in this form,

$$E = E_0 (1 - \gamma\tau) \tag{2.5}$$

where, E_0 is the value of the Young's modulus at reference temperature i.e. $\tau = 0$ and γ is the slope of the variation of E with τ. The modulus variation (2.6) become

$$E = E_0 [1 - \alpha(1 - x/a)] \tag{2.6}$$

where, $\alpha = \gamma\tau_0 (0 \leq \alpha < 1)$ thermal gradient. It is assumed that thickness also varies linearly in one direction as shown below:

$$h = h_0 (1 + \beta_1 x / a) \tag{2.7}$$

where, β_1 are taper parameters in x direction respectively and $h=h_0$ at x=0.Put the value of E & h from equation (2.6) & (2.7) in the equation (2.2), one obtain

$$D_1 = [E_0 [1 - \alpha(1 - x/a)] h_0^3 (1 + \beta_1 x/a)^3] / 12(1 - v^2) \tag{2.8}$$

Rayleigh-Ritz technique is applied to solve the frequency equation. In this method, one requires maximum strain energy must be equal to the maximum kinetic energy. So it is necessary for the problem under consideration that

$$\delta (V^* - T^*) = 0 \tag{2.9}$$

for arbitrary variations of W satisfying relevant geometrical boundary conditions.
Since the plate is assumed as clamped at all the four edges, so the boundary conditions are

$$\left.\begin{array}{ll} W = W_{,x} = 0 & , x = 0, a \\ W = W_{,y} = 0 & , y = 0, a \end{array}\right\} \tag{2.10}$$

Now assuming the non-dimensional variables as

$$X = x / a, \overline{W} = W / a, \overline{h} = h / a \tag{2.11}$$

The kinetic energy T* and strain energy V* are [2]

$$T^* = (1/2)\rho p^2 \overline{h_0} a^5 \int_0^1 \int_0^1 [(1+\beta_1 X)\overline{W^2}] dY dX \tag{2.12}$$

and

$$\begin{aligned} V^* = Q \int_0^1 \int_0^1 & [1-\alpha(1-X)](1+\beta_1 X)^3 \{(\overline{W}_{,XX})^2 + (\overline{W}_{,YY})^2 \\ & + 2v \overline{W}_{,XX} \overline{W}_{,YY} + 2(1-v)(\overline{W}_{,XY})^2\} dY dX \end{aligned} \tag{2.13}$$

where, $Q = E_0 h_0{}^3 a^3 / 24(1-v^2)$

Using equations (2.13) & (2.14) in equation (2.10), one get

$$(V^{**} - \lambda^2 T^{**}) = 0 \tag{2.14}$$

where,

$$\begin{aligned} V^{**} = \int_0^1 \int_0^1 & [1-\alpha(1-X))](1+\beta_1 X)^3 \{(\overline{W}_{,XX})^2 + (\overline{W}_{,YY})^2 \\ & + 2v \overline{W}_{,XX} \overline{W}_{,YY} + 2(1-v)(\overline{W}_{,XY})^2\} dY dX \end{aligned} \tag{2.15}$$

and

$$T^{**} = \int_0^1 \int_0^1 [(1+\beta_1 X)\overline{W^2}] dY dX \tag{2.16}$$

Here, $\lambda^2 = 12\rho(1-v^2)a^2 / E_0 h_0{}^2$ is a frequency parameter. Equation (2.16) consists two unknown constants i.e. A_1 & A_2 arising due to the substitution of W. These two constants are to be determined as follows

$$\partial(V^{**} - \lambda^2 T^{**}) / \partial A_n \quad , \text{n=1, 2} \tag{2.17}$$

On simplifying (2.17), one gets

$$bn_1 A_1 + bn_2 A_2 = 0 \quad , \text{n =1, 2} \tag{2.18}$$

where, bn_1, bn_2 (n=1,2) involve parametric constant and the frequency parameter.

For a non-trivial solution, the determinant of the coefficient of equation (2.18) must be zero. So one gets, the frequency equation as

$$\begin{vmatrix} b_{11} b_{12} \\ b_{21} b_{22} \end{vmatrix} = 0 \tag{2.19}$$

With the help of equation (2.19), one can obtains a quadratic equation in λ^2 from which the two values of λ^2 can found. These two values represent the two modes of vibration of frequency i.e. λ_1(Mode1) & λ_2(Mode2) for different values of taper constant and thermal gradient for a clamped plate.

3. Result and Discussion

All calculations are carried out with the help of Latest Matrix Laboratory computer software. Computation has been done for frequency of visco-elastic square plate for different values of taper constant β_1 and thermal gradient α, at different points for first two modes of vibrations have been calculated numerically.

In Fig 1, It is clearly seen that value of frequency decreases as value of thermal gradient increases from 0.0 to 1.0 for β1 = 0.0 for both modes of vibrations. In Fig 2, It is evident that frequency decreases continuously as thermal gradient increases, β1=0.4 respectively with the two modes of vibration.

In Fig 3, Also it is obvious to understand the decrement in frequency for β1= 0.6 But it is also noticed that value of frequency is increased with the increment in β1. In Fig 4, Increasing value of frequency for both of the modes of vibration is shown for increasing value of taper constant β_1 from 0.0 to 1.0 and α=0.4 respectively. Note that value of frequency increased.

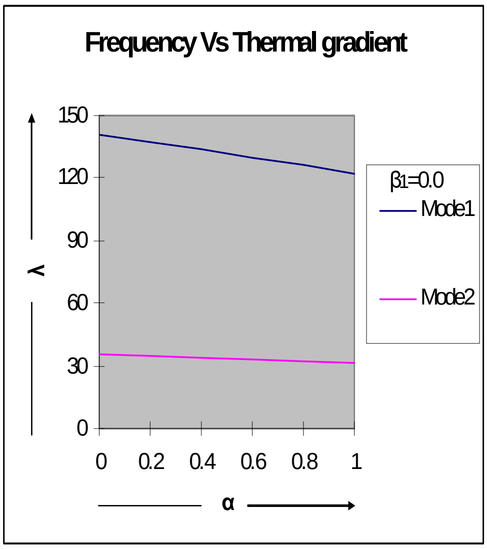

Fig 1. Frequency vs. thermal gradient at β_1=0.0

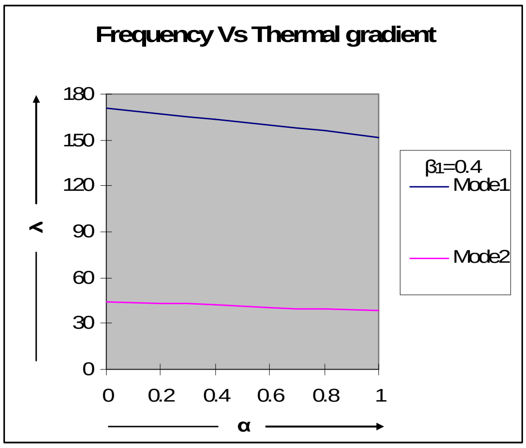

Fig 2. Frequency vs. thermal gradient at β_1=0.4

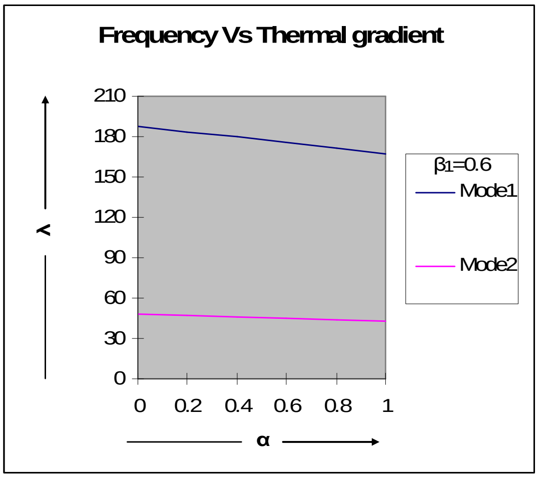

Fig 3. Frequency vs. thermal gradient at β_1=0.6

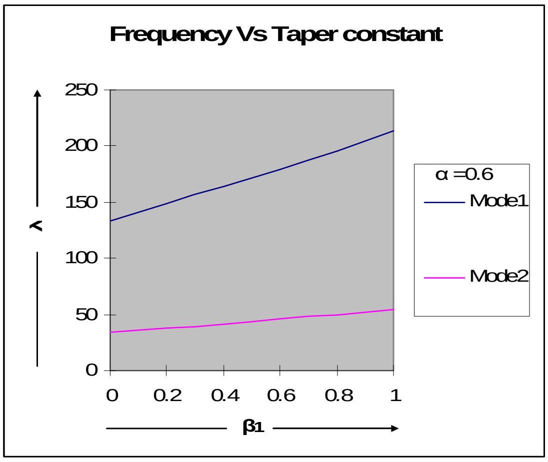

Fig 4. Frequency Vs Taper constant at α=0.4

4.Conclusion

Results of present paper are compared with paper [9]. It is interesting to note that value of frequency has greater value in this paper as compared to [9]. So, main aim for our research is to develop a theoretical mathematical model for scientists and design engineers so that they can make a use of it with a practical approach, for the welfare of the human beings as well as for the advancement of technology.

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