International Journal of Engineering & Applied Sciences (IJEAS)
Vol.4, Issue 1(2012)17-30

EFFECT OF INFLOW CONDITIONS ON UNDER-EXPANDED SUPERSONIC JETS

M.M. Ashraful Alam[1], *Toshiaki Setoguchi* [2]
[1] *Department of Mechanical Engineering, Khulna University of Engineering & Technology, Khulna-9203, Bangladesh, E-mail: rana94.kuet@gmail.com*
[2] *Institute of Ocean Energy, Saga University, 1 Honjo-machi, Saga 840-8502, Japan*
[1] Corresponding author

Abstract

The present work involves numerical simulations to investigate the effect of inflow condition on aerodynamic and shock characteristics of under-expanded turbulent jets from sonic nozzle. The TVD finite volume method was carried out and two equation k-ε turbulence model was used to model the turbulent stresses of the compressible flows in the present simulations. The jet pressure ratio was settled from 1.893 to 6.0 for generating perfectly expanded to moderately high under-expanded jets. The pressure and Mach number distributions on jet axis, and the flowfield structure was visualized by density distributions. The potential core and supersonic flow lengths were also measured to make a quantitative investigation on the jet structure. The effect of inflow condition at nozzle inlet was found to be pronounced resulting in the sonic line moved upstream of the nozzle throat. Moreover, numerically predicted results were compared with the experimental data to validate the present numerical code.

Keywords: Compressible flow; numerical analysis; potential core; shock wave; supersonic flow lengths

1. INTRODUCTION

Supersonic circular and planar jets have occupied attention of the researchers for over half a century. Understanding the characteristics of supersonic jets is critical to the optimization of thrust generation for rockets, gas turbines and so on. Moreover, supersonic jets have many diverse fields of engineering applications, such as in supersonic aircrafts, jet propulsion thrust vectoring, fuel injectors for supersonic combustion, soot blower device, coating technology for structural materials and so on. A great deal of researches have been conducted theoretically, experimentally and numerically to obtain detailed flow features of supersonic jets [1-7]. It has been well known that the major structure of supersonic jet is determined by nozzle pressure ratio and nozzle configuration. Much effort has been devoted to the major characteristics of supersonic jets. According to these previous works, the under-expanded supersonic jet is specified by its barrel shock structure, Mach disk location, jet boundary configuration, velocity decay and supersonic length, etc., which are usually determined by jet pressure ratios [8-15]. Especially, many works have been done to investigate the detail of Mach disk structure; since it is important in the determination of major characteristics of supersonic jets [4, 16, 17]. Moreover, ideally-expanded supersonic jets have been studied through a large number of experimental, analytical and numerical investigations by many researchers over the past years. Most of studies pointed out the effect of nozzle divergence angle [5], Pitot pressure distributions [6]; while some studies proposed empirical formulae [7] on the spreading rate of the turbulent jets.

In the present study, a numerical work was conducted to investigate the effect of inflow conditions on the aerodynamic and shock characteristics of supersonic turbulent jets from sonic nozzle. The pressure and Mach number distributions in the axial direction were determined, and flowfield structure was visualized by density distributions. The potential core and supersonic flow lengths were also measured to make a quantitative investigation on the jet structure. Moreover, the numerically predicted results of the correctly-expanded supersonic free jets from convergent-divergent nozzle were compared with the experimental data to validate the present numerical code.

2. NUMERICAL ANALYSIS

2.1. Mathematical modeling

Computational Fluid Dynamic (CFD) investigations were performed to investigate the effect of inflow conditions on aerodynamic and shock characteristics of under-expanded supersonic turbulent jets. Flow under the study was treated as compressible, viscous, unsteady and turbulent. The governing equations are given by the conservation forms of mass, momentum and energy. The mass averaged, time-dependent Navier-Stokes equations were employed to mathematically model the present problem. The resulting equations are expressed in an integral form:

$$\boldsymbol{\Gamma} \frac{\partial}{\partial t}\left(\int_{v} \boldsymbol{Q}\, dV \right) + \oint \left[\boldsymbol{F} - \boldsymbol{G} \right] dA = 0 \tag{1}$$

where $\boldsymbol{F}$ and $\boldsymbol{G}$ are the inviscid and viscous flux vectors in standard conservation form and $\boldsymbol{Q}$ is the dependent vector of primary variables.

$$\boldsymbol{F} = \left[\rho v, \rho v v_x + p\hat{i}, \rho v v_y + p\hat{j}, \rho v v_z + p\hat{k}, \rho v H \right]^T \tag{2}$$

$$\boldsymbol{G} = \left[0, \tau_{xi}, \tau_{yi}, \tau_{zi}, \tau_{ij} v_j + q \right]^T \tag{3}$$

$$\boldsymbol{Q} = \left[p, v_x, v_y, v_z, T \right]^T \tag{4}$$

Here ρ, v and p are the density, velocity, and pressure of the fluid, respectively. τ is the viscous stress tensor, and q is the heat flux.
In the above equations, H is total enthalpy per unit mass and is related to the total energy E by $H = E + p/\rho$, where E includes both internal and kinetic energies. The preconditioning matrix $\boldsymbol{\Gamma}$ is included in Eq. (1) to provide an efficient solution of the present two-dimensional compressible flow. This matrix is given by

$$\boldsymbol{\Gamma} = \begin{bmatrix} \theta & 0 & 0 & 0 & \rho_T \\ \theta v_x & \rho & 0 & 0 & \rho_T v_x \\ \theta v_y & 0 & \rho & 0 & \rho_T v_y \\ \theta v_z & 0 & 0 & \rho & \rho_T v_z \\ \theta H - \delta & \rho v_x & \rho v_y & \rho v_z & \rho_T H + \rho C_p \end{bmatrix} \tag{5}$$

where ρ_T is the derivative of density with respect to temperature at constant pressure. Here, δ = 1 for an ideal gas and δ = 0 for an incompressible fluid. The parameter θ is defined as

$$\theta = 1/U_r^2 - \left(\rho_T H + \rho C_p \right) \tag{6}$$

In Eq. (6), the reference velocity U_r is chosen such that eigenvalues of the system remain well conditioned with respect to the convective and diffusive timescales and C_p is the specific heat at constant pressure.

To close the governing equations, the standard k-ε turbulent model [18, 19] was employed in computations. Robustness, economy, and reasonable accuracy for a wide range of turbulent flows explain its popularity in fluid flow and heat transfer simulations. It is a semi-empirical model, and the derivation of the model equations relies on phenomenological considerations and empiricism.

2.2. Numerical modeling

The preconditioned governing equations were discretized spatially using a cell-centered finite volume scheme, in which the physical domain is subdivided into numerical cells and integral equations are applied to each cells. A fully implicit method was implemented on the present multi-block spatial domain. The convective fluxes were formulated using the Roe's flux difference splitting scheme [20], and the third-order accuracy of this scheme was conceived from the original MUSCL (Monotone Upstream Centered Schemes for Conservation Laws) [21] by blending a central differencing scheme and second-order upwind scheme. For the time derivatives in the governing equations, an implicit multistage time stepping scheme [22, 23], which is advanced from time t to time $t + \Delta t$ with a 2nd order Euler backward scheme for physical time and implicit pseudo-time marching scheme for inner iteration, was used.

2.3. Computational conditions

Under-expanded supersonic turbulent jet flows driven by the cylindrical straight nozzle with exit diameter of ϕD_e=12.7 mm (characteristic length) were considered in the present computation. The nozzle is composed of a convergent curved entrance wall which has a radius of curvature R=D_e and following straight wall with a length of 0.4D_e. This is exactly the same as that used by Addy [4]. The jet pressure ratio was defined as the ratio of total pressure p_0 at nozzle supply to back pressure p_b (=p_0/p_b). In the present study, p_0/p_b was varied from 1.893 to 6.0. The upstream total temperature T_0 and total pressure p_0 were assumed to be maintained constant at 293.15K and 101.3kPa, respectively, through the whole computations.

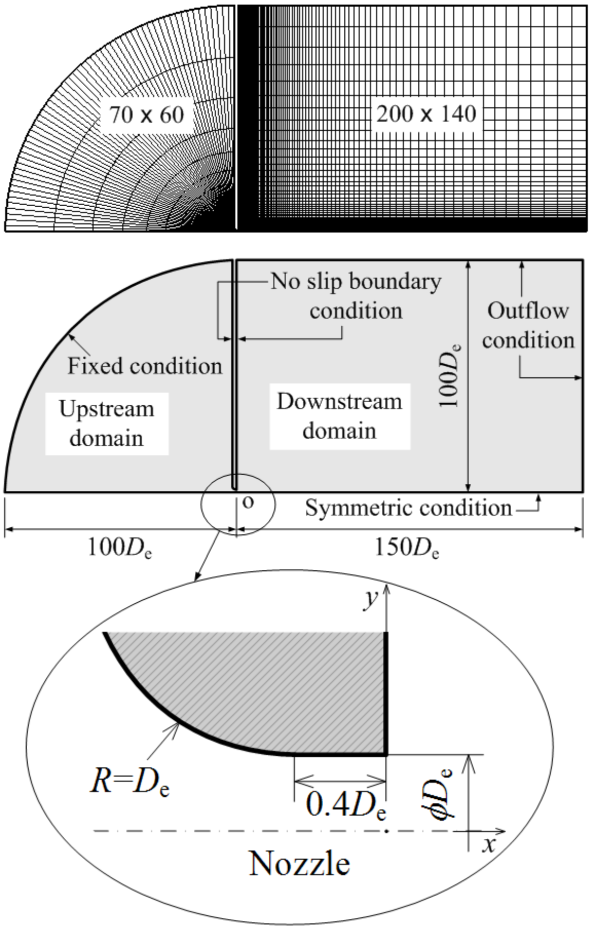


Fig. 1 Schematics of computational domain & boundary condition

Two cases of inflow conditions at upstream of the computational domain were studied in the present simulation works. In case 1, fixed boundary conditions were applied to upstream boundary of the computational domain i.e. at upstream of the nozzle inlet; while chocked flow condition (M_e=1) at nozzle exit was employed as inflow condition for case 2. The outlet static pressure was used at exit of the computational domain. Adiabatic and no-slip boundary conditions were applied to solid wall surfaces. The symmetric condition was used at the boundary of nozzle center line. The upstream domain in the present computations extends to the distance of 100D_e upstream from the nozzle exit for case 1, while the downstream domain covers the region of 150D_e and 100D_e from the nozzle exit in the x- and y-directions, respectively. This wide computational domain was required to ensure the computational domain independent solutions.

The computational domain and boundary conditions used for simulating supersonic turbulent jets from sonic nozzle in the present study are illustrated in Fig. 1. The computational domain was meshed with quadrilateral elements with map scheme to yield a regular, structured grid. The grid nodes were placed in such a way that there were enough nodes near the throat and wall regions in order to capture the higher variable gradients accurately. The computational grids closest to the nozzle walls are located at $0.0003D_e$ away from the walls. Several preliminary computations have been performed on different computational grid meshes to investigate the grid-independent solutions. According to these results, the number of grids applied is 70×60 in the nozzle region and 200×140 in the jet plume region. The solution was declared as converged when the residual for each variable becomes less than the chosen convergence criterion. In present simulations, the convergence criteria for conserved variables were set to a value of 10^{-4}. Another convergence criterion is to check the conserved quantities directly through the computational boundaries. The net mass flux was investigated when there was an applicable imbalance through computational boundaries.

3. RESULTS & DISCUSSION

The validation of present computational code in predicting the effect of inflow conditions on aerodynamic and shock characteristics of supersonic turbulent jets was investigated through the experimental and numerical studies of correctly-expanded supersonic jet flows driven by a convergent-divergent nozzle. The nozzle which designed based upon the method of characteristics with a design Mach number of M_d=2.0 was considered in the present validation work. The nozzle has a throat diameter of ϕD_t=20 mm, an exit diameter of ϕD_e=26 mm and a straight section near the exit, as schematically shown in Fig. 2. In this validation work, p_0/p_b was kept constant at 7.8 to get nearly correct expansion at nozzle exit. The upstream total temperature T_0 and total pressure p_0 were assumed to be constant at 298.15 K and 101.3 kPa, respectively.

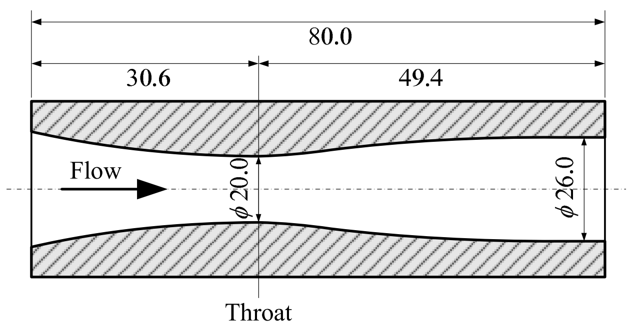


Fig. 2 Schematic view of convergent-divergent nozzle (M_d=2.0)

Figure 3 shows the comparison of predicted schlieren image of correctly-expanded supersonic jet with experimental data [24]. Figures 3(a) and 3(b) show the computed schlieren image and schlieren picture obtained by the experiment, respectively. At nozzle pressure ratio of p_0/p_b=7.8, the predicted jet Mach number is about M_j=2.0 and the flow is ideally-expanded at nozzle exit. In this case, as the pressure at nozzle exit is nearly matched to the back pressure, no shock cell structure is found in the jet. However, the weak compression waves (C), which observed in Fig. 3, are due to the boundary layer effect along nozzle interior wall. The jet boundary is nearly parallel to the jet axis. The predicted schlieren image is nearly the same as visualized by the experiment. From comparison of the computed and experimental results, it is found that the present computation predicts well the correctly-expanded supersonic jets.

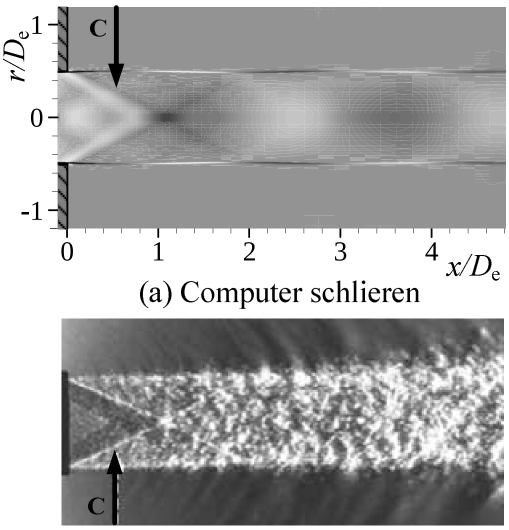


Fig. 3 Comparison of numerical schlieren image with experimental data (p_0/p_b=7.8)

Numerically predicted density contours of supersonic turbulent jets issued from sonic nozzle at pressure ratios p_0/p_b of 1.893, 2.50 and 6.0 for case 1 are shown in Figs. 4(a-c). Similarly, Figs. 5(a-c) show the jet flowfields for case 2. In these figures, the jet flowfields evolve from correct-expansion to moderately high under-expansion. An increase in the nozzle pressure ratio p_0/p_b makes the stream wise extent of the shock cells that roughly indicated the increment in the length of the 'supersonic core'. For case 2, the jet was found correctly-expanded at nozzle exit (M_e=1) in Fig. 5(a); while slightly over-expanded jet was found in Fig. 4(a) after employing the fixed boundary conditions at upstream of the computational domain (case 1). This clearly noticed a strong influence of inflow conditions on predicting the jet structure. In Fig. 5(c), small Mach disk was found in the jet flowfield at moderately high under-expansion condition (p_0/p_b=6.0). On the contrary, for case 1, jet becomes moderately under-expanded and Mach disk could not be found in the flowfield even if at same pressure ratio due to the change in the inflow condition, as shown in Fig. 4(a).

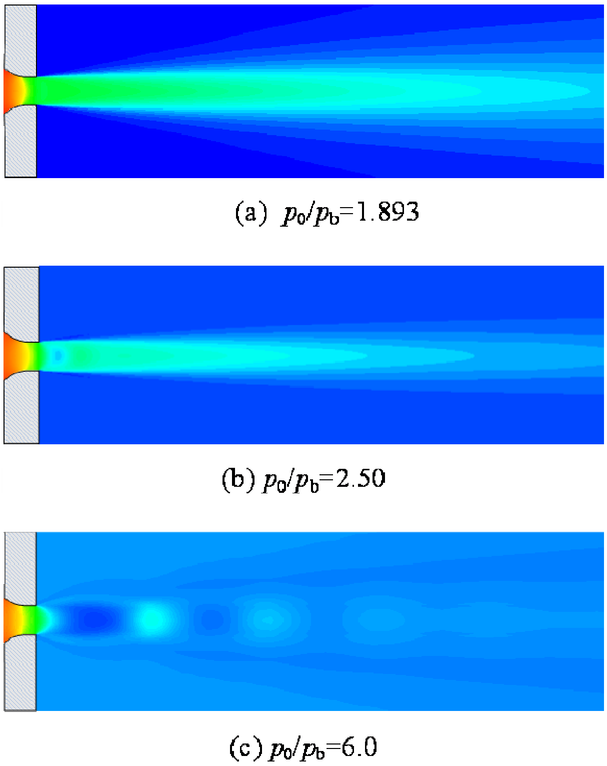
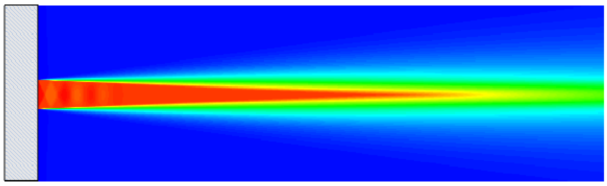
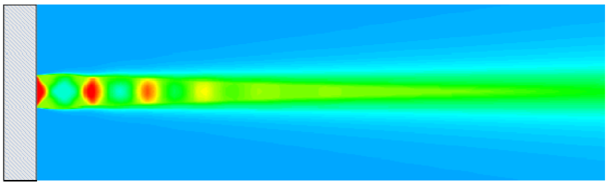


Fig. 4 Density contours under different operating conditions (Case 1)

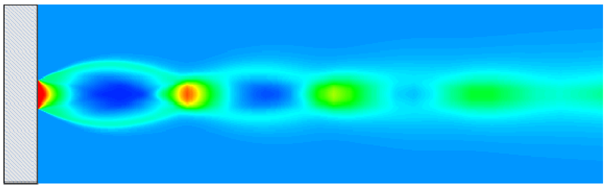
Jet centerline static pressure distributions for cases of 1 and 2 are presented in Fig. 6. The pressures under each operating condition (p_0/p_b) exhibit unique characteristics. The pressure variation shows the characteristic quasi-periodic structure, due to the presence of shock cells in the flowfield of jets at weak to moderately high under-expansion conditions. Moreover, as the nozzle pressure ratio p_0/p_b increases, the flowfield is observed to be stretched, where the shock cell spacing increases, leading to an increase in the length over which shocks are present. This phenomena show an agreement with the experimental observation of Addy [4]. Furthermore, longer shock cells were found for case 2 in comparison with the case 1.



(a) p_0/p_b=1.893, correctly-expanded jet



(b) p_0/p_b=2.50, weakly under-expanded jet



(c) p_0/p_b=6.0, moderately high under-expanded jet

Fig. 5 Density contours under different operating conditions (Case 2)

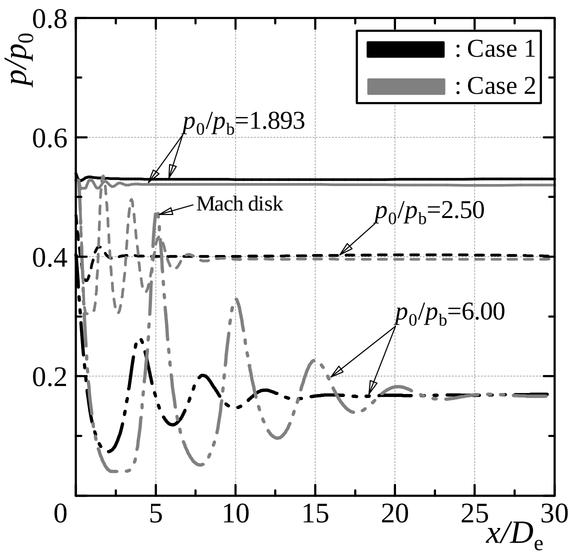


Fig. 6 Static pressure distributions on jet axis

Mach number distributions on jet axis for cases of 1 and 2 are illustrated in Fig. 7. The influence of inflow conditions on the numerical simulations of supersonic turbulent jets from sonic nozzle can be clarified clearly from this figure. At pressure ratio p_0/p_b=6.0, small Mach disk is appeared in the flowfield for case 2; while jet become moderately under-expanded for case 1 – no Mach disk in the shock cell. It is mentioned that inflow condition has strong influence on predicting the potential core length of jet. However, for case 2, the potential core length at a given pressure ratio (e.g. p_0/p_b=2.5) was found larger in comparison with the case 1.

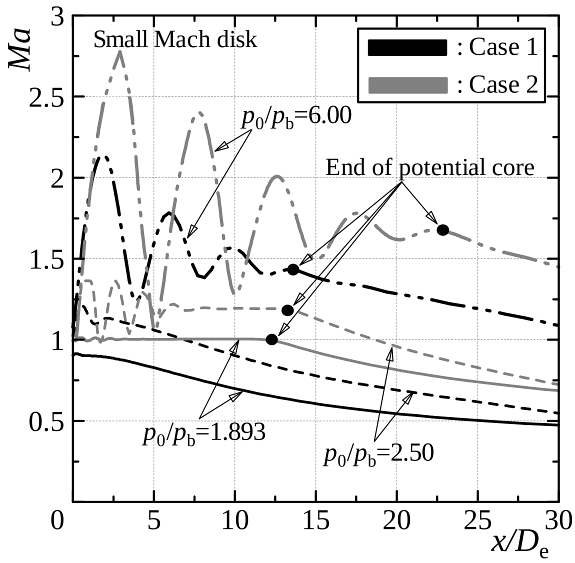


Fig. 7 Mach number distributions on jet axis

Potential core and supersonic flow lengths under different operating conditions (p_0/p_b) were measured from the Mach contours and distributions of Mach number on jet axis, and presented in Fig. 8. Potential core length x_c and supersonic flow length x_s are normalized by the nozzle exit diameter D_e. Both the potential core x_c/D_e and supersonic flow length x_s/D_e increases with the increase in the nozzle pressure ratio p_0/p_b. Here, it is mentioned that no supersonic flow length was found for case 1 due to the effect of inflow condition. Potential core x_c/D_e and supersonic flow length x_s/D_e were found lager for case 2 in comparison with the case 1.

Figure 9 shows the distribution of sonic lines near the nozzle exit for cases of 1 and 2. Since, chocked condition (M_e=1) was applied for case 2 the sonic lines under different operating condition (p_0/p_b) were found co-linear at nozzle exit. On the contrary, though the Mach number at nozzle exit should be theoretically 1, the sonic line moves to the upstream region from the nozzle exit for case 1 due to the influence of inflow conditions in simulating turbulent jets. Moreover, supersonic core at a given nozzle pressure ratio p_0/p_b was found larger in radial direction for case 2 in comparison with the case 1. Furthermore, this figure also evident that the increase in the nozzle pressure ratio expands the jet configuration outward.

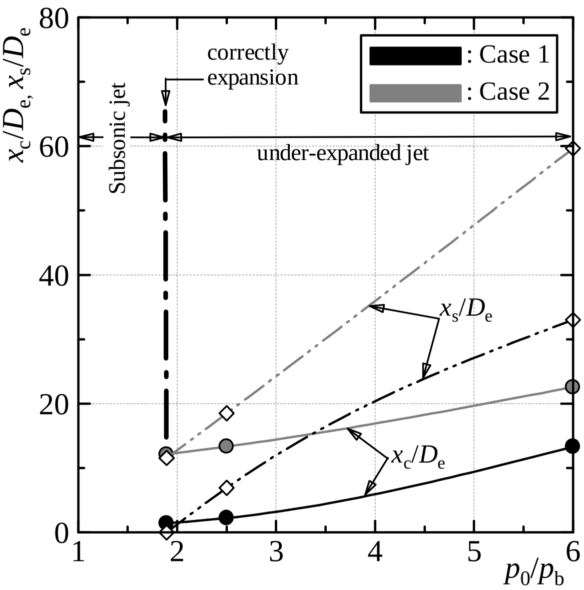


Fig. 8 Potential core and supersonic flow lengths under different operating conditions

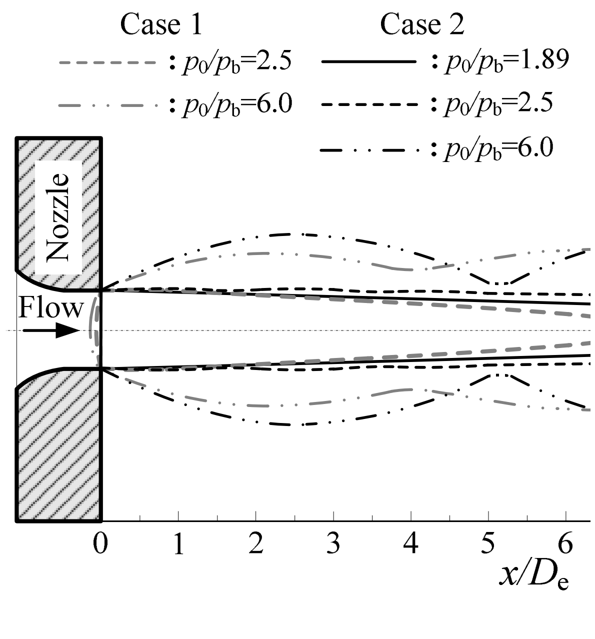


Fig. 9 Distributions of sonic lines near nozzle exit

5. CONCLUSIONS

In the present study, two-dimensional axisymmetric turbulent jet flows from sonic nozzle were investigated under different operating conditions (p_0/p_b).

The unsteady, compressible, Navier-Stokes equations have been solved numerically to simulate the effect of inflow conditions on the aerodynamic and shock characteristics of supersonic jets. The numerically predicted results of the correctly-expanded supersonic free jets from convergent-divergent nozzle were compared with the experimental data to validate the present numerical code. A great influence of inflow conditions on the aerodynamic and shock characteristics of supersonic turbulent jet were found. The conclusions are summarized as follows: the increase in the pressure ratio increases the shock cell spacing, potential core and supersonic flow lengths. The sonic line moves to the upstream region from nozzle exit in case of using fixed boundary conditions at upstream of the computational domain. Potential core and supersonic flow length were found lager when chocked condition was applied at nozzle exit as inflow condition. Supersonic core at chocked inflow condition was found larger in the radial direction. Moreover, outward expansion of jet configuration was found with the increase in nozzle pressure ratio.

6. REFERENCES

[1] D'anbrosio, D., DeSocio, L.M. and Gaffuri, G., Physical and Numerical Experiments on an Under-Expanded Jet. *Meccanica*, 34, 267–280, 1999.

[2] Love, E.S., Grigsby, C.E., Lee, L.P. and Woodling, M.J., Experimental and theoretical studies of axisymmetric free jets. *NASA TR R*-6, 1959.

[3] Frauenberger, J.H. and Forister, J.G., The axial decay and radial spread of a supersonic jet exhausting into air at rest. *Aeronaut. Q.*, 12, 131-149, 1961.

[4] Addy, A.L., Effects of axisymmetric sonic nozzle geometry on Mach disk characteristics. *AIAA J.*, 19(1), 121-122, 1981.

[5] Kashitani, M., Miyazato, Y., Masuda, M. and Matsuo, K., Numerical and experimental investigations of supersonic jets from sootblower nozzle. *JSME Intl. J.*, *Series B*, 41(2), 375-380, 1998.

[6] Katanoda, H., Miyazato, Y., Masuda, M. and Matsuo, K., Pitot pressures of correctly-expanded and under-expanded free jets from axisymmetric supersonic nozzles. *Shock Waves*, 10, 95-101, 2000.

[7] Kashitani, M., Yamaguchi, Y., Miyazato, Y., Masuda, M. and Matsuo, K., Mean-flow properties on high and low Reynolds number ideally

expanded supersonic jets. *AIAA*-2001-1053, 2001.

[8] Villermaux, E. and Rehab, H., Mixing in Coaxial Jets. *J. Fluid Mech.*, 425, 161–185, 2000.

[9] Sheeran, W. and Dosanjh, D., Observations on Jet Flows from a Two-Dimensional, Under-expanded, Sonic Nozzle. *AIAA J.*, 6(3), 540–542, 1968.

[10] Chang, I.S. and Chow, W.L., Mach Disk from Under-expanded Axisymmetric Nozzle Flow. *AIAA J.*, 12(8), 1079-1082, 1974.

[11] Kim, H.D. and Shin, H.S., Numerical Study on Under-Expanded Jets through a Supersonic Nozzle (Part 2). *J. Korea Society of Mech. Eng., Series B*, 20(6), 1994-2004, 1996.

[12] Kim, H.D. and Lee, J.S., An Experimental Study on Supersonic Jet Issuing from Gas Atomizing Nozzle (Part 1). *J. Korea Society of Mech. Eng., Series B*, 20(2), 677-709, 1996.

[13] Mate, B., Graur, I.A., Elizarova, T., Chirokov, I., Tejeda, G., Fernandez, J.M. and Montero, S., Experimental and Numerical Investigation of an Axisymmetric Supersonic Jet. *J. Fluid Mech.*, 426, 177-197, 2001.

[14] Abbett, M., Mach Disk in Under-expanded Exhaust Plumes. *AIAA J.*, 9(3), 512-514, 1971.

[15] Davidor, W. and Penner, S.S., Shock Stand-off Distance and Mach Disk Diameters in Under-expanded Sonic Jets. *AIAA J.*, 9(8), 1651-1652, 1971.

[16] Eastman, D.W. and Radtke, L.P., Location of the Normal Shock Wave in the Exhaust Plume of a Jet. *AIAA J.*, 1(4), 918-919, 1963.

[17] Crist, S. Sherman, P.M. and Glass, D.R., Study of the Highly Under-expanded Sonic Jet. *AIAA J.*, 4(1), 68-71, 1966.

[18] Launder, B.E. and Spalding, D.B., *Lectures in Mathematical Models of Turbulence*, Academic Press, London, England, 1972.

[19] Launder, B.E. and Spalding, D.B., The Numerical Computation of Turbulent Flows. *Comp. Meth. Appl. Mech. Eng.*, 3, 269-289, 1974.

[20] Roe, P.L., Approximate Riemann solvers, parameter vectors, and difference schemes. *J. Comput. Phys.*, 43, 357-372, 1981.

[21] van Leer, B., Towards the ultimate conservative difference scheme V. A second order sequel to Godunov's method. *J. Comput. Phys.*, 32, 101-136, 1981.

[22] Pandya, S.A., Venkateswaran, S. and Pulliam, T.H., Implementation of dual-time procedures in overflow. *Technical Report AIAA*-2003-

0072, 2003.
[23] Turkel, E. and Vatsa, V.N., Choice of variables and preconditioning for time dependent problems. *Technical Report AIAA*-2003-3692, *16th AIAA Comp. Fluid Dyn. Conf., Orlando, Florida, June* 2003.
[24] Kweon, Y.H., Miyazato, Y., Aoki, T., Kim, H.D. and Setoguchi, T., Experimental investigation of nozzle exit reflector effect on supersonic jet. *Shock Waves*, 15, 229-239, 2006.

7. NOMENCLATURE

Symbol	**Meaning**	**Unit**
D	nozzle diameter	(m)
M	Mach number	(-)
p	pressure	(Pa)
T	temperature	(K)
x	axial distance	(m)
Sub-script	**Meaning**	
0	stagnation	
b	ambient	
c	potential core	
d	design	
e	nozzle exit	
s	supersonic flow	
t	nozzle throat	