# ANALYSIS OF THICK PLATES ON ELASTIC FOUNDATION BY BACK-PROPAGATION ARTIFICIAL NEURAL NETWORK USING ONE PARAMETER FOUNDATION MODEL 

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#### Abstract

In this study, the purpose of this paper is prediction of nondimensional maximum displacement and bending moments of thick plates on Winkler-type elastic foundation using Artificial Neural Network. For this purpose, training and testing database were created by using a computer software, coded in Fortran, based on Finite Element Model. An eight-noded (PBQ8) quadrilateral finite element based on Mindlin plate theory and Winkler foundation model are adopted for the finite element solution. Nondimensional subgrade reaction modulus, span/thickness ratio of the plate and aspect ratio of the plate were considered as input parameters and nondimensional vertical displacement and bending moments of thick plates were considered as output parameters. It is seen that the solutions by ANN agree very well with the solutions by FEM, and ANN significantly reduces analysis time.


Keywords: Artificial neural network, back propagation, Thick Plate, Elastic Foundation, Winkler Model

## 1. Introduction

Plates on elastic foundation are found wide applications in practical engineering structures. A lot of numerical methods such as finite difference method, finite element method and boundary element method have been widely used for solving these problems. But these methods require more computational time. So Artificial Neural Network has attracted a lot of attention in the solution of these kinds of problems.

In general, the analysis of this problem is based on the incorporation of the foundation reaction into the corresponding differential equation of plates. Although various models have been introduces to consider interaction between the plate and its foundation, Winkler model often is preferred because of its simplicity and providing good result especially under the point loads. On the other hand, Kirchhoff thin plate theory is used in most of studies on platefoundation interaction in technical literature. But, the effect of shear deformation thorough the plate thickness have been ignored in the classical plate theory. However the effect of the shear deformation becomes important as the thickness of the plate increases. Therefore Mindlin plate element taking shear deformations into account is used in this study.

Artificial Neural Networks are used in many areas such as financial, industrial, military, health and engineering since it can actually realize a lot of functions such as estimation, classification, data mapping, data filtering, recognition and matching, identifying and interpreting. Many studies demonstrating successful outcomes with ANN approach can be also found in the area of civil engineering [1-6].

In this paper, the authors designed a new ANN architecture for thick plates on Winkler-type elastic foundation and obtained dimensionless factors of displacement and moments of the
thick plate using dimensionless parameters such as nondimensional subgrade reaction modulus, span/thickness ratio of the plate and aspect ratio of the plate.

## 2. Artificial Neural Network (ANN)

ANNs are computer programs inspired from human brain. Human brain has about $10^{11}$ nerve cells called neuron. Information is processed in neurons in human brain. Hence a neuron is defined as main component of ANN models. A biological neuron shown in Fig. 1 composed of four main parts. Exterior signals come from other neurons or environments are received by input path of a neuron. This path called as dendrites ${ }^{(1)}$. Received exterior signals by dendrites are summed in the cell body (soma) ${ }^{(2)}$. If summed signals are greater than the threshold level of the neuron, the cell body produces an impulse and sends it to the path of a neuron (axon) ${ }^{(3)}$. End of axon is splits up to many branches. All branches connect to many dendrites of other neurons through a junction called synapses ${ }^{(4)}$ [7].


Figure 1. Main parts of a biological nerve cell
Artificial neurons have input wires, connection weights, activation functions and output wires instead of the dendrites, the synapses, the cell body (soma) and the axon in a biological neuron respectively. Received input signals ( $x_{\mathrm{i}}$ ) are summed after weighted by synaptic weights ( $w_{\mathrm{ij}}$ ) between neurons by Eq. 1 and an output is produced by activation function in a mathematical neuron. Outputs produced by activation function in artificial neurons are either used as an input or results for next layer neurons or output layer respectively. Generally Threshold, Piecewise linear, Gaussian and Sigmoid function are used as activation function in ANN studies. In this study, sigmoid function defined by Eq.2, multi layer feed forward neural network shown in Fig. 2 and back propagation called generalized delta rule were used as activation function, network topology and learning algorithm respectively [7].

$$
\begin{align*}
& u_{j}=\sum_{i=1}^{n} w_{i j} x_{i}  \tag{1}\\
& f\left(u_{j}\right)=\frac{1}{1+e^{-\beta\left(u_{j}-b_{j}\right)}} \tag{2}
\end{align*}
$$

Here, $u_{j}$ is summation of $n$ inputs for neuron $j, w_{i j}$ is synaptic weight between neurons $i$ and $j$, $x_{i}$ is input for neuron $j, f\left(u_{j}\right)$ is output of neuron $\mathrm{j}, \beta$ is slope parameter and $b_{j}$ is bias value of neuron $j$.


Figure 2. (a) A feedforward ANN and (b) a neuron of artificial neural network
There are an input layer, one or more hidden layer(s) and an output layer in a multi layer feed forward neural network. Each layer has neuron(s). Numbers of input layer neurons are the same with numbers of input variables of problem. Similarly, numbers of output layer neurons are the same with the numbers of output variables of problem. Number(s) of hidden layer(s) and number(s) of neuron(s) in hidden layer(s) must be well determined by researcher. Generally, trial and error approach based on type of problem, computational speed and computational accuracy was used in determinations of number(s) of hidden layer(s) and number(s) of neuron(s) in hidden layer(s).Initial values of synaptic weights and bias of neural network allocated randomly at the beginning of network training. These values are recalculated according to errors between produced network outputs and desired outputs during backward propagation of network error(s). Network training is continued until network errors for all training examples become lesser than desired error level. In general, validation of trained neural network model is done empirically using test examples different from training examples. Mostly, error calculations such as mean absolute error, mean squared error, root mean squared error and mean absolute percent error (MAPE) are used in the validation studies of developed ANN model. Mean absolute percent error (MAPE) calculation given in Eq. 3 was preferred for validation of developed ANN model in this study.

$$
\begin{equation*}
M A P E=M e a n\left(\sum_{i=1}^{n} \sum_{j=1}^{m} \frac{\left|t_{i j}-y_{i j}\right|}{y_{i j}} .100\right) \tag{3}
\end{equation*}
$$

## 3. Plates on Elastic Foundation and Training of ANN

The main equation of plates on Winkler elastic foundation is

$$
\begin{equation*}
D \nabla^{4} w+k w=q \tag{4}
\end{equation*}
$$

Here, $w, D, k$ and $q$ are displacement of the plate in vertical direction, flexural rigidity of the plate, subgrade reaction modulus of the soil and external load.

In this study, a new ANN architecture designed by the authors for thick plates on Winklertype elastic foundation is used. Training sets and test sets are obtained by using a computer program coded in Fortran. Computer program is based on FEM. 8-noded (PBQ8) quadrilateral rectangular finite elements based on Mindlin theory is used to develop the element stiffness matrix. More details can be found in reference [8].

As a numerical example, a plate freely resting on Winkler-type foundation subjected to external concentrated load is considered to demonstrate the accuracy and the efficiency of ANN formulation. Poisson's ratio of plate is 0.30 . Aspect ratio of the plate is taken as $1.0,1.5$ and 2.0. The ratio of the span length to the plate thickness is considered as $40,20,15,10,5$ and 4 for each aspect ratio. The nondimensional modulus of subgrade reaction is taken as 1,2 , $3,4,5,6,7,8,9,10,11,12,13,14$ and 15 for each the ratio of span length to plate thickness. $16 \times 16$ mesh sizes are used in the analysis.

## Training and Testing Data:

The nondimensional modulus of subgrade reaction and factors for displacement and bending moments are used and they are defined as follows,

$$
\begin{align*}
& K=l_{x}(k / D)^{0.25} \\
& a_{m}=w D /\left(P l_{x}^{2}\right)  \tag{5}\\
& c_{x}=M_{x} / P \\
& c_{y}=M_{y} / P
\end{align*}
$$

where $l_{\mathrm{x}}$ is the short span, $D$ is flexural rigidity of the plate and $P$ is concentrated load.
ANN is trained with the dimensionless maximum displacement and bending moments obtained from the FEM analysis as outputs and the related parameters such as nondimensional subgrade reaction modulus, aspect ratio and span/thickness ratio as inputs.

Table 1. Training examples for ANN

|  | Inputs |  |  | Outputs |  |  | Inputs |  |  |  | Outputs |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No | K | $l_{x} / h$ | $l_{x} / l_{y}$ | $1000 . a_{m}$ | 100.cx | 100.cy | No | K | $l_{x} / h$ | $l_{x} / l_{y}$ | $1000 . a_{m}$ | 100.cx | 100.c. ${ }^{\text {y }}$ |
| 1 | 1 | 40 | 1.0 | 625.74 | 37.24 | 37.24 | 22 | 13 | 20 | 1.0 | 1.05 | 23.25 | 23.25 |
| 2 | 2 | 40 | 1.0 | 66.68 | 36.97 | 36.97 | 23 | 1 | 15 | 1.0 | 927.79 | 40.40 | 40.40 |
| 3 | 4 | 40 | 1.0 | 8.96 | 34.05 | 34.05 | 24 | 2 | 15 | 1.0 | 69.42 | 40.13 | 40.13 |
| 4 | 5 | 40 | 1.0 | 5.55 | 31.67 | 31.67 | 25 | 3 | 15 | 1.0 | 19.09 | 39.12 | 39.12 |
| 5 | 6 | 40 | 1.0 | 3.76 | 29.48 | 29.48 | 26 | 4 | 15 | 1.0 | 9.69 | 37.13 | 37.13 |
| 6 | 7 | 40 | 1.0 | 2.71 | 27.70 | 27.70 | 27 | 5 | 15 | 1.0 | 6.22 | 34.69 | 34.69 |
| 7 | 8 | 40 | 1.0 | 2.05 | 26.23 | 26.23 | 28 | 6 | 15 | 1.0 | 4.41 | 32.44 | 32.44 |
| 8 | 9 | 40 | 1.0 | 1.62 | 24.98 | 24.98 | 29 | 7 | 15 | 1.0 | 3.33 | 30.58 | 30.58 |
| 9 | 10 | 40 | 1.0 | 1.32 | 23.88 | 23.88 | 30 | 8 | 15 | 1.0 | 2.65 | 29.02 | 29.02 |
| 10 | 11 | 40 | 1.0 | 1.09 | 22.89 | 22.89 | 31 | 10 | 15 | 1.0 | 1.87 | 26.47 | 26.47 |
| 11 | 1 | 20 | 1.0 | 873.71 | 39.91 | 39.91 | 32 | 11 | 15 | 1.0 | 1.63 | 25.35 | 25.35 |
| 12 | 2 | 20 | 1.0 | 68.79 | 39.64 | 39.64 | 33 | 12 | 15 | 1.0 | 1.45 | 24.32 | 24.32 |
| 13 | 3 | 20 | 1.0 | 18.71 | 38.64 | 38.64 | 34 | 13 | 15 | 1.0 | 1.30 | 23.34 | 23.34 |
| 14 | 4 | 20 | 1.0 | 9.33 | 36.68 | 36.68 | 35 | 14 | 15 | 1.0 | 1.18 | 22.42 | 22.42 |
| 15 | 5 | 20 | 1.0 | 5.89 | 34.26 | 34.26 | 36 | 15 | 15 | 1.0 | 1.08 | 21.54 | 21.54 |
| 16 | 7 | 20 | 1.0 | 3.02 | 30.21 | 30.21 | 37 | 1 | 10 | 1.0 | 971.45 | 40.77 | 40.77 |
| 17 | 8 | 20 | 1.0 | 2.36 | 28.69 | 28.69 | 38 | 2 | 10 | 1.0 | 70.68 | 40.50 | 40.50 |
| 18 | 9 | 20 | 1.0 | 1.91 | 27.38 | 27.38 | 39 | 3 | 10 | 1.0 | 20.14 | 39.45 | 39.45 |


| 19 | 10 | 20 | 1.0 | 1.60 | 26.22 | 26.22 | 40 | 4 | 10 | 1.0 | 10.69 | 37.41 | 37.41 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 20 | 11 | 20 | 1.0 | 1.37 | 25.16 | 25.16 | 41 | 5 | 10 | 1.0 | 7.17 | 34.91 | 34.91 |
| 21 | 12 | 20 | 1.0 | 1.19 | 24.17 | 24.17 | 42 | 6 | 10 | 1.0 | 5.31 | 32.59 | 32.59 |
| Table 1continued |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | Inputs |  |  | Outputs |  |  | Inputs |  |  |  | Outputs |  |  |
| No | K | $l,{ }_{2} /$ | $l_{1} / l_{v}$ | 1000.am | 100.cx | 100.c. ${ }^{\text {c }}$ | No | K | $l_{2} /$ /h | $l_{2} / l_{v}$ | $1000 . a_{m}$ | 100.c. ${ }^{\text {x }}$ | 100.c. ${ }^{\text {c }}$ |
| 43 | 7 | 10 | 1.0 | 4.20 | 30.64 | 30.64 | 94 | 7 | 20 | 1.5 | 2.98 | 30.17 | 30.16 |
| 44 | 8 | 10 | 1.0 | 3.48 | 29.00 | 29.00 | 95 | 8 | 20 | 1.5 | 2.34 | 28.68 | 28.70 |
| 45 | 9 | 10 | 1.0 | 2.99 | 27.55 | 27.55 | 96 | 9 | 20 | 1.5 | 1.90 | 27.38 | 27.40 |
| 46 | 10 | 10 | 1.0 | 2.64 | 26.23 | 26.23 | 97 | 10 | 20 | 1.5 | 1.59 | 26.22 | 26.23 |
| 47 | 11 | 10 | 1.0 | 2.37 | 25.00 | 25.00 | 98 | 11 | 20 | 1.5 | 1.36 | 25.16 | 25.16 |
| 48 | 13 | 10 | 1.0 | 1.98 | 22.74 | 22.74 | 99 | 12 | 20 | 1.5 | 1.19 | 24.17 | 24.17 |
| 49 | 14 | 10 | 1.0 | 1.84 | 21.69 | 21.69 | 100 | 13 | 20 | 1.5 | 1.05 | 23.25 | 23.25 |
| 50 | 15 | 10 | 1.0 | 1.72 | 20.69 | 20.69 | 101 | 1 | 15 | 1.5 | 627.51 | 40.11 | 46.60 |
| 51 | 1 | 5 | 1.0 | 1004.80 | 41.00 | 41.00 | 102 | 2 | 15 | 1.5 | 54.11 | 39.76 | 44.95 |
| 52 | 2 | 5 | 1.0 | 76.57 | 40.69 | 40.69 | 103 | 3 | 15 | 1.5 | 17.90 | 38.62 | 40.96 |
| 53 | 3 | 5 | 1.0 | 25.76 | 39.48 | 39.48 | 104 | 4 | 15 | 1.5 | 9.45 | 36.67 | 37.10 |
| 54 | 4 | 5 | 1.0 | 16.01 | 37.16 | 37.16 | 105 | 5 | 15 | 1.5 | 6.05 | 34.42 | 34.35 |
| 55 | 5 | 5 | 1.0 | 12.19 | 34.36 | 34.36 | 106 | 6 | 15 | 1.5 | 4.31 | 32.33 | 32.26 |
| 56 | 6 | 5 | 1.0 | 10.06 | 31.73 | 31.73 | 107 | 8 | 15 | 1.5 | 2.64 | 29.02 | 29.03 |
| 57 | 7 | 5 | 1.0 | 8.70 | 29.46 | 29.46 | 108 | 9 | 15 | 1.5 | 2.19 | 27.68 | 27.69 |
| 58 | 8 | 5 | 1.0 | 7.75 | 27.45 | 27.45 | 109 | 10 | 15 | 1.5 | 1.87 | 26.47 | 26.48 |
| 59 | 9 | 5 | 1.0 | 7.04 | 25.63 | 25.63 | 110 | 11 | 15 | 1.5 | 1.63 | 25.36 | 25.36 |
| 60 | 10 | 5 | 1.0 | 6.48 | 23.94 | 23.94 | 111 | 12 | 15 | 1.5 | 1.45 | 24.32 | 24.32 |
| 61 | 11 | 5 | 1.0 | 6.01 | 22.35 | 22.35 | 112 | 13 | 15 | 1.5 | 1.30 | 23.34 | 23.34 |
| 62 | 12 | 5 | 1.0 | 5.61 | 20.85 | 20.85 | 113 | 14 | 15 | 1.5 | 1.18 | 22.42 | 22.42 |
| 63 | 13 | 5 | 1.0 | 5.26 | 19.43 | 19.43 | 114 | 15 | 15 | 1.5 | 1.08 | 21.54 | 21.54 |
| 64 | 14 | 5 | 1.0 | 4.94 | 18.09 | 18.09 | 115 | 1 | 10 | 1.5 | 657.04 | 40.48 | 46.97 |
| 65 | 1 | 4 | 1.0 | 1012.5 | 41.03 | 41.03 | 116 | 2 | 10 | 1.5 | 55.36 | 40.13 | 45.30 |
| 66 | 2 | 4 | 1.0 | 80.9 | 40.68 | 40.68 | 117 | 3 | 10 | 1.5 | 18.97 | 38.96 | 41.27 |
| 67 | 3 | 4 | 1.0 | 29.94 | 39.35 | 39.35 | 118 | 4 | 10 | 1.5 | 10.46 | 36.96 | 37.37 |
| 68 | 4 | 4 | 1.0 | 19.95 | 36.84 | 36.84 | 119 | 5 | 10 | 1.5 | 7.00 | 34.65 | 34.56 |
| 69 | 5 | 4 | 1.0 | 15.87 | 33.85 | 33.85 | 120 | 6 | 10 | 1.5 | 5.22 | 32.48 | 32.40 |
| 70 | 6 | 4 | 1.0 | 13.50 | 31.04 | 31.04 | 121 | 7 | 10 | 1.5 | 4.16 | 30.61 | 30.59 |
| 71 | 7 | 4 | 1.0 | 11.92 | 28.59 | 28.59 | 122 | 8 | 10 | 1.5 | 3.47 | 28.99 | 28.99 |
| 72 | 8 | 4 | 1.0 | 10.77 | 26.41 | 26.41 | 123 | 9 | 10 | 1.5 | 2.99 | 27.54 | 27.55 |
| 73 | 10 | 4 | 1.0 | 9.12 | 22.58 | 22.58 | 124 | 11 | 10 | 1.5 | 2.37 | 25.00 | 25.00 |
| 74 | 11 | 4 | 1.0 | 8.48 | 20.86 | 20.86 | 125 | 12 | 10 | 1.5 | 2.16 | 23.84 | 23.84 |
| 75 | 12 | 4 | 1.0 | 7.92 | 19.25 | 19.25 | 126 | 13 | 10 | 1.5 | 1.98 | 22.74 | 22.74 |
| 76 | 13 | 4 | 1.0 | 7.41 | 17.74 | 17.74 | 127 | 14 | 10 | 1.5 | 1.84 | 21.69 | 21.69 |
| 77 | 14 | 4 | 1.0 | 6.95 | 16.33 | 16.33 | 128 | 15 | 10 | 1.5 | 1.72 | 20.69 | 20.69 |
| 78 | 15 | 4 | 1.0 | 6.51 | 15.00 | 15.00 | 129 | , | 5 | 1.5 | 681.54 | 40.73 | 47.19 |
| 79 | 2 | 40 | 1.5 | 51.96 | 36.60 | 41.75 | 130 | 2 | 5 | 1.5 | 61.45 | 40.34 | 45.42 |
| 80 | 3 | 40 | 1.5 | 17.08 | 35.48 | 37.82 | 131 | 3 | 5 | 1.5 | 24.65 | 39.02 | 41.19 |
| 81 | 4 | 40 | 1.5 | 8.72 | 33.58 | 34.02 | 132 | 4 | 5 | 1.5 | 15.81 | 36.77 | 37.08 |
| 82 | 5 | 40 | 1.5 | 5.36 | 31.39 | 31.32 | 133 | 5 | 5 | 1.5 | 12.05 | 34.13 | 34.00 |
| 83 | 6 | 40 | 1.5 | 3.66 | 29.38 | 29.30 | 134 | 6 | 5 | 1.5 | 9.99 | 31.64 | 31.52 |
| 84 | 7 | 40 | 1.5 | 2.67 | 27.67 | 27.65 | 135 | 7 | 5 | 1.5 | 8.67 | 29.42 | 29.37 |
| 85 | 8 | 40 | 1.5 | 2.04 | 26.22 | 26.24 | 136 | 8 | 5 | 1.5 | 7.74 | 27.44 | 27.42 |
| 86 | 9 | 40 | 1.5 | 1.61 | 24.98 | 25.00 | 137 | 9 | 5 | 1.5 | 7.04 | 25.63 | 25.62 |
| 87 | 10 | 40 | 1.5 | 1.31 | 23.88 | 23.89 | 138 | 10 | 5 | 1.5 | 6.48 | 23.94 | 23.93 |
| 88 | 11 | 40 | 1.5 | 1.09 | 22.89 | 22.89 | 139 | 11 | 5 | 1.5 | 6.01 | 22.35 | 22.35 |
| 89 | 1 | 20 | 1.5 | 591.30 | 39.62 | 46.10 | 140 | 12 | 5 | 1.5 | 5.61 | 20.85 | 20.85 |
| 90 | 2 | 20 | 1.5 | 53.54 | 39.27 | 44.46 | 141 | 14 | 5 | 1.5 | 4.94 | 18.09 | 18.09 |


| 91 | 3 | 20 | 1.5 | 17.52 | 38.14 | 40.48 | 142 | 15 | 5 | 1.5 | 4.65 | 16.82 | 16.82 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 92 | 5 | 20 | 1.5 | 5.71 | 33.99 | 33.92 | 143 | 1 | 4 | 1.5 | 688.43 | 40.77 | 47.21 |
| 93 | 6 | 20 | 1.5 | 3.99 | 31.93 | 31.86 | 144 | 2 | 4 | 1.5 | 65.95 | 40.35 | 45.36 |
| Tabloe1 continued |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | Inputs |  |  | Outputs |  |  |  | Inputs |  |  | Outputs |  |  |
| No | K | $l_{x} / h$ | $l_{x} / l_{y}$ | 1000.am | 100.cx | 100.cy | No | K | $l_{x} / h$ | $l_{x} / l_{y}$ | $1000 . a_{m}$ | 100.cx | 100.cy |
| 145 | 3 | 4 | 1.5 | 28.89 | 38.93 | 40.98 | 190 | 13 | 15 | 2.0 | 1.30 | 23.34 | 23.34 |
| 146 | 4 | 4 | 1.5 | 19.77 | 36.48 | 36.73 | 191 | 14 | 15 | 2.0 | 1.18 | 22.42 | 22.42 |
| 147 | 5 | 4 | 1.5 | 15.75 | 33.65 | 33.48 | 192 | 15 | 15 | 2.0 | 1.08 | 21.54 | 21.54 |
| 148 | 6 | 4 | 1.5 | 13.44 | 30.96 | 30.82 | 193 | 1 | 10 | 2.0 | 510.33 | 40.82 | 52.76 |
| 149 | 8 | 4 | 1.5 | 10.77 | 26.40 | 26.36 | 194 | 2 | 10 | 2.0 | 52.65 | 40.17 | 47.46 |
| 150 | 9 | 4 | 1.5 | 9.87 | 24.42 | 24.40 | 195 | 3 | 10 | 2.0 | 18.77 | 38.80 | 40.86 |
| 151 | 10 | 4 | 1.5 | 9.12 | 22.58 | 22.57 | 196 | 4 | 10 | 2.0 | 10.25 | 36.87 | 37.10 |
| 152 | 11 | 4 | 1.5 | 8.48 | 20.86 | 20.86 | 197 | 5 | 10 | 2.0 | 6.93 | 34.63 | 34.53 |
| 153 | 12 | 4 | 1.5 | 7.92 | 19.25 | 19.25 | 198 | 6 | 10 | 2.0 | 5.20 | 32.48 | 32.41 |
| 154 | 13 | 4 | 1.5 | 7.41 | 17.74 | 17.74 | 199 | 7 | 10 | 2.0 | 4.15 | 30.61 | 30.59 |
| 155 | 14 | 4 | 1.5 | 6.95 | 16.33 | 16.32 | 200 | 8 | 10 | 2.0 | 3.47 | 28.99 | 28.99 |
| 156 | 15 | 4 | 1.5 | 6.51 | 15.00 | 15.00 | 201 | 9 | 10 | 2.0 | 2.99 | 27.54 | 27.55 |
| 157 | 1 | 40 | 2 | 335.95 | 37.28 | 48.94 | 202 | 10 | 10 | 2.0 | 2.64 | 26.23 | 26.23 |
| 158 | 3 | 40 | 2.0 | 16.87 | 35.30 | 37.40 | 203 | 12 | 10 | 2.0 | 2.16 | 23.84 | 23.84 |
| 159 | 4 | 40 | 2.0 | 8.50 | 33.48 | 33.74 | 204 | 13 | 10 | 2.0 | 1.98 | 22.74 | 22.74 |
| 160 | 5 | 40 | 2.0 | 5.29 | 31.38 | 31.30 | 205 | 14 | 10 | 2.0 | 1.84 | 21.69 | 21.69 |
| 161 | 6 | 40 | 2.0 | 3.64 | 29.38 | 29.33 | 206 | 15 | 10 | 2.0 | 1.72 | 20.69 | 20.69 |
| 162 | 7 | 40 | 2.0 | 2.66 | 27.67 | 27.66 | 207 | 1 | 5 | 2.0 | 530.59 | 41.05 | 52.99 |
| 163 | 8 | 40 | 2.0 | 2.04 | 26.22 | 26.24 | 208 | 2 | 5 | 2.0 | 58.8 | 40.37 | 47.54 |
| 164 | 9 | 40 | 2.0 | 1.61 | 24.98 | 25.00 | 209 | 3 | 5 | 2.0 | 24.47 | 38.89 | 40.78 |
| 165 | 10 | 40 | 2.0 | 1.31 | 23.88 | 23.89 | 210 | 4 | 5 | 2.0 | 15.63 | 36.69 | 36.81 |
| 166 | 11 | 40 | 2.0 | 1.09 | 22.89 | 22.89 | 211 | 5 | 5 | 2.0 | 12.00 | 34.12 | 33.95 |
| 167 | 1 | 20 | 2.0 | 460.49 | 39.96 | 51.85 | 212 | 6 | 5 | 2.0 | 9.98 | 31.64 | 31.52 |
| 168 | 2 | 20 | 2.0 | 50.86 | 39.31 | 46.62 | 213 | 7 | 5 | 2.0 | 8.67 | 29.42 | 29.37 |
| 169 | 3 | 20 | 2.0 | 17.31 | 37.97 | 40.07 | 214 | 8 | 5 | 2.0 | 7.74 | 27.44 | 27.42 |
| 170 | 4 | 20 | 2.0 | 8.87 | 36.12 | 36.37 | 215 | 9 | 5 | 2.0 | 7.04 | 25.63 | 25.62 |
| 171 | 6 | 20 | 2.0 | 3.97 | 31.93 | 31.88 | 216 | 10 | 5 | 2.0 | 6.48 | 23.94 | 23.93 |
| 172 | 7 | 20 | 2.0 | 2.98 | 30.18 | 30.17 | 217 | 11 | 5 | 2.0 | 6.01 | 22.35 | 22.35 |
| 173 | 8 | 20 | 2.0 | 2.34 | 28.68 | 28.70 | 218 | 12 | 5 | 2.0 | 5.61 | 20.85 | 20.85 |
| 174 | 9 | 20 | 2.0 | 1.90 | 27.38 | 27.40 | 219 | 13 | 5 | 2.0 | 5.26 | 19.43 | 19.43 |
| 175 | 10 | 20 | 2.0 | 1.59 | 26.22 | 26.23 | 220 | 15 | 5 | 2.0 | 4.65 | 16.82 | 16.82 |
| 176 | 11 | 20 | 2.0 | 1.36 | 25.16 | 25.16 | 221 | 1 | 4 | 2.0 | 537.15 | 41.07 | 53.00 |
| 177 | 12 | 20 | 2.0 | 1.19 | 24.17 | 24.17 | 222 | 2 | 4 | 2.0 | 63.37 | 40.38 | 47.45 |
| 178 | 13 | 20 | 2.0 | 1.05 | 23.25 | 23.25 | 223 | 3 | 4 | 2.0 | 28.71 | 38.80 | 40.58 |
| 179 | 1 | 15 | 2.0 | 487.81 | 40.46 | 52.37 | 224 | 4 | 4 | 2.0 | 19.60 | 36.42 | 36.46 |
| 180 | 2 | 15 | 2.0 | 51.41 | 39.80 | 47.11 | 225 | 5 | 4 | 2.0 | 15.71 | 33.64 | 33.43 |
| 181 | 3 | 15 | 2.0 | 17.7 | 38.45 | 40.54 | 226 | 6 | 4 | 2.0 | 13.44 | 30.96 | 30.82 |
| 182 | 4 | 15 | 2.0 | 9.23 | 36.58 | 36.83 | 227 | 7 | 4 | 2.0 | 11.90 | 28.56 | 28.49 |
| 183 | 5 | 15 | 2.0 | 5.98 | 34.41 | 34.32 | 228 | 9 | 4 | 2.0 | 9.87 | 24.42 | 24.40 |
| 184 | 6 | 15 | 2.0 | 4.30 | 32.34 | 32.28 | 229 | 10 | 4 | 2.0 | 9.12 | 22.58 | 22.57 |
| 185 | 7 | 15 | 2.0 | 3.29 | 30.55 | 30.54 | 230 | 11 | 4 | 2.0 | 8.48 | 20.86 | 20.86 |
| 186 | 9 | 15 | 2.0 | 2.19 | 27.68 | 27.69 | 231 | 12 | 4 | 2.0 | 7.92 | 19.25 | 19.25 |
| 187 | 10 | 15 | 2.0 | 1.87 | 26.47 | 26.48 | 232 | 13 | 4 | 2.0 | 7.41 | 17.74 | 17.74 |
| 188 | 11 | 15 | 2.0 | 1.63 | 25.35 | 25.36 | 233 | 14 | 4 | 2.0 | 6.95 | 16.33 | 16.32 |
| 189 | 12 | 15 | 2.0 | 1.45 | 24.32 | 24.32 | 234 | 15 | 4 | 2.0 | 6.51 | 15.00 | 15.00 |

Table 2. Testing examples for ANN

|  | Inputs |  |  | Outputs |  |  | Inputs |  |  |  | Outputs |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No | K | $l_{x} / h$ | $l_{x} / l_{y}$ | $1000 . a_{m}$ | 100.cx | 100.cy | No | K | $l_{x} / h$ | $l_{x} / l_{y}$ | $1000 . a_{m}$ | $100 . c_{x}$ | 100.cy |
| 1 | 3 | 40 | 1.0 | 18.26 | 35.99 | 35.99 | 10 | 10 | 10 | 1.5 | 2.64 | 26.23 | 26.23 |
| 2 | 6 | 20 | 1.0 | 4.09 | 32.04 | 32.04 | 11 | 13 | 5 | 1.5 | 5.26 | 19.43 | 19.43 |
| 3 | 9 | 15 | 1.0 | 2.19 | 27.68 | 27.68 | 12 | 7 | 4 | 1.5 | 11.90 | 28.56 | 28.49 |
| 4 | 12 | 10 | 1.0 | 2.16 | 23.84 | 23.84 | 13 | 2 | 40 | 2.0 | 49.45 | 36.63 | 43.85 |
| 5 | 15 | 5 | 1.0 | 4.65 | 16.82 | 16.82 | 14 | 5 | 20 | 2.0 | 5.64 | 33.98 | 33.89 |
| 6 | 9 | 4 | 1.0 | 9.87 | 24.42 | 24.42 | 15 | 8 | 15 | 2.0 | 2.63 | 29.02 | 29.03 |
| 7 | 1 | 40 | 1.5 | 425.80 | 36.94 | 43.38 | 16 | 11 | 10 | 2.0 | 2.37 | 25.00 | 25.00 |
| 8 | 4 | 20 | 1.5 | 9.09 | 36.21 | 36.65 | 17 | 14 | 5 | 2.0 | 4.94 | 18.09 | 18.09 |
| 9 | 7 | 15 | 1.5 | 3.29 | 30.54 | 30.53 | 18 | 8 | 4 | 2.0 | 10.77 | 26.40 | 26.36 |

As can be seen form Table 1 and Table 2, variables in training and testing data range from bottom and top limit presented in Table 3.

Table 3. Range of variables in training and testing database

| Parameter | Range |
| :--- | :---: |
| $K$ | $1 \sim 15$ |
| $l_{x} / h$ | $4 \sim 40$ |
| $l_{x} / l_{y}$ | $1 \sim 2$ |
| $1000 . a_{m}$ | $1.05 \sim 1012.53$ |
| $100 . c_{x}$ | $15 \sim 41.07$ |
| $100 . c_{y}$ | $15 \sim 53$ |

## Determination of ANN architecture:

In order to obtain better results, different ANN architectures with different training parameters were evaluated. After all initial performance evaluations, the ANN architecture shown Fig. 5 was selected for this study. As seen from Fig.5, selected ANN architecture has four layers. There are 3, 6, 6 and 3 neurons in input layer, $1^{\text {st }}$ hidden layer, $2^{\text {nd }}$ hidden layer, and output layer respectively.


Figure 5. Selected multi layered ANN architecture

## Training of the ANN Model：

Training set consists of 234 examples．Connection weights were selected randomly between 0 and 1 by computer program coded using Visual Basic Program for ANN studies．Training cycle was released and computer program stopped when the error tolerance was less than or equal 0.02 for each output neuron．These adjusted neural weights of the ANN obtained in this study are presented in Table 4，Table 5 and Table 6 ．Testing of the ANN was performed by using these neural weights．

Table 4．Neural weights between input layer and 1st hidden layer

|  | $1{ }^{\text {st }}$ hidden Layer Neurons |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $1^{\text {st }}$ <br> Neuron | $2^{\text {nd }}$ <br> Neuron | $3^{\mathrm{rd}}$ <br> Neuron | $4^{\text {th }}$ <br> Neuron | $5^{\text {th }}$ <br> Neuron | $6^{\text {th }}$ Neuron |
| $\cdots \quad \approx 1^{\text {st }}$ Neuron | －4．595572047 | 13.62461668 | 0.002271442 | －0．186683768 | －0．579188860 | 0.497921499 |
| 言 | －1．063877289 | －2．634195212 | －0．339157413 | －9．399162491 | －2．640729082 | 0.398394236 |
| ニージ $3^{\text {rd }}$ Neuron | 1.249631080 | 3.607865589 | 0.365054228 | 4.929387044 | －2．724773911 | 0.400441316 |

Table 5．Neural weights between 1st hidden layer and 2nd hidden layer

|  | $1^{\text {st }}$ <br> Neuron | $\begin{gathered} 2^{\text {nd }} \\ \text { Neuron } \end{gathered}$ | $\begin{gathered} 1^{\text {st }} \text { hidden Lay } \\ 3^{\text {rd }} \\ \text { Neuron } \end{gathered}$ | yer Neurons $4^{\text {th }}$ <br> Neuron | $\begin{gathered} 5^{\text {th }} \\ \text { Neuron } \end{gathered}$ | $\begin{gathered} 6^{\text {th }} \\ \text { Neuron } \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\begin{gathered} -13.622287097 \\ 0.160024803 \\ 0.313706558 \\ -11.497594319 \\ 0.211745981 \\ 0.321021580 \end{gathered}$ | $\begin{gathered} -21.73100132 \\ 3.085699294 \\ -4.722953948 \\ -4.690059786 \\ 8.352726805 \\ -5.965550321 \end{gathered}$ | $\begin{gathered} -20.675363672 \\ 1.5834471895 \\ 23.863177116 \\ 11.793048979 \\ 1.224423228 \\ 5.296759553 \end{gathered}$ | $\begin{gathered} -36.327177279 \\ 5.7477940208 \\ -5.712931638 \\ 3.562264369 \\ 5.163831319 \\ 7.701932878 \end{gathered}$ | $\begin{gathered} 43.200190512 \\ 0.794813577 \\ 17.860070787 \\ 8.539661329 \\ 1.380598400 \\ -4.466205448 \end{gathered}$ | $\begin{gathered} -2.678836309 \\ -2.988741439 \\ -14.858651510 \\ 7.842248344 \\ -7.847671248 \\ 8.580680428 \end{gathered}$ |

Table 6．Neural weights between 2nd hidden layer and output layer

|  |  | $2^{\text {nd }}$ Hidden Layer Neurons |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $1^{\text {st }}$ <br> Neuron | $\begin{gathered} 2^{\text {nd }} \\ \text { Neuron } \end{gathered}$ | $3^{\mathrm{rd}}$ <br> Neuron | $4^{\text {th }}$ <br> Neuron | $\begin{gathered} 5^{\text {th }} \\ \text { Neuron } \end{gathered}$ | $\begin{gathered} 6^{\text {th }} \\ \text { Neuron } \end{gathered}$ |
| 产 | $1^{\text {st }}$ Neuron | 22.670805986 | 6.639261213 | 14.395055499 | －1．348652056 | 2.378231287 | 5.603706477 |
| 者 | $2^{\text {nd }}$ Neuron | 0.122379446 | 0.365607907 | －1．270902076 | －2．624017238 | 0.752479848 | 1.969575712 |
| ${ }_{0}^{2}$ | $3^{\text {rd }}$ Neuron | 0.177936912 | 6.280252671 | －3．414687034 | －1．561338369 | $-2.335396363$ | 0.518774672 |

## Testing of the ANN Model：

18 testing examples given in Table 2 were used in order to evaluate the performance of the trained ANN Model．The absolute percent errors between testing examples results and the ANN model outputs were calculated and given in Table 7.

As seen from this table, maximum percent errors of three $a_{\mathrm{m}}, \mathrm{c}_{\mathrm{x}}$ and $\mathrm{c}_{\mathrm{y}}$ factors were calculated as $4.398,4.141$ and 1.940 percent respectively. Also the mean absolute percent error (MAPE) given in Eq. 3 were calculated in Table 7. for $a_{\mathrm{m}}, \mathrm{c}_{\mathrm{x}}$ and $\mathrm{c}_{\mathrm{y}}$ outputs.

As can be seen the results, it can be said that trained ANN model was showed quite good performance for testing sets.

Table 7. Percent Errors between test examples and ANN Outputs

| No | $1000 . a_{m}$ |  |  | 100.cx |  |  | 100.cy |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Test <br> Data | ANN Outputs | MAPE <br> Values | Test <br> Data | ANN Outputs | MAPE <br> Values | Test <br> Data | ANN Outputs | MAPE <br> Values |
| 1 | 18.26 | 19.100 | 4.398 | 35.99 | 34.604 | 4.002 | 35.99 | 35.641 | 0.976 |
| 2 | 4.09 | 4.022 | 1.681 | 32.04 | 32.512 | 1.452 | 32.04 | 31.756 | 0.894 |
| 3 | 2.19 | 2.224 | 1.533 | 27.68 | 28.371 | 2.435 | 27.68 | 28.370 | 2.433 |
| 4 | 2.16 | 2.146 | 0.645 | 23.84 | 23.495 | 1.467 | 23.84 | 24.045 | 0.854 |
| 5 | 4.65 | 4.660 | 0.227 | 16.82 | 17.546 | 4.141 | 16.82 | 16.753 | 0.397 |
| 6 | 9.87 | 9.908 | 0.385 | 24.42 | 23.827 | 2.487 | 24.42 | 24.491 | 0.292 |
| 7 | 425.8 | 429.721 | 0.912 | 36.94 | 37.675 | 1.952 | 43.38 | 44.215 | 1.888 |
| 8 | 9.09 | 8.912 | 1.994 | 36.21 | 35.658 | 1.548 | 36.65 | 35.952 | 1.940 |
| 9 | 3.29 | 3.312 | 0.680 | 30.54 | 31.279 | 2.362 | 30.53 | 30.641 | 0.363 |
| 10 | 2.64 | 2.633 | 0.244 | 26.23 | 25.815 | 1.607 | 26.23 | 26.573 | 1.293 |
| 11 | 5.26 | 5.248 | 0.229 | 19.43 | 19.410 | 0.100 | 19.43 | 19.294 | 0.703 |
| 12 | 11.9 | 12.191 | 2.390 | 28.56 | 28.885 | 1.127 | 28.49 | 29.172 | 2.337 |
| 13 | 49.45 | 48.184 | 2.627 | 36.63 | 36.419 | 0.577 | 43.85 | 43.917 | 0.152 |
| 14 | 5.64 | 5.616 | 0.417 | 33.98 | 33.825 | 0.457 | 33.89 | 33.285 | 1.817 |
| 15 | 2.63 | 2.669 | 1.475 | 29.02 | 29.364 | 1.173 | 29.03 | 28.814 | 0.749 |
| 16 | 2.37 | 2.363 | 0.267 | 25.00 | 24.228 | 3.184 | 25.00 | 25.026 | 0.105 |
| 17 | 4.94 | 4.993 | 1.064 | 18.09 | 18.296 | 1.129 | 18.09 | 18.045 | 0.247 |
| 18 | 10.77 | 11.025 | 2.320 | 26.40 | 26.211 | 0.719 | 26.36 | 26.569 | 0.787 |
| Maximum Error \% |  |  | 4.398 |  |  | 4.141 |  |  | 1.940 |
|  |  |  | 1.083 |  |  | 1.670 |  |  | 1.011 |

## 4. Conclusions

The aim of this study is the prediction of the nondimensional maximum displacement and bending moments of the plate on Winkler-type elastic foundation using artificial neural networks. Following conclusions can be drawn from this study.

- The success of an ANN model depends on the training data and the structure of network. The ANN model presented in this study are valid only for the range of data given Table 3 for $v=0.3$ and concentrated load. A future enrichment of the used database would increase the authority of the proposed method.
- The developed ANN model could not only make a good estimation of maximum displacement and bending moment of thick plate on Winkler-type elastic foundation but also significantly reduce modeling and computational time.
- It is note that the errors between ANN and FEM results decrease as the training database enlarges.
- This study can be enlarged for various Poisson ratios and load conditions.


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