International Journal of Engineering & Applied Sciences (IJEAS)
Vol.4, Issue 4(2012)26-40

NUMERICAL SIMULATION OF VIBRATION OF NON-HOMOGENEOUS PLATES OF VARIABLE THICKNESS

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Recived:28 December 2012

Abstract

Differential Quadrature Method (DQM) is used to analyse free transverse vibrations of non-homogeneous orthotropic rectangular plates of variable thickness. A new model to represent the non-homogeneity of the plate material has been taken which incorporates earlier models. Following Lévy approach i.e the two parallel edges are simply supported, the fourth-order differential equation governing the motion of such plates of variable thickness has been solved for different combinations of clamped, simply-supported and free-edge boundary conditions. Effect of non- homogeneity together with other plate parameters such as orthotropy, aspect ratio and foundation modulus on the natural frequencies has been studied for the first three modes of vibration. Numerical results are presented to illustrate the method and demonstrate its efficiency. Normalized displacements are presented for specified plates for all the three boundary conditions.

***Keywords*:** DQM; orthotropy; variable thickness; non-homogeneity; elastic foundation.

1. Introduction

The vibration analysis of plate type structures are considered in two main stages: one in formulation of a mathematical model for a given physical problem and the second in the solution of the model. With the development of computer technology, various numerical methods have been used to solve different types of problem in engineering and science which are described by the differential equations. These equations are either linear or non-linear and in most cases, their closed form solutions are not possible. As a result, various numerical techniques such as Frobenius method [1], finite-difference method [2], simple polynomial approximation method [3], Galerkin's method [4], Rayleigh-Ritz method [5], Harmonic differential quadrature [6], characteristic orthogonal polynomial method [7], quintic splines method [8], finite element method [9], Chebyshev collocation method [10, 11] and Generalised differential quadrature method [12] etc. have been employed to study the vibrational behaviour of plates of various geometries. The numerical methods, such as finite difference and finite element method require fine mesh size to obtain accurate results but are computationally expensive. The method of quintic splines, characteristic orthogonal polynomials and Frobenius method require a large number of terms for plates of variable thickness.

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Differential quadrature method (DQM), which requires few grid points for desired accuracy was introduced by Bellman et al.[13] and generalised and simplified subsequently by Quan and Chang[14, 15] and Shu and Richards[16] has emerged as a distinct numerical technique during last two decades. This has encouraged researchers working in structural mechanics to study of vibrational behaviour of plates of various geometries using differential quadrature method [17- 24], to mention a few.In several technological applications, plate type structural components are subjected to high temperature environmental conditions which results in non-homogeneity of the material due to physical composition i.e due to use of fibers with different modulii along two perpendicular directions and having different strength properties or by design [25]. An extensive review of all available models to represent non-homogeneity has been given in the references [8, 23]. In a recent paper, Seema et al. [24] analysed vibration of non-homogeneous orthotropic rectangular plates resting on Winkler foundation by assuming $E_x = E_1 e^{\mu X}$, $E_y = E_2 e^{\mu X}$ *and* $\rho = \rho_0 e^{\beta X}$. In this model, both the Young's modulii are assumed to depend upon the same parameter μ for which there is no physical basis. A new model is assumed where $E_x = E_1 e^{\mu X}$, $E_y = E_2 e^{\mu X}$ $(\mu_1 \neq \mu_2)$ *and* $\rho = \rho_0 e^{\beta X}$ which incorporates earlier models [8, 24]. The primary objective of this work is to obtain free vibration frequencies of non-homogeneous orthotropic rectangular plates of linear, parabolic and quadratic thickness variations on the basis of new model representing the non-homogeneity of the plate material. The present theoretical investigation will be of practical interest to design engineers.

2. Mathematical formulation

Consider a non-homogeneous orthotropic rectangular plate of length *a*, breadth *b*, thickness *h* (*x*) varying along x- direction only and density ρ (*x, y*) such that the middle surface of the plate is *z* = 0 and the origin is at one of the corners of the plate. The *z*-axis is taken perpendicular to the plate. The *x*- and *y*- axes are taken along the principal directions of orthotropy.

The differential equation which governs the transverse free vibration of such plates is given by [24]

$$D_x \frac{\partial^4 w}{\partial x^4} + D_y \frac{\partial^4 w}{\partial y^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + 2\frac{\partial H}{\partial x}\frac{\partial^3 w}{\partial x \partial y^2} + 2\frac{\partial H}{\partial y}\frac{\partial^3 w}{\partial y \partial x^2} + 2\frac{\partial D_x}{\partial x}\frac{\partial^3 w}{\partial x^3} + 2\frac{\partial D_y}{\partial y}\frac{\partial^3 w}{\partial y^3}$$
$$+ \frac{\partial^2 D_x}{\partial x^2}\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 D_y}{\partial y^2}\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 D_1}{\partial y^2}\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 D_1}{\partial x^2}\frac{\partial^2 w}{\partial y^2}$$
$$+ 4\frac{\partial^2 D_{xy}}{\partial x \partial y}\frac{\partial^2 w}{\partial y \partial x} + \rho h \frac{\partial^2 w}{\partial t^2} + k_f w = 0, \qquad (1)$$

where $D_x = E_x h^3 / 12(1 - \nu_x \nu_y)$, $D_y = E_y h^3 / 12(1 - \nu_x \nu_y)$, $D_{xy} = G_{xy} h^3 / 12$, $D_1 = E^* h^3 / 12$, $H = D_1 + 2D_{xy}$, $w(x, y, t)$ is the transverse deflection, *t* the time, ρ the mass density and E_x, E_y, ν_x, ν_y and G_{xy} are material constants and $E_x \nu_y = E_y \nu_x$ and $E^* = E_x \nu_y / 12(1 - \nu_x \nu_y) = E_y \nu_x / 12(1 - \nu_x \nu_y)$.

The thickness of the plate is assumed to vary in *x*-direction only i.e. *h* = *h*(*x*) and the two opposite edges $y = 0$ and $y = b$ are taken to be simply supported (Lévy approach). For a harmonic solution, the displacement w is expressed as

$$w(x, y, t) = \bar{w}(x) \sin(p\pi y / b) e^{i\omega t} \qquad (2)$$

where, p is a positive integer and ω the circular frequency in radians.

Furthermore, for elastically non-homogeneous material, it is assumed that Young's modulii E_x, E_y and density ρ are functions of space variable x only. Following [26, 27], the shear modulus is $G_{xy} = \sqrt{E_x E_y} / 2(1+\sqrt{\nu_x \nu_y})$.

Introducing non-dimensional variables, $X = x/a, Y = y/b, \bar{h} = h/a, W = \bar{w}/a$, equation (1) reduces to

$$
\begin{aligned}
&\bar{h}^3 E_x W^{iv} + [2(\bar{h}^3 E_x' + 3\bar{h}^2 \bar{h}' E_x)] W''' \\
&+ [(6\bar{h}\bar{h}'^2 + 3\bar{h}^2 \bar{h}'')E_x + 6\bar{h}^2 \bar{h}' E_x' + \bar{h}^3 E_x'' - 2\lambda^2 \bar{h}^3 (E^* + 2G_{xy})(1-\nu_x \nu_y)] W'' \\
&- [2\lambda^2 \{3\bar{h}^2 \bar{h}'(\nu_y E_x + 2(1-\nu_x \nu_y) G_{xy}) + \bar{h}^3 (\nu_y E_x' + 2(1-\nu_x \nu_y) G_{xy}')\}] W' \\
&+ [\lambda^4 \bar{h}^3 E_y - \lambda^2 \nu_y \{\bar{h}^3 E_x'' + 6\bar{h}^2 \bar{h}' E_x' + (6\bar{h}\bar{h}'^2 + 3\bar{h}^2 \bar{h}'')E_x\} \\
&- 12(1-\nu_x \nu_y)(\rho \bar{h} a^2 \omega^2 - a k_f)] W = 0
\end{aligned}
\tag{3}
$$

where $\lambda^2 = p^2 a^2 \pi^2 / b^2$ and primes denote differentiation with respect to X.

Taking quadratic variation in thickness i.e. $\bar{h} = h_0 (1+\alpha_1 X + \alpha_2 X^2)$ [28] and assuming new model for non-homogeneity of the plate material.

$$E_x = E_1 e^{\mu_1 X}, \quad E_y = E_2 e^{\mu_2 X} \text{ and } \rho = \rho_0 e^{\beta X}, \tag{4}$$

equation (3) reduces to

$$A_0 W^{iv} + A_1 W''' + A_2 W'' + A_3 W' + A_4 W = 0 \tag{5}$$

$$
\begin{aligned}
&A_0 = 1, A_1 = 2(\mu_1 + \frac{3(\alpha_1 + 2\alpha_2 X)}{(1+\alpha_1 X + \alpha_2 X^2)}), \\
&A_2 = \frac{6(\alpha_1 + 2\alpha_2 X)^2}{(1+\alpha_1 X + \alpha_2 X^2)^2} + \frac{6\alpha_2}{(1+\alpha_1 X + \alpha_2 X^2)} + \frac{6\mu_1(\alpha_1 + 2\alpha_2 X)}{(1+\alpha_1 X + \alpha_2 X^2)} + \mu_1^2 \\
&- 2\lambda^2 (\nu_y + \frac{\sqrt{E_2/E_1}\; e^{(\mu_2-\mu_1)X/2}}{(1+\sqrt{\nu_x \nu_y})}(1-\nu_x \nu_y)), \\
&A_3 = -2\lambda^2 [\frac{3(\alpha_1 + 2\alpha_2 X)}{(1+\alpha_1 X + \alpha_2 X^2)} \{\nu_y + \frac{\sqrt{E_2/E_1}\; e^{(\mu_2-\mu_1)X/2}}{(1+\sqrt{\nu_x \nu_y})}(1-\nu_x \nu_y)\} \\
&+ \{\mu_1 \nu_y + \frac{(\mu_1+\mu_2)\sqrt{E_2/E_1}\; e^{(\mu_2-\mu_1)X/2}}{2(1+\sqrt{\nu \nu_y})}(1-\nu_x \nu_y)\}] \\
&A_4 = \lambda^4 (E_2/E_1) e^{(\mu_2-\mu_1)X} - \lambda^2 \nu_y \{\mu_1^2 + \frac{6\mu_1(\alpha_1 + 2\alpha_2 X)}{(1+\alpha_1 X + \alpha_2 X^2)} + \frac{6(\alpha_1 + 2\alpha_2 X)^2}{(1+\alpha_1 X + \alpha_2 X^2)^2} + \frac{6\alpha_2}{(1+\alpha_1 X + \alpha_2 X^2)}\} \\
&- \frac{\Omega^2}{(1+\alpha_1 X + \alpha_2 X^2)^2} e^{(\beta-\mu_1)X} + \frac{12K}{h_o^3 (1+\alpha_1 X + \alpha_2 X^2)^3} e^{-\mu_1 X}
\end{aligned}
$$

$K = a\, k_f (1-\nu_x \nu_y)/E_1$, $\quad \Omega^2 = 12\rho_0 (1-\nu_x \nu_y)\, a^2 \omega^2 / E_1 h_0^2$.

Here h_0, ρ_0 are thickness and density of the plate at $X = 0$, μ_1 and μ_2 the non-homogeneity parameters, α_1 and α_2 the taper parameters, β the density parameter and E_1, E_2 the Young's modulii in proper directions at
$X = 0$.

The equation (5) together with the boundary conditions at the edges $X = 0$ and $X = 1$ gives rise to a two-point boundary value problem with variable coefficients, whose closed form solution is not possible. An approximate solution is obtained by employing differential quadrature method.

3. Method of Solution: Differential Quadrature Method

A brief description of DQM is as follows:
Let $X_1, X_2, \ldots, X_m$ be the m grid points in the applicability range [0, 1] of the plate. According to the DQM, the n^{th} order derivative of $W(X)$ with respect to X can be expressed discretely at the point X_i as

$$\frac{d^n W(X_i)}{dX^n} = \sum_{j=1}^{m} c_{ij}^{(n)} W(X_j) \ , \qquad n=1, 2, 3, 4 \text{ and } i=1, 2, \ldots, m \tag{6}$$

where $c_{ij}^{(n)}$ are the weighting coefficients associated with the nth order derivative of W(X) w. r. to X at discrete point X_i. Following Shu [29, pages 31, 35] are given by

$$c_{ij}^{(1)} = \frac{M^{(1)}(X_i)}{(X_i - X_j) M^{(1)}(X_j)} \quad , \qquad i, \ j=1,2,\ldots,m \ ; \quad i \neq j \tag{7}$$

$$M^{(1)}(X_i) = \prod_{\substack{j=1 \\ j\neq i}}^{m} (X_i - X_j) \, , \tag{8}$$

$$c_{ij}^{(n)} = n\left(c_{ii}^{(n-1)} c_{ij}^{(1)} - \frac{c_{ij}^{(n-1)}}{X_i - X_j} \right) \quad for \ \ i, j = 1,2,\ldots,m, \ j \neq i \quad and \ n = 2,3,4 \tag{9}$$

$$c_{ii}^{(n)} = -\sum_{\substack{j=1 \\ j\neq i}}^{m} c_{ij}^{(n)} \qquad for \ \ i = 1,2,\ldots,m \qquad and \ \ n = 1,2,3,4 \, . \tag{10}$$

Discretizing equation (5) at grid points X_i, $i = 3, 4, \ldots, m-2$, it reduces to,

$$A_0 W^{iv}(X_i) + A_{1,i} W'''(X_i) + A_{2,i} W''(X_i) + A_{3,i} W'(X_i) + A_{4,i} \ W(X_i) = 0. \tag{11}$$

Substituting for $W(X)$ and its derivatives at the i^{th} grid point in the equation (11) and using relations (6) to (10), equation (11) becomes

$$\sum_{j=1}^{m} (A_0 c_{ij}^{(4)} + A_{1,i} c_{ij}^{(3)} + A_{2,i} c_{ij}^{(2)} + A_{3,i} c_{ij}^{(1)}) W(X_j) + A_{4,i} W(X_i) = 0. \tag{12}$$

For $i = 3,4,\ldots, (m-2)$, one obtains a set of $(m-4)$ equations in terms of unknowns $W_j (\equiv W(X_j)), \ j = 1,2,\cdots, m,$ which can be written in the matrix form as

$$[B][W^*] = [0] \tag{13}$$

where B and W^* are matrices of order (m-4) x m and (m x 1) respectively.
Here, the (m-2) internal grid points chosen for collocation, are the zeros of shifted Chebyshev polynomials of order (m-2) with orthogonality range [0, 1] given by

$$X_{k+1} = \frac{1}{2}[1+\cos(\frac{2k-1}{m-2}\frac{\pi}{2})] \ , \qquad k = 1, 2, \ldots, m\text{-}2 \tag{14}$$

4. Boundary Conditions and Frequency Equations

The three sets of different combinations of boundary conditions namely, C-C, C-S and C-F have been considered here, where C, S and F stand for clamped, simply supported and free edge, respectively and first symbol denotes the condition at the edge X = 0 and second symbol at the edge X = 1.By satisfying the relations

$$W = \frac{dW}{dX} = 0\,; W = \frac{d^2W}{dX^2} - (E^*/E_x^*)\lambda^2 W = 0\,; \quad and$$

$$\frac{d^2W}{dX^2} - (E^*/E_x^*)\lambda^2 W = \frac{d}{dX}\left(E_x \bar{h}^3 \left\{ \frac{d^2W}{dX^2} - \lambda^2 \nu_y W \right\} \right) - 4\lambda^2(1-\nu_x\nu_y)G_{xy}\bar{h}^3 \frac{dW}{dX} = 0\,,$$

for clamped, simply supported and free edge conditions, respectively, a set of four homogeneous equations in terms of unknown W_j are obtained. These equations together with field equation (12) give a complete set of m homogeneous equations in m unknowns. For C-C plate this set of equations can be written as

$$\begin{bmatrix} B \\ B^{CC} \end{bmatrix} [W^*] = [0] \tag{15}$$

where B^{CC} is a matrix of order 4×m.
For a non-trivial solution of equation (15), the frequency determinant must vanish and hence,

$$\begin{vmatrix} B \\ B^{CC} \end{vmatrix} = 0\ . \tag{16}$$

Similarly for C-S and C-F plates, the frequency determinants can be written as

$$\begin{vmatrix} B \\ B^{CS} \end{vmatrix} = 0 \quad \text{and} \qquad \begin{vmatrix} B \\ B^{CF} \end{vmatrix} = 0 \tag{17,18}$$

respectively.

5. Numerical results and Discussions

The frequency equations (16-18) provide the values of the frequency parameter Ω. The lowest three roots of frequency equations (16-18) have been obtained using bisection method to investigate the effect of various plate parameters such as non-homogeneity, orthotropy, thickness variation, aspect ratio and foundation modulus on the frequency parameter Ω for p = 1. The values of various parameters are taken as follows: Winkler foundation parameter K=0.0(0.02) 1.0; non-homogeneity parameters μ_1= -0.5(0.1) 1.0; μ_2 = -0.5(0.1) 1.0; density parameter β = -0.5(0.1) 1.0; taper parameters α_1= -0.5 (0.1) 1.0; α_2 = -0.5 (0.1) 1.0 such that $\alpha_1 + \alpha_2$ > -1 and aspect ratio a/b = 0.5 (0.5)2.0 for C-C, C-S and C-F boundary conditions. The elastic constants for the plate material 'ORTHO1' Biancolini [30] are taken as $E_1 = 1\times10^{10}\ MPa$, $E_2 = 5\times10^9\ MPa$, $\nu_x = 0.2$, $\nu_y = 0.1$. The thickness h_0 at the edge X = 0 has been taken as 0.1.

To choose the appropriate number of grid points m, convergence studies have been carried out for different sets of plate parameters. The convergence graphs for first three modes of vibration for specified plate i.e a/b = 1, K= 0.02, μ_1= 0.5, μ_2 = -0.5, β= 0.5, α_1 = -0.5 and α_2 = 0.5 are presented in Figs. 2(a-c) for C-C, C-S and C-F plates, respectively. It is observed that frequency parameter converges with increasing number of grid points. For convergence of frequency parameter in lower modes less number of grid points is needed than for the higher ones. A consistent improvement in the values of frequency parameter Ω is observed with the increasing values of m. In all the computations m = 20 has been fixed to achieve four decimal accuracy.

Numerical results are presented in Figs. (3-10). It is found that the value of frequency parameter Ω for a C-S plate is greater than that for a C-F plate but less than that for a C-C plate for the same set of values of plate parameters in all the three modes of vibration.

Figs. 3(a-c) show the plots of frequency parameter Ω with the increasing values of non-homogeneity parameter μ_1 for aspect ratio a/b = 1.0, foundation parameter K = 0.02, taper parameters α_1 = 0.5, and α_2 = -0.3, 0.3, density parameter β = -0.5, 0.5 and μ_2 = 0.5 for all the three plates vibrating in fundamental, second and third mode. The frequency parameter Ω is found to increase with the increasing values of non-homogeneity parameter μ_1 for all the three plates considered here. The rate of increase of Ω with μ_1 is more pronounced in case of C-C plate as compared to C-S and C-F plates. Also, the rate of increase of Ω with increasing values of μ_1 in all the three plates becomes higher and higher with increase in the number of modes.

Figs. 4(a-c) depict the variation of frequency parameter Ω with the increasing values of non-homogeneity parameter μ_2 for aspect ratio a/b = 1.0, foundation parameter K = 0.02, taper parameters α_1 = 0.5, and α_2 = -0.3, 0.3, density parameter β = -0.5, 0.5 and μ_1 = 0.5 for all the three plates vibrating in fundamental, second and third mode, respectively. It is observed that the behaviour of non-homogeneity parameter μ_2 on frequency parameter Ω is almost the same as that of the behaviour of μ_1, except that the rate of increase of frequency parameter Ω with the increasing values μ_2 is smaller than that of μ_1.

Figs. 5(a-c) show the plots of frequency parameter Ω with the increasing values of density parameter β for aspect ratio a/b=1.0, foundation parameter K= 0.02, taper parameters α_1 = 0.5, α_2 = -0.3, 0.3, and non-homogeneity parameters μ_1= 0.5, μ_2 = -0.5, 0.5 for all the three plates. It is observed that the frequency parameter Ω decreases with the increasing values of density parameter β for all the three boundary conditions considered here. The rate of decrease of Ω with β is more pronounced in the case of C-C plate as compared to C-S and C-F plates other plate parameters being fixed. Also, the rate of decrease in second mode is higher as compared to first mode but smaller than that in third mode.

Fig. 6(a) depicts the effect of aspect ratio a/b on frequency parameter Ω for density parameter β = 0.5, foundation parameter K= 0.02, non-homogeneity parameters μ_1= -0.5, 0.5, μ_2 = 0.5and taper parameters α_1 = 0.5 and α_2 = -0.3, 0.3 for all the three plates vibrating in fundamental mode. It is observed that the frequency parameter Ω increases with the increasing values of aspect ratio a/b for all the three plates considered here. The rate of increase of Ω with a/b is more prominent for a/b>1 than that for a/b<1. The behavior is almost same in case of second and third mode of vibration as that of first mode. The rate of increase of Ω with a/b increases with the increase in number of modes, Figs. 6 (b) and (c).

The effect of taper parameter α_1 on the frequencies for the first mode of vibration has been shown in Fig. 7(a) for a/b = 1.0, β = 0.5, α_2 = 0.0 (linear thickness variation), K= 0.0, 0.02, μ_1 = -0.5, 0.5 and μ_2 = 0.5 for C-C, C-S and C-F plates, respectively. It is seen that, in the absence of elastic foundation (K=0.0), the frequency parameter Ω is found to increase continuously with the increasing values of taper parameter α_1 for all the plates. However, in the presence of an elastic foundation (K=0.02), the frequency parameter Ω is found to increase with the increasing values of α_1 for C-C and C-S plates. In case of C-F plate for μ_1 = -0.5, 0.5, the frequency parameter Ω first decreases and then increases with local minima in the vicinity of (α_1 = -0.1, μ_1 = -0.5) and also in case of (α_1 = -0.3, μ_1= 0.5). Figs. 7 (b, c) show that when the plate is vibrating in second and third mode, the frequency parameter Ω increases with the increasing values of taper parameter α_1 for all the three boundary conditions. The rate of increase of frequency parameter Ω with taper parameter α_1 increases with the increase in the number of modes.

Figs. 8(a-c) show the plots of frequency parameter Ω versus taper parameter α_2 for a/b = 1.0, β = 0.5, α_1 = 0.0 (parabolic thickness variation), K= 0.0, 0.02, μ_1= -0.5, 0.5 and μ_2 = 0.5 for plates vibrating in fundamental, second and third modes, respectively. It is observed that the behavior of the frequency parameter Ω with taper parameter α_2 is almost the same as that with taper parameter α_1 for all the three plates in all the mode considered here except that there exist local minima in the vicinity of α_2 = 0.4 for μ_1= -0.5 which shifts towards α_2 = 0.3 for μ_1= 0.5 for C-F plate with the incorporation of K=0.02 in the fundamental mode.

The graphs of the frequency parameter Ω versus foundation parameter K have been given in Figs. 9 (a-c) for a/b =1(for square plate), β = -0.5, α_1 = 0.5, α_2 = 0.5, μ_1= -0.3, 0.3 and μ_2 = -0.3, 0.3. For plates vibrating in the first mode, Fig. 9 (a) shows that the frequency parameter Ω is found to increase with the increasing values of the foundation parameter K for all the three boundary conditions. The rate of increase of Ω with K increases with the order of the boundary conditions C-C, C-S and C-F whatever be the values of other plate parameters. A similar behaviour is observed for the second and third modes of vibration as shown in the Figs. 9 (b, c) except that the rate of increase of the frequency parameter Ω goes on decreasing with the increase in the order of modes.

Mode shapes have been computed for two values of non-homogeneity parameters μ_1 = -0.5 and μ_2 = 0.5, taper parameters α_1 = 0.5 and α_2 = - 0.3, 0.3, β = -0.5, 0.5 and K = 0.02 for a square plate i.e. a/b = 1 for all the three boundary conditions. Figures 10 (a-c) show normalised transverse displacements for all the three modes of vibrations. The nodal lines are found to shift towards the edge X = 0 with increase in α_2 while the nodal lines are seen to shift towards the edge X=1 with increase in β.

A comparison of our results for homogeneous (μ_1=μ_2= 0, β=0), isotropic (E_2 / E_1=1) plate of uniform thickness (α_1= α_2 = 0) with those results obtained by other methods has been presented in tables 1 and 2 for υ = 0.3. Table 6.2 shows a comparison of our results for p=1 with approximate values obtained by quintic spline technique [8], Chebyshev collocation method [10], Frobenius method [32] and exact values from Leissa [31]. As a special case, the results have also been computed for p = 2 and compared with those obtained by quintic spline technique [8], Chebyshev collocation method [10] and Frobenius method [32], for a/b = 0.5, 1.0 and are presented in table 2. Excellent agreement of results shows the computational efficiency and accuracy of the present method.

Table 1: Comparison of frequency parameter Ω for isotropic (E_2/E_1=1), homogeneous ($\mu_1 = \mu_2 = \beta = 0$) plates of uniform thickness ($\alpha_1 = \alpha_1 = 0.0$) for p= 1 and υ= 0.3.

Boundary Conditions		*K*=0.0				*K*=0.01			
	a/b	**0.5**		**1.0**		**0.5**		**1.0**	
	Ref.\Mode	**I**	**II**	**I**	**II**	**I**	**II**	**I**	**II**
C-C	Liessa [31]	—	—	28.946	69.320	—	—	—	—
	Lal et al.[10]	23.816	63.635	28.951	69.327	26.214	64.472	30.954	70.187
	Jain & Soni [32]	23.816	63.535	28.951	69.327	—	—	—	—
	Lal and Dhanpati [8]	23.820	63.603	28.950	69.380	26.219	64.539	30.953	70.239
	Present	**23.815**	**63.5345**	**28.950**	**69.3270**	**26.2142**	**64.472**	**30.954**	**70.1872**
C-S	Liessa [31]	—	—	23.646	58.641	—	—	—	—
	Lal et al.[10]	17.332	52.098	23.646	58.646	20.503	53.237	26.060	59.661
	Jain & Soni [32]	17.332	52.097	23.646	58.646	—	—	—	—
	Lal and Dhanpati [8]	17.335	52.150	23.647	58.688	20.506	53.288	26.061	59.702
	Present	**17.3318**	—	**23.6363**	**58.6464**	**20.5034**	**53.2372**	**26.0605**	**59.6607**
C-F	Liessa [31]	—	63.635	12.680	—	—	—	—	—
	Lal et al.[10]	5.704	24.944	12.687	33.065	12.351	27.243	16.762	34.839
	Lal and Dhanpati [8]	5.703	24.949	12.684	33.064	12.350	27.248	16.760	34.831
	Present	**5.7039**	**24.9438**	**12.6874**	**33.0651**	**12.3505**	**27.3432**	**16.7621**	**348325**

Table 2:.Comparison of frequency parameter Ω for isotropic (E_2/E_1=1), homogeneous ($\mu_1 = \mu_2 = \beta = 0$) of uniform thickness ($\alpha_1 = \alpha_1 = 0.0$) for K = 0.0, p= 2 and υ= 0.3.

Boundary Conditions	***a/b***	**0.5**			**1.0**		
	Ref.\ Mode	**I**	**II**	**III**	**I**	**II**	**III**
C-C	Lal et al.[10]	28.9508	69.3270	129.0951	54.7431	94.5853	154.7754
	Jain & Soni [32]	28.9508	69.3270	129.0956	54.7430	94.5852	154.7757
	Lal and Dhanpati [8]	28.9499	69.3796	129.3675	54.7312	94.5927	154.9509
	Present	**28.9509**	**28.9509**	**129.0956**	**54.7431**	**94.5853**	**154.7757**
C-S	Lal et al.[10]	23.6464	58.6464	113.2377	51.6742	86.1350	140.8484
	Jain & Soni [32]	23.6463	58.6463	113.2281	51.6742	86.1344	140.8455
	Lal and Dhanpati [8]	23.6468	58.6880	113.4541	51.6700	86.1493	141.0035
	Present	**23.6463**	**58.6464**	**113.8281**	**51.6743**	**86.1345**	**140.8456**

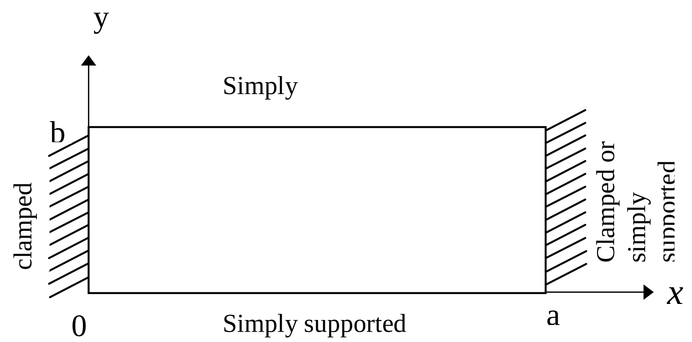

Fig. 1: Boundary Condotions

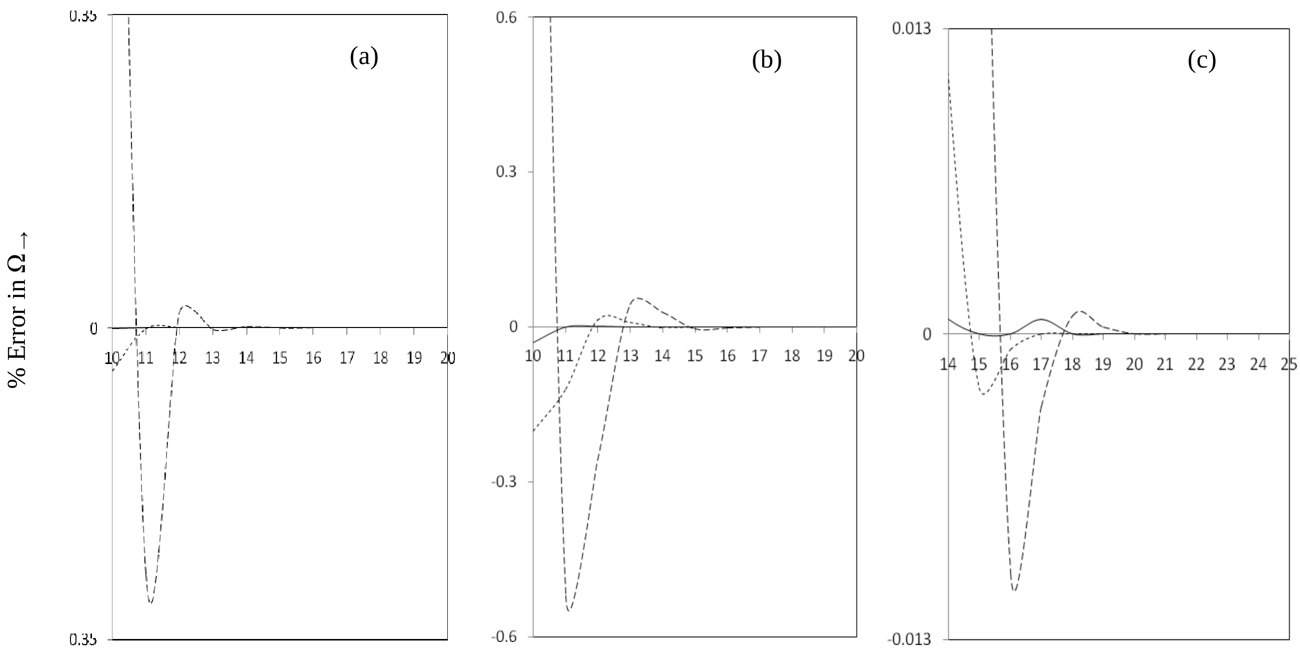

Fig. 2: Percentage error in frequency parameter Ω; (a) C-C plate (b) C-S plate and (c) C-F plate, for a/b= 1.0, K= 0.02, μ_1 = 0.05, μ_2 = -0.5, β= 0.5, α_1 = -0.5, α_2 = 0.5, ———, first mode, -------, second mode, – – – –, third mode. % error = $[(\Omega_m - \Omega_{20})/\ \Omega_{20}] \times 100$.

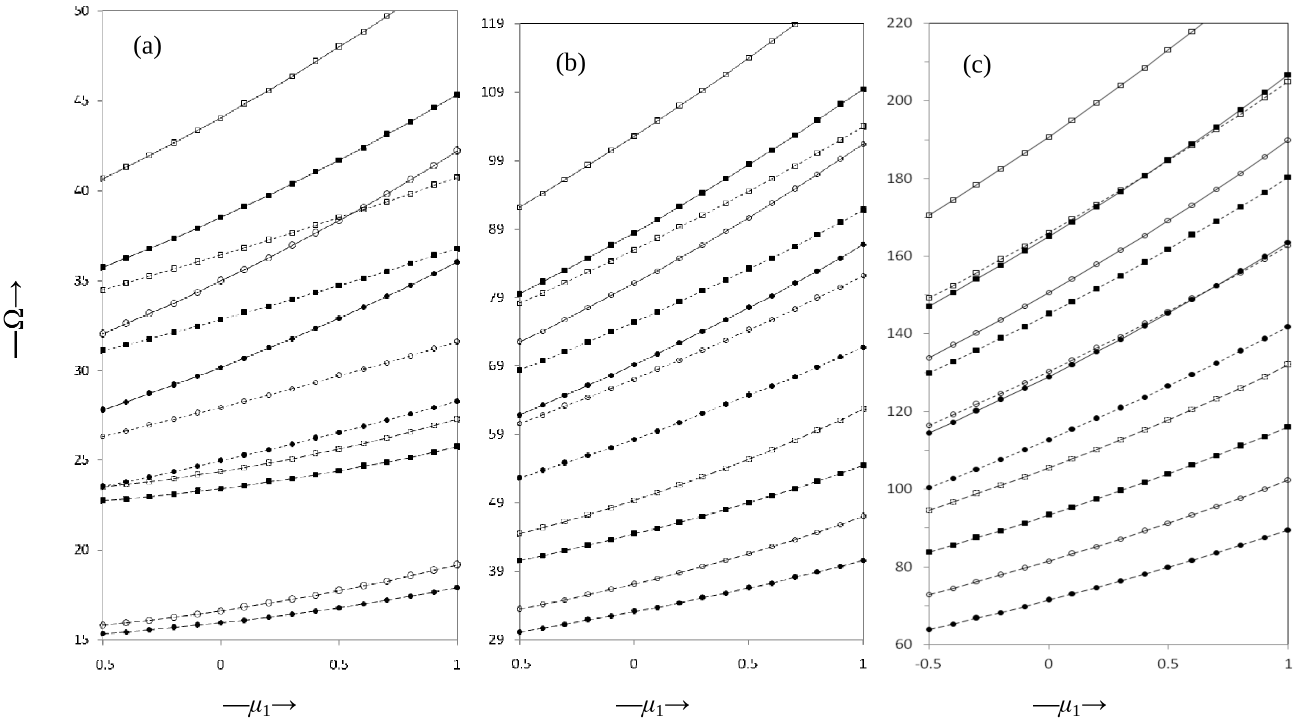

Fig. 3: Frequency parameter for C-C, C-S and C-F plates vibrating in (a) first mode (b) second mode and (c) third mode, for a/b = 1, μ_2 = 0.5, K=0.02, α_1= 0.5. ————, C-C; ----------, C-S; – – – –, C-F; ■, β = -0.5, α_2= -0.3; □, β = -0.5, α_2= 0.3 ; •, β = 0.5, α_2= -0.3;o, β = 0.5, α_2= 0.3.

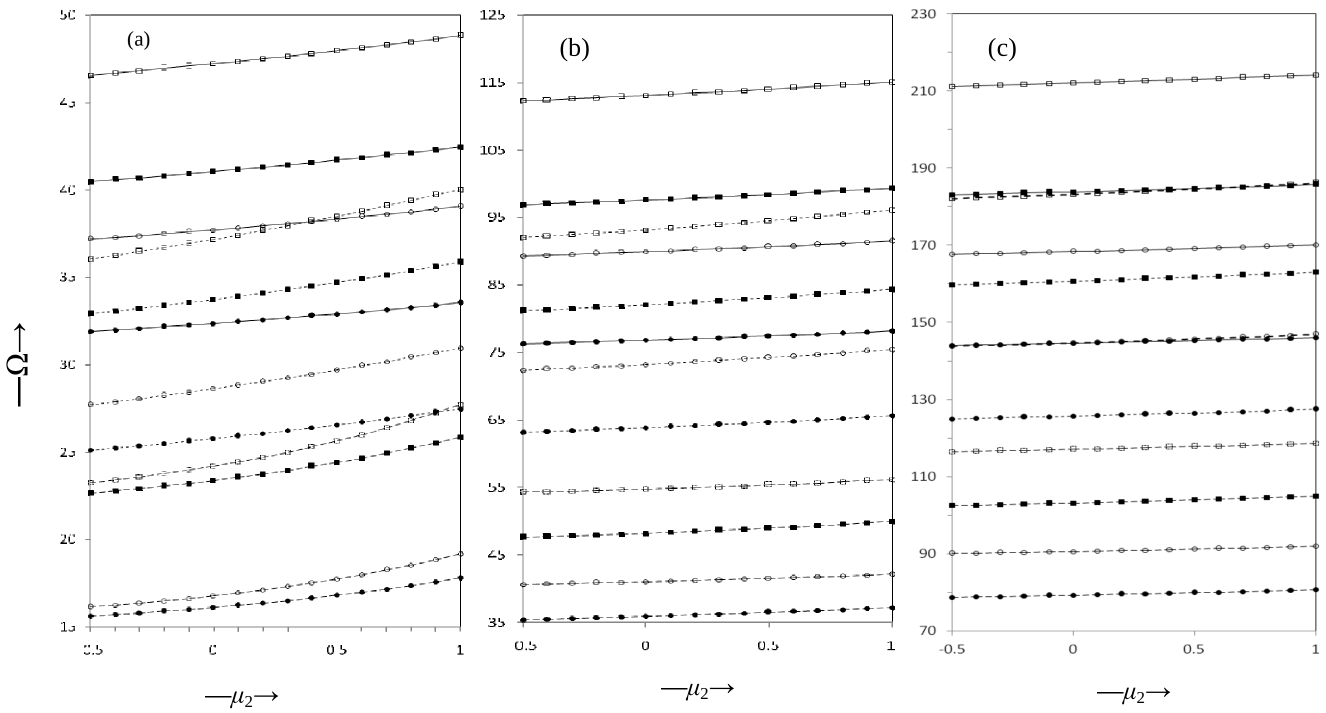

Fig. 4: Frequency parameter for C-C, C-S and C-F plates vibrating in (a) first mode (b) second mode and (c) third mode, for a/b = 1, μ_1 = 0.5, K=0.02, α_1= 0.5. ———, C-C; --------, C- S; – – – –, C-F; ■, β = -0.5,α_2= -0.3; □, β = -0.5, α_2= 0.3 ; •, β = 0.5, α_2= -0.3; o, β = 0.5, α_2= 0.3.

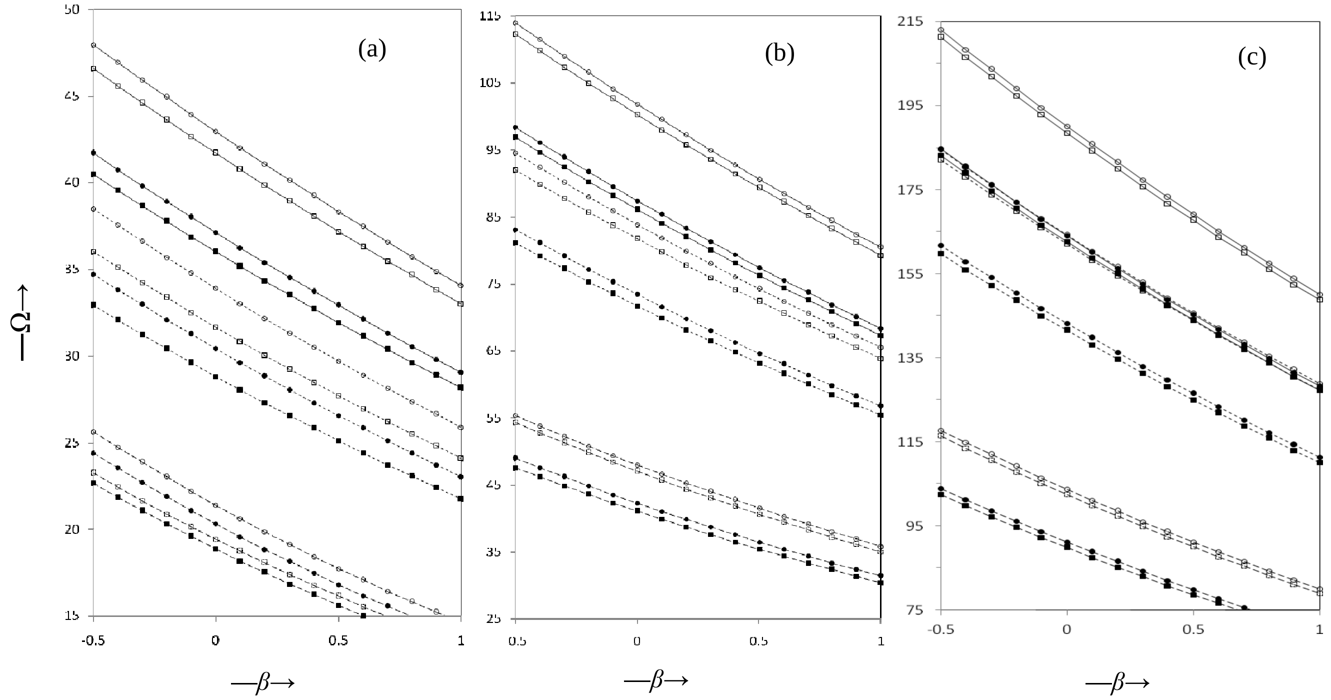

Fig. 5: Frequency parameter for C-C, C-S and C-F plates vibrating in (a) first mode (b) second mode and (c) third mode, for a/b = 1, μ_1 = 0.5, K=0.02, α_1= 0.5. ———, C-C; ---------, C-S; – – – –, C-F; ■, μ_2 = -0.5, α_2= -0.3; □, μ_2 = -0.5, α_2= 0.3 ; ●, μ_2 = 0.5, α_2= -0.3; o, μ_2 = 0.5, α_2= 0.3.

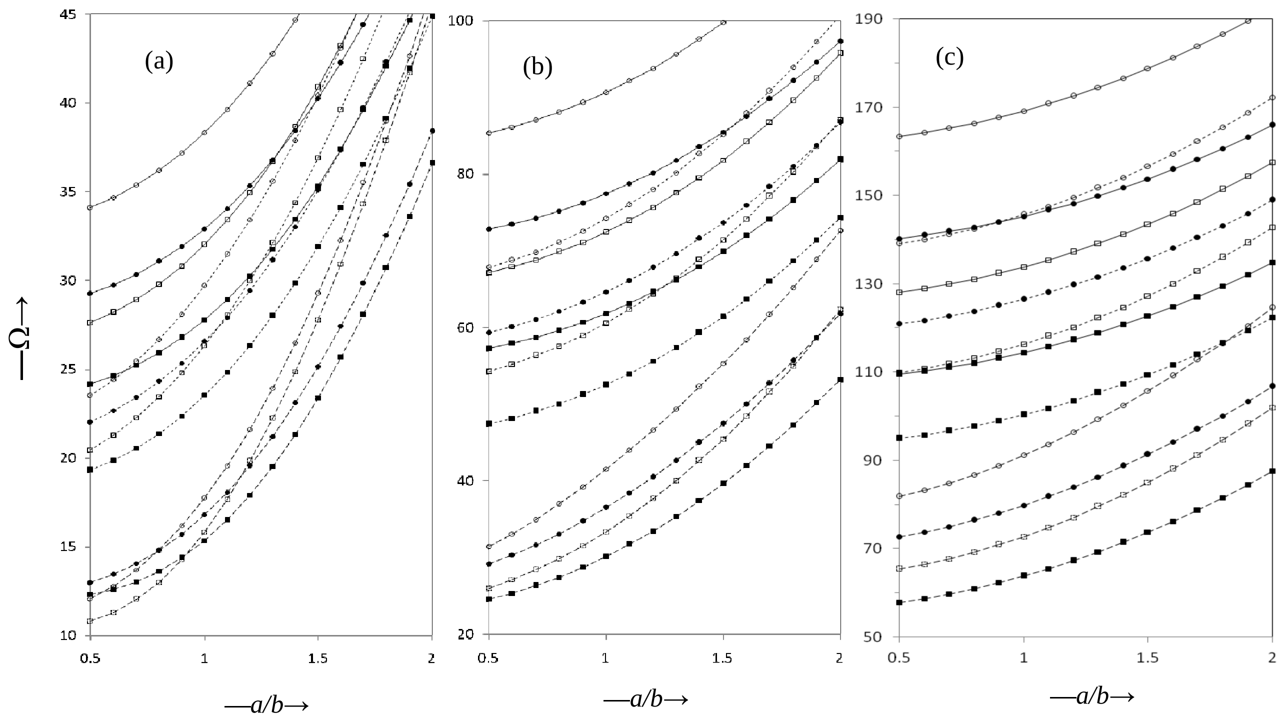

Fig. 6: Frequency parameter for C-C, C-S and C-F plates vibrating in (a) first mode (b) second mode and (c) third mode, for β = 0.5, μ_2 = 0.5, K=0.02, α_1= 0.5. ———, C-C; ---------, C-S; – – – – C-F; ■, μ_1 = -0.5,α_2= -0.3; □, μ_1 = -0.5, α_2= 0.3 ; ●, μ_1 = 0.5, α_2= -0.3;o, μ_1 = 0.5, α_2= 0.3.

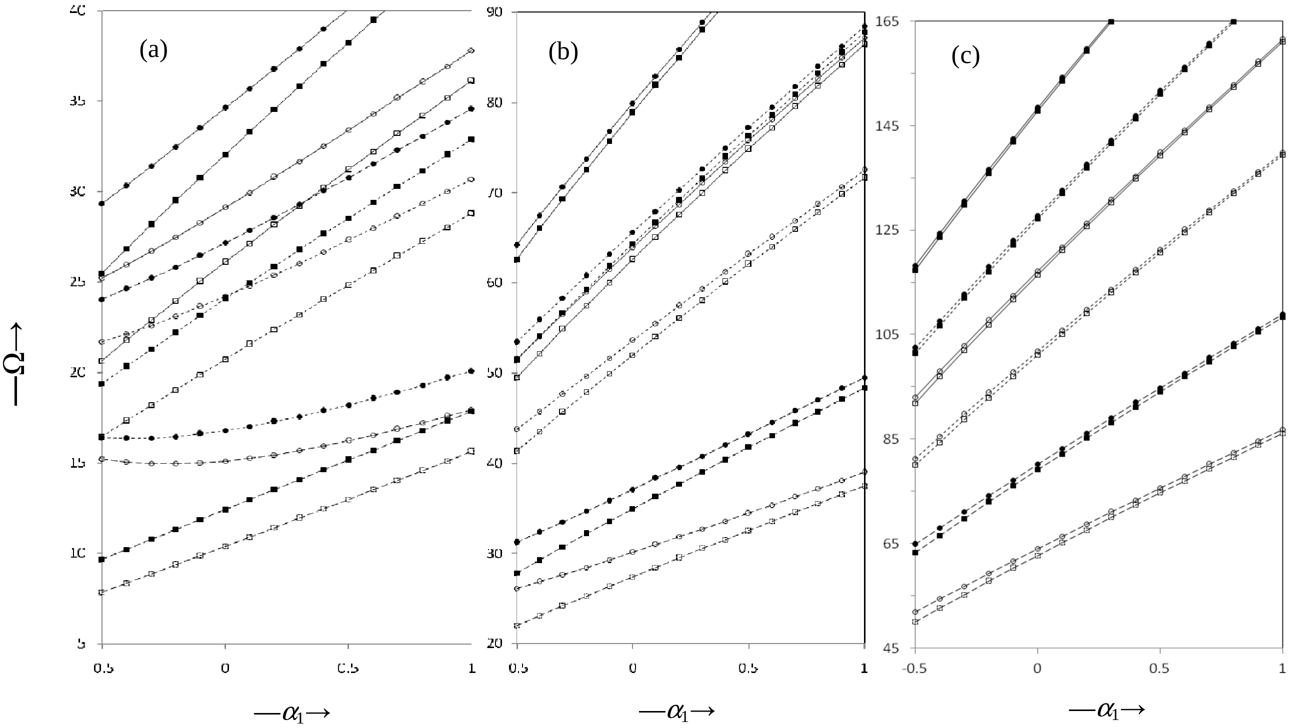

Fig. 7: Frequency parameter for C-C, C-S and C-F plates vibrating in (a) first mode (b) second mode and (c) third mode, for *a/b*= 1, β = 0.5, μ_2 = 0.5, α_2= 0.0. ———, C-C; ---------, C-S; – – – –, C-F; ■, μ_1 = -0.5, *K*=0.00; □, μ_1 = 0.5, *K*=0.0; •, μ_1 = -0.5, *K*=0.02; o, μ_1 = 0.5, *K*=0.02.

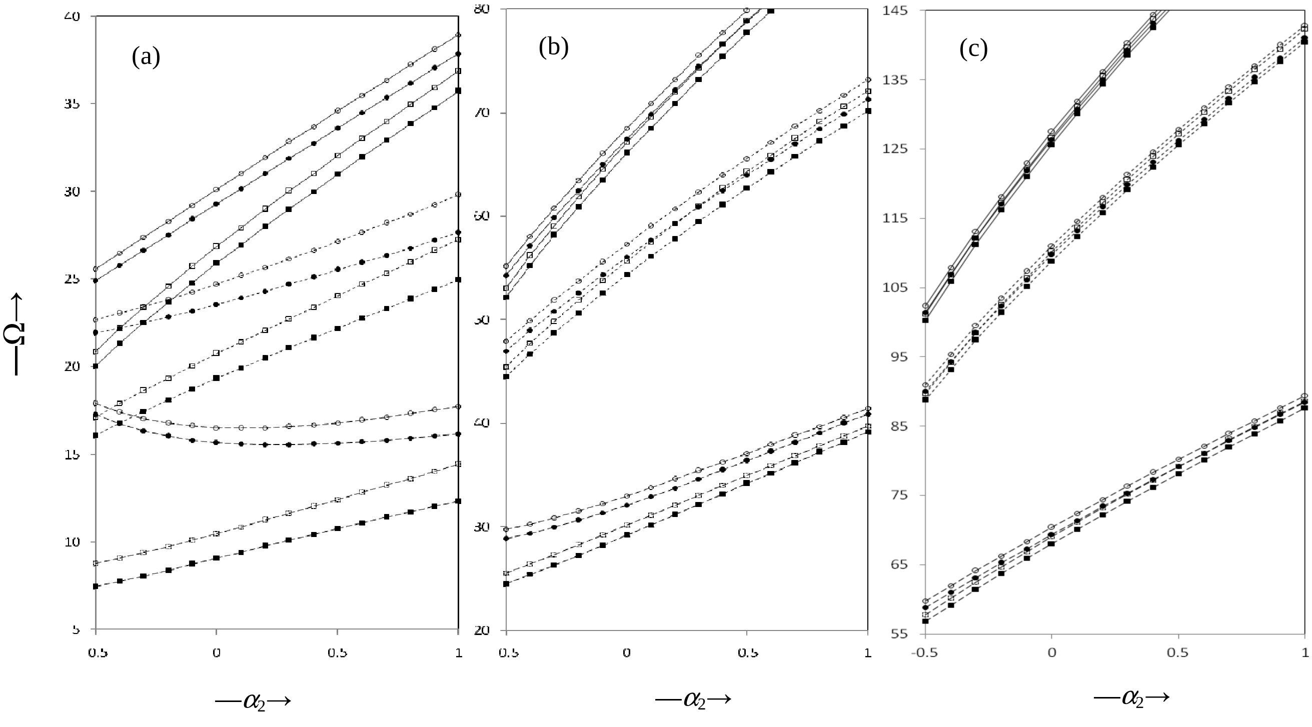

Fig. 8: Frequency parameter for C-C, C-S and C-F plates vibrating in (a) first mode (b) second mode and (c) third mode, for *a/b*= 1, β = 0.5, μ_2 = 0.5, α_1= 0.0. ———, C-C; ----------, C-S; – – – – C-F; ■, μ_1 = -0.5, *K*=0.00; □, μ_1 = 0.5, *K*=0.0; •, μ_1 = -0.5, *K*=0.02; o, μ_1 = 0.5, *K*=0.02.

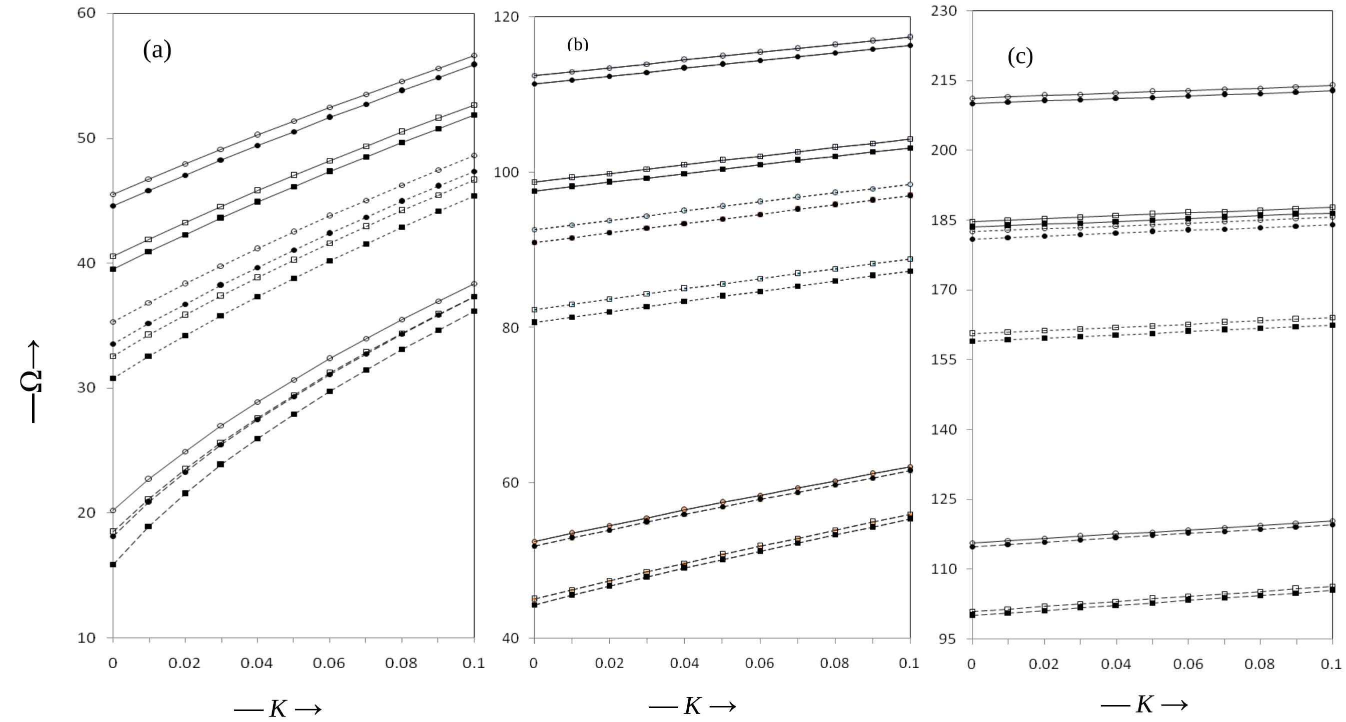

Fig. 9: Frequency parameter for C-C, C-S and C-F plates vibrating in (a) first mode (b) second mode and (c) third mode, for a/b= 1,β = -0.5, α_1 = 0.5, α_2= 0.5. ———, C-C; ---------, C-S; – – – –, C-F; ■, μ_1 = -0.3, μ_2 = -0.3; □, μ_1 = -0.3, μ_2 = 0.3; ●, μ_1 = 0.3, μ_2 = -0.3; o, μ_1 = 0.3, μ_2 = 0.3.

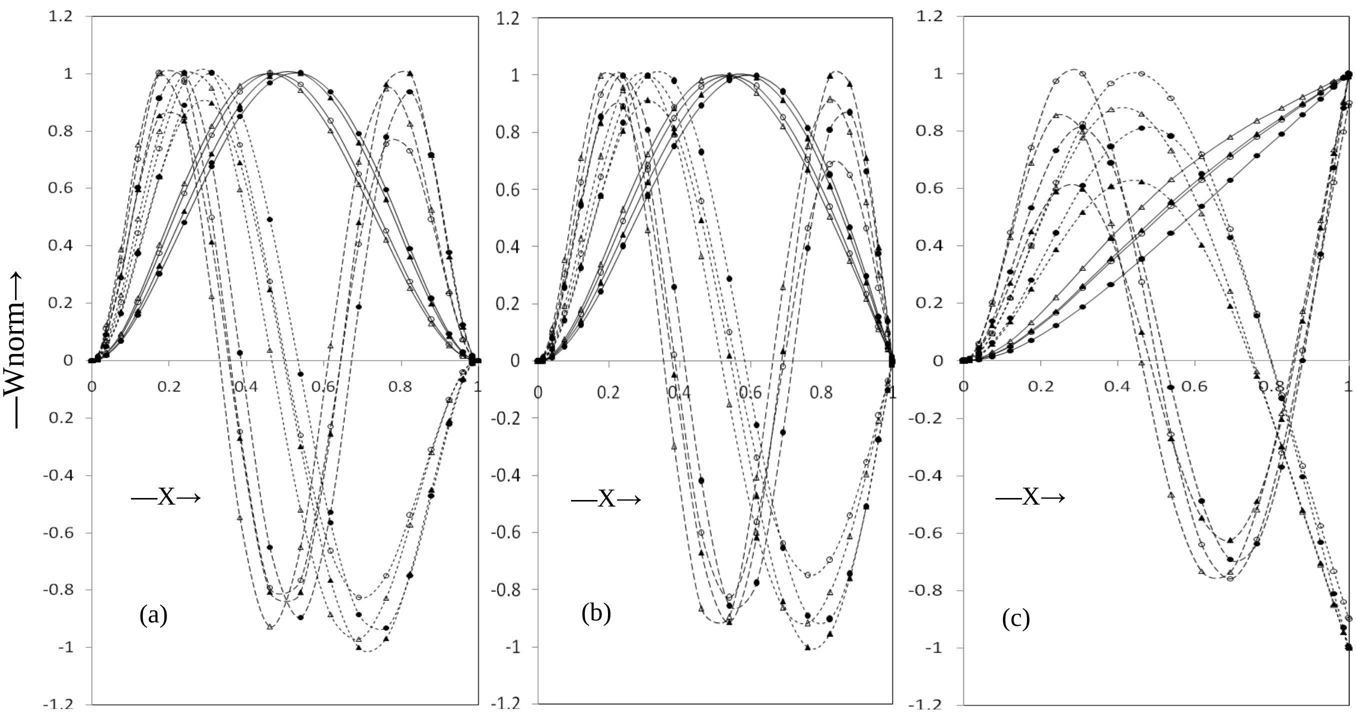

Fig. 10: Normalized displacements for the first three modes of vibration for (a) C-C plate (b) C-S plate and (c) C-F plate, for a/b=1.0, α_1 =0.5, K = 0.02, μ_1 = -0.5, μ_2 = 0.5. ———, first mode; ………, second mode; ----------, third mode.
△, β = -0.5; ○, β = 0.5; ▲, ●, α_2= -0.3; △,○, α_2= 0.3.

6. Conclusions

DQM method has been employed for obtaining natural frequencies of orthotropic rectangular plates on the basis of a new model to approximate the non-homogeneity of the plate material. The effect of various parameters such as non-homogeneity, orthotropy, aspect ratio and foundation modulus on the frequency parameter has been studied for linear as well as parabolic thickness variation. A comparison of results with those available for special cases have also been presented, which are in good agreement.

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