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COMPARISON OF FREE VIBRATION AND BUCKLING BEHAVIOUR OF CROSS-PLY LAMINATED PLATES WITH ISOTROPIC AND ORTHOTROPIC PLATES

*Bahar Uymaz**

[*]*Namık Kemal University, Department of Mechanical Engineering, Corlu, Tekirdag, Turkey*
[*]*E-mail address: buymaz@nku.edu.tr, web page: http://www.nku.edu.tr*

Abstract

In the present study Navier method with generalized shear deformation theory for exponential model which proposed by Aydoğdu [31] is used to determine the natural frequencies and critical buckling loads of elastic plates. According to the model the transverse shear strains through the thickness direction of the plate are distributed exponential and the theory accounts the rotary inertia. The convergence and comparison studies demonstrate the accuracy and correctness of the present study. The results are obtained for comparing the anti-symmetric and symmetric cross-ply laminated plates with isotropic and orthotropic plates for simply supported boundary condition. The material anisotropy, plate geometry (side-side, side-thickness), variation of higher frequencies, and variation of vibration and axial buckling mode shapes are compared.

Key words: Cross-ply laminated plates, orthotropic plates, generalized shear deformation plate theory, free vibration, axial buckling.

1.Introduction

Composites are generally used because they have desirable properties which could not be achieved by either of the constituent materials acting alone. The most common example is the fibrous composite consisting of reinforcing fibers embedded in a matrix material. In the continuous fiber composite laminate individual continuous fiber/matrix laminae are oriented in the required directions and bonded together to form a laminate [1]. Each layer can be considered as a homogeneous, orthotropic material having a value of Elasticity modulus considerably greater in the longitudinal directio than in transverse direction [2]. Despite the difficulties in determining the mechanical properties of laminated composite structures due to the complex nature of those in comparison with traditional materials make important studying about this subject due to their high specific strength and stiffness. However, investigation of natural frequencies and critical buckling loads gives an idea about dynamic properties and stability characteristics of the system, respectively. Laminated composite plates are commonly used structural elements many engineering applications such as aviation, automobiles, marine and submarine vehicles, aerospace etc. Hence these structures attract great attention by researchers to investigate the elastic behaviors of laminated plates such as bending [3-8], buckling [9-10] and free vibration analysis [11,15].

This paper provides a contribution in rich literature as well-known for bending, vibration and buckling analysis of laminated plates with a comparison of isotropic, orthotropic and laminated plates in each others. In most applications, the laminate thickness of laminated composite plates which are bonded together to form a laminate with desired thickness and stiffness is small compared to the planar dimensions composites. Therefore, two-dimensional theories are used to analyze laminated plates for stresses, usually. The two-dimensional theories are obtained from the three-dimensional elasticity theory by making assumptions concerning the variation of displacements and/or stresses through the thickness of the laminate. In the literature, there are many studies of laminated plates with three-dimensional plate theories [16-17] and with displacement based shear deformation plate theories such as classical laminated plate theory [18,21] and various higher order shear deformation theories [22,25] or generalized higher order shear deformation plate theory [26,30]. In this study, the exponential model [31] of generalized shear deformation plate theory which based on assumed displacement expansions plate theory is considered.

The elastic plates are compared in terms of material anisotropy, plate thickness, side-to-side ratio and mode shapes. In the buckling problem, the elastic plates are compared in terms of different loading conditions. The mode shapes giving information for geometrical character of the vibration and buckling behavior are plotted for considered elastic plates.

2. Equations of Laminated Plates

A rectangular plate which has a length *a*, a width *b* and a constant thickness *h* is considered. The plate geometry and dimensions are defined with respect to a Cartesian coordinate system (x,y,z), the origin of the coordinate system is placed at the geometric center of the plate. The coordinate parameters are such that $-a/2 \leq x \leq a/2$, $-b/2 \leq x \leq b/2$, $-h/2 \leq x \leq h/2$ and the corresponding displacement components U, V and W along the x, y and z directions, respectively.

The plate is assumed to be constructed of arbitrary number, N, of linearly elastic orthotropic layers. Thus, the state of stress in the k-th layer is given by the generalized Hooke's law as follows

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{bmatrix}^k = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & 0 & 0 & 0 \\ \overline{Q}_{12} & \overline{Q}_{22} & 0 & 0 & 0 \\ 0 & 0 & \overline{Q}_{44} & 0 & 0 \\ 0 & 0 & 0 & \overline{Q}_{55} & 0 \\ 0 & 0 & 0 & 0 & \overline{Q}_{66} \end{bmatrix}^k \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} \tag{1}$$

Where $\overline{Q}_{ij}^{(k)}$ are the reduced and transformed material stiffness of the k-th layer and defined as follows

$$
\begin{aligned}
\overline{Q}_{11} &= Q_{11}\cos^4\theta + Q_{22}\sin^4\theta + 2(Q_{12}+2Q_{66})\cos^2\theta\sin^2\theta \\
\overline{Q}_{12} &= (Q_{11}+Q_{22}-4Q_{66})\cos^2\theta\sin^2\theta + Q_{12}(\cos^4\theta+\sin^4\theta) \\
\overline{Q}_{22} &= Q_{11}\sin^4\theta + Q_{22}\cos^4\theta + 2(Q_{12}+2Q_{66})\cos^2\theta\sin^2\theta \\
\overline{Q}_{44} &= Q_{44}\cos^4\theta + Q_{55}\sin^4\theta \\
\overline{Q}_{55} &= Q_{44}\sin^4\theta + Q_{55}\cos^4\theta \\
\overline{Q}_{66} &= (Q_{11}+Q_{22}-2Q_{12}-2Q_{66})\cos^2\theta\sin^2\theta + Q_{66}(\cos^4\theta+\sin^4\theta)
\end{aligned} \tag{2}
$$

Where the angle θ is referred to as lamination angle and Q_{ij} are reduced material stiffness and defined as follows

$$
\begin{aligned}
&Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}}, \qquad Q_{12} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}}, \qquad Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}}, \\
&Q_{44} = G_{23}, \qquad Q_{55} = G_{13}, \qquad Q_{66} = G_{12}
\end{aligned} \tag{3}
$$

In the higher order shear deformation theories the transverse normal stress is neglected because the virtual strain energy of this stress is zero due to the fact that kinematically consistent virtual strain must be zero, (ε_z=0), [32]. Thus, the infinitesimal strain components ε_{ij} (i,j=x,y,z) are defined as follows

$$
\varepsilon_x = U,_x, \qquad \varepsilon_y = V,_y, \qquad \gamma_{yz} = V,_z + W,_y, \qquad \gamma_{xz} = U,_z + W,_x, \qquad \gamma_{xy} = U,_y + V,_x \tag{4}
$$

According to the generalized shear deformable shell theory which presented by Soldatos and Timarci [29], and Timarci and Soldatos [30], the displacement field of the plate is assumed as follows

$$
\begin{aligned}
U(x,y,z;t) &= u(x,y;t) - zw,_x + f(z)u_1(x,y;t), \\
V(x,y,z;t) &= v(x,y;t) - zw,_y + f(z)v_1(x,y;t), \\
W(x,y,z;t) &= w(x,y;t),
\end{aligned} \tag{5}
$$

Where the displacement components U, V and W are the corresponding components along the x, y and z directions, respectively. And where u, v, w, u_1 and v_1 are the five unknown displacement functions of middle surface of the plate while f represent shape function determining the distribution of the transverse shear strains and stresses along the thickness. Depends on the selection of the shape function f(z) the shear deformation theory corresponds such as the classical plate theory (CPT) in which the displacement field is selected so as to satisfy the Kirchhoff hypothesis, first order shear deformation plate theory (FSDPT) of Timoshenko, parabolic shear deformation plate theory (PSDPT) of Reddy and general exponential shear deformation plate theory (ESDPT) of Aydoğdu [31] which is also used in the present study and they are given in Table 1. Hence the strain components are written in terms of the displacement components as follows

$$
\begin{aligned}
&\varepsilon_x = u,_x - zw,_{xx} + f(z)u_1,_x \,, \\
&\varepsilon_y = v,_y - zw,_{yy} + f(z)v_1,_y \,, \\
&\gamma_{yz} = f' v_1, \\
&\gamma_{xz} = f' u_1, \\
&\gamma_{xy} = u,_y + v,_x + z(-2w,_{xy}) + fu_1,_y + fv_1,_x
\end{aligned} \tag{6}
$$

where a prime denotes the derivative with respect to z and ,x=∂/∂x.

Table 1. Definitions of shear functions considered in the study.

Corresponding Plate Theory	f(z)
CPT	0
FSDPT	z
PSDPT	$z(1\text{-}4z^2/3h^2)$
ESDPT	$z3^{\frac{-2(z/h)^2}{\ln 3}}$

The force and moment resultants are defined as follows

$$
\begin{aligned}
&\left(N_x^c, N_y^c, N_{xy}^c\right) = \int_{-h/2}^{h/2} \left(\sigma_x, \sigma_y, \tau_{xy}\right)^k dz, \qquad \left(M_x^c, M_y^c, M_{xy}^c\right) = \int_{-h/2}^{h/2} \left(\sigma_x, \sigma_y, \tau_{xy}\right)^k z dz, \\
&\left(M_x^a, M_{xy}^a\right) = \int_{-h/2}^{h/2} \left(\sigma_x, \tau_{xy}\right)^k f(z) dz, \qquad \left(M_y^a, M_{yx}^a\right) = \int_{-h/2}^{h/2} \left(\sigma_y, \tau_{yx}\right)^k f(z) dz, \\
&\left(Q_x^a, Q_y^a\right) = \int_{-h/2}^{h/2} \left(\tau_{xz}, \tau_{yz}\right)^k f' dz
\end{aligned} \tag{7}
$$

By substituting the stress-strain relations into the definitions of the force and moment resultants in accordance with the generalized shear deformable shell theory, the constitutive relations equations are obtained as follows

$$
\begin{bmatrix} N_x^c \\ N_y^c \\ N_{xy}^c \\ M_x^c \\ M_y^c \\ M_{xy}^c \\ M_x^a \\ M_y^a \\ M_{xy}^a \\ M_{yx}^a \end{bmatrix}
=
\begin{bmatrix}
A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 & B_{11} & B_{12} & 0 & 0 \\
A_{12} & A_{12} & 0 & B_{12} & B_{22} & 0 & E_{12} & E_{22} & 0 & 0 \\
0 & 0 & A_{66} & 0 & 0 & B_{66} & 0 & 0 & E_{66} & E_{66} \\
B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 & F_{11} & F_{12} & F_{16} & 0 \\
B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 & F_{12} & F_{22} & 0 & 0 \\
0 & 0 & B_{66} & 0 & 0 & D_{66} & 0 & 0 & F_{66} & F_{66} \\
E_{11} & E_{12} & 0 & F_{11} & F_{12} & F_{16} & H_{11} & H_{12} & 0 & 0 \\
E_{12} & E_{22} & 0 & F_{12} & F_{22} & 0 & H_{12} & H_{22} & 0 & 0 \\
0 & 0 & E_{66} & 0 & 0 & F_{66} & 0 & 0 & H_{66} & H_{66} \\
0 & 0 & E_{66} & 0 & 0 & F_{66} & 0 & 0 & H_{66} & H_{66}
\end{bmatrix}
\begin{bmatrix} u,_x \\ v,_y \\ u,_y + v,_x \\ -w,_{xx} \\ -w,_{yy} \\ -2w,_{xy} \\ u_{1,x} \\ v_{1,y} \\ u_{1,y} \\ v_{1,x} \end{bmatrix}
$$

$$\begin{bmatrix} Q_y^a \\ Q_x^a \end{bmatrix} = \begin{bmatrix} A_{44} & 0 \\ 0 & A_{55} \end{bmatrix} \begin{bmatrix} v_1 \\ u_1 \end{bmatrix} \tag{8}$$

In these definitions, the resultants and strains denoted with a superscript ‘c’ are the conventional ones of the classical plate theories whereas the remaining ones with a superscript ‘a’ are additional quantities incorporating the transverse shear deformation effects.

The extensional, coupling, bending and transverse shear rigidities in accordance with generalized shear deformable shell theory are given as follows

$$\begin{gathered} \left(A_{ij}, B_{ij}, D_{ij}\right) = \int_{-h/2}^{h/2} Q_{ij}^{(k)} \left(1, z, z^2\right) dz, \qquad \left(E_{ij}, F_{ij}, H_{ij}\right) = \int_{-h/2}^{h/2} Q_{ij}^{(k)} \left(f, z.f, f^2\right) dz, \\ A_{kl} = \int_{-h/2}^{h/2} Q_{kl}^{(k)} \left(f'\right)^2 dz, \qquad (i, j = 1,2), \qquad (k, l = 4,5), \qquad (\ )' = d(\ )/dz. \end{gathered} \tag{9}$$

Applying Hamilton and minimum potential energy principles [33] the governing equations of the considered plate are obtained as follows

$$\begin{aligned} & N_{x,x}^c + N_{xy,y}^c = \left(\rho_0 u - \rho_1 w,_x + \rho_0^{-11} u_1\right),_{tt} \\ & N_{y,y}^c + N_{xy,x}^c = \left(\rho_0 v - \rho_1 w,_y + \rho_0^{-21} v_1\right),_{tt} \\ & M_{x,xx}^c + M_{y,yy}^c + 2M_{xy,xy}^c + q + N_x^e w,_{xx} + N_y^e w,_{yy} + 2N_{xy}^e w,_{xy} = \\ & \qquad \left(\rho_0 w - \rho_1 v,_y - \rho_2 \left(w,_{yy} + w,_{xx}\right) + \rho_1^{-11} u_{1,x} + \rho_1^{-21} v_{1,y} + \rho_1 u,_x\right),_{tt} \\ & M_{x,x}^a + M_{xy,y}^a - Q_x^a = \left(\rho_0^{-11} u - \rho_1^{-11} w,_x + \rho_0^{-12} u_1\right),_{tt} \\ & M_{y,y}^a + M_{yx,x}^a - Q_y^a = \left(\rho_0^{-21} v - \rho_1^{-21} w,_y + \rho_0^{-22} v_1\right),_{tt} \end{aligned} \tag{10}$$

Where $q(x,y,t)$ is transverse load, N_x^e, N_y^e, N_{xy}^e are the constant in-plane edge loads and the inertia terms ρ_i and ρ_i^{-lm} s are defined as follows

$$\rho_i = \int_{-h/2}^{h/2} \rho z^i dz, \qquad (i = 0,1,2), \qquad \rho_i^{-lm} = \int_{-h/2}^{h/2} \rho z^i f^m dz, \qquad (i = 0,1; \quad m = 1,2). \tag{11}$$

and ρ is the mass per unit volume.

The boundary conditions at the edges of the plate are obtained as a result of Hamilton’s principle and they are given in Table 2.

Table 2. The boundary conditions at the edges of the plate.

at x = ± a/2	at y = ± b/2
either u or N_x^c prescribed	either v or N_y^c prescribed,
either v or N_{xy}^c prescribed	either u or N_{xy}^c prescribed,
either w or $M_{x,x}^c + 2M^c_{xy,y}$ prescribed	either w or $M_{y,y}^c + 2M^c_{yx,x}$ prescribed,
either $w,_x$ or M_x^c prescribed	either $w,_y$ or M_y^c prescribed,
either u_1 or M_x^a prescribed	either u_1 or M_{yx}^a prescribed,
either v_1 or M_{xy}^a prescribed	either v_1 or M_y^a prescribed.

The Navier type solution is obtained for simply supported boundary condition of considered plates and considered boundary condition is defined as follows

$$
\begin{aligned}
N_x^c = v = w = M_x^c = M_x^a = v_1 = 0 \qquad \text{at } x = \pm a/2 \\
N_y^c = u = w = M_y^c = M_y^a = v_1 = 0 \qquad \text{at } y = \pm b/2
\end{aligned}
\tag{12}
$$

2.1.Solution Procedure Of Free Vibration

The proposed displacement model for solution which satisfies considered boundary condition and the governing equations is given as follows

$$
\begin{aligned}
(u, au_1) &= (A, D)\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\sin\omega t, \\
(v, bv_1) &= (B, E)\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}\sin\omega t, \\
w &= C\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\sin\omega t.
\end{aligned}
\tag{13}
$$

Where m and n are half-wave numbers along x and y directions, respectively, and ω is radial frequency. According to the free vibration problem the transverse load term (q(x;t)) and the external force terms (N_x^e, N_y^e, N_{xy}^e) are set to zero.

For free vibration analysis the displacement field components which given with Eq.(13) are substituted into governing equations which given with Eq.(10) and this process is leaded to an eigenvalue equation which given as follows

$$
\{[K] - \Omega_n^2 [M]\}\{[X_{mn}]\} = \{[0]\} \tag{14}
$$

where K and M are stiffness and inertia matrices, respectively, Ω_n is the free vibration frequency parameter and X_{mn} is the column vector of unknown coefficients of series (14). For a given a pair of m and n with certain geometrical and material properties of the plate, the solution of this eigenvalue problem predicts five natural frequencies for vibration problem. The non-dimensional frequency parameter is defined as follows

$$
\Delta_n^2 = \Omega_n^2\left(\frac{a^2 b^2}{\pi^4 h D_0}\right), \qquad D_0 = \frac{E_2 h^3}{12(1-\nu^2)}, \qquad \Omega_n^2 = \rho\omega^2 \tag{15}
$$

2.2.Solution Procedure Of Buckling

The proposed displacement model for solution which satisfies considered boundary condition and the governing equations is given as follows

$$(u, au_1) = (A, D)\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b},$$
$$(v, bv_1) = (B, E)\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}, \qquad (16)$$
$$w = C\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}.$$

According to the axial buckling problem the transverse load term (q(x;t)) and the external shear force term (N_{xy}^e) are set to zero. The in-plane axial forces (N_x^e , N_y^e) are negative for compressive forces and positive for tensile forces. As in the case of axial buckling the in-plane forces are defined as follows

$$\boldsymbol{N}_x^e = -N_0, \quad \boldsymbol{N}_x^e = -\delta N_0, \quad \delta = \frac{\boldsymbol{N}_y^e}{\boldsymbol{N}_x^e} \qquad (17)$$

where δ is a non-dimensional load parameter which corresponds the loading conditions. The value of δ is 0, 1 and -0.5 when the plate is subjected to the uniaxial compression along the x axis, the biaxial compression and the tensile loading in the y direction while the plate is under compression along the x direction, respectively.

For buckling analysis the displacement field components which given with Eq.(13) are substituted into governing equations which given with Eq.(10) and this process is leaded to an eigenvalue equation which given as follows

$$\{[K] - \boldsymbol{N}_a[M]\}\{[X_{mn}]\} = \{[0]\} \qquad (18)$$

where K and M are stiffness and geometric matrices, respectively, N_a is the critical buckling load parameter and X_{mn} is the column vector of unknown coefficients of series (17). For a given a pair of m and n with certain geometrical and material properties of the plate, the solution of this eigenvalue problem predicts five critical buckling loads for buckling problem. The non-dimensional critical buckling load parameter is defined as follows

$$\boldsymbol{N}_a = N_0\left(\frac{a^2}{\pi^2 D_0}\right), \quad D_0 = \frac{E_2 h^3}{12(1-\nu^2)} \qquad (18)$$

3. Numerical Results

The cross-ply laminated plates are compared from the point of free vibration and buckling behavior under in-plane axial forces with isotropic and orthotropic plates. The analysis is performed based on a generalized shear deformable shell theory using Navier type solution. Hence, it is considered simply supported boundary condition. The material properties of laminated plates which used in the present study are given in Table 3.

Table 3. Comparison of non-dimensional fundamental frequency parameter of simply supported isotropic and orthotropic plates, ($\Omega = \omega_{11}\left(b^2/\pi^2\right)\sqrt{\rho h / D_{22}}$).

a/b	Method	Isotropic		E_1/E_2=3		E_1/E_2=10	
		a/h=100	a/h=10	a/h=100	a/h=10	a/h=100	a/h=10
0.5	Ref.[32]	4.999	4.900	7.669	7.517	13.072	12.814
	Present Study	4.998	4.973	7.669	7.630	13.074	13.008
1.0	Ref.[32]	2.000	1.984	2.541	2.521	3.672	3.643
	Present Study	1.997	1.982	2.539	2.519	3.671	3.641
2.0	Ref.[32]	1.250	1.244	1.342	1.336	1.499	1.491
	Present Study	1.248	1.224	1.341	1.315	1.496	1.466
3.0	Ref.[32]	1.111	1.106	1.145	1.139	1.183	1.178
	Present Study	1.109	1.067	1.140	1.099	1.178	1.135

The frequency parameters, the critical buckling loads and mode shapes of vibration and buckling are obtained for different plate geometry and material anisotropy according to vibration and axial loading conditions.

In order to establish the validity of the present study comparison results are presented in Table 4-9. In Table 4, comparison of non-dimensional fundamental frequency parameter of isotropic and orthotropic plates is given. In Table 5, comparison of non-dimensonal frequency parameters for higher modes of laminated plates according to the classical plate theory is given. In table 6, comparison of critical buckling load of isotropic plates under uniaxial compression (δ=0) along the x-axis is given. In Table 7-9, comparison of critical buckling load of orthotropic and laminated plates are given for uniaxial compression along the x-axis and biaxial compression (δ=1). The results are in good agreement.

Table 4. Comparison of non-dimensonal frequency parameters of simply supported laminated plates according to the classical plate theory (a/b=1, E_1/E_2=20).

(m,n)	Solution Method	(0º)	(0º /90º)	$(0º/90º)_2$	$(0º/90º)_s$
(1,1)	Ref.[34]	4.847	0.990	1.386	2.638
	Present Study	4.845	0.984	1.385	2.636
(1,2)	Ref.[34]	6.781	2.719	3.913	4.917
	Present Study	6.778	2.716	3.911	4.915
(1,3)	Ref.[34]	11.111	5.789	8.456	9.637
	Present Study	11.105	5.784	8.450	9.632
(2,1)	Ref.[34]	18.193	2.719	3.913	9.354
	Present Study	18.188	2.716	3.911	9.352
(2,2)	Ref.[34]	19.388	3.959	5.547	10.554
	Present Study	19.381	3.956	5.544	10.549
(2,3)	Ref.[34]	22.153	6.702	9.507	13.826
	Present Study	22.141	6.695	9.501	13.818

Table 5. Comparison of critical buckling load of simply supported isotropic plates under uniaxial compression along the x-axis (δ=0) for different a/b ratios.

a/b	0.5	1.0	1.5
Ref.[32]	6.250	4.000	4.340†
Present Study	6.249	3.999	4.339†

† Denotes change to the next higher mode.

Table 6. Comparison of critical buckling load of simply supported orthotropic plates under uniaxial compression along the x-axis (δ=0) for different a/b ratios.

a/b	Solution Method	E_1/E_2=1	E_1/E_2=3	E_1/E_2=10	E_1/E_2=25
0.5	Ref.[32]	6.250	14.708	42.737	102.750
	Present Study	6.619	14.699	42.729	102.739
1.0	Ref.[32]	4.000	6.458	13.488	28.495
	Present Study	4.369	6.449	13.479	28.489
2.0	Ref.[32]	$4.000^{(2,1)}$	$6.458^{(2,1)}$	8.987	12.745
	Present Study	$4.369^{(2,1)}$	$6.449^{(2,1)}$	8.979	12.739
3.0	Ref.[32]	$4.000^{(3,1)}$	$6.042^{(2,1)}$	$9.182^{(2,1)}$	14.273
	Present Study	$4.369^{(3,1)}$	$6.039^{(2,1)}$	$9.179^{(2,1)}$	14.269

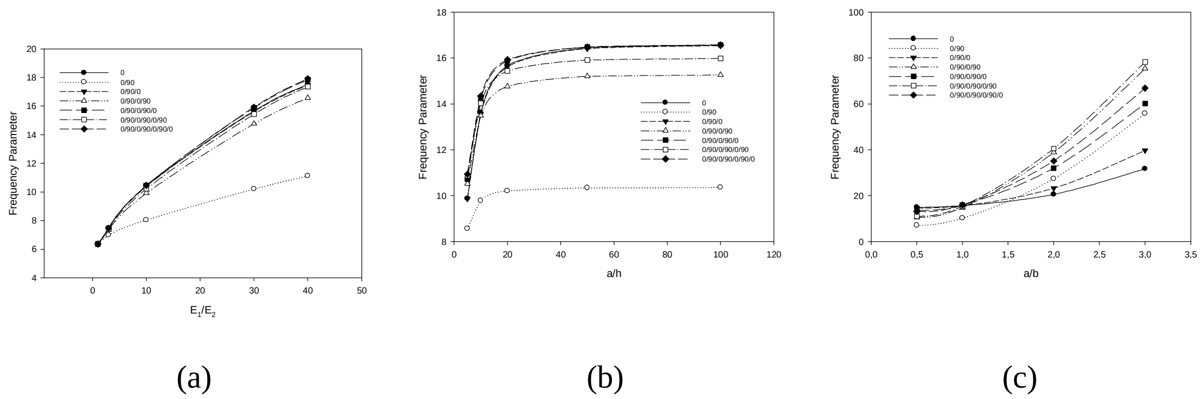
Table 7. Comparison of critical buckling load of simply supported orthotropic plates under biaxial compression (δ=1) for different a/b ratios, $N = N_{cr} b^2 / (\pi^2 D_{22})$.

a/b	Solution Method	E_1/E_2=1	E_1/E_2=3	E_1/E_2=10	E_1/E_2=25
0.5	Ref.[32]	5.000	11.767	$25.427^{(1,3)}$	$40.784^{(1,4)}$
	Present Study	5.299	11.759	$25.419^{(1,3)}$	$40.779^{(1,4)}$
1.0	Ref.[32]	2.000	3.229	6.744	$10.196^{(1,2)}$
	Present Study	2.179	3.219	6.739	$10.189^{(1,2)}$
2.0	Ref.[32]	1.250	1.442	1.798	2.549
	Present Study	1.319	1.439	1.789	2.539
3.0	Ref.[32]	1.111	1.179	1.260	1.427
	Present Study	1.139	1.169	1.249	1.419

Table 8. Comparison of critical buckling load of simply supported laminated plates under uniaxial compression along the x-axis and biaxial compression for different a/b ratios, (0º/90º /0º/90º).

a/b	Solution Method	E_1/E_2=5	E_1/E_2=10	E_1/E_2=20	E_1/E_2=40
Uniaxial compression (δ=0)					
0.5	Ref.[34]	4.705	4.157	3.828	3.647
	Present Study	4.699	4.149	3.819	3.639
1.0	Ref.[34]	2.643	2.189	1.923	1.778
	Present Study	2.639	2.179	1.919	1.769
$1.5^{(2,1)}$	Ref.[34]	2.955	2.487	2.211	2.061
	Present Study	2.949	2.479	2.209	2.059
Biaxial compression (δ=1)					
0.5	Ref.[34]	3.764	3.325	3.062	2.917
	Present Study	3.759	3.319	3.059	2.909
1.0	Ref.[34]	1.322	1.095	0.962	0.889
	Present Study	1.319	1.089	0.959	0.879
1.5	Ref.[34]	1.009	0.860	0.773	0.725
	Present Study	0.999	0.859	0.769	0.719

Firstly, effects of the material anisotropy, the plate thickness and the side-to-side ratio on the number of layer of cross-ply laminated plates are investigated in Fig.1. It is observed that the frequency parameters increase with increasing the material anisotropy ratio (E_1/E_2), the side-to-side ratio (a/b) and the plate becoming thinner. The variation of frequency with a/h ratio is sharply for a/h<20 values and after the value of a/h>20 the variation of frequency is slightly. When the effect of the material anisotropy and the thickness of the plate on the freqeuncy parameter is investigated, it is seen that the frequency values of the two-layer cross-ply plate are smaller than the others with big differences.



(a) (b) (c)

Figure 1: Variation of frequency parameter with number of layers of cross-ply laminated plates for (a) different orthotropy degrees (a/h=20, a/b=1); (b) different a/h ratios (E1/E2=30, a/b=1); (c) different a/b ratios (E1/E2=30, a/h=20).

Comparisons of anti-symmetric and symmetric cross-ply laminated plates with isotropic and orthotropic plates in terms of free vibration behaviour are given in Fig.2-5. As usual, frequency values of considered plates increase with increasing the material anisotropy, the plate thickness and the length of the plate and also frequency values of symmetric cross-ply and orthotropic plates are very closely each others. According to the material anisotropy and the plate thickness, symmetric cross-ply and orthotropic plates are more rigid but according to the plate length anti-symmetric plates are more rigid and frequency values of symmetric cross-ply laminated plates are bigger than frequency values of orthotropic plates. In all profiles, isotropic plates have minimum frequency values. In the higher modes, it is seen that the variations of frquency values increase wavy and frequency values of -symmetric cross-ply laminated and orthotropic plates are very closely each others but orthotropic plates have more rigidity in terms of symmetric cross-ply laminated plates and anti-symmetric cross ply laminated plates.

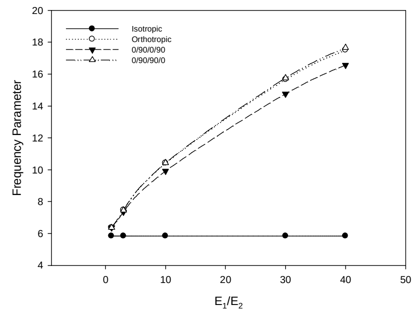
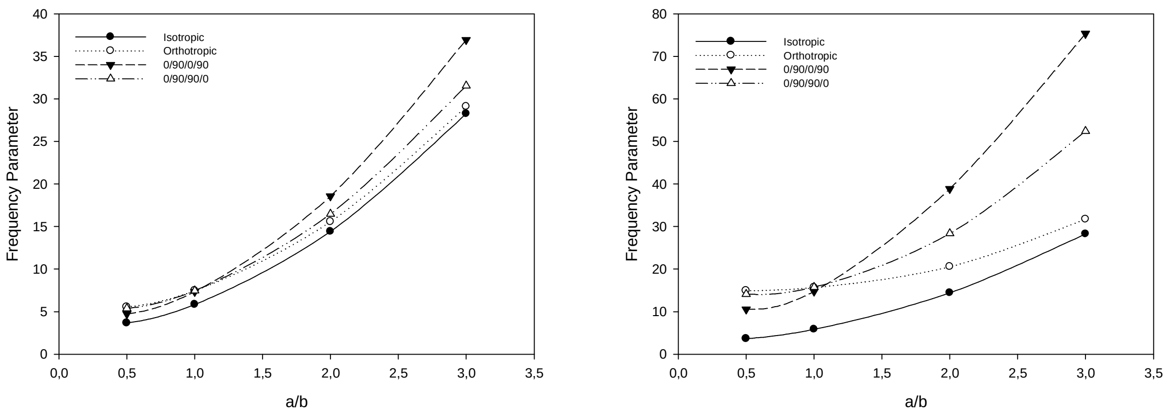
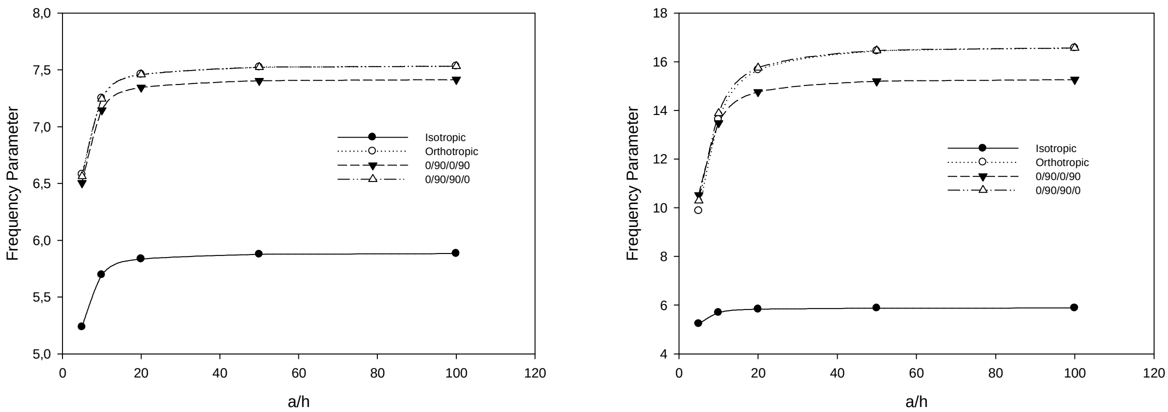


Figure 2: Variation of frequency parameter with orthotropy degree for different elastic plates (a/b=1 a/h=20).



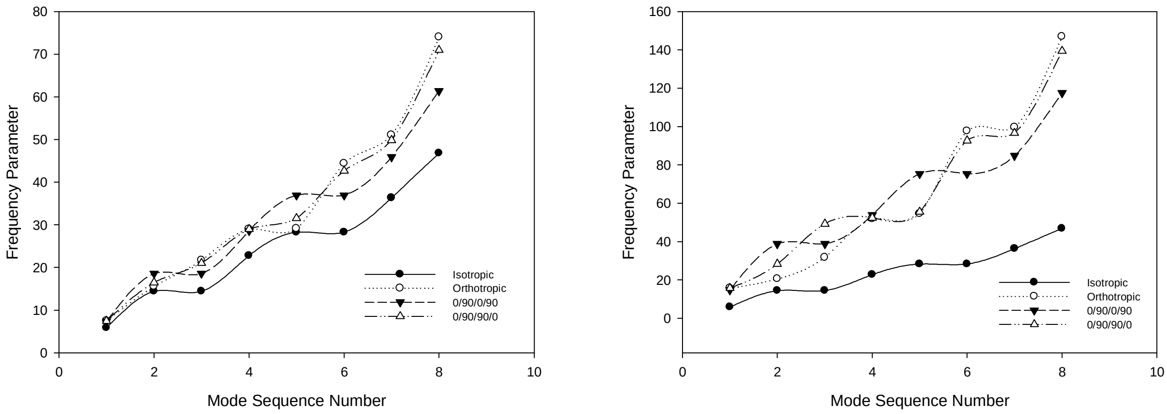
(a) E1/E2=3 (b) E1/E2=30

Figure 3: Variation of frequency parameter with a/b ratios and with ortotropy degrees for different elastic plates (a/h=20).



(a) E1/E2=3 (b) E1/E2=30

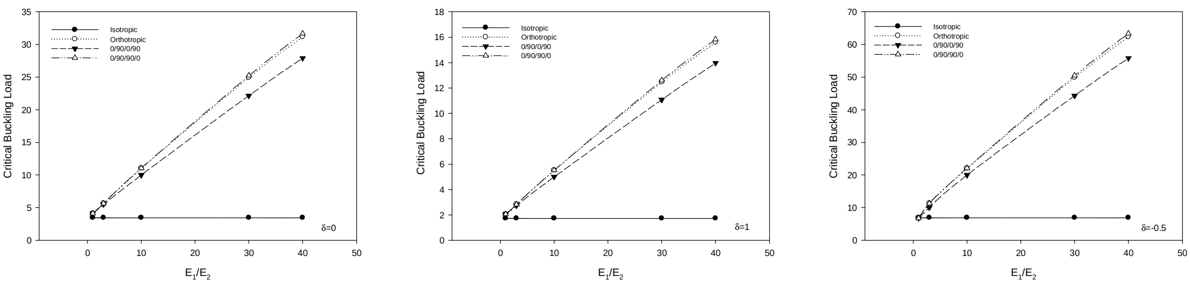
Figure 4: Variation of frequency parameter with a/h ratios and with ortotropy degrees for different elastic plates (a/b=1).



(a) E1/E2=3 (b) E1/E2=30

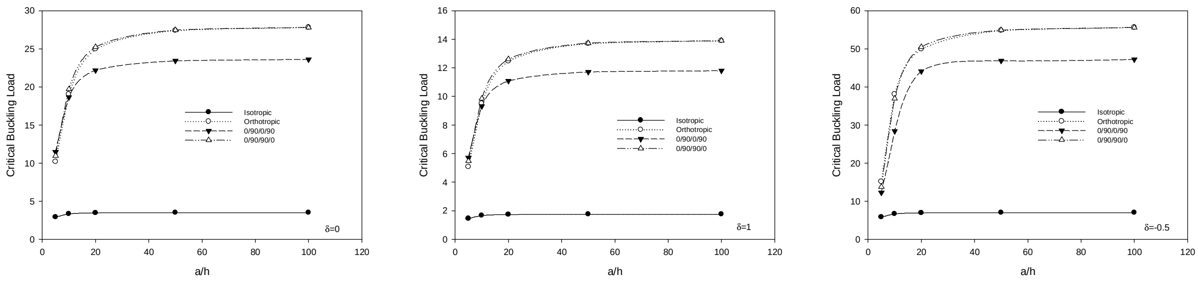
Figure 5: Variation of higher frequency parameter with mode sequence numbers and with ortotropy degrees for different elastic plates (a/b=1, a/h=20).

Comparisons of anti-symmetric and symmetric cross-ply laminated plates with isotropic and orthotropic plates in terms of buckling behaviour under various buckling load types are given in Fig.6-9. As expected all of considered plates have maximum value of critical buckling load when the plate is subjected to uniaxial compression along the x axis (δ=0) and have minimum value critical buckling load when the plate is subjected to bi-axial compression load (δ=1). According to the variation of material anisotropy critical buckling loads of considered plates are almost linearly increasing and critical buckling loads of symmetric cross-ply laminated and orthotropic plates are very closely each others and critical buckling loads of them are bigger than anti-symmetric laminated plates. It is observed that critical buckling loads increase with the plate becoming thinner. The variation of buckling load with a/h ratio is sharply for a/h<20 values and after the value of a/h>20 the variation of buckling load is slightly. According to the variation of the side-to-side ratio, it is seen that buckling loads increase with increasing of the plate length. However, critical buckling loads of orthotropic plates decrease with increasing of the plate length for bi-axial compression loading. In terms of the variation of the plate length anti-symmetric plates more stable than the others. In all profiles, isotropic plates have minimum critical buckling loads. In the higher buckling modes, it is seen that the variations of critical buckling loads of considered plates are slightly until sixth mode and next it is observed that critical buckling loads of anti-symmetric laminated plates increase sharply for uniaxial compression loading. For bi-axial compression and tensile loading in the y direction while the plate is under compression along the x direction, critical buckling loads of orthotropic plates higher than symmetric laminated plates, of anti-symmetric laminated plates plates and of isotropic plates, respectively.



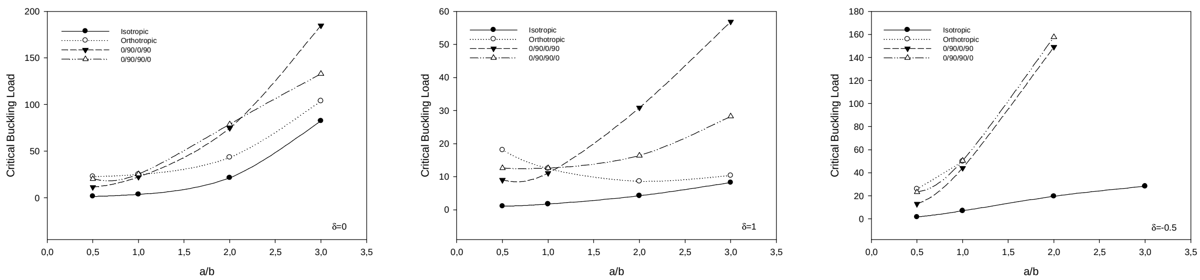
(a) (b) (c)

Figure 6: Variation of critical buckling load with orthotropy degrees for different elastic plates which subjected to considered loading conditions (a/b=1 a/h=20).



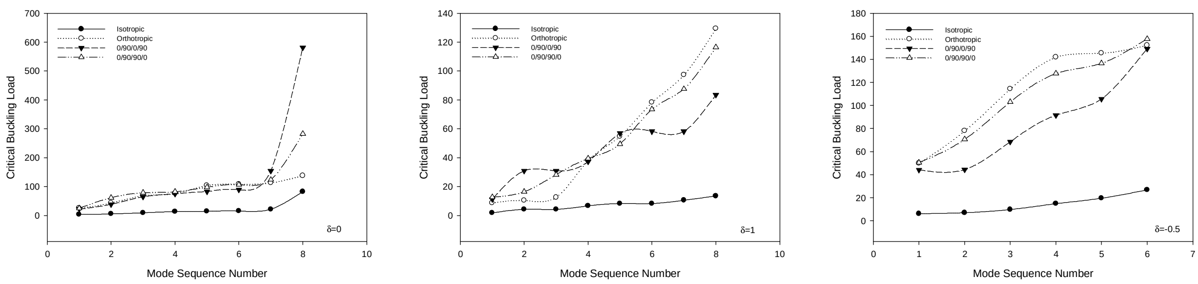
(a) (b) (c)

Figure 7: Variation of critical buckling load with a/h ratios for different elastic plates which subjected to considered loading conditions (E1/E2=30 a/b=1).



(a) (b) (c)

Figure 8: Variation of critical buckling load with a/b ratios for different elastic plates which subjected to considered loading conditions (E1/E2=30 a/h=20).



(a) (b) (c)

Figure 9: Variation of higher critical buckling load with mode sequence numbers for different elastic plates which subjected to considered loading conditions (E1/E2=30 a/b=1 a/h=20).

The vibration mode shapes of the first six frequency parameters of considered elastic plates are given in Fig. 10-16 for transverse displacement field component (w). The mode shapes are unknown coefficients corresponding to eigenvectors of eigenvalue problem which is given in Eq. 14. From this equation, firstly the eigenvalues which corresponding the natural frequencies are obtained. Hence, the eigenvectors which corresponding natural frequencies and so unknown coefficients are obtained. Thus, substituting the coefficients in the Eq. (13) the mode shapes can be drawn. It is observed that isotropic, orthotropic and symmetric laminated plates have the same mode arrangements when the value of material anisotropy (E_1/E_2) is 3. When the value of material anisotropy is 30, the mode arrangements of symmetric laminated and orthotropic plates are varying for higher modes. The mode arrangements of anti-symmetric laminated plates are different from the other plates and there is no variation with the mode arrangements of anti-symmetric laminated plates with the variation of the material anisotropy.

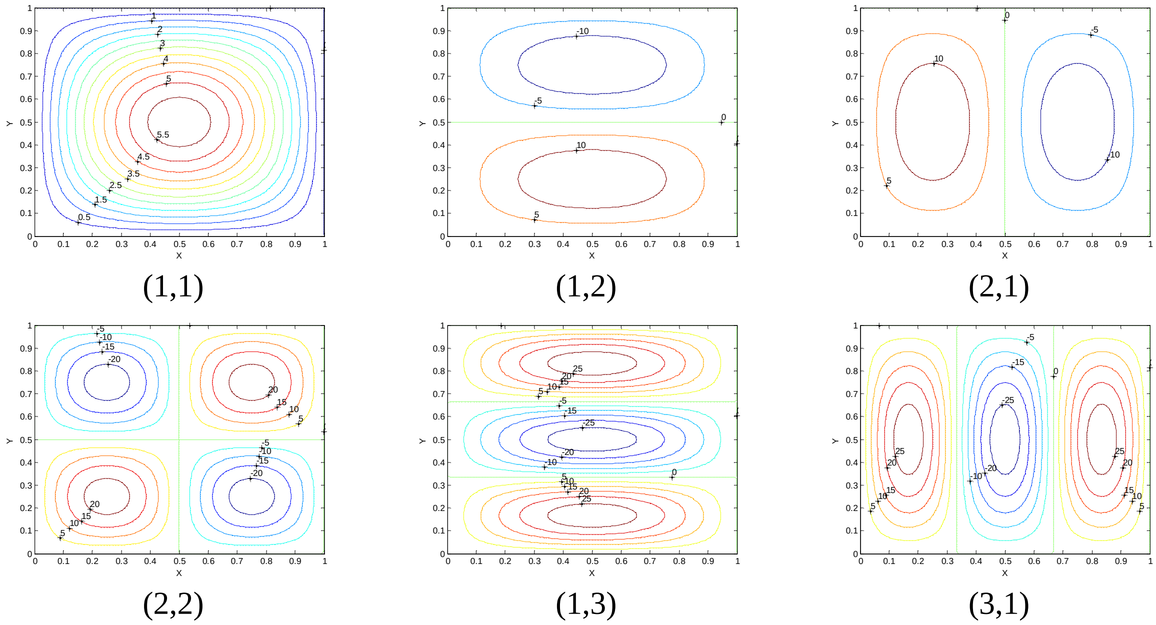


Figure 10: Mode shapes of free vibration modes of isotropic plates, (a/b=1, a/h=20)

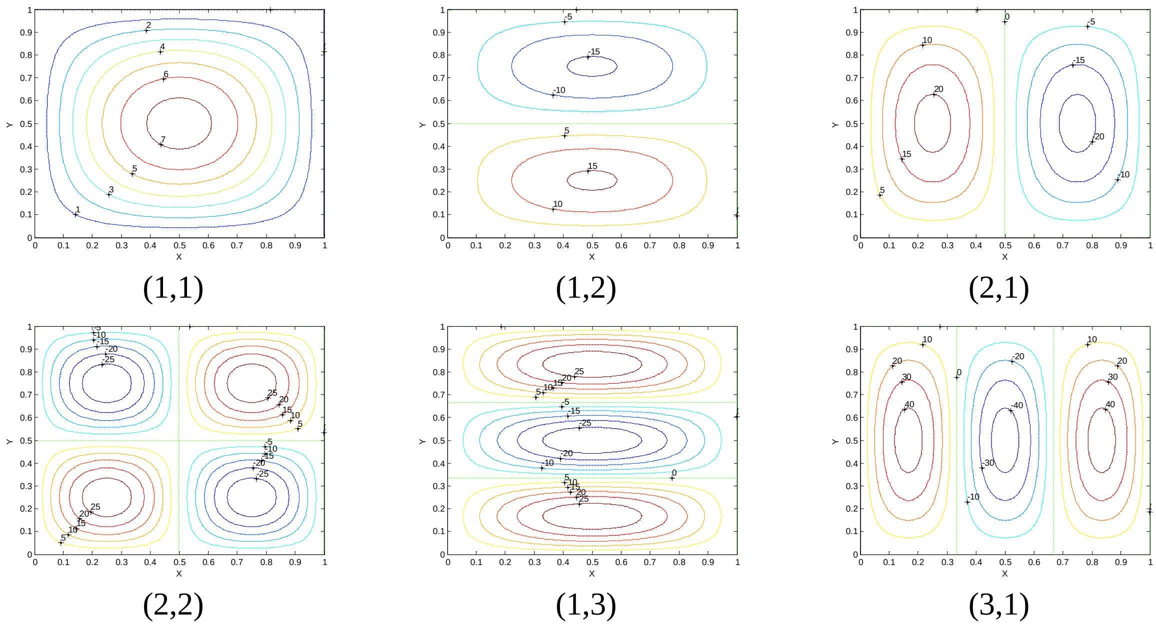


Figure 11: Mode shapes of free vibration modes of orthotropic plates, (a/b=1, a/h=20, E_1/E_2=3).

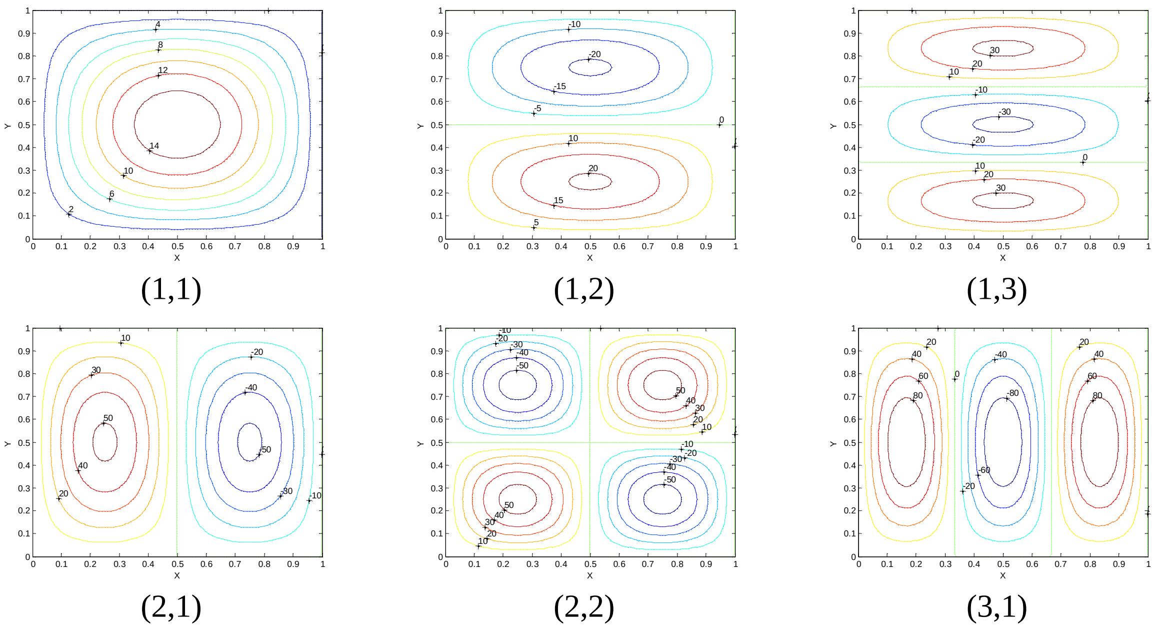


Figure 12: Mode shapes of free vibration modes of orthotropic plates, (a/b=1, a/h=20, E_1/E_2=30).

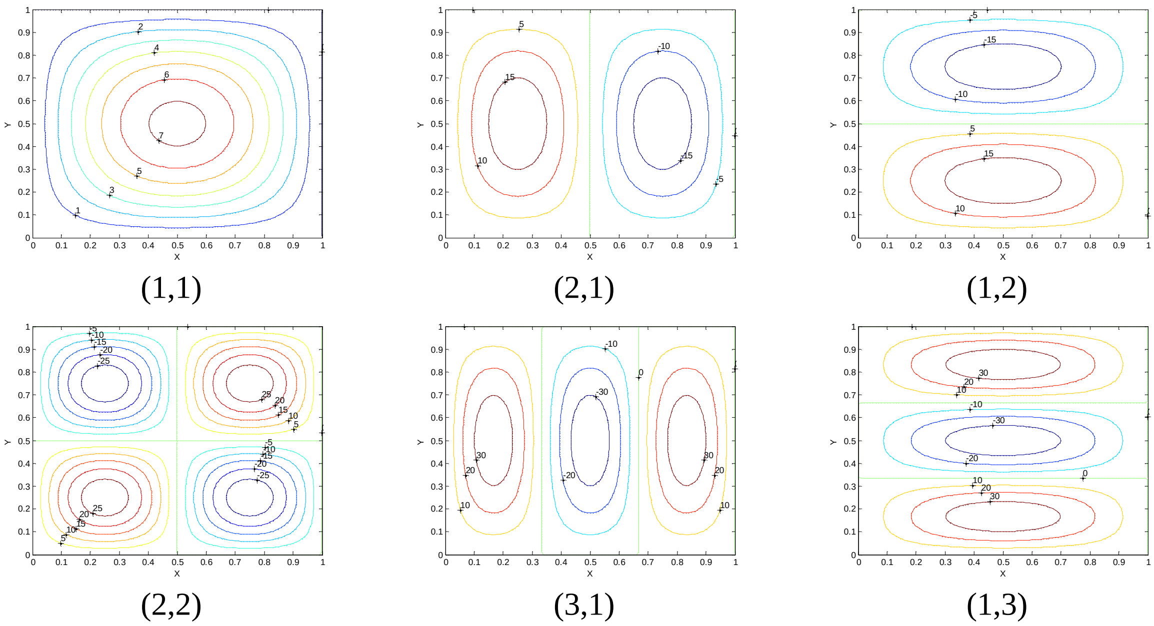


Figure 13: Mode shapes of free vibration modes of anti-symmetric cross-ply laminated plates, (a/b=1, a/h=20, E_1/E_2=3, 0/90/0/90).

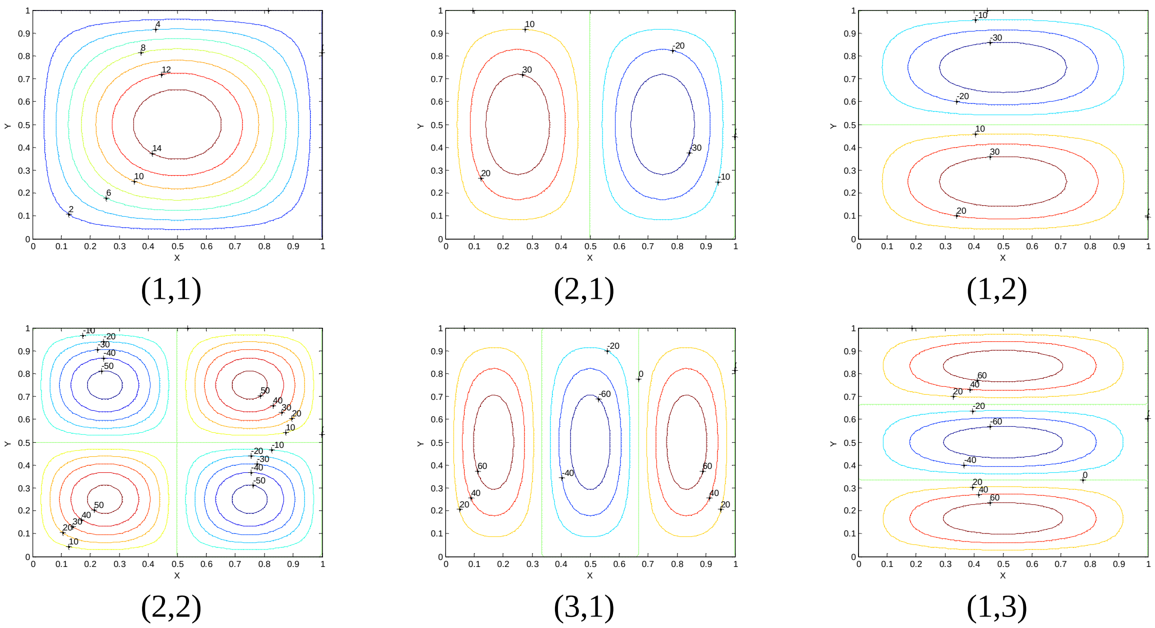


Figure 14: Mode shapes of free vibration modes of anti-symmetric cross-ply laminated plates, (a/b=1, a/h=20, E_1/E_2=30, 0/90/0/90).

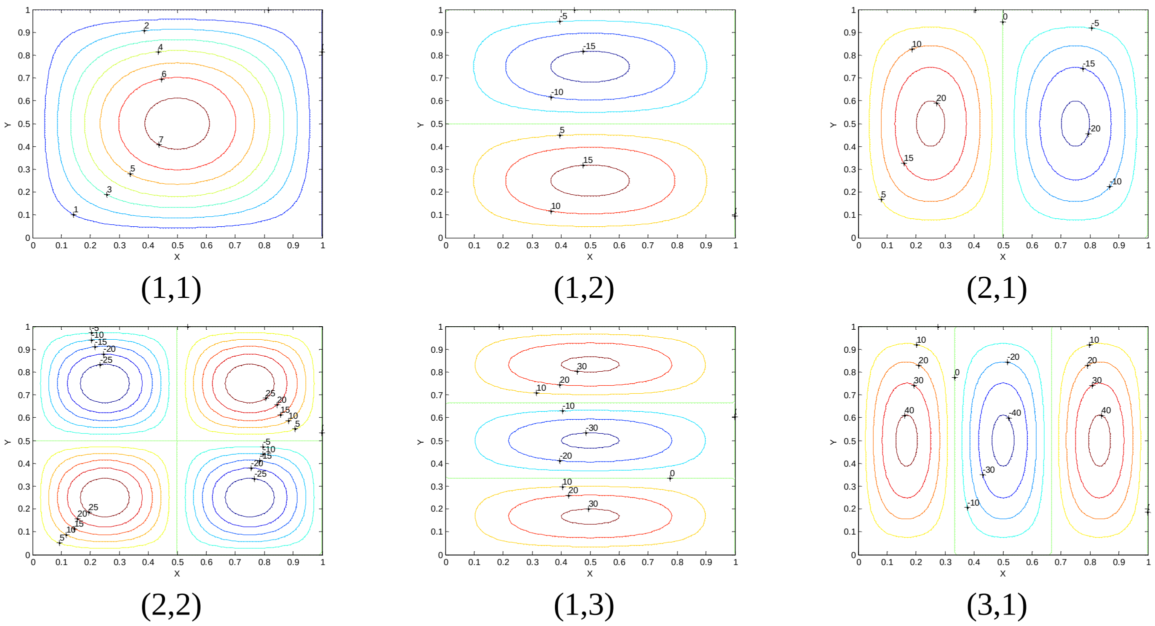


Figure 15: Mode shapes of free vibration modes of symmetric cross-ply laminated plates, (a/b=1, a/h=20, E_1/E_2=3, 0/90/90/0).

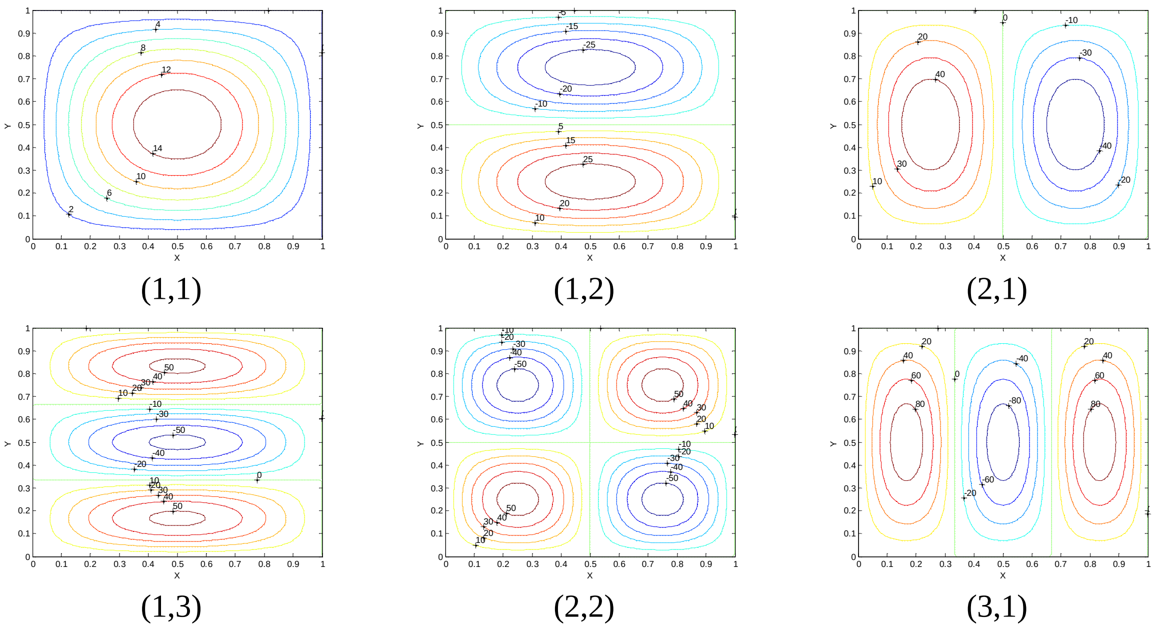
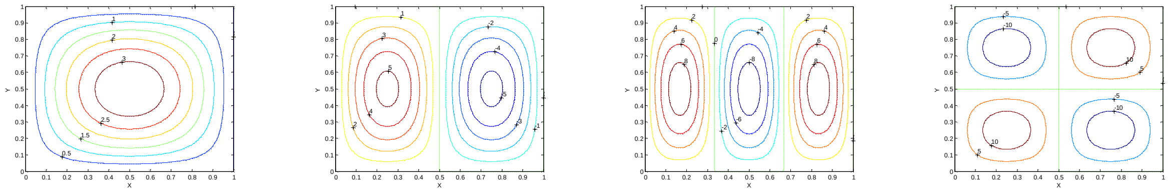
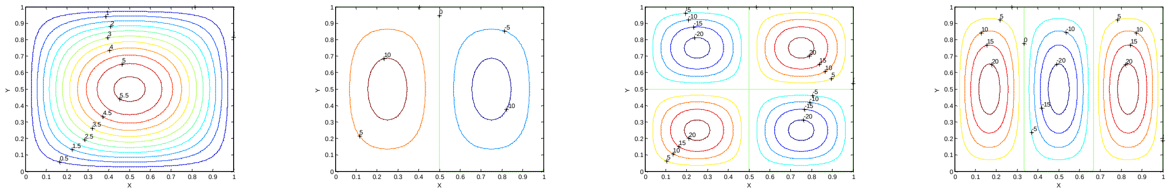


Figure 16. Mode shapes of free vibration modes of symmetric cross-ply laminated plates, (a/b=1, a/h=20, E_1/E_2=30, 0/90/90/0).

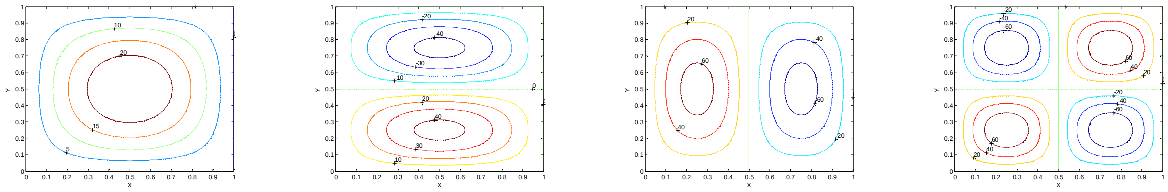
The buckling mode shapes of the first four critical buckling loads of considered elastic plates which obtained with Navier method are given in Fig. 17-19 for transverse displacement field component (w). It is observed that the mode arrangements of symmetric laminated and anti-symmetric laminated plates have the same variation at the same material anisotropy. Critical buckling loads increase with increasing material anisotropy, but also wave numbers of plates increase even if at lower modes.



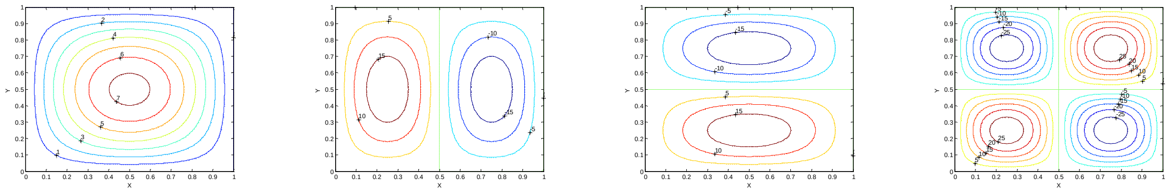
isotropic plate



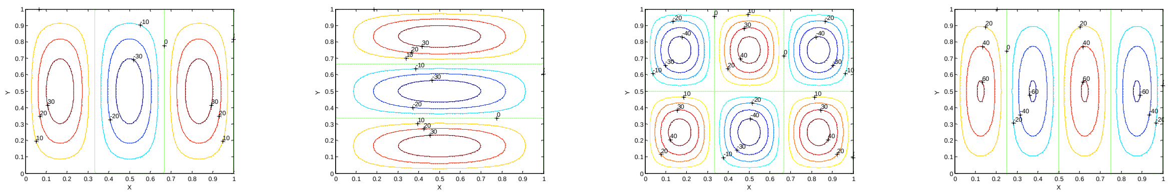
orthotropic plate (E_1/E_2=3)



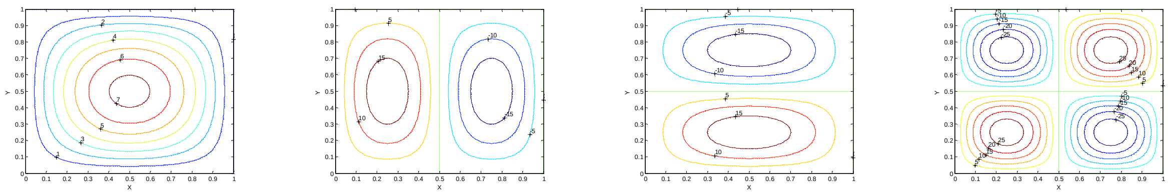
orthotropic plate (E_1/E_2=30)



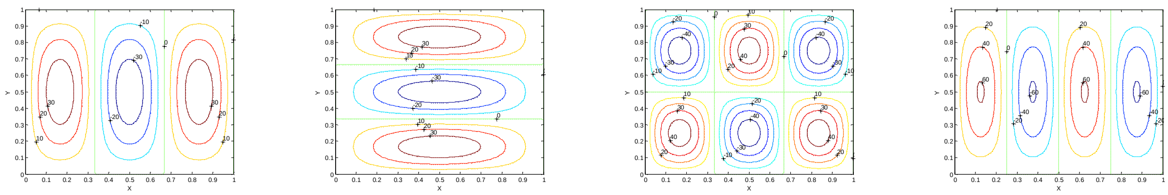
anti-symmetric cross-ply laminated plate (0/90/0/90, E_1/E_2=3)



anti-symmetric cross-ply laminated plate (0/90/0/90, E_1/E_2=30)



symmetric cross-ply laminated plate (0/90/90/0, E_1/E_2=3)



symmetric cross-ply laminated plate (0/90/90/0, E_1/E_2=30)

Figure 17: Mode shapes of axial buckling modes of different elastic plates which subjected to uniaxial compression along the x-axis, (a/b=1, a/h=20).

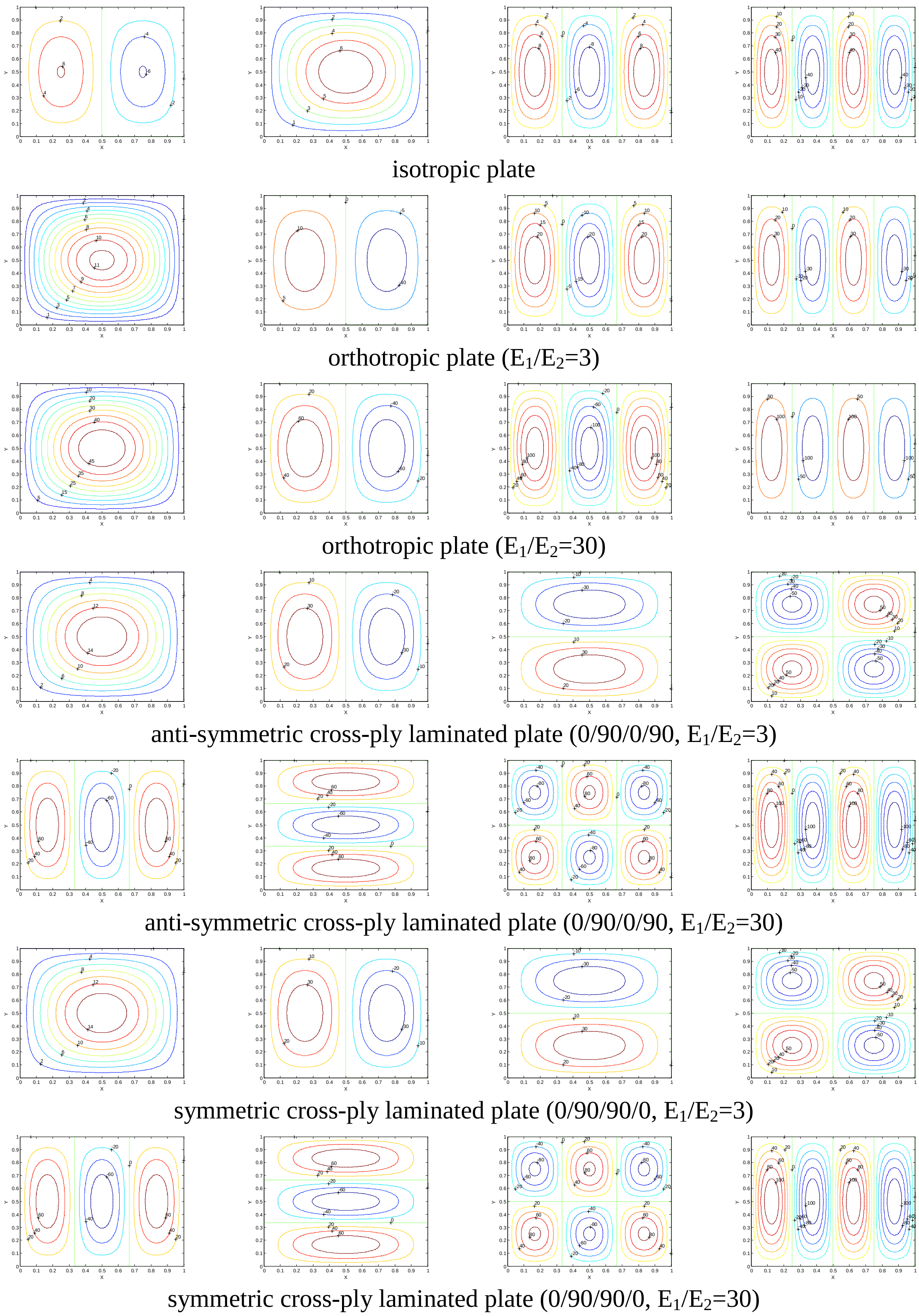
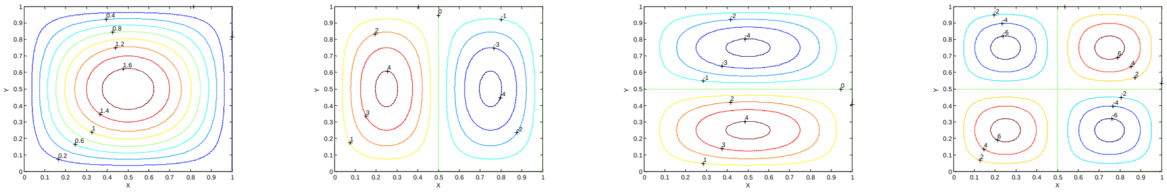
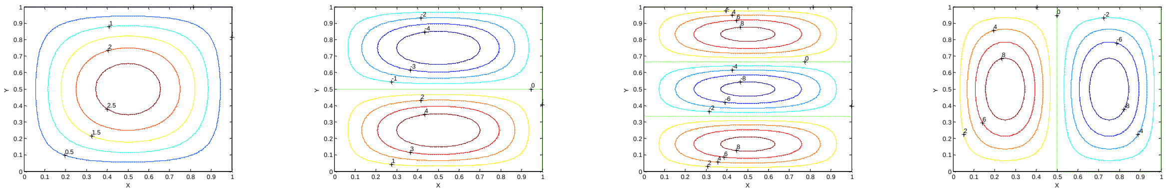


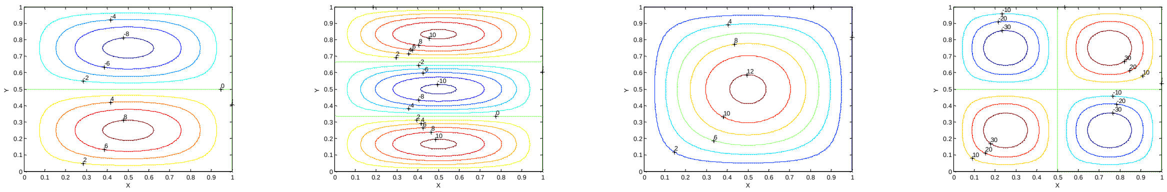
Figure 18: Mode shapes of axial buckling modes of different elastic plates which subjected to tensile loading in the y direction while the plate is under compression along the x direction, (a/b=1, a/h=20).



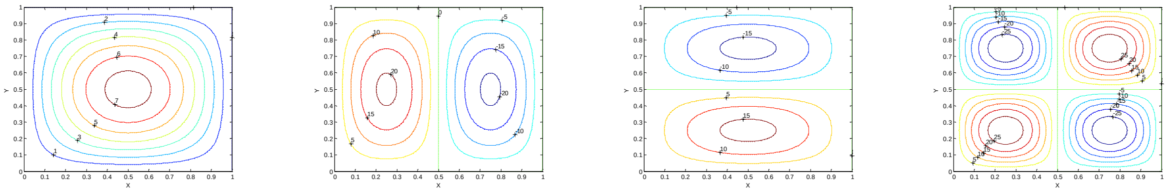
isotropic plate



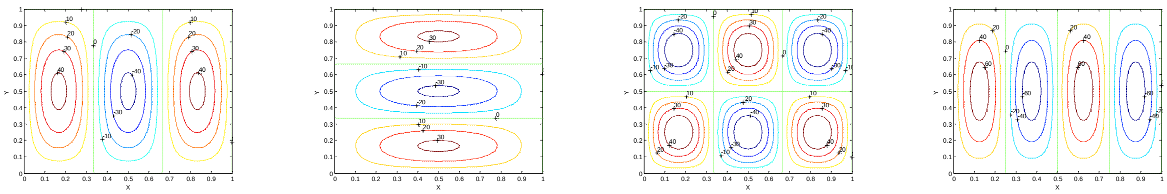
orthotropic plate (E_1/E_2=3)



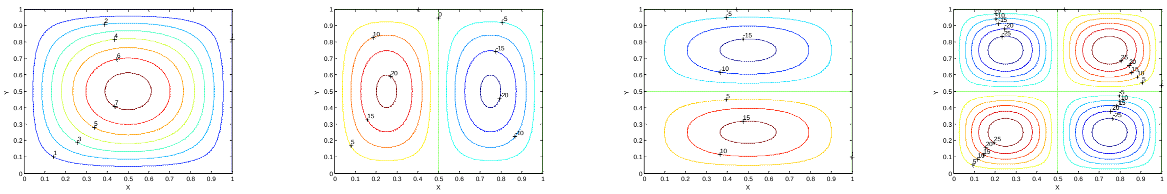
orthotropic plate (E_1/E_2=30)



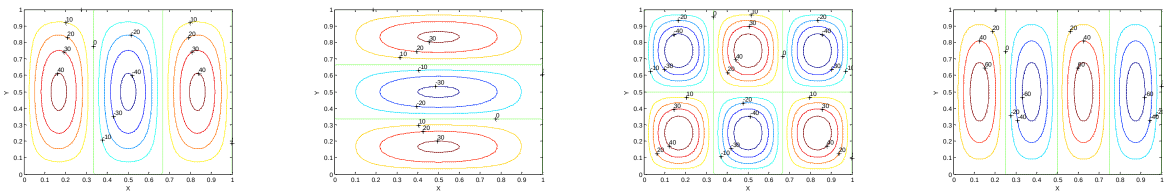
anti-symmetric cross-ply laminated plate (0/90/0/90, E_1/E_2=3)



anti-symmetric cross-ply laminated plate (0/90/0/90, E_1/E_2=30)



symmetric cross-ply laminated plate (0/90/90/0, E_1/E_2=3)



symmetric cross-ply laminated plate (0/90/90/0, E_1/E_2=30)

Figure 19: Mode shapes of axial buckling modes of different elastic plates which subjected to biaxial compression, (a/b=1, a/h=20).

4.Conclusions

The objective of the study is to do a relative analysis with isotropic, orthotropic and laminated plates in terms of free vibration and axial buckling behaviours. In the study an analytical solution is carried based on a higher order shear deformation theory for simply supported boundary condition.
According to the variation of material anisotropy and plate thickness, it is observed that frequency values and critical buckling loads of orthotropic and symmetric laminated plates are very closely each others and higher than the values of anti-symmetric laminated plates with increasing material anisotropy and decreasing plate thickness. According to the variation of side-to-side ratio, it is observed that when the values of frequency and critical buckling loads ranging from the higher value to the less value, they are belong to anti-symmetric laminated, symmetric laminated, orthotropic and isotropic plates, respectively. In all conditions, isotropic plates have minimum frequency value and critical buckling loads. It is seen that the frequency values and the critical buckling loads increasingly increase with increasing material anisotropy, decreasingly increase with increasing side-to-side ratio and sharply increase until a critical value (a/h=20) and after the critical value the variation is slightly with decreasing plate thickness.

It is observed that the mode arrangements vary considering isotropic plates with increasing material anisotropy. In the buckling problem, it is seen that the variation of validation is not only for mode arrangement as in the vibration problem but also for nodal point number.

5.References

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