International Journal of Engineering & Applied Sciences (IJEAS)
Vol.7, Issue 4(2015)34-44

A Simple Buckling Analysis of Aorta Artery

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Received date: December 2015
Accepted date:January 2016

Abstract

Aortas are the largest artery in the body and they carry the blood away which is pumped from the heart. Aorta artery is also the artery which is affected by the highest blood pressure. Its stability has a vital importance to humans and animals. The loss of stability in arteries may lead to arterial tortuosity and kinking. This situation causes to blackouts and serious permanent health problems. In this article, the buckling analysis of aorta artery is investigated by using Euler-Bernoulli beam theory for different boundary conditions. The aorta artery is modeled as a cylindrical tube with different average diameters. Results are presented in figures and table.

Keywords: Aorta artery, buckling, Euler-Bernoulli.

1. Introduction

Aorta artery is first mechanically modeled and its stability under blood pressure was studied to determine the critical buckling loads by Han in 2007 [1]. His researches showed that arteries may buckle and become tortuous due to reduced axial strain, hypertensive pressure, and a weakened wall. [2-9]. Han also has investigated the critical buckling by using nonlinear elastic thick-walled cylindrical model with residual stress which was developed by Han [3]. On the other hand, collapse of the vessel lumen and, the bent buckling of tubular arteries was recently reported [10-13]. The buckling of aorta artery causes additional wall stress and it leads to affect blood flow and cause extra loads to itself. In this study the simple classical Euler-Bernoulli buckling theory will be used in order to calculate the buckling loads of aorta artery for four different boundary conditions. In literature many researches about the buckling theory for small-scaled and micro-scaled structures [14-46] and rod, beam, plate, shell models are being used in order to determine the vibration of continuous systems. [47-52]

2. Buckling analysis of aorta artery

The demonstration of aorta artery and its continuum model are shown in Fig.(1) and Fig.(2). In order to calculate the critical buckling load of the model, Euler-Bernoulli beam theory is used. For modeling, L is the length of microtubules, R_i and R_o is inner and outer radius, D_{avg} average diameter, t thickness, E Young's modulus.

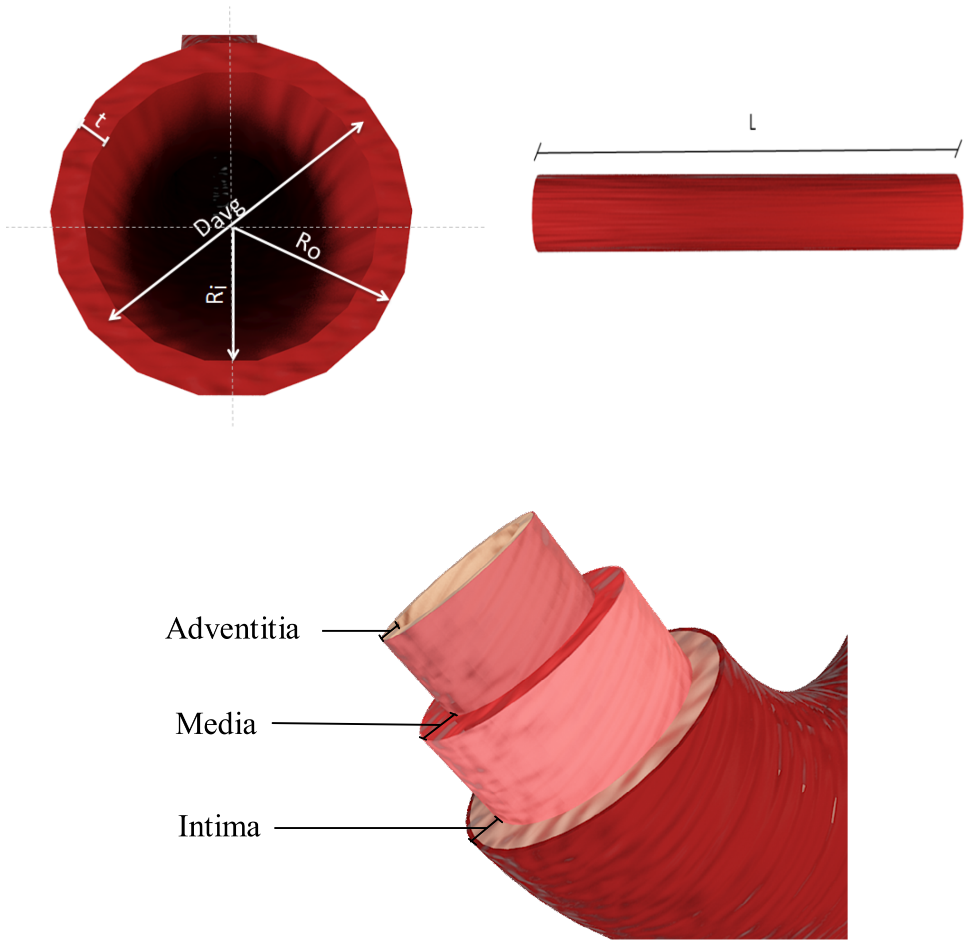


Fig. 1. Demonstration of aorta artery

Arteries are composed of three layers. These layers are intima, media and adventitia respectively from inside to outside of vessels (Fig.1). Intima is the intermost layer of the artery wich is covering the lumen side of vessels and it is composed of endothelial cells and lines the entire circulatory system, from the heart and the large arteries all the way down to the very tiny capillary beds. The intima layer also contain extracellular matrix and a supporting layer of collagenous tissue. Endothelial cells sorted in a single layer along the lumen side. Media is the muscular middle layer of the arteries and veins. It is composed of smooth muscle layers. Adventitia is outermost layer of vessels surrounding the media layer. It is mainly composed of collagen and, in arteries, is supported by external elastic lamina.

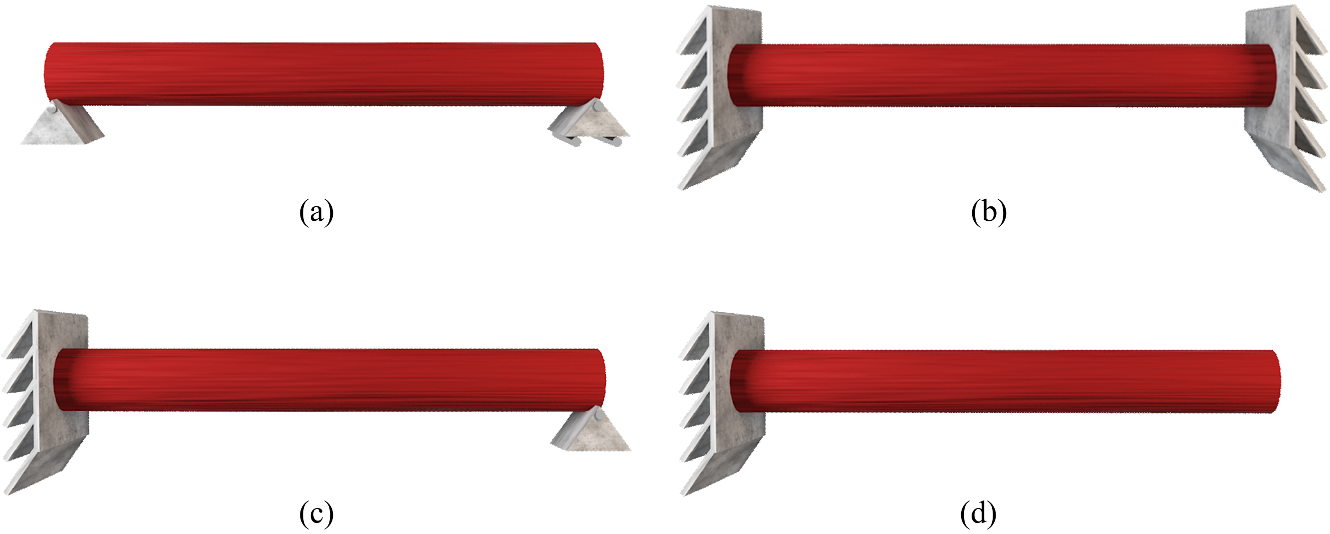


Fig. 2. Continuum model of aorta artery with variable boundary conditions

The aorta artery has a curved cylindrical shaped structure *in vivo*. In this study the structure is modeled as homogenous, straight cylindrical shaped tube. Critical buckling loads are investigated for simply supported (Fig. 2(a)), clamped (Fig. 2(b)), popped (Fig. 2(c)) and cantilever (Fig. 2(d)) boundary conditions.

3. Euler-Bernoulli formulation

The buckling equation of a beam is:

$$EI\frac{d^4y}{dx^4} + P\frac{d^2y}{dx^2} = 0 \tag{1}$$

If setting $\alpha^2 = \dfrac{P}{EI}$, Eq.(1) can be simplified as:

$$y^{iv} + \alpha^2 y'' = 0 \tag{2}$$

If setting $y = e^{rx}$, Eq.(2) can be simplified as:

$$Br^4e^{rx} + \alpha^2 Br^2 e^{rx} = 0 \tag{3}$$

By reducing Eq.(3), we can obtain:

$$r^4 + \alpha^2 r^2 = 0 \tag{4}$$

Solving Eq.(4), the result is:

$$r^2 = -\alpha^2$$
$$r_{1,2} = 0 \qquad \text{and} \qquad r_{3,4} = \pm i\alpha \tag{5}$$

$r_{1,2}$ and $r_{3,4}$ are two pairs of single complex root of Eq.(4).

By substitution roots into Eq.(5) and solving it, we obtain:

$$y = C_1 \sin \alpha x + C_2 \cos \alpha x + C_3 x + C_4 \tag{6}$$

C_1, C_2, C_3, C_4 are constants which can be obtained from boundary conditions. The first order derivative of Eq.(6) is:

$$y' = C_1 \alpha \cos \alpha x - C_2 \alpha \sin \alpha x + C_3 \tag{7}$$

The second order derivative of Eq.(6) is:

$$y'' = -C_1 \alpha^2 \sin \alpha x - C_2 \alpha^2 \cos \alpha x \tag{8}$$

The third order derivative of Eq.(6) is:

$$y''' = -C_1 \alpha^3 \cos \alpha x + C_2 \alpha^3 \sin \alpha x \tag{9}$$

For a beam which is Clamped-Free supported, the boundary conditions would be as followed:

$$y(0) = y'(0) = 0\,, \qquad y''(l) = y'''(l) + \alpha^2 y'(l) = 0 \tag{10}$$

By substituting boundary conditions into Eq.(6), Eq.(7), Eq.(8) and Eq.(9) we obtain:

$$y(0) = C_2 + C_4 = 0 \tag{11}$$
$$y'(0) = C_1 \alpha + C_3 = 0 \tag{12}$$
$$y''(l) = -C_1 \alpha^2 \sin \alpha l - C_2 \alpha^2 \cos \alpha l = 0 \tag{13}$$
$$y'''(l) + \alpha^2 y'(l) = C_3 \alpha^2 = 0 \tag{14}$$

As it is mentioned above C_1, C_2, C_3, C_4 are constants and we can obtain those constant by using Eq.(11), Eq.(12), Eq.(13) and Eq.(14). The solution is obtained as follow:

$$\alpha^5 \cos(\alpha l) = 0 \tag{15}$$

There are 2 possibilities which make the Eq.(15) equal to zero.

$$\alpha^5 = 0 \tag{16}$$

$$\cos(\alpha l) = 0 \tag{17}$$

By substituting $\alpha^2 = \dfrac{P}{EI}$ into Eq.(17) we can obtain:

$$\cos\left(\sqrt{\frac{P}{EI}}l\right) = 0 \text{, so } \sqrt{\frac{P}{EI}}l = n\frac{\pi}{2} \tag{18}$$

So the buckling load can be obtained via this formula:

$$P = \frac{n^2\pi^2 EI}{4l^2} \tag{19}$$

For a beam which is simply supported at both ends, the boundary conditions would be as followed:

$$y(0) = y''(0) = 0, \qquad y(l) = y''(l) = 0 \tag{20}$$

By substituting boundary conditions into Eq.(6) and Eq.(8) we obtain:

$$y(0) = C_2 + C_4 = 0 \tag{21}$$

$$y''(0) = -C_2\alpha^2 = 0 \tag{22}$$

$$y(l) = C_1 \sin\alpha l + C_2 \cos\alpha l + C_3 l + C_4 = 0 \tag{23}$$

$$y''(l) = -C_1\alpha^2 \sin(\alpha l) - C_2\alpha^2 \cos(\alpha l) = 0 \tag{24}$$

As it is mentioned above C_1, C_2, C_3, C_4 are constants and we can obtain those constant by using Eq.(21), Eq.(22), Eq.(23) and Eq.(24). The solution is obtained as follow:

$$-\alpha^4 \sin(\alpha l) = 0 \tag{25}$$

There are 2 possibilities which make the Eq.(15) equal to zero.

$$-\alpha^4 = 0 \tag{26}$$

$$\sin(\alpha l) = 0 \tag{27}$$

By substituting $\alpha^2 = \dfrac{P}{EI}$ into Eq.(27) we can obtain:

$$\sin\left(\sqrt{\frac{P}{EI}}l\right) = 0 \text{, so } \sqrt{\frac{P}{EI}}l = n\pi \tag{28}$$

So the buckling load can be obtained via this formula:

$$P = \frac{n^2 \pi^2 EI}{l^2} \tag{29}$$

For a beam which is Clamped-Simple supported at ends, the boundary conditions would be as followed:

$$y(0) = y'(0) = 0, \qquad y(l) = y''(l) = 0 \tag{30}$$

By substituting boundary conditions into Eq.(6) and Eq.(8) we obtain:

$$y(0) = C_2 + C_4 = 0 \tag{31}$$

$$y'(0) = C_1\alpha + C_3 = 0 \tag{32}$$

$$y(l) = C_1 \sin \alpha l + C_2 \cos \alpha l + C_3 l + C_4 = 0 \tag{33}$$

$$y''(l) = -C_1\alpha^2 \sin(\alpha l) - C_2\alpha^2 \cos(\alpha l) = 0 \tag{34}$$

As it is mentioned above C_1, C_2, C_3, C_4 are constants and we can obtain those constant by using Eq.(31), Eq.(32), Eq.(33) and Eq.(34). The solution is obtained as follow:

$$\alpha[\sin(\alpha l) - \alpha l \cos(\alpha l)] = 0 \tag{35}$$

By arranging the transcendent Eq.(35) we obtain:

$$\tan(\alpha l) = \alpha l \tag{36}$$

The result of the Eq.(36) is $\alpha l = 4.49$, by substituting $\alpha^2 = \frac{P}{EI}$ into Eq.(36) we can obtain:

$$P = \alpha^2 EI \tag{37}$$

So the buckling load can be obtained via this formula:

$$P = 2.05\frac{n^2 \pi^2 EI}{l^2} \tag{38}$$

For a beam which is fixed at both ends, the boundary conditions would be as followed:

$$y(0) = y'(0) = 0, \qquad y(l) = y'(l) = 0 \tag{39}$$

By substituting boundary conditions into Eq.(6) and Eq.(8) we obtain:

$$y(0) = C_2 + C_4 = 0 \tag{40}$$

$$y'(0) = C_1\alpha + C_3 = 0 \tag{41}$$

$$y(l) = C_1 \sin \alpha l + C_2 \cos \alpha l + C_3 l + C_4 = 0 \tag{42}$$

$$y'(l) = C_1 \alpha \cos \alpha l - C_2 \alpha \sin \alpha l + C_3 \tag{43}$$

As it is mentioned above C_1, C_2, C_3, C_4 are constants and we can obtain those constant by using Eq.(31), Eq.(32), Eq.(33) and Eq.(34). The solution is obtained as follow:

$$\sin(\frac{1}{2}\alpha l) = 0 \tag{44}$$

By arranging the transcendent Eq.(44) we obtain:

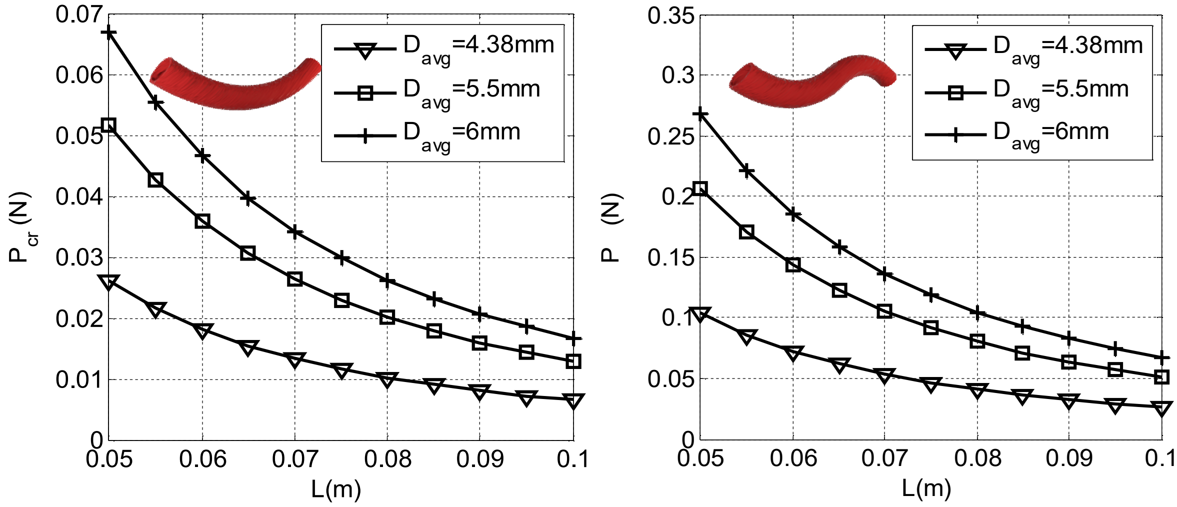
$$\frac{1}{2}\alpha l = \pi \tag{45}$$

So the buckling load can be obtained via this formula:

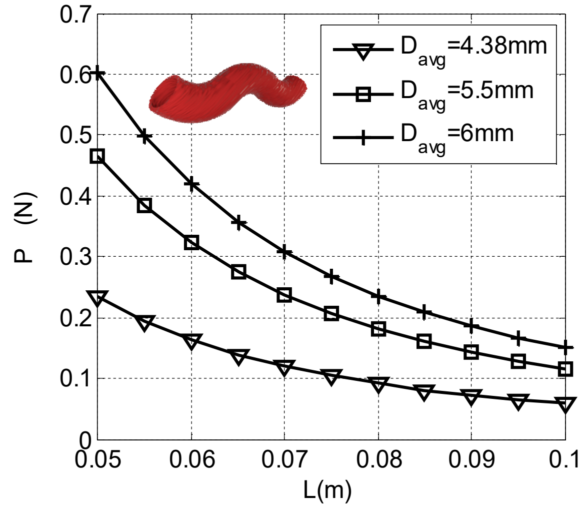
$$P = 4\frac{n^2 \pi^2 EI}{l^2} \tag{46}$$

4. Numerical Examples

In this study, the buckling of aorta artery with various boundary conditions is investigated via classical Euler-Bernoulli beam theory. Some of the results which are showing the buckling loads for simply supported aorta arteries are in Figure (3). These results show the decreasing of buckling load as the length of the artery increases. Three different average diameter is taken into account (D_{avg}=4.38mm, D_{avg}=5.5mm and D_{avg}=6mm).The elasticity modulus is E=200kPa [1], the thickness is t=1mm, the moment of inertia is $I = \pi t R_{avg}^3$.($R_{avg} = D_{avg}/2$)



(a) (b)



(c)

Fig.3. Variation of buckling load of Aorta Artery for different average diameters for first 3 modes respectively.

Boundary conditions	15	25	*L/r* 45	65	85
C-F	0.0151	0.0054	0.0017	0.0008	0.0005
S-S	0.0604	0.0217	0.0067	0.0032	0.0019
C-S	0.1237	0.0445	0.0137	0.0066	0.0039
C-C	0.2414	0.0869	0.0268	0.0129	0.075

Table 1. The buckling load of aorta artery (N) for different boundary conditions and L/r ratio

The influences of the length on the buckling load for simply supported aorta artery for first three modes are illustrated in Figs. 3(a, b, c), respectively. In Fig. 3(a) the buckling loads are critical buckling load since it is for the first mode. As it can be seen in Figs. 2 (a,b,c,d), the buckling load is investigated for simply supported, clamped, propped and cantilever boundary conditions respectively. Fig.3(a, b, c) shows that the buckling load is decreasing dramatically with the increasing of length for all three modes. In Table 1. It can be seen that the highest buckling load is for Clamped boundary condition and lowest L/r ratio. As the L/r ratio increase, the artery is buckling under lower loads.

5. Concluding remarks

Buckling analysis of aorta artery is investigated for variable boundary conditions. Present equations from literature [14] are used in order to calculate the critical buckling loads. Results are presented in figures and table. The maximum buckling load is found for clamped case and lowest L/r ratio. On the other hand the minimum buckling load is for Clamped-Free case and highest L/r ratio.

Acknowledgements

The financial support of the Scientific Research Projects Unit of Akdeniz University is gratefully acknowledged.

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