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Nonlocal Finite Element Formulation for Vibration

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Abstract

Vibration formulation is presented for axially compressed nano beam embedded in elastic matrix. The effect of length scale is investigated using nonlocal elasticity theory. The governing equations are obtained by using the Hamilton's principle. Finite element formulations have been achieved for nonlocal Euler–Bernoulli beam theory. Global stiffness and mass matrix are obtained.

Keywords: Nonlocal elasticity, Euler Bernoulli beam theory, Finite Element Formulation.

1. Introduction

Nanoscience and nanotechnology has enabled the opening of a new era in many areas (chemical, medicine, engineering, electronics etc.). For keep up with technology fast, the correct solution method which considers the size effect is the most important factor. Experimental research is very difficult and expensive. Some methods such as Hybrid atomistic–continuum mechanics and related to the atomic modeling; molecular dynamics [1-3], tight-binding molecular dynamics, the density functional theory take into account the size effect. Therefore, various theories have been developed that gives importance to effects of small scale such as strain gradient theory [4,5], modified couple stress theory [6-9], couple stress elasticity theory [10-13], nonlocal elasticity theory[14-15]. Nonlocal elasticity theory of Eringen is the most widely used among them. According to the nonlocal elasticity theory of Eringen [14-15], the stress at any reference point is effecting the whole body which not depends only on the strains at this point but also on strains at all points of the body. This definition of the Eringen's nonlocal elasticity is based on the atomic theory of lattice dynamics, and some experimental observations on phonon dispersion. Nonlocal theory considers long-range interatomic interaction and yields to results dependent on the size of a body [14-16]. Applying first the nonlocal elasticity theories to nanotechnology is by Peddieson et al. [17] and Sudak [18]. Nanostructures with nonlocal elasticity theory have been studied for different type (numerical and analytical) solution with contributions continuum mechanics by finite element method [19-25], by finite difference method [26-27] by differential transform method [28-30], by differential quadrature method [31-34], and by analytical solution [35-53]. As shown in Fig. 1 the beam is lying on Winkler and Pasternak foundation.

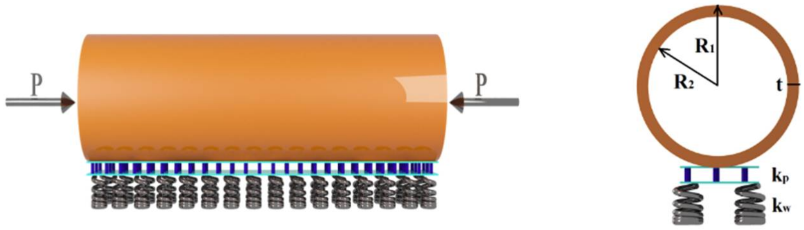

Fig. 1. Continuum model of beam

2. Nonlocal Euler-Bernoulli Beam Model for Vibration

The stress at any reference point in the beam depends on the strains at this point and also on strains at all points of the body according to Eringen's nonlocal elasticity theory [14,15]. The main equations for a homogenous and isotropic elastic continuum body can be stated as:

$$\sigma_{ij,j} = 0, \tag{1}$$

$$\sigma_{i,j}(x) = \int_V \alpha(|x - x'|, \chi) C_{ijkl} \varepsilon(x') dV(x'), \tag{2}$$

$$\varepsilon_{ij}(x') = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), \tag{3}$$

where σ_{ij} is the nonlocal stress tensor, ρ is the mass density of the body, u is the displacement vector at a reference point x in the body, $C_{ijkl}(x')$ is the classical (Cauchy) or local stress tensor at any point x' in the body, $\varepsilon_{ij}(x')$ is the linear strain tensor at point x' in the body, t is denoted as time, V is the volume occupied by the elastic body, $\alpha|x - x'|$ is the distance in Euclidean form, λ and μ are the Lame constants. $\alpha|x - x'|$ is the nonlocal kernel which defines the impact of strain at point x' on the stress at point x in body. The nonlocal constitutive formulation is

$$[1 - (e_0 a)^2 \nabla^2]\sigma_{ij} = C_{ijkl} \tag{4}$$

The displacement components based on the Euler-Bernoulli beam theory may be written as [36-37]:

$$u = -z\frac{\partial w}{\partial x}, \quad v = 0 \quad w = w(x,t) \tag{5}$$

where 'w' is the transverse displacement. The strain-displacement equations for Euler-Bernoulli beam is given by

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} = -z\frac{\partial^2 w}{\partial x^2}(x,t) \quad \varepsilon_{yy} = \varepsilon_{zz} = \varepsilon_{xy} = \varepsilon_{xz} = \varepsilon_{yz} = 0 \tag{6}$$

Consider the stress-strain relation for Euler-Bernoulli beam is given by

$$\sigma_{xx} = -Ez\frac{\partial^2 w}{\partial x^2}(x,t) \quad \sigma_{yy} = \sigma_{zz} = \tau_{xy} = \tau_{xz} = \tau_{yz} = 0 \tag{7}$$

According to Eq. (4), the nonlocal stress-strain equations for beam can be written as [36-37]

$$\sigma_{xx} - (e_0 a)^2 \frac{\partial^2 \sigma_{xx}}{\partial x^2} = E\varepsilon_{xx}\,,\ \sigma_{yy} = 0\,,\ \sigma_{zz} = 0 \tag{8a}$$

$$\tau_{xy} - (e_0 a)^2 \frac{\partial^2 \tau_{xy}}{\partial x^2} = 0\,,\ \tau_{yx} = 0\ \tau_{xz} = \tau_{zx} = 0\,,\ \tau_{yz} = \tau_{zy} = 0 \tag{8b}$$

The generalized Hamilton's principle has the form

$$\delta\int_0^t \left[T - (U - W)\right]dt = 0 \tag{9}$$

The strain and kinetic energies of the classic Euler-Bernoulli beam are equal to

$$U = \frac{1}{2}\int_V \sigma_{xx}\varepsilon_{xx}\,dV \tag{10}$$

$$T = \frac{1}{2}\rho\int_V \left[\left(\frac{\partial w}{\partial t}\right)^2\right]dV \tag{11}$$

The work done by the axial compressive force, Winkler foundation modulus (k_w) and Pasternak foundation modulus (k_g) can be expressed as

$$W = \frac{1}{2}\int_0^L \left[\left(P - k_g\right)\left(\frac{\partial w}{\partial x}\right)^2 - k_w (w)^2\right]dx \tag{12}$$

Substitution of Eqs.(10)-(12) into Eq.(9), acquired

$$\int_0^t \left[\int_0^L \rho A\frac{\partial w}{\partial t}\delta\frac{\partial w}{\partial t}dx - \left(-M\delta\left(\frac{\partial^2 w}{\partial x^2}\right)dx\right) + \left(\left(P - k_g\right)\frac{\partial w}{\partial x}\delta\frac{\partial w}{\partial x} - k_w w\delta w\right)dx\right]dt = 0 \tag{13}$$

When Eq.(13) under the double integral equal to zero under the double integral, differential equations of motion,

$$\frac{\partial^2 M}{\partial x^2} = \rho A\frac{\partial^2 w}{\partial t^2} + \left(P - k_g\right)\frac{\partial^2 w}{\partial x^2} + k_w w \tag{14}$$

The nonlocal moment resultants for beam can be obtained via (8a) as

$$M_x - (e_0 a)^2 \frac{\partial^2 M_x}{\partial x^2} = -EI \frac{\partial^2 w}{\partial x^2} \tag{15}$$

Substitution of Eq.(14) in to Eq. (15) leads to

$$M = (e_0 a)^2 \left(\rho A \frac{\partial^2 w}{\partial t^2} + (P - k_g) \frac{\partial^2 w}{\partial x^2} + k_w w \right) - EI \frac{d^2 w}{dx^2} \tag{16}$$

Finally, by substituting Eq.(16) into Eq.(13), we obtained governing equations for nonlocal Euler-Bernoulli beam [21,22,28,34]

$$\rho A \frac{\partial^2 w}{\partial t^2} - \frac{\partial^2}{\partial x^2} \left[(e_0 a)^2 \left(\rho A \frac{\partial^2 w}{\partial t^2} + (P - k_g) \frac{\partial^2 w}{\partial x^2} + k_w w \right) - EI \frac{\partial^2 w}{\partial x^2} \right] - (k_g - P) \frac{\partial^2 w}{\partial x^2} + k_w w = 0 \tag{17}$$

The Euler-Bernoulli beam element is a beam with four degrees of freedom (DOF) and has two end nodes: 1 and 2. The node displacement vector

$$w^e = \begin{bmatrix} w_1 & \theta_1 & w_2 & \theta_2 \end{bmatrix} \tag{18}$$

By multiplying shape function (ϕ) and discretized displacements at nodes ($[w(t)]^{ne}$) of an element we obtain the displacement of element ($w(x,t)^e$)

$$w(x,t)^e = [\phi][w(t)]^{ne} \qquad \ddot{w}(x,t)^e = [\phi][\ddot{w}(t)]^{ne} \tag{19}$$

To solve the equations the 'Hermitian cubic shape functions' are used. Dimensionless natural coordinate can be stated as below

$$\xi = \frac{2x}{L} - 1 \tag{20}$$

where L is the element length. Coordinate ξ has a range of $\xi = -1$ at node 1 ($x = 0$) to $\xi = +1$ at node 2 ($x = L$) and $d\xi/dx = 2/L$. The shape functions in terms of ξ are

$$\phi = \begin{Bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{Bmatrix} = \begin{Bmatrix} \frac{1}{4}(1-\xi)^2(2+\xi) \\ \frac{1}{8}L(1-\xi)^2(1+\xi) \\ \frac{1}{4}(1+\xi)^2(2-\xi) \\ -\frac{1}{8}L(1+\xi)^2(1-\xi) \end{Bmatrix} \tag{21}$$

By using the shape functions (Eq.(21)) and dimensionless natural coordinates (Eq.(20)) , the stiffness matrix becomes

$$K^1 = \int_{-1}^{1} \left[EI \frac{16}{L^4} \frac{\partial^2 \phi^T}{\partial \xi^2} \frac{\partial^2 \phi}{\partial \xi^2} \frac{L}{2} \partial \xi \right] = \int_{-1}^{1} \frac{8EI}{L^3} \begin{Bmatrix} \phi_1'' \\ \phi_2'' \\ \phi_3'' \\ \phi_4'' \end{Bmatrix} \begin{bmatrix} \phi_1'' & \phi_2'' & \phi_3'' & \phi_4'' \end{bmatrix} d\xi = \int_{-1}^{1} \frac{8EI}{L^3} \begin{bmatrix} \phi_1''\phi_1'' & \phi_1''\phi_2'' & \phi_1''\phi_3'' & \phi_1''\phi'' \\ \phi_2''\phi_1'' & \phi_2''\phi_2'' & \phi_2''\phi_3'' & \phi_2''\phi'' \\ \phi_3''\phi_1'' & \phi_3''\phi_2'' & \phi_3''\phi_3'' & \phi_3''\phi'' \\ \phi_4''\phi_1'' & \phi_4''\phi_2'' & \phi_4''\phi_3'' & \phi_4''\phi'' \end{bmatrix} d\xi$$

$$K^1 = \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \tag{22}$$

Similarly

$$K^2 = (e_0 a)^2 \frac{(k_g - P)}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \tag{23}$$

$$K^3 = \frac{(k_g - P)}{30L} \begin{bmatrix} 36 & 3L & -36 & 3L \\ 3L & 4L^2 & -3L & -L^2 \\ -36 & -3L & 36 & -3L \\ 3L & -L^2 & -3L & 4L^2 \end{bmatrix} \tag{24}$$

$$K^4 = \frac{k_w}{420} \begin{bmatrix} 156L & 22L^2 & 54L & -13L^2 \\ 22L^2 & 4L^3 & 13L^2 & -3L^3 \\ 54L & 13L^2 & 156L & -22L^2 \\ -13L^2 & -3L^3 & -22L^2 & 4L^3 \end{bmatrix} \tag{25}$$

$$K^5 = \frac{(e_0 a)^2 k_w}{30L} \begin{bmatrix} 36 & 3L & -36 & 3L \\ 3L & 4L^2 & -3L & -L^2 \\ -36 & -3L & 36 & -3L \\ 3L & -L^2 & -3L & 4L^2 \end{bmatrix} \tag{26}$$

Also, the mass matrix can be given as

$$M^1 = \frac{\rho A}{420}\begin{bmatrix} 156L & 22L^2 & 54L & -13L^2 \\ 22L^2 & 4L^3 & 13L^2 & -3L^3 \\ 54L & 13L^2 & 156L & -22L^2 \\ -13L^2 & -3L^3 & -22L^2 & 4L^3 \end{bmatrix} \quad (27)$$

$$M^2 = \frac{(e_0 a)^2 \rho A}{30L}\begin{bmatrix} 36 & 3L & -36 & 3L \\ 3L & 4L^2 & -3L & -L^2 \\ -36 & -3L & 36 & -3L \\ 3L & -L^2 & -3L & 4L^2 \end{bmatrix} \quad (28)$$

$$K = K^1 + K^2 + K^3 + K^4 + K^5, \quad M = M^1 + M^2 \quad (29)$$

Finally, the vibration of Euler-Bernoulli beam can be expressed as

$$\det\left|K - \omega^2 M\right| = 0 \quad (37)$$

The vibration is obtained by using above eigenvalue equation.

3. Concluding remarks

The finite element formulation for nonlocal beam is briefly derived via stiffness and mass matrix. The detailed background of these derivations will be presented and some results provided in next manuscript of present authors..

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