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LEARNING MATHEMATICAL STRUCTURES[1]

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Abstract

Students built, imagined and analyzed mathematical structures in courses like origami, math and art. A rhombicosidodecahedron has a 3-4-5-4 patterned structure, an origami crease pattern is a set of geometrical relationships between side, length with unequal expressions of x and ys. In mathematics courses; graphing functions via geogebra. working with complex structures as of (IzI/(Imz)+Iz-5I), solving a 4th order polynomial by approximation are all structural. Some mathematical structures and the way students learn and think with them is the focus. It is mainly a review study from many practical and experimental researches over many years. Seeing all different structures as a structural unity we may conclude somethings about the students' actions on learning with those structures. When students develop an Islamic pattern via ruler and compass, they carry an algorithm to analyze structure. Insight develops over time by meaningful experience which is a timely developing systematic awareness of structural properties of different bases. Number 8 by Kandinsky, can be analyzed by students to see the geometrical relationships and the Euclidean structure it carries. 116. Sonnet by Sheakespeare may include Proof by Contradiction. Each structure is unique but they carry commonalities like perspective, focus, factors(variables), unity glue, context. All these phases are analyzed for learning mathematical structures.

Keywords: Mathematical structures; modelling; mathematical thinking

MATEMATİKSEL YAPILARI ÖĞRENME

Özet

Origami, matematik ve sanat gibi derslerde matematiksel yapıları analiz edilmiştir hayal edilmektedir ve inşa edilmektedir. Bir Rhombicosidodecahedron'un 3-4-5-4 örüntü yapısı vardır, bir origami kat izi geometrik ilişkilerden örülüdür. Matematik derslerinde; geogebra ile fonksiyonların grafiğini çizilmiştir, zorlu karmaşık yapılarla uğraşılmıştır, 4. Dereceden polinomlar yaklaşıklıkla çözülmeye çalışılmıştır. Matematiksel yapıların örnekleri ve öğrencilerin onlarla nasıl düşündüğü ve öğrendiği burada odak noktası olduğu ve uzun yıllar süren araştırmalardan edinilmiş sonuçların paylaşılacağı bir derleme çalışması olarak düşünülebilir. Bütün o farklı yapıları bir yapısal bütünlük altında düşünmeye başlanıldığında öğrencilerin bu yapılarla nasıl öğrendiğini anlamamıza yardımcı olacağı düşünülmektedir. İslami bir örüntüyü pergel ve cetvel ile oluşturduğumu zaman bir algoritma izlenilmektedir ve burada amaç yapıyı çözmeye yöneliktir. Öngörü zamanla oluşmaktadır ve bunun için anlamlı deneyim gerekmektedir. Ve bunun için de farklı tabanlarda yapısal özelliklerin sistematik farkındalığı gerekebilir. Kandinsky'nin "Sayı 8" eseri, öğrenciler tarafından, taşıdığı Öklid yapısı ve geometrik ilişkiler açısından incelenebilir. Sheakespeare'in 116. Sonesi "Olmayana ergi" içerebilir. Her yapı tektir ama bazı ortak noktalar içerebilir: perspektif, odak, değişkenler, birleşme yapıştırıcısı ve bağlam gibi. Yapıların öğrenilmesinde bu aşamalar incelenmeye çalışılmıştır.

Anahtar Sözcükler: Matematiksel yapılar; modelleme; matematiksel düşünce

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1. INTRODUCTION

Besides structuralism as a philosophy, mathematical structures are thought as not important. In reality, if a student understands the value of a mathematical structure, teacher can expect much more regarding understanding and learning of that mathematical topic (Stanford Encyclopedia of Philosophy, 2022). In many mathematical sub disciplines, there are many sub structures to think about, to analyze and to construct. Not even in sub disciplines but in many interdisciplinary focuses, one may end up with mathematical structures to search for the relationships and properties of structures. There are some theories as of van Hiele (1986) specific to geometrical thinking and learning, but theories on learning mathematical structures do not exist.

1.1. Definition of a Structure

From Oxford dictionary, the definition of a structure is given as "the arrangement of and relations between the parts or elements of something complex, an object constructed from several parts". Hence, the relations gluing is important and the glued parts are important. In Figure 1, a rhombicosidodecahedron from modular origami can be seen. The photo is taken from the inside out to be able to see the structural relationships. The structure has a 3-4-5-4 patterned relationship and can be seen if looked carefully to the squares, triangles and pentagons around a corner (Oxford Learner's dict, 2022).

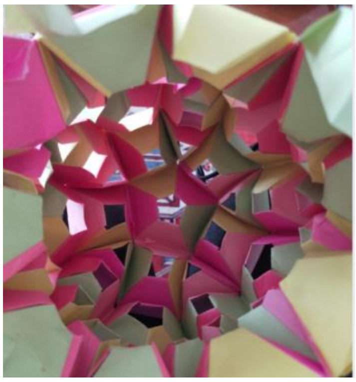

Figure 1. Rhombicosidodecahedron from inside

A mathematical structure is a mathematical object, a whole made up of parts, a patternistic skeleton, an object with special mathematical characteristics, having a mathematical unification, having mathematical glue of parts unifying, with an Underlying mathematical pattern with distinct specialties. They can have a structural baseline. One may need to understand the 1st, last and the middle elements. A pattern is a structure within, a structure related. Following example are taken from a bunch of studies of ourselves, regarding student explanations of their understandings. For the time being, structures were not the main focus. Throughout the time, it all came to a similar conclusion. And this is where, this new study is flourished (Çeziktürk,2004 2019a, 2019b, 2019c, 2019d, 2019e, 2020a, 2020b; Çeziktürk-Kipel, 2013, 2015, 2017, 2018a, 2018b; Çeziktürk-Kipel ve Özdemir, 2016; Çeziktürk-Kipel ve Yavuz, 2019; Çeziktürk, İnce, Karadeniz, Kenar ve Yalım, 2019; Çeziktürk-Kipel ve Köklü, 2019; Durası, Yalın, Karadeniz, Yalçıntuğ, Şahinler, İnce, Yasin, Şahin, Kenar ve Çeziktürk, 2019; Hangül ve Çeziktürk, 2020; Kerpiç, Ulusoy ve Çeziktürk-Kipel, 2018; Yazıcı ve Çeziktürk-Kipel, 2018; Yıldızhan ve Çeziktürk-Kipel, 2019).

1.1.1. Origamic structures:

In origami, a one-piece origami is a structure. A modular origami creation is a structure but all modules are structural baselines. Origami crease patterns talks for themselves. Ona flat sheet of paper, a bunch of geometrical relationships with interesting corner points. Sometimes an origami may be from a different piece of paper other than a square like a rectangular cut paper. For example, a pentagon may be folded from a rectangular sheet. Some origami structures are kinetic in other words dynamic (Figure 2). Hence, they are movable with an infinite rotation from inside out like "Origami fireworks" (Çeziktürk-Kipel, 2017; Çeziktürk-kipel, 2018a; Çeziktürk, İnce, Karadeniz, Kenar ve Yalım, 2019; Yazıcı ve Çeziktürk-Kipel, 2018).

Figure 2. Example origami structures: all modular

Videos, instructions (if new pieces are there) may be difficult to follow. Some delay time is needed between reentry with the structure.

1.1.2. Structures from Math and art:

Mona Lisa by Da Vinci had a color structure, and when one looked from each side the girl on the canvas could be seen with a different mood due to the color structure of the painting. But also, some form of golden ratio was included both as golden rectangle and as the face beauty to obey. In the painting of Number 8 by Kandinsky; a lof of geometrical concepts could be seen. Anybody looking to the painting could see different relationships as much as common ones. Sides, parallel lines, intersecting lines, points, circles, circular arcs, triangles, etc. We are not sure what he was suggesting but he somehow set the baseline with his book on from Points to Lines to Planes

İs it new /original/not seen anywhere before? Originality is a hinder to get over. systematic building of knowledge helps understanding structures. Ahmet Gunestekin's painting (Figure 3) can be analyzed by this point regarding the geometrical relationships (Çeziktürk-Kipel, 2015; Çeziktürk-Kipel, Özdemir, 2016; Kerpiç, Ulusoy ve Çeziktürk-Kipel, 2018a; Çeziktürk-Kipel, Yavuz, 2019; Çeziktürk, 2019d; Durası, Yalın, Karadeniz, Yalçıntuğ, Şahinler, İnce, Yasin, Şahin, Kenar ve Çeziktürk , 2019; Yıldızhan ve Çeziktürk-Kipel, 2019) .

Figure 3. Ahmet Güneştekin's work

1.1.3. Structures from Mathematics courses:

In Analytic Geometry

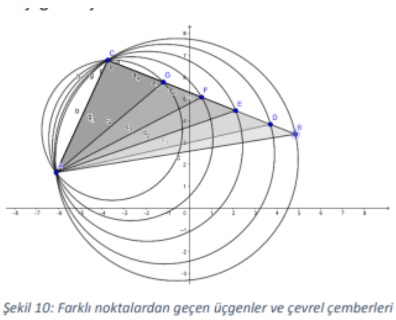

Figure 4. Example structure from analytic geometry

A cube, a conic section, a line, a point, an intersection of planes, a graph of an algebraic function are all structures from analytic geometry. All these sometimes intersect, sometimes unite but always related. Hence, systematic cognitive (schematic) building of knowledge is required to fit a new structure into existing arena (Figure 4) (Çeziktürk, 2004; Çeziktürk-Kipel, 2013; Çeziktürk, 2019c; Çeziktürk, 2020b; Hangül ve Çeziktürk, 2020).

In Complex Analysis,

Complex functions, complex numbers, complex integral, complex differentiation, length of a complex number are all somehow complex structures (Çeziktürk- Kipel, 2018b; Çeziktürk, 2019a). One of the best aesthetic formulas of mathematics: Euler formula gives us

$e^{i\theta} = \cos\theta + i \sin\theta$:   0,1,i, π , -1

In Numerical Analysis,

1. soru

i	xi	yi	xiyi	xi2	(yi-orty)2	yi-a0-a1xi	
1	1,25	10,82	13,525	1,5625	23,85101	1,609845	
2	2,53	8,53	21,5809	6,4009	6,727539	0,32298	
3	3,24	6,35	20,574	10,4976	0,171189	-1,30059	
4	5,16	5,27	27,1932	26,6256	0,443889	-0,87589	
5	5,82	4,12	23,9784	33,8724	3,298764	-1,50865	
6	7,1	4,05	28,755	50,41	3,557939	-0,57551	
7	8,07	6,2	50,034	65,1249	0,069564	2,334676	
8	10,25	2,15	22,0375	105,0625	14,33569	-0,00686	toplam
	43,42	47,49	207,678	299,5564	52,45559	-4E-15	
ortalama	5,4275	5,93625					

a1=(8*d13-b13*c13)/(8*e13-b13*b13)   Sy=   2,737454

-0,7837   sy/x=   #SAYI!   0 a0=c14-(a17*b14)

10,18978   y=10,18978+(-0,7837)x

Figure 1. Solution of first problem in H1

Figure 5. Example structure from numerical analysis

Algorithms to calculate error, algorithms to find roots of a function exist in numerical analysis. Some structures are not certain. They work with approximations and guessing (Figure 5). Actually many structures are like that. Some negligence is always necessary (Çeziktürk, 2019e).

1.1.4. Structures from Mathematics and Literature:

Let me not to the marriage of true minds
Admit impediments. Love is not love
Which alters when it alteration finds,
Or bends with the remover to remove.
O no! it is an ever-fixed mark
That looks on tempests and is never shaken;
It is the star to every wand'ring bark,
Whose worth's unknown, although his height be taken.
Love's not Time's fool, though rosy lips and cheeks
Within his bending sickle's compass come;
Love alters not with his brief hours and weeks,
But bears it out even to the edge of doom.
If this be error and upon me prov'd,
I never writ, nor no man ever lov'd.

Figure 6. 116. Sonnet of Shakespeare

Mathematical structures in poems are very interesting pieces for students. One can check Divan Literature to see connections, some special poets' poems like Özdemir Asaf, Nazım Hikmet, Tevfik Fikret, etc. In literature, there are two very well-known literature pieces that students should both read and analyze: Alice in Wonderland by Lewis Carroll and Flatland by Edwin A. Abbott. Systematic building of knowledge helps understanding structures. In Flatland, the value of 3D can be understood by the analysis of the structure of 2D, and some concepts like infinite smalls, ratio and proportion etc. can be found in Alice in Wonderland that is written for children of all ages. Students like surprises, and these two books are full of surprises. If we let them write poems and see the value of the structure, they may exercise and experience different original structures (Çeziktürk-Kipel, Köklü, 2019; Çeziktürk, 2020a).

The poem at Figure 6 is of William Sheakespeare; Sonnet number 116. The last two sentences are believed to be "proof by contradiction" of mathematical proof structures.

1.1.5. Structures from Islamic Geometry:

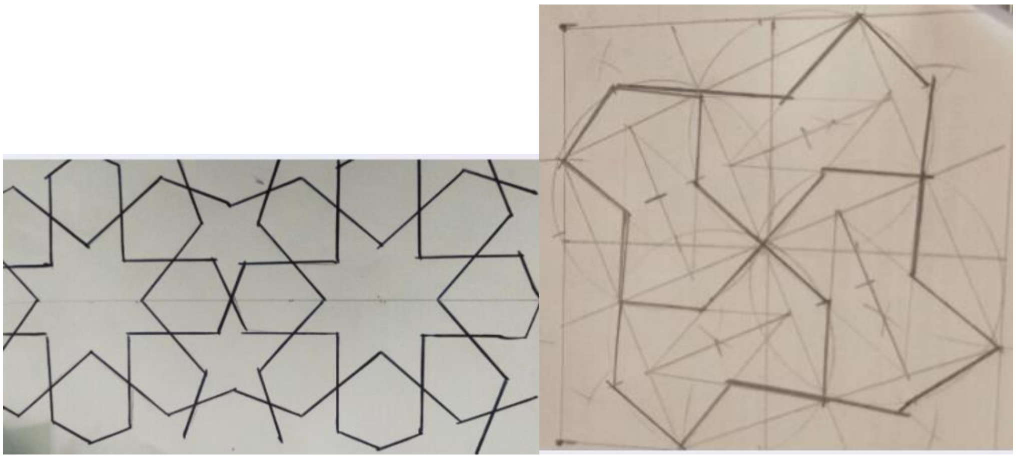

Figure 7. Algorithmic and geometric structure of islamic tilings

Hidden infinity lines, Stars with many different rays, Vertical and horizontal symmetry, Rotational symmetry and Reflection are possible structures arise while constructing an example Islamic pattern (Figure 7). Algorithm is teachable, certain, sometimes enables some little errors like unwanted lines etc. But if the student goes back and checks and evaluates the structure again sometimes there are some u turns for correction (Cezikturk-Kipel, 2018a; Kerpiç, Ulusoy ve Çeziktürk-Kipel, 2018).

1.1.6. Structures from Vedic Math and Soroban:

Different structures may be a result of different cultures. Vedic math is a Hindu culture mathematics from old times (Çeziktürk, 2019b). There are 16 sutras and these sutras work as mathematical sentences and rules. Sometimes different structures may be a result of different way of thinking. When we analyzed their thinking, most of our students said that why would we need to add or multiply differently than we used to? Actually we do not need but we could learn from different ways of thinking. Because most of the time we memorize and we do not actually understand why we do something as we do always. It was very interesting to see that even high school mathematics education students did have difficulty with understanding the ways of Hindus. +, _, x Operations are universal but how we multiply may be different. Students hesitate when they first see it, but then they like challenge.

Here, underneath we see a simplified way of solving a second degree equation by Vedic mathematics.

For example, in Vedic mathematics (old Hindu mathematics);

To solve,

$\frac{x^2+2x+7}{x^2+3x+5}=\frac{x+2}{x+3}$ one needs to notify the resemblance of factors 2 and 3 in the second rational to the first rational of equations. If those numbers used there, one can solve this kind of rational equations very easily. All one needs to do is to solve for;

$\frac{x+2}{x+3}=\frac{7}{5}$. Also notice that 7 and 5 are taken from the factors of the x^0 from the first rational. The result becomes -11/2 but interesting is to make students understand why this structure Works and other do not. That is the beneficial idea of mathematical structures of Vedic mathematics. Somehow, they make you to check if there could be other special mathematical structures that needs to be analyzed.

1.1.7. Structures from Impossible figures:

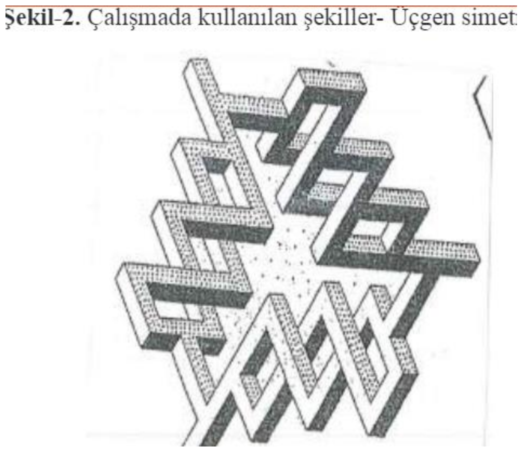

Figure 8. Impossible figures

These figures (Figure 8) are available on the Internet as pages of an old book named as "Impossible Figures" from Dover books. A page with isometric dots are given for the students to redraw the above figure to the isometric dots given. It is challenging since they are impossible in 3D reality. But they are possible in a 2D drawing (Çeziktürk-Kipel, 2015). When we checked for what kind of corrections students make with these kinds of structures, it turned out that mostly on the corners and turning points they were having problems drawing. Actually, these places were the problems parts that were producing impossibility. If one looks carefully, can see three planes intersecting each other at the turning points or at the corners. When somebody was said about impossibility, they expect something odd but they are not ready of where this impossibility may occur.

- İmpossibility as a structure

«not suitable for insight at the

first time» ones are difficult to learn

1.1.8. Structures from: Cross-stitching, Knitting and Crotchet

Figure 9. Structure concept at scaled crocheting

In the cross-stitching, a "x" is the structure one makes with needle and rope. Where would take out the needle and where would place the needle is the structure of cross-stitching and experts know that a good cross-stitch can be understood by reversing it seeing the back stitches.

In the above photos (Figure 9), crocheting for play babies are actually a structure building exercise. Besides scaling, and coloring it with different variations of patterns are challenging and structure teaching. Calculating where to locate empty space and where to put the pattern is of course is an interesting fact and play.

2. METHOD

2.1. Method

This study is a review study from many splendid studies that were carried around this topic without knowing where it was leading. A review study is systematically combining the results of at least two or more studies on the findings, results and evaluations without a specific method and by different techniques (Yılmaz, 2021). In this year, a structural pattern was flourished and arose like a sun out of the clouds. In all parts and wholes, there were some ties and patternistic structures. And these structures were coming together as a whole pattern at the end.

2.2. Data Collection and Analysis Techniques

This is some sort of a phenomenological research. It is qualitative but supported with quantitative studies. It is phenomenology since, we have not started with this aim up. It all flourished from the data and small studies over many years.

Hence, in some way this study could be named as structural phenomenology. Because, it is about body building, it is about relations between variables. Glue sometimes became mathematics, sometimes it was intuition, sometimes it was previous experience with those structures or parts of those structures. Sometimes technology was a glue, and sometimes just hand-eye coordination (Table 1).

Table 1. What is structural phenomenology being about?

glue	Body building/relations between variables, real glue, attaches, geometry, technology, hand-eye coordination, translations between representations combines pieces in special form
Parts	Variables, modules, problems, particular representations, needs to be understood well!
Whole	Model, example, topic, context bounded
Perspective	Author, artist, researcher, origamist, whose perspective is that?

Some new wordings came into the consideration such as: structural periphery, structural monogamy, structural resemblance, deconstructivism, structural rope, structural ties, and structural bell. All these new wordings somehow shaped how the data is analyzed as well to build the new sub structure for learning with mathematical structures.

Students by this process somehow learn to write mathematics. Symbolic math is a well-formed structure. Some symbols need technological infrastructure. Symbols themselves are structures that we are not familiar with the history of them. For example;

- Lambda, phi, integral sign, root sign, ∀ε, ∃! δ such that… As in limit of a function… For many of these symbols, we are not even sure about where do they come from. But they form a structure, an actually a holistic structure hence it is important to use them properly.

3. FINDINGS

There are some wordings came out of the process over the years: multidisciplinary, prior knowledge, experience with structures, possibility, structural collaboration, approximate structures, systematic building of knowledge, originality, role of insight, time delay between, hidden pattern rules. Structures are strict: but may be formed by approximations. Structures may collapse to build new structures. Structures do not accept so much deformation-spills out false assumptions. Structures are interesting for math makers /doers. Especially if the structures are familiar or if they are totally unfamiliar. Hence, experience with structures is important but originality is a possibility producing specialty. Structures are for us to know them /to understand them. Structures are soft in case of building a new structure from old one hence prior knowledge is important. Structures are hygienic since most structures live isolated. STEM project artifacts are a good example of multidisciplinary for structures. Some structures collaborate to build new structures. Time delay between building structure phases are a need since insight develops over time by more and more experience. Some hidden patterns may help structural phenomenology since, hidden patterns are both interesting and pattern producing in general. Even in Geogebra, some structures are approximate but we see them as a whole without holes. For example, a circle is full of points but we see a whole rope.

In the process, we have learned somethings:

- **Structural periphery:** boundaries are important since they somehow define the structure.
- **Structural monogamy:** any structure is unique even the ones unwanted and unexpected from the process.
- **Structural resemblance:** In mathematics we call it as isomorphism and if we want to understand a structure, it is better to find a structure resembling to the original one but much more simple.
- **Deconstructivism:** Sometimes it is a need to deconstruct. It helps to see our falsities and possible problems.
- **Structural rope:** Sometimes, it can be a glue. Sometimes, it is just a folder to consist some sort of structures or so.
- **Structural ties/nodes:** Like human brain, each link works by itself, but in case of a collapse, remaining parts sometimes holds the full piece.
- **Structural bell /symmetry, wholeness, aesthetic, reality:** Golden ratio, beauty, systematic touch, etc.

Here it may be a good example to give a structural analysis of a structural phenomenology. In Analytic Geometry, for example, one could identify the phases as followed: analysis, links, examples, periphery/boundaries, difficulties, unwanted stop, start over, other possibilities (Table 2). This may be an example of structural phenomenology in pure mathematics courses.

Table 2. Structural learning for Analytic Geometry

Analysis	What are the parts of a function; terms, variables, special functions: rational functions, absolute valued functions, square root, polynomials, trigonometric functions
Links	Roots of a function, y and x intercepts, domain, range, sections, middle points, lines, curves, surfaces
Examples	Similarities to earlier examples
Periphery, boundaries	Limit, asymptotes, tangent lines, derivatives, maxima minima points, inclination points
Difficulties	Undefined functions, undefined regions, undefined points, not equal limits and derivatives from left and right
Unwanted Stop	Time delay is needed, some other time come back again
Start over	Go back and see what is left
Other possibilities	New structures flourishing
Result	Graphic, solution of the problem, classification of the function, etc.

Regarding the examples that we have given from review studies of ours, Islamic Geometry is a special structure to be thought and learnt. It is good for geometrical thinking, it is good for working with DGS software, and it is good for understanding some concepts like infinity, symmetry, and transformational geometry, parallel and intersecting lines, common points and planes. In Table 3, one can find the same structural phenomenology phases as of Analytic Geometry but this time with Islamic geometry specialties.

Table 3. Structural learning for Islamic Geometry

Analysis	Infinite loops, infinite line cracks, parallel lines, symmetry lines, repeating patterns,
Links	Smallest repeating pattern, geometric relations occurring/flourishing, (some approximations may be needed)
Examples	Recall earlier examples, what is differently? What is same? Star patterns, medallions, border examples, door patterns etc., libraries
Periphery, boundaries	Boundaries of the smallest repeating pattern, boundaries of the big pattern to be developed
Difficulties	Not clear geometric relationships, not clear patterns etc.
Unwanted Stop	Problem with proceeding. Stop and start over.
Start over	Go back and see what is left: a symmetry, a line, a point, a parallelism etc.
Other possibilities	Proceed and see where it goes. Sometimes a wrong pattern can be teaching something.
Result	Resulting pattern/isomorphism to the original pattern started with.

Similar phases can be found in Origami learning especially with modular origami and model building processes. In Table 4, phases of structural phenomenology are applied to origami, as can be seen.

Table 4. Structural learning for Origami models

Analysis	Modules, one piece paper, geometric structures under a folding pattern
Links	Geometric pattern of the model. Eg. 3-4-5-4 of rhombicosidodecahedron
Examples	What it resembles, which earlier model?
Periphery, boundaries	Total number of modules on a corner, on whole model, boundaries of the starting paper (if rectangle; what is a and b?)
Difficulties	Originalities, newness, new folds, cuts
Unwanted Stop	Not easy to continue, a mountain to get over with
Start over	Begin from scratch and see what should be different, what is problematic
Other possibilities	Does it go to the another model, do it? does it go to a similar version of the model, do it. See the possibilities, It teaches for some other time
Result	Correct result, or any result. To see the faulty models. If correct is found by mistake just understand the problem.

These examples for structural phenomenology phases can be increased. However, for the time being, it may be a better idea to explain where did those phases come about. In building 3-4-5-4 patterned Rhombicosidodecahedron and pentagon from a rectangular paper strip, those simple foot steps are taken. In reality, those phases can be listed under some categories as well: analysis, links and examples are for acquaintance analysis; difficulties, unwanted stop and start over for experience related phases; and other possibilities and the result are the artifact related phases. From above list periphery and boundaries are in between of the acquaintance analysis and the experience phases. In other words, somehow periphery and boundaries sets us some boundaries that we should obey and stick with. Unwanted stop is a point where the learner feels helpless and stuck. Most of the time without a proper guidance, this is the phase where the learner stops any kind of learning experience with these structures. But proper guidance enables the learner to see that it is an “unwanted” stop hence, it is like a bus stop. There is a next step one can take to avoid confusion. Time delay happens mostly here. If the learner has some sort of a experience with these kind of unwanted stops from any structure, there is a possibility that he /she may never stop from learning other structures. But, if there is no previous experience with this unwanted stop, there may exist some falling from whole phase and not going any further (Figure 11).

4. DISCUSSION AND RESULT

4.1. Discussion

Learning structures are part of the learning process. However, in most of the theories not so much explanation is given for them. Here, in this study, we have examples for three structure learning situations: analytic geometry from pure mathematics, modular origami, and Islamic patterns. Even though all these have some sort of different phase but they also come under some naming as of: analysis, links, examples, periphery/boundaries, difficulties, unwanted stop, start over, other possibilities and result (Figure 10). Other possibilities should be supported by the teacher so that new and original structures may flourish. Unwanted stops should be supported by proper guidance by the teacher so that learner would understand that even the teacher passes through these phase sometimes. Time lapse is a protector here. Gives the time to the learner to refresh the whole learning process.

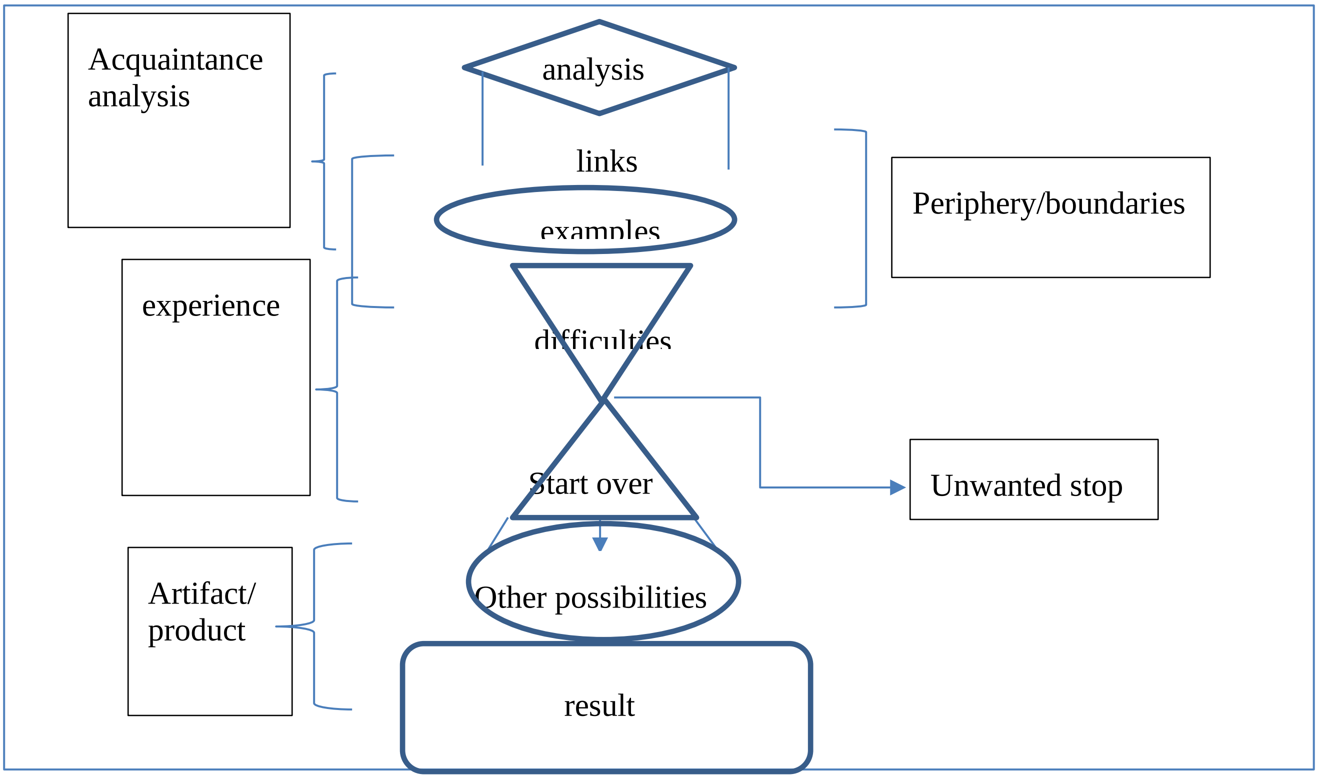

Figure 10. Structural phenomenology phases

This structural phenomenology is slightly different than Piaget's, or Bloom's taxonomy or even van Hiele 's (van Hiele, 1986; Stanford Encyclopedia of Philosophy, 2022) levels because it starts with something from higher levels. But analysis here, is an experiential analysis hence not so difficult for the learned to proceed. And it enables part to whole thinking. In the experience phase, the learner is much aware of the canvas that he or she is studying with. Because the boundaries are already set forward. For example, the learner understands that a change in the boundaries would result in a different experience and different artifact and even other possibilities.

The same phases may be applied to all other structural learning contexts: to Vedic mathematics, impossible figures, Shakespeare's sonnets, other pure math examples, in Painting and even in crocheting in scales. It is important to note in here that this is an ongoing Project and not fully finished yet. Hence, there may be some corrections and updates possible in the process of more structural phenomenology examples. And another important thing to consider is the study group being preservice teachers for all of these studies. Hence, there could be different groups and there could be different answers possibly. Nevertheless, since we are dealing with learners and since preservice teachers would act like teachers sometime after, hence it may be a good idea to start with them. Study is also restricted to the little research studies Cezikturk carries, hence there may be some objectivity issues.

4.2. Limitations

This is a review study of many splendid studies of the author herself. Hence, it may carry some part of subjectivity issue with the method itself. However, it is a beginning study of this kind in order. Meanwhile, it is supposed to be a line of research and it may end up with a theory building in the future studies. It should be thought that way.

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