Cilt / Volume 57 | Sayı / Issue 2 | 365-392

VERİMLİLİK DERGİSİ
JOURNAL OF PRODUCTIVITY

Modelling Strategy of Airline Tankering with Nonlinear Programming*

Niyazi Cem GÜRSOY[1], Nesrin ALPTEKİN[2]

ABSTRACT

Purpose: This study aims to reduce the fuel costs, which constitute the largest share of total cost that airline companies have, with extra fuel transportation.

Methodology: A nonlinear programming model has been developed for tankering application that takes advantage of the different fuel prices at the airports. General Reduced Gradient Method (GRG) is used to solve the generated nonlinear programming problem.

Findings: In the application part of study, two applications have been studied on and one of them is parametrical and other assumes İstanbul as a hub airport in order to analyze the effect of flight distance, load rate, fuel price difference between the airports and altitude of cruise flight on tankering and cost. Although the load rate is high and the fuel price difference is low, flights have been conducted between the centers which have short flight distances, the model allowed to tanker. It was observed that when the amount of fuel recommended by the problem result was taken for the designed scenarios, the fuel consumption increased by 2.5-3% compared to the trips without tankering. Despite the increase in fuel consumption, it has been found that a total fuel cost can be saved of 1% to 47% for round trips.

Originality: The efficiency of the original optimization model created with non-linear modeling was developed and tested for various scenarios.

Keywords: Tankering, Airline Fuel Management, Nonlinear Programming, General Reduced Gradient Method, Optimization.

JEL Codes: C61, L52, L93, O21.

Doğrusal Olmayan Programlama ile Havayolu Fazladan Yakıt Taşıma Stratejisinin Modellenmesi

ÖZET

Amaç: Bu çalışmanın amacı havayolu firmalarının en büyük maliyetini oluşturan yakıt maliyetlerini fazladan yakıt taşıma ile azaltmaktır.

Yöntem: Havalimanlarında yakıt fiyatlarının farklı olmasından faydalanan fazladan yakıt alma uygulaması için bir doğrusal olmayan programlama modeli geliştirilmiştir. Oluşturulan doğrusal olmayan programlama problemini çözmek için Genel İndirgenmiş Gradyan Metodundan yararlanılmıştır.

Bulgular: Çalışmanın uygulama kısmında uçuş mesafesinin, doluluk oranının, uçulan merkezler arasındaki yakıt fiyat farkının ve düz uçuş yüksekliğinin fazladan yakıt taşıma ve maliyet üzerine etkisini analiz etmek amacıyla parametrik ve İstanbul'u merkez alan iki uygulama yapılmıştır. Uçuş mesafesi kısa olan merkezler arasında gerçekleşen seferlerde doluluk oranı yüksek ve yakıt fiyat farkı az olsa da model fazladan yakıt taşımaya izin vermiştir. Tasarlanan senaryolar için problem sonucunun önerdiği miktarlarda yakıt alındığında, fazladan yakıt alınmadan gerçekleştirilen seferlere kıyasla uçağın yakıt sarfiyatının %2,5-3 artırdığı gözlemlenmiştir. Bu yakıt sarfiyatının artışına rağmen gidiş-dönüş seferleri için toplam yakıt maliyetinden %1-%47 arasında tasarruf sağlanabileceği tespit edilmiştir.

Özgünlük: Doğrusal olmayan modelleme ile oluşturulan orijinal optimizasyon modelinin verimliliği çeşitli senaryolar geliştirilerek test edilmiştir.

Anahtar Kelimeler: Fazladan Yakıt Taşıma, Havayolu Yakıt Yönetimi, Doğrusal Olmayan Programlama, Genel İndirgenmiş Gradyan Metodu, Eniyileme.

JEL Kodları: C61, L52, L93, O21.

---

* This study is derived from the Master's Thesis titled "Modeling Strategy of Airline Tankering with Nonlinear Programming and Application" which is conducted by Niyazi Cem GÜRSOY at Anadolu University Graduate School of Social Sciences under the supervision of Prof. Dr. Nesrin Alptekin.

[1] Lecturer, University of Turkish Aeronautical Association, Faculty of Air Transportation, Department of Flight Training, Ankara, Turkey, ncgursoy@thk.edu.tr, 0000-0003-2743-5314 (*Corresponding Author*).

[2] Prof. Dr., Anadolu University, Business Administration Faculty, Department of Business Administration, Eskişehir, Turkey, nesrinesen@anadolu.edu.tr, 0000-0002-8967-8955.

DOI: 10.51551/verimlilik.1065007
Research Article | Submitted Date: 30.01.2022 | Accepted Date: 02.04.2023
Cite: Gürsoy, N.C. and Alptekin, N. (2023). "Modelling Strategy of Airline Tankering with Nonlinear Programming", *Verimlilik Dergisi*, 57(2), 365-392.

1. INTRODUCTION

Since the start of flights with engine-driven aircraft in the early 20th century, the air transport industry has developed very rapidly and today it has an important place in our lives because it is a fast and reliable method of transportation for people to reach each other. Due to the fact that aviation industry is inevitable, the efficiency and productivity of service providers such as stakeholders; airports, airline companies, air traffic etc. is of utmost important. Since in the supply of air transport, the costs are higher compared to all other modes of transport and it has been mandatory to manage it effectively in order to obtain sufficient profit margins.

The sustainability of the air transport industry depends on the success of the airline companies. Success of airline that is the only organization that carries out the passenger transportation business and provides transportation services to people depends on many factors such as their profitability, operational performance, quality etc. The airline's profitability directly related with its costs and how effectively those costs are managed.

In order to reduce fuel costs, which is one of the biggest costs of airlines' direct operating cost, they apply an operational strategy that reduces the total fuel cost by carrying the optimum amount of extra fuel by taking advantage of the fuel price differences at the airports. In the implementation of this procedure that is called fuel tankering or fuel transportation, many factors needed taking into account such as the structural features of the aircraft, daily fuel prices, load factor, air temperature, flight altitude, wind, etc. How much extra fuel will be transported is generally determined by flight planning software in airline companies. In this study, a nonlinear programming model that can be applied to an airline network with N different destinations is proposed. The model, which recommends how much fuel will be taken by meeting the legal requirements at the points where the aircraft should buy fuel, aims to optimize the total fuel cost.

Generalized Reduced Gradient (GRG) method, which is used for solving constrained nonlinear programming problems, is chosen to solve the research model that gives the tankering amount. The GRG method, which has been widely used to solve many problems for more than 20 years, has been preferred because it has been tested many times and proved that it has a robust algorithm (Lasdon et al., 1978; Lasdon et al., 1974). The disadvantage of GRG, which have gradient-based solution technic, is that it can increase the processing load because it tests that the constraints whether they are in the feasible region or not in each iteration. If the problem is modeled properly, it is very comfortable for the solution to reach the global optimum. MS Excel solver using GRG2 algorithm is used to solve the research problem.

The implementation of the research problem is based on the Airbus A319 aircraft model and the Flight Crew Operating Manual (FCOM) of this aircraft is used for fuel burn estimates. In the model, it is assumed that the airline flight network structure is accepted as a hub and spoke in such a way that aircraft take off from the origin airport to the destination and returns to the origin airport. The first application is made parametrically and the aircraft load factors are accepted as 70%, 80%, 90% and 100% in the created scenario. The flight distances between airports are tested at 300nm, 500nm, 700nm, 900nm, 1100nm, 1300nm, and 1500nm. The altitudes chosen in the scenario during flights are evaluated as 29.000ft, 31.000ft, 33.000ft, 35.000ft, 37.000ft, and 39.000ft. The fuel price ratio between the centers flown is determined as 50%, 60%, 70%, 80%, and 90%. In all scenarios created, problems have been successfully solved and results have been found. In the other application, in order to show the fair fuel price advantages of İstanbul, it is thought that an airline based in İstanbul will fly to seven different centers with medium-range flight distance. The load factors for flights to Sofia, Athens, Florence, Bratislava, Baghdad, Tehran, and Tripoli were taken as 70%, 80%, 90%, and 100% as in the case study, and level flight altitude are 29,000ft, 31,000ft, 33,000ft, Tested at 35.000ft, 37.000ft and 39.000ft. The created problems are successfully resolved on Excel and the results are interpreted.

With this study, a nonlinear programming model has been proposed that may support to optimize the amount of fuel purchase from airports. In the experiments, the most suitable conditions for tankering have been investigated with different scenarios first parametrically then with a designed case scenario. It is stated that İstanbul is an attractive hub in terms of fuel costs, and it is founded that airlines can save money with the tankering procedure on medium-haul flights to the extent that the difference in fuel price allows. İstanbul is a convenient and advantageous center for tankering to put in place.

Fuel is one of the biggest factors affecting the economy, strategy, profitability, and efficiency of airline companies. According to IATA data, the total billed fuel expense of the global airline industry was $ 188 billion in 2019, and when the average barrel price is $ 65.0 (barrel Brent), it corresponds to 27.7% of operational expenses (IATA, 2019a). As can be seen in the Table 1 according to Turkish Airlines fuel price analysis, although the ratio of fuel costs in operational expenses has been irregular over the years, it did not fall below 25%.

Table 1. Turkish Airlines fuel cost

Costs	*2015*	*2016*	*2017*	*2018*	*2019*
Fuel Expenses (Million $)	2,997	2,673	2,866	3,768	3,873
Fuel Consumption (000 Tons)	4,272	4,693	4,847	5,275	5,525
Average Change of Unit Cost (%)	-30	-19	4	20.7	-1.8
Fuel Expense Rate in Operational Expenses (%)	30	26	28	32	35

Source: Turkish Airlines (2019)

The fuel costs of the airline business vary depending on the market value of the jet fuel, the network structure of the airline, the weight of aircraft, the direction and intensity of the wind which is exposed to during the flight, air pressure, density and temperature, and finally the cost index used during the flight. Airlines use operational practices and financial instruments in order to reduce fuel costs. Effective determination of the center of gravity of the aircraft, prevention of excess weight, proper flight planning and selection of the most suitable route, single engine shutdown during taxi, keeping turnaround times short for efficient use of the auxiliary power unit and finally tankering the optimum amount of fuel by taking advantage of the different fuel prices at the airports have been implemented by the airlines. are among their fuel-saving operational procedures (Airbus, 2004). It is known that aircraft used in airline fleets for a long time consume more fuel, and therefore one of the reasons for lowering the average age of the fleet by modernizing them is to reduce fuel consumption. When it comes to significantly lowering the cost of air transportation, it is seen to be an effective procedure to calculate the optimum amount of tankering to be put into the aircraft during the flight planning phase. As a result, companies will be able to employ their resources as effectively as possible and boost their profitability. Over the long run, with the help of efficient operational procedures their position will be improved, giving them a competitive edge over rivals in the market.

Airlines can also make hedging agreements with fuel suppliers in order to minimize the risk arising from the fluctuation of fuel prices. Financial instruments that can be used to hedge the risk of fuel price can be listed as forward contracts, futures contracts, call options, collars, and swaps (Tuncer and Aydoğan, 2019).

In October 2003, with the elimination of the DGCA 1996-year decisions which prevents private airlines to the entry into the domestic market, the competitive environment in the domestic market is ensured and therefore the pace of development of commercial civil aviation transport operations in Turkey has been accelerated. It is aimed to make the market attractive for airline companies by reducing airport usage fees, reducing or removing some of the additional taxes, and giving incentives to companies that want to establish airlines with the regulations (Gerede and Orhan, 2015). With the developments after 2003, more than 50% of Turkish Airlines' shares were offered to the public. One of the incentives given by the government to the development of the airline industry in Turkey has been also for fuel used in aircraft. With the decision taken by the Council of Ministers in 2011, the Special Consumption Tax (SCT) used for air fuels is applied as zero[3]. In another law related to the taxation of air fuels, fuel and oils to be used by aircrafts in international flights are exempted from import tax in the Customs Law[4]. Although Turkey has very few oil resources, the air-fuel prices in Turkey are more affordable than many other countries because of the incentives provided by the government. Air fuel prices due to the incentives generated by oil or raw material resources, as in Turkey in some countries it may be more economical.

Many airline companies around the world have made use of the fuel price differences in various countries and regions as an opportunity and have managed to reduce their fuel costs by carrying extra fuel. According to the study conducted by EUROCONTROL (European Air Navigation Safety Organization), it was estimated that every 2.1 million of the annual 10 million flights made in countries within the ECAC (European Civil Aviation Conference) applying tankering; It was estimated that 1.6 million flights overall implemented full tankering with 16.5% and 0.45 million flights performing partial tankering. As a result, with tankering, fuel consumption increased by 136 kg per flight, while on average 126 € was saved in cost per flight (EUROCONTROL, 2019).

Aircraft manufacturers have developed tables and calculations based on fuel prices in the flight operations manual to determine how much to tanker. Yet these tables and calculations are rarely used for today's aircraft. The airline company states that the amount of tankering to be carried in the operation manuals should be based on the flight planning, and in the notes that the company conveyed to the flight crew the tankering can be considered as holding fuel if the fuel price difference is too much, and if there is no suitable fuel at the destination tankering should be performed.

---

[3] Republic of Turkey, Council of Ministers Decision No. 1435, Official Gazette. 25/02/2011. Number: 27857, art. one.

[4] Republic of Turkey, Article 176 of Customs Law No. 4458.

Airlines develop their own flight planning systems or outsource it. The factors affecting the airline company's outsourcing can be summarized as the current economic status of the company, the company's ownership, capital status, age, and service level (Rutner and Brown, 1999). Outsourcing of the flight planning system is more common in newly established airlines. Tankering amount is determined by algorithms in flight planning software and no information has been obtained about which type of optimization techniques are used in it.

It is seen that airlines those flights are planned to Turkey or from Turkey may found Turkey’s airports attractive to save money with tankering application because of the incentives that government had been proposed such as taxation absence for carbon dioxide produced by aircraft and available tax regulations which hinder the increase of the fuel costs. This study shows the strategies that airline companies can follow in reducing fuel costs and aims to provide a model that can encourage airlines to develop tankering strategies as part of their flight planning system.

In this study, firstly, the importance of fuel cost in airline companies and what kind of strategies they have developed in management to deal with fuel cost are explained. Studies done to calculate the optimum tankering amount have been compiled in the literature review section. The GRG2 algorithm used to solve the research problem is explained in the third section. The definition and modeling of the research problem are given in the fourth chapter. Two different experiments were designed for the analysis of the proposed model. In one of the experiments, flights to 7 different centers based in İstanbul were designed and the assumed situations in the flights were explained. In the other experimental design, it was assumed that the aircraft flew at different load rates and distances between 300-1300 nm. In line with the assumptions accepted in the analysis part of the experiment, the most effective tankering application was investigated at which load rates and at which distances. Finally, in the conclusion part, the results obtained from the study were summarized and in which cases the tankering application was efficient was examined.

2. LITERATURE REVIEW

It has been observed that not many studies have been done in the literature on tankering. When crude oil prices increased after the oil crisis in 1973, it had reflected in airlines as a doubling of fuel costs. Therefore, airlines have had to develop fuel management strategies. Tankering as a fuel management strategy, firstly Darnell and Loffin (1977) tried to optimize the fuel cost by considering the fuel management strategy of National Airlines. Developed the linear programming model based on fuel prices, fuel availability, fuel burn rate, flight data, and extra fuel cost in order to determine the amount of fuel to be purchased, from which station and vendor. National Airlines was able to save multi-million dollars thanks to the fuel management strategy developed.

Stroup and Wollmer (1992) worked on a linear programming model that minimizes fuel cost by carrying extra fuel. In their proposed fuel management model, they based the flight schedules of airlines, fuel prices, stop restrictions and supplier restrictions. They tried to solve the problem by handling the airline's single flight and the entire network. McDonnels Douglas aircraft company used the fuel management policy developed with the proposed model to estimate the possible profitability of various aircraft types. As a result of the implementation of the model, they concluded that fuel costs can be reduced up to 5-6%.

Abdelghany et al. (2005) developed a nonlinear model that aims to reduce the cost by purchasing excess fuel from airports where fuel prices are cheap in order to reduce the fuel cost. They used the GRG2 algorithm to solve the model. In the model, the objective function is not considered as single flight, but by considering the whole flight network; they included the cost of the fuel purchased, the cost of remaining fuel in the fuel tank of the aircraft before fueling, and the maintenance cost arising from the increased weight due to tankering. They got the solution of the models using Excel. At the end of the study, they concluded that tankering is more effective with efficient aircrafts with low fuel burn rate, savings can be achieved by tankering on short-haul flights and high maintenance costs may limit tankering amount.

Guerreiro Fregnani et al. (2013) tried to optimize the amount of tankering with a linear programming model based on the domestic flight network in Brazil. They wrote the objective function in the model to minimize the total fuel cost. In the experiment, a regionally used aircraft type has been considered and airline network structure is like an aircraft was taking off from a single point, stopping by 12 different airports and returning to the airport of origin. The data inputs of the model were taken from the aircraft operation manual to determine the amount of fuel that the aircraft will consume without tankering in the designed routes. With the model they proposed, although it was observed that 1% increase on fuel consumption, 5% savings were achieved in the total fuel cost.

Hubert et al. (2015) analyzed the use of the tankering strategy of the US Air Force in air operations. Using historical data from the US Air Force, they investigated how much they could save from costs with tankering in the future for war and peacetime. According to the results of their studies, the most economical aircraft models were C-5, C-17, and C 130. They stated that with the additional fuel tanks to the aircraft to buy extra fuel, tankering capacity increased and the fuel cost savings could increase up to 460%. Finally, they stated that the fuel savings achieved with tankering has been also vary depending on the scenario applied.

Deo et al. (2020), studied tankering strategy with taking into account the cost index determined by the airline and embodied optional intermediate fueling stops. They applied integer linear programming based on airline network structure to solve tankering problem. They applied a case study on a network of sixteen airports with different fuel prices and seventeen different intermediate refueling stations to demonstrate the benefits of determining the optimum amount of tankering using the flight cost index and intermediate refueling stops. According to the results of the study, they observed a 3% cost reduction compared to traditional methods when fuel transportation strategy was applied to the optimized flight routes within the multi-stop flight network. They argued that the fuel cost for aircraft carrying cargo or passengers on long-haul flights can be reduced with an intermediate refueling stop.

When the studies on tankering optimization were examined, it was seen that, except for one study, the others used the linear programming model. In some studies, the flight network is reflected in the model. It has been determined that there is no detailed parametric study showing the effectiveness of the tankering application in the literature, and that the flight network which is hub&spoke is not integrated into the model. With this study, an experiment was created on how to make tankering application in airlines using hub&spoke system and the amount of tanking fuel was optimized with nonlinear programming. With the parametric application, it has been shown that how much tankering can be done at which level flights, at which load rates, at what distance, which has not been mentioned before in the literature.

3. GENERALIZED REDUCED GRADIENT (GRG) METHOD

Generalized Reduced Gradient (GRG) method is used to solve the studied non-linear tankering strategy model. The Reduced Gradient (RG) method was first presented by Wolfe in 1967 which is based on the simple variable elimination technique for equality constrained problems (Arora, 2017). Later, Abadie and Carpentier (1969) developed the RG method and proposed the Generalized Reduced Gradient (GRG) method as a solution method for nonlinear problems with inequality constraints. In 1974, Lasdon explained the GRG method with a few changes, making its principles and logic suitable for computer programs (Lasdon et al., 1974). In GRG method and the RG method, the variables are divided into basic (dependent) and non-basic (independent) variables as similar to the in the solution of linear problems with the simplex method. Instead of using penalty functions, the GRG method changes the inequality constraints so that the required change in basic variables can be calculated directly with non-basic variables (Frank et al., 2012). By adding a slack variable to the inequality constraints, it is transformed into an equality constraint and a nonlinear equally constrained model is obtained. Then, the total incremental change in the objective function can be calculated with the generalized reduced gradient defined from the objective function, considering both the basic and non-basic variables.

One of the main difficulties in solving nonlinear programming problems is deciding which of inequality constraints are active in the solution or which is not (Nocedal and Wright, 2006). Active constraints will remain fully active even in small search movements while applying steps in the GRG method. If some active constraints are not fully met due to the nonlinearity of the constraint function, the Newton-Raphson method is used to satisfy the constraints. In this respect, GRG is similar to the Gradient Projection method (Arora, 2017: 592).

The GRG method has been theoretically studied and Rao has been taken as reference (Rao, 2009: 412–418). A nonlinear programming problem is shown in Equations 1-5.

$$Minimize\ f(X) \tag{1}$$

$$h_j(X) \leq 0,\ j = 1,2,\dots,m \tag{2}$$

$$l_k(X) = 0,\ k = 1,2,\dots,l \tag{3}$$

$$x_i^{(l)} \leq x_i \leq x_i^{(u)},\ i = 1,2,\dots,n \tag{4}$$

$$X = \{X_1\ X_2 \dots X_n\}^T \tag{5}$$

In the equations given symbols defined as $f(X)$ objective function, $h_j(X)$ inequality constraints, $l_k(X)$ equality constraints, X set of design variables, $x_i^{(l)}$ and $x_i^{(u)}$ are the lower and upper limits of the design variables. All constraints should be given as equality for the solution of the proposed GRG models. For this reason, a non-negative slack variable is added to the inequality constraint in Equation 2. Thus, the lower limit of the design variables is 0 and the upper limit becomes a very large number (infinite) and the model is found as follows.

$$Minimize\ f(X) \tag{6}$$

$$h_j(X) + x_{n+j} = 0,\ j = 1,2,\dots,m \tag{7}$$

$$l_k(X) = 0,\ k = 1,2,\dots,l \tag{8}$$

$$x_i^{(l)} \le x_i \le x_i^{(u)},\ i = 1,2,\dots,n \tag{9}$$

$$x_{n+j} \ge 0,\ j = 1,2,\dots,m \tag{10}$$

Thus, it turns into a model with n + m variables $(x_1, x_2, \dots x_n, x_{n+1}, \dots, x_{n+m})$. Equations 11-13 is obtained when the problem is edited.

$$Minimize\ f(X) \tag{11}$$

$$g_j(X) = 0,\ j = 1,2,\dots,m+1 \tag{12}$$

$$x_i^{(l)} \le x_i \le x_i^{(u)},\ i = 1,2,\dots,n+m \tag{13}$$

The GRG method is based on the view that variables are eliminated using equality constraints. Thus, theoretically one variable $x_i\ (i = 1,2,\dots,n+m)$ can be reduced by one variable for each of the m + 1 equality constraints given in Equations 7 and 8. In order to do this, it is appropriate to divide n + m design variables into two sets arbitrarily as follows.

$$X = \begin{Bmatrix} Y \\ Z \end{Bmatrix} \tag{14}$$

$$Y = \begin{Bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{n-1} \end{Bmatrix} = Basic\ or\ Independent\ Variables \tag{15}$$

$$Z = \begin{Bmatrix} z_1 \\ z_2 \\ \vdots \\ z_{m+1} \end{Bmatrix} = Non-basic\ or\ Dependent\ Variables \tag{16}$$

If the first partial derivatives of the objective and constraint functions are taken, the following Equations 17 and 18 are obtained.

$$df(X) = \sum_{i=1}^{n-1} \frac{\partial f}{\partial y_i} dy_i + \sum_{i=1}^{m+1} \frac{\partial f}{\partial z_i} dz_i = \nabla_Y^T f dY + \nabla_Z^T f dZ \tag{17}$$

$$dg_i(X) = \sum_{j=1}^{n-1} \frac{\partial g_i}{\partial y_j} dy_j + \sum_{j=1}^{m+1} \frac{\partial g_i}{\partial z_j} dz_j \tag{18}$$

Short representation is as in Equations 19-25.

$$dg = [C]dY + [D]dZ \tag{19}$$

where,

$$\nabla_Y f = \begin{Bmatrix} \frac{\partial f}{\partial y_1} \\ \frac{\partial f}{\partial y_2} \\ \vdots \\ \frac{\partial f}{\partial y_{n-1}} \end{Bmatrix} \tag{20}$$

$$\nabla_Z f = \begin{Bmatrix} \frac{\partial f}{\partial z_1} \\ \frac{\partial f}{\partial z_2} \\ \vdots \\ \frac{\partial f}{\partial z_{m+1}} \end{Bmatrix} \tag{21}$$

$$[C] = \begin{bmatrix} \frac{\partial g_1}{\partial y_1} & \cdots & \frac{\partial g_1}{\partial y_{n-1}} \\ \vdots & \ddots & \vdots \\ \frac{\partial g_{m+1}}{\partial y_1} & \cdots & \frac{\partial g_{m+1}}{\partial y_{n-1}} \end{bmatrix} \tag{22}$$

$$[D] = \begin{bmatrix} \frac{\partial g_1}{\partial z_1} & \cdots & \frac{\partial g_1}{\partial z_{m+1}} \\ \vdots & \ddots & \vdots \\ \frac{\partial g_{m+1}}{\partial z_1} & \cdots & \frac{\partial g_{m+1}}{\partial z_{m+1}} \end{bmatrix} \tag{23}$$

$$dY = \begin{Bmatrix} dy_1 \\ dy_2 \\ \vdots \\ dy_{n-1} \end{Bmatrix} \tag{24}$$

$$dZ = \begin{Bmatrix} dz_1 \\ dz_2 \\ \vdots \\ dz_{m+1} \end{Bmatrix} \tag{25}$$

If it is assumed that all the constraints of the vector X elements satisfy $g(X) = 0$, it must correspond to $dg = 0$ in order to maintain its feasibility in $X + dX$. If $dg = 0$ is in Equation 19, Equation 25 is obtained.

$$dZ = -[D]^{-1}[C]dY \tag{26}$$

If the change in the objective function caused by the change in the vector $\boldsymbol{X}$ specified in Equation 17 is written using the Equation 26, Equation 27 is obtained.

$$df(\boldsymbol{X}) = (\nabla_{\boldsymbol{Y}}^{T} f - \nabla_{\boldsymbol{Z}}^{T}[D]^{-1}[C])d\boldsymbol{Y} \tag{27}$$

In other words, the Generalized Reduced Gradient notation is presented in Equations 28 and 29.

$$\frac{df}{dY}(X) = G_R \tag{28}$$

$$G_R = \nabla_Y f - ([D]^{-1}[C])^T \nabla_Z f \tag{29}$$

The Generalized Reduced Gradient vector is shown in Equation 29. Geometrically, the GRG can be called the projection of the original n-dimensional gradient defined over the (n-m) dimensional feasible region by the basic variables.

If an unconstrained multivariable problem is minimized, its gradient must be set to zero to reach the optimum (Erdoğan and Alptekin, 2006). Similarly, for a constrained problem, when its reduced gradient equals zero, it will take its smallest value. This situation can be justified in the same way that the minimum value of the problem satisfies the Kuhn – Tucker conditions. The Generalized Reduced Gradient (G_R) can be used to generate the search direction (S) to reduce the value of the constrained objective function, similar to the ∇f gradient that can be used to generate a search direction (S) for an unconstrained function. A suitable step length (λ) value should be chosen to minimize the (f) value along the search direction (S). For each λ value, the dependent variable vector Z is updated using Equation 26. Equation 26 also determines the step size with a linear approach to the nonlinear problem, so constraints in λ values may not be found exactly equal to zero ($dg \neq 0$). When Y kept constant, Equation 30 is obtained. If dg in Equation 30 is replaced in Equation 19, the following equation is obtained. The dZ value obtained by Equation 31 is used to update the Z value as given in Equation 32.

$$g(X) + dg(X) = 0 \tag{30}$$

$$dZ = [D]^{-1}(-g(X) - [C]dY) \tag{31}$$

$$Z_{update} = Z_{current} + dZ \tag{32}$$

The updated X vector is tested in constraints and the process of using Equation 31 is continued until the dZ value is sufficiently small. For Equation 31 dZ, Newton's equations can be considered as a method of solving simultaneously. The algorithm of the GRG method is given below (Rao, 2009: 416–418).

1. *Basic and non-basic variables are determined:* First, initial trial vector X is determined. Using the guidelines below, the basic and non-basic variables Y and Z of the problem are determined. The non-basic variables matrix $[D]$ is determined such that its determinant is not 0. Since the elements of the X matrix will be adjusted through iterative processes to provide the feasible region, any element of X equal to the lower and upper bounds of the problem should first be taken as the basic

variable. Slack variables added to the constraints to ensure equality, since they are linear terms, should be taken as non-basic variables. But if the initial value of any nonbasic variable is 0 (the lower limit of the slack variable), it should be taken as the basic variable.

2. *Calculate the generalized reduced gradient:* The GRG is determined using Equation 29. The derivatives in Equation 29 can be taken as numbers when necessary.
3. *Test for convergence:* If all components of the GRG are close to zero, it is considered to be sufficiently convergent and the current X vector is considered to be the optimum solution to the problem. The following test given in Equation 33 can be used for this.

$$||G_R|| \leq \varepsilon \tag{33}$$

where ε is a small number. If this relation is not satisfied, proceed to Step 4.

4. *Determine the search direction:* The GRG can be used to find the appropriate search direction (S) as if the gradient of objective functions of unconstrained problems was found. Techniques such as Steepest Descent, Fletcher – Reeves, Davidon – Fletcher – Powell or Broydon – Fletcher – Goldfarb – Shanno can be used here. In the steepest descent method, the S vector is determined as follows (Equation 34).

$$S = -G_R \tag{34}$$

5. *Find the minimum along the search direction:* An estimate is made for the step length (λ). When considering the basic variables Equation 35 is used and the ith element of the S vector s_i represents. Similarly, considering the non-basic variables, the Equation 36 is obtained by using $dY = \lambda S$ in Equation 26.

$$\lambda = \begin{cases} \frac{y_i^{(u)} - (y_i)_{old}}{s_i} & if\ s_i > 0 \\ \frac{y_i^{(l)} - (y_i)_{old}}{s_i} & if\ s_i < 0 \end{cases} \tag{35}$$

$$T = -[D]^{-1}[C]S \tag{36}$$

Equation 37 is used and t_i represents the i'th element of the T vector.

$$\lambda = \begin{cases} \frac{z_i^{(u)} - (z_i)_{old}}{t_i} & if\ t_i > 0 \\ \frac{z_i^{(l)} - (z_i)_{old}}{t_i} & if\ t_i < 0 \end{cases} \tag{37}$$

The λ_1 value obtained from Equation 35 can enable some basic variables to reach their upper or lower limits. Similarly, the value of λ_2 obtained from Equation 37 can make some non-basic variables reach their lower or upper limits. The smaller of λ_1 or λ_2 can be used as the upper limit λ value to initiate a one-dimensional minimization process. The vector $\boldsymbol{X}_{new}$ is found using Equation 38.

$$X_{new} = \begin{Bmatrix} Y_{old} + dY \\ Z_{old} + dZ \end{Bmatrix} = \begin{Bmatrix} Y_{old} + \lambda^* S \\ Z_{old} + \lambda^* T \end{Bmatrix} \tag{38}$$

If the vector X_{new} found with the λ^* step is not feasible, Y_{new} is kept constant and the modified Z_{new} is obtained by using Equation 31 and $dZ = Z_{new} - Z_{old}$. Finally, when the convergence is proven using Equation 31, the following X_{new} (Equation 39) is obtained and goes to Step 1.

$$X_{new} = \begin{Bmatrix} Y_{old} + \Delta Y \\ Z_{old} + \Delta Z \end{Bmatrix} \tag{39}$$

To solve the tankering problem proposed in this study, MS Excel solver plugin based on GRG2 algorithm written by Lasdon et al. (1978) was used. The written GRG2 code has been used for nonlinear programming models for years; It has proven to be one of the robust and reliable approaches.

The standard Microsoft Excel Solver has a limit of 200 decision variables and 100 constraints (in addition to the limits on variables) for nonlinear problems. Premium Solver Platform can run up to 500 decision variables and 250 constraints for nonlinear problems. The LSGRG code written by Smith and Lasdon (1992) for large-scale problems is based on a powerful GRG method. The solver they developed uses sparse matrix storage methods, advanced techniques to select the basis or prevent degenerations, "crashing" collision methods to quickly reach a feasible solution, and other algorithmic methods adapted for large problems. LSGRG solver can solve problems with 12,000 variables, 12,000 constraints, including the boundaries of the variables (Smith and Lasdon, 1992).

There are studies in the literature that develop a new optimization approach inspired by the GRG method or make changes, additions to some steps of this method. Rudd et al. (2013) studied problems dealing with the optimum state and control course for a multi-scale dynamic system composed of many dynamic systems or vehicles. They presented a Generalized Reduced Gradient Method for problems with stochastic differential equations defined as small intermediary systems with distributed optimal control points.

Rudd et al. (2017) inspired by the GRG method and developed an indirect method that optimizes for very large robotic systems and dispersed optimal control points in complex environments. They showed that the GRG method is significantly more efficient than classical optimal control methods with their complex calculations. They have used their proposed method for very large robotic systems to navigate environments where there are obstacles, and they are exposed to external forces and abuses. The result of the study shows that the method significantly improves its performance compared to current direct distributed optimal control and stochastic gradient methods.

Toplu and Körpe (2018) have developed an optimization solver using the GRG method. The developed model is applied to a problem and compared with different solvers. According to their study, the developed solver 700 seconds faster than the sequential second order solver in order to solve a problem.

4. PROBLEM DEFINITION and FORMULATION

4.1. Problem Statement

The fact that aviation fuel prices have different prices in different centers has been seen as an opportunity to reduce the fuel cost, which has the largest share in the costs of airline companies. In order to control fuel costs marginally and to save money, a fuel management problem that deals with reducing the cost by purchasing extra fuel from where the fuel is cheap has been tried to be solved. In this study, a nonlinear programming model aimed at minimizing the fuel cost was developed in order to determine the amount of tankering that is optimized to the extent permitted by the performance and capacity of the aircraft for airline companies that operate flights to airports where fuel prices vary. In order to solve the proposed model, an application was performed with MS Excel Solver using the Generalized Reduced Gradient Method (GRG).

The nonlinear programming model was used in the study because it was necessary to calculate the cost of the fuel that was still in the aircraft before it refueled when it landed at a different location. This dependent variable, abbreviated as c_i', was calculated with the weighted average technique as in Equation 47. This approach, which is used to improve the accuracy of the cost computation in the problem, has been tested to produce the most precise results. On the other hand, as in many studies in the literature (Stroup and Wollmer, 1992; Fregnani et al. 2013; Deo et al. 2020), it is seen that tankering problems can be modelled and solved by linear programming model.

In this study, firstly, different jet fuel prices, different flight distances and different cruising altitude levels were applied to the model and the amount of savings by tankering was tried to be parametrically tested. İstanbul has been accepted as the center for the other application of the study. Based on the aviation fuel prices dated 14.02.2020, it will be accepted that flights are organized to the cities of Bratislava, Florence, Athens, Sofia, and Baghdad, where the fuel price is higher than İstanbul, and to Tehran and Tripoli, where the air fuel price is lower. The application of the research problem and its results will be shown by trying to include the airline load factor, flight altitude, aircraft type, flight network structure, flight safety and limits determined by the relevant national and international laws and regulations.

4.2. Notation

The notations and objective function determined for the coefficients and variables to be used in the research problem model are as follows:

Decision variable

x_i: fuel amount loaded at i airport

Parameters

y_i: the amount of fuel remaining in the aircraft after landing from the i airport to the $i+1$ airport

c_i: fuel cost at airport i

c_i': the cost of remaining fuel from the previous flight in the aircraft fuel tank before refueling at airport i

p_i: payload weight loaded from airport i

f_{max}: fuel tank capacity

OEW: Operating Empty Weight - the total weight of the aircraft ready for service without fuel, cargo and passengers

$MTOW_i$: Maximum Take-off Weight of the aircraft from the airport i (calculated based on the flight distance from i airport to $i+1$ airport)

MLW: Maximum Landing Weight

$BLOCK_i$: total fuel weight of the aircraft in the parking position after refueling at airport i

$TRIP_i$: the estimated trip fuel weight spent from the i airport to the $i+1$ airport

$TRIPcont_i$: contingency fuel weight for trip loaded from i airport for $i+1$

HLD_i: The estimated holding fuel weight, which must be loaded with the regulation taken due to the traffic in the $i+1$ airport or for other reasons

ALT_i: fuel weight taken for a planned flight from airport i to another airport due to inability to land at airport $i+1$ or missed approach (alternate fuel)

$TAXI_i$: The estimated weight of fuel consumed during taxi from parking position to runway take-off point at airport i and from runway landed at $i+1$ airport to parking position

APU_i: The weight of fuel consumed to operate the APU in the parking position after loading fuel at airport i and after landing at the airport $i+1$ until the fuel is loaded in the parking position

$TANKER_i$: tankering fuel weight loaded from airport i to airport $i+1$ to minimize cost

$TANKERM_i$: structural maximum tankering capacity determined for the flight distance from airport i to airport $i+1$ and p_i.

4.3. Problem Modelling

The objective function (Equation 40) is used to minimize the fuel cost. Fuel cost calculation in objective function was created by considering the current fuel prices at the airport and the amount of fuel remaining in the aircraft after each flight. The main reason for using the nonlinear programming model is that the coefficient c_i' in the objective function is not linear.

$$Minimize\ Z = \sum_{i=1}^{N} c_i x_i + c_i' y_i \quad i = 1,2 \ldots .N \tag{40}$$

In Equation 41, the maximum take-off weight is determined according to the fuel estimation by considering the flight distance to the airport $i+1$. In cases where the flight distance is short, $MTOW$ may be higher. If the $MTOW$ amount determined by the aircraft manufacturer for short-haul flights is taken as a criterion and full capacity fuel is taken, fuel may have to be discharged to meet the structural landing weight before landing.

$$OEW + x_i + y_i + p_i \leq MTOW_i \tag{41}$$

The landing weight of the aircraft is specified in the manuals for each aircraft. The constraint in the Equation 42 is written by assuming that it consumes trip fuel until it is reduced from the take-off weight. The maximum structural landing weight here is valid for all flights.

$$OEW + x_i + y_i + p_i - (TRIP_i) \leq MLW \tag{42}$$

Adequate fuel constraint Equation 43 has been written to prevent the fuel taken on board exceeding the total capacity of the storage tanks.

$$x_i + y_i \leq f_{max} \tag{43}$$

$BLOCK_i$ i specified in Equation 45 is the amount of fuel that should be in the aircraft parking position at the airport after loading fuel. The inequality showing that the sum of the fuel already existing in the aircraft and taken at the airport to which the aircraft is flown is more than the $BLOCK_i$ is given in Equation 44.

$$x_i + y_i \geq BLOCK_i \tag{44}$$

$$BLOCK_i = TRIP_i + TRIPcont_i + HLD_i + ALT_i + TAXI_i + APU_i \tag{45}$$

In Equation 46, the calculation of the fuel remaining in the plane after landing at the airport $i+1$ is shown.

$$y_{i+1} = x_i + y_i - TRIP_i \tag{46}$$

The price calculation of the fuel remaining in the aircraft is shown in Equation 47. The cost of fuel remaining in the aircraft is calculated by calculating the weighted average.

$$c_i' = \frac{c_i x_i + (c_{i-1})(y_i)}{x_i + y_i} \tag{47}$$

Since it is assumed that the aircraft does not hold due to any missed approach or traffic, the fuel constraint that should be in the aircraft after landing at the airport $i + 1$ is shown in Equation 48.

$$y_{i+1} \geq ALT_i + HLD_i \tag{48}$$

In order to reach $i + 1$ airport using Equation 49, it is found how much more fuel will be taken on top of the required fuel specified in Equation 45.

$$TANKER_i = (x_i + y_i) - BLOCK_i \tag{49}$$

The amount of tankering loaded from airport i to airport $i + 1$ will vary depending on the structural weight constraints of the aircraft, the flight distance and the payload to be loaded. Therefore, the following Equation 50 tankering constraint is added to the model.

$$TANKER_i \leq TANKERM_i \tag{50}$$

Equation 51 is added to the model because the amount of fuel purchased at the airports ($i = 1,2,3 \ldots N$) is not negative.

$$x_i \geq 0 \quad i = 1,2,3 \ldots N \tag{51}$$

In order to show the research problem, a parametric experiment is first applied to the model. This model first flight distances of 300nm, 500nm, 700nm, 900nm, 1100nm, 1300nm and 1500nm; assuming the load factors as 70%, 80%, 90% and 100%; at 29.000ft, 31.000ft, 33.000ft, 35.000ft, 39.000ft flight levels of cruising flight altitude; fuel price differences are 50%, 60%, 70%, 80%, and 90%. Later, a real-life case is studied and it was tested by selecting centers with short and medium flight distances such as Bratislava, Athens, Sofia, Florence, Baghdad, Tehran, Tripoli where there are fuel price differences by taking İstanbul as the center.

MS Excel solver plug-in using GRG2 algorithm was used to solve the model. The convergence value is taken as = 0.000001. The computer used to solve the problems has a 64 bit operating system and 8 GB of RAM and the solution of the problems took an average of 0.5 seconds.

4.4. Problem Limitations

Every part of the aircraft needs to be changed and maintained after certain hours of flight. Increasing the weight carried in airplanes can cause more wear of some parts during landings and take-offs and additional maintenance costs. Maintenance costs due to increased weight and increased carbon dioxide emission tax costs due to tankering results in excess weight are not included in the model.

In the literature, in studies about tankering, the flight network was designed and optimized on a network basis (Abdelghany et al., 2005; Deo et al., 2020; Stroup and Wollmer, 1992). The airline network structure in Turkey is generally based on hub-and-spoke. The developed model is designed in a way that it may apply tankering optimization for n different destinations within the network structure determined by the airline. However, for the research, it is assumed that the airline company's network structure is hub-and-spoke form, so the flights between the two centers were considered as the origin and destination. Thus, it is assumed that the aircraft took off from the origin to destination and come back again to the origin.

The fuel estimation data to be included in the model is calculated using FCOM (Flight Crew Operating Manual). There is 200 kg. between weights in the tables referenced for the fuel estimates. By taking the average of these values, the fuel consumption estimation accuracy has been reduced to 50 kg. The application of the model is limited due to the fact that the variability of weather conditions cannot be reflected on the problem.

5. EXPERIMENTAL DESIGN

A319 type passenger aircraft is taken as a basis for the application of the study and some features of Airbus A319 are given in Table 2. First, it is assumed that the airports flown are at sea level and the air temperature is at standard atmospheric (ISA) conditions, that is, 15 ° C at sea level and the wind speed is 0 kt. The distances to the alternate airport have been taken as 100nm and the cruise flight level to alternate airport is 1500ft. Normally, flight altitude levels can change during cruise flight. However, in this study, in order to provide uniformity, it was assumed that the aircraft go to the destination point without changing the flight level after reaching the cruising flight altitude and the fuel estimation calculations in the FCOM were made accordingly. The conditions assumed by FCOM for fuel estimates are given below:

- The fuel estimates made for take-off were accepted as the speed of the aircraft increased as 250kt/300kt/M.78[5] respectively, from the moment the aircraft released the brake.
- In the fuel estimates made for cruise flight, it is assumed that the aircraft travels at a speed of M.78, the distance from the center of gravity to the aerodynamic center of the wings is 33%, the air temperature ISA, the air conditioners are operating normally, the anti-ice system is turned off and the highest thrust is applied for cruise flight.
- The speed of the aircraft during landing is assumed to be M.78/300kt/250kt respectively.
- For the fuel estimates made for the alternate airport, take-off speed is 250kt/300kt/M.78, and landing flight speed is M.78/300kt/250kt, respectively, the distance of the center of gravity to the aerodynamic center of the wings is 33%, the air temperature ISA, the air conditioners are operating normally and -ice system is assumed to be closed.
- In the fuel estimates for holding, it is assumed that the speed of the green dot and the clean configuration (flaps and landing gear are in the stowed position). Green dot speed gives the best lift and resistance ratio when all motors are running (De Baudus and Castaigns 2016). At the same time in holding, it is assumed
- that the distance of the center of gravity from the aerodynamic center of the wings is 33%, the air conditioners are operating normally and the anti-ice system is turned off. The fuel allocated for holding is planned to wait 30 minutes at an altitude of 1500ft above the alternate airport in the aircraft operations of ICAO's Annex 6, Article 4.3.6.3. (ICAO, 2018:4-12).
- Contingency fuel for the possibility of deviation from the planned route during the cruise is taken as 5% of the cruise fuel according to Annex 6 standards (ICAO, 2018: 4-11).
- It has been assumed that the aircraft spend an average of 12 minutes for taxi maneuvers. During the taxi, it is stated that the aircraft consumes 10kg of fuel per minute.
- It has been stated that the fuel consumption will be 130kg per hour while the APU is running on the ground. The time A319 spends on the ground is given as approximately 100 minutes (Schonland, 2019). In the study, it has been assumed that the aircraft operate APU for 1 hour at the airport.

Table 2. Some features of Airbus A319

Airbus A319 Type of Weight Limitations	*Weight Limits*
Engine Type	CFM-56-5B
Maximum Takeoff Weight (MTOW)	68.000 kg
Maximum Landing Weight (MLW)	61.000 kg
Maximum Zero Fuel Weight (MZFW)	57.000 kg
Minimum Weight	35.400 kg
Operational Empty Weight (OEW)	41.400 kg
Maksimum Fuel Capacity (MAXF)	18.729 kg

Source: Airbus (2005)

A319 type aircraft produced by Airbus company was used in the study. It has similar characteristics with A319, A320, and A321 types belonging to the A320 family. The characteristics of the model used in this study of A319, which is a member of the A320 family, which is the second most used passenger plane used since 1988. To find the operational empty weight, the seat, cabin systems, fire tubes, etc. that the airplane operator adds to the aircraft for operation. The total weight of such tools is taken as 6000 kg. Operational empty weight may vary depending on the seat design of the airline company and many factors, and in this study, 41,000 kg was accepted in this study. The number of seats in the FCOM used is given as 156. The total weight of a passenger and his/her baggage is considered to be 100 kg. It was assumed that the airline did not carry any cargo other than the passenger baggage on the plane and the payload weights were determined according to the load factors.

In the case study part of the study, a scenario (Table 3) was developed by considering the possible flights of an airline company using İstanbul Airport as a central airport (hub). It has been assumed that flights will be made to 7 different airports from İstanbul where jet fuel prices and distances are different. Fuel prices dated 14.02.2020 were used (aeroportos.weebly.com, 2020). The distances of the flights are determined by the routes created from the skyvector.com site according to the city from which they depart

---

[5] The abbreviation M stands for Mach number and is the ratio of the velocity of a mass in motion to the velocity of sound under the conditions of existing gravity. When M = 1, the speed of the airplane becomes equal to the speed of sound. M.78 means 78% of the speed of sound.

from İstanbul and can be seen in Annex 7-13 (skyvector.com, 2020). The departure and return flight distances of the planes are considered equal. Jet A1 fuel density is taken 0.785 kg/ L, thus the price per kilogram of Jet fuel has been obtained.

Table 3. Flight information to be used in the scenario

Airport	*IATA Code*	*ICAO Code*	*Fuel Price (USD/kg)*	*Distance (nm)*
İstanbul	ISL	LTFM	0.72	-
Bratislava	ISL - BTS	LZIB	1.94	645.1
Florence	ISL - FLR	LIRQ	1.65	907.4
Athens	ISL - ATH	LGAV	1.70	317.8
Sofia	ISL - SOF	LBSF	1.54	257.6
Teheran	ISL - THR	OIII	0.63	1122.7
Baghdad	ISL - SDA	ORBI	1.85	955.4
Tripoli	ISL - MJI	HLLM	0.45	995

It has been deemed to be a fuel supplier for the airports to be operated. It is assumed that there is no restriction on the amount of fuel to be purchased from the airports. Each center has the capacity to provide sufficient fuel supply. It has been accepted that there are no emergency incidents, missed approach, holding due to traffic and landing at an alternate airport during the planned flights. Therefore, it will be assumed that emergency, contingency, alternate aerodrome, and holdings fuels remain in the fuel tank after the aircraft lands. Load factor may vary depending on the flights made by the airline companies, the business model applied and the demand. According to the report of IATA, the annual occupancy rate for domestic flights for the whole world is 83.8% and the annual occupancy rate for international flights is 85.5% (IATA, 2019b). Since the passenger aircraft is considered in this study, the load factor is accepted as 70%, 80%, 90%, and 100%. The take-off weight of the aircraft was determined according to the load factors selected (Table 4).

5.1. Forecasting Amount of Flight Fuel Burn

A flight plan was made and it was estimated how much fuel the aircraft would consume according to the distance and weight to be traveled, before putting the model created for the research problem into the solver, FCOM (Flight Crew Operations Manual) prepared by the aircraft manufacturer was used for the estimation.

Table 4. Total fuel to be loaded by distances (FL290)

Load Factor/ Flight Distance	*Total Fuel Amount to Be Loaded (kg)*			
	70%	*80%*	*90%*	*100%*
300 nm	4.726	4.858	4.908	5.055
500 nm	6.091	6.189	6.267	6.315
700 nm	7.404	7.502	7.548	7.575
900 nm	8.519	8.553	8.600	8.835
1100 nm	9.768	9.900	10.069	10.096
1300 nm	11.047	11.283	11.329	11.386

The take-off weight limits of the aircraft will be determined according to the distance to be flown, before determining the amount of tankering. Since if the plane takes the highest take-off weight as the limit and reaches the destination with full capacity fuel, the landing weight of the plane will be above the limit when the plane reaches the destination will have to dump fuel on the airport where it will land. Fuel consumption estimation was made by using the table in FCOM (Table 5).

Table 5. The amount of fuel required for various flights from İstanbul (FL330)

	Total Fuel Amount to Be Loaded (kg)			
Load Factor / Flight Distance from İstanbul	*70%*	*80%*	*90%*	*100%*
Sofia (257nm)	4.463	4.509	4.588	4.634
Athens (317nm)	4.779	4.876	4.923	5.002
Bratislava (645nm)	6.616	6.714	6.813	6.840
Florence (900nm)	7.982	8.132	8.230	8.258
Baghdad (955nm)	8.413	8.500	8.598	8.625
Tripoli (995nm)	8.560	8.710	8.809	8.888
Tehran (1122nm)	9.365	9.419	9.491	9.570

Take-off weight was determined according to different aircraft load factors, different flight distances and different cruise flight altitude levels and then fuel consumption was estimated. When flying in FL290, the total amount of fuel that the aircraft should take according to different flight distances and load factors is given in Table 4. After the amount of tankering was determined by processing the data into the model and solving the problem, a further fuel consumption estimation was made to see the effect of carrying extra fuel on fuel consumption.

The fuel calculations made for given so far are at 300 nm, 500 nm, 700 nm, 900 nm, 1100 nm, 1300 nm flight distances, FL290, FL310, FL330, FL350, and FL390 cruise flight altitude levels and 70%, 80%, 90%, and 100% load factors. In addition, the same flight levels and load factors is used to calculate for flights to seven destinations, taking İstanbul as the center. In Table 5, the total amount of fuel required to be loaded in flights to different centers in FL330 is given.

6. EXPERIMENTAL ANALYSIS

6.1. Problem Solving

As mentioned in the research problem, the flight is planned to start from the center, go to another destination and return to the central airport. In Equation 54, the ratio of the fuel price at the airport of origin (c_1) to the fuel price at the destination airport (c_2) is calculated as a percentage.

$$100\ X\ \left(\frac{c_1}{c_2}\right) \tag{54}$$

According to the model given when starting the solution of the problem, the amount of fuel remaining in the aircraft at the airport (1st airport) that flight comes from unknown airport accepted as comes from previous airport that aircraft will be land. Thus $y_1 = ALT_2 + HLD_2$ equation added the model and while calculating c_1', c_0 assumed as equals to c_2 $(c_0 = c_2)$ then c_1' was found as in Equation 55.

$$c_1' = \frac{c_1x_1 + c_2y_1}{x_1 + y_1} \tag{55}$$

Fuel consumption was determined in scenarios created for flights with different load factors and at different distances. The model of the problem, which aims to minimize the aircraft fuel cost by tankering in flights between centers with different fuel prices, has been solved in MS Excel by taking into account the aircraft limits given in Table 2.

During the solution of nonlinear problem, two different results were encountered one of them allows tankering and the other does not allow tankering. Two answer report examples are examined. First, in Table 6, it is stated that the tankering is allowed at 900nm flight distance and at cruise flight altitude 33000ft. An answer report was given for the problem with 70% load factor and 50% fuel price difference. As can be seen in the problem answer report given in Table 6, the model allows the aircraft to buy 10,641 kg of fuel at the airport of origin and only 1,198 kg of fuel on return. It appears that all of the tankering capacity is used. According to the response report, the first of the binding constraints is the constraint on the total amount of fuel that must be taken from the destination airport to the airport of origin. The BLOCK1 restrictor is non-binding because there is 4721 kg capacity freedom from origin to destination. It reaches the upper limit of tankering restrictor as seen in this amount.

As seen in Table 7, the Lagrange multipliers of the binding constraints in the model are given. The Lagrange multiplier tells how the change of the constrained resource will affect the objective function value, in other words, it expresses the marginal value of the resource. In this solution, where tankering is allowed to be transported, an increase of 1 kg in the amount of fuel to be loaded on return from the destination will increase the total fuel cost by $ 2.24 for round trips. Likewise, 1kg increase in the tankering capacity of the aircraft will reduce the total fuel cost by approximately 0.6 $.

The solution to a problem solution that is not allowed tankering is given in Table 8. The problem scenario has been determined as 700 nm flight distance, cruise flight altitude 29,000 ft, fuel price difference 80% and load factor 70%.

Table 6. Problem answer report that allows tankering

Solution Time: 0.406 Seconds. Iterations: 1 Subproblems: 0			
Solver Options			
Max Time Unlimited, Iterations Unlimited, Precision 0,000001, Use Automatic Scaling Convergence 0.00001, Population Size 100, Random Seed 0, Derivatives Central, Require Bounds Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%, Assume NonNegative			
Objective Cell (Min)			
Original Value	*Final Value*		
26344,2	19171.07445		
Variable Cells			
Name	*Original Value*	*Final Value*	*Integer*
x1	15000	10641.67	Contin
x2	1198.334	1198.334	Contin
Constraints			
Name	*Cell Value*	*Status*	*Slack*
MTOW1	65023.9	Not Binding	776.1
MTOW2	60302.234	Not Binding	5497.766
MLW1	59103.9	Not Binding	1896.1
MLW2	54382.234	Not Binding	6617.766
MFW1	12703.9	Not Binding	6025.415
MFW2	7982.234	Not Binding	10746.766
BLOCK1	12703.9	Not Binding	4721.666
BLOCK2	7982.234	Binding	0
LEFT1	6783.9	Not Binding	4721.666
LEFT2	3102.234	Not Binding	1040
TANKER	4722	Binding	0

Table 7. Problem sensitivity report allowing tankering

Adjustable Cells		
Name	*Final Value*	*Reduced Gradient*
x1	10641.67	0
x2	1198.334	0
Constraints		
Name	*Final Value*	*Lagrange Multiplier*
MTOW1	65023.9	0
MTOW2	60302.23	0
MLW1	59103.9	0
MLW2	54382.23	0
MFW1	12703.9	0
MFW2	7982.234	0
BLOCK1	12703.9	0
BLOCK2	7982.234	2.245887
LEFT1	6783.9	0
LEFT2	3102.234	0
TANKER	4721.666	-0.59903

Problem answer report example in Table 8 shows that not to allow tankering. Since factors affecting the fuel consumption such as flight distance, altitude, load factor, and aircraft fuel burning rate of the departure are thought to be the same with return flights, the optimum fuel amount to be taken from the origin and destination airports were equal. After the total fuel constraints to be loaded on the aircraft (BLOCK1, BLOCK2) have been satisfied, no more fuel was allowed to be loaded to keep the cost low. Table 9 shows Lagrange multiplier values and reduced gradient values of the binding constraints. The higher destination fuel price in the problem caused the Lagrange multiplier values of the total fuel constraints to differ. While 1 kg increase in the total amount of fuel required to be loaded at the destination affects the cost 1.67 $, this value is 0.62 $ at the origin. In the origin, the fuel constraint in the aircraft before the start of the flight was created by assuming that the aircraft came from the 2nd airport. The price of the remaining fuel in tank was effective in finding the optimum result, as it was calculated by weighted average calculation.

Table 8. Problem answer report that not allows tankering

Solution Time: 0.656 Seconds.			
Iterations: 3 Subproblems: 0			
Solver Options			
Max Time Unlimited, Iterations Unlimited, Precision 0,000001, Use Automatic Scaling			
Convergence 0.00001, Population Size 100, Random Seed 0, Derivatives Central, Require Bounds			
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%, Assume NonNegative			
Objective Cell (Min)			
Original Value	*Final Value*		
34704.99	21991.16896		
Variable Cells			
Name	*Original Value*	*Final Value*	*Integer*
x1	10000	5342.5	Contin
x2	5342.499574	5342.5	Contin
Constraints			
Name	*Cell Value*	*Status*	*Slack*
MTOW1	59724.434	Not Binding	5675.566
MTOW2	59724.43399	Not Binding	5675.56601
MLW1	54381.934	Not Binding	6618.066
MLW2	54381.93399	Not Binding	6618.06601
MFW1	7404.434	Not Binding	11324.881
MFW2	7404.43399	Not Binding	11324.56601
BLOCK1	7404.434	Binding	0
BLOCK2	7404.43399	Binding	0
LEFT1	2061.934	Binding	0
LEFT2	3046.93399	Not Binding	984.9999899
TANKER	0	Not Binding	5029.566

Table 9. Problem sensitivity report not allows tankering

Adjustable Cells		
Name	*Final Value*	*Reduced Gradient*
x1	5342.5	0
x2	5342.5	0
Constraints		
Name	*Final Value*	*Lagrange Multiplier*
MTOW1	59724.43	0
MTOW2	59724.43	0
MLW1	54381.93	0
MLW2	54381.93	0
MFW1	7404.434	0
MFW2	7404.434	0
BLOCK1	7404.434	0.6187
BLOCK2	7404.434	1.675591
LEFT1	2061.934	1.110617
LEFT2	3046.934	0
TANKER	0	0

6.2. Analysis of Parametric Solution Results

It has been investigated how tankering is affected by flight distances, fuel price differences, cruise flight altitude and different load factors with the scenarios and problems established parametrically. The findings are presented in Figure 1 and Figure 2.

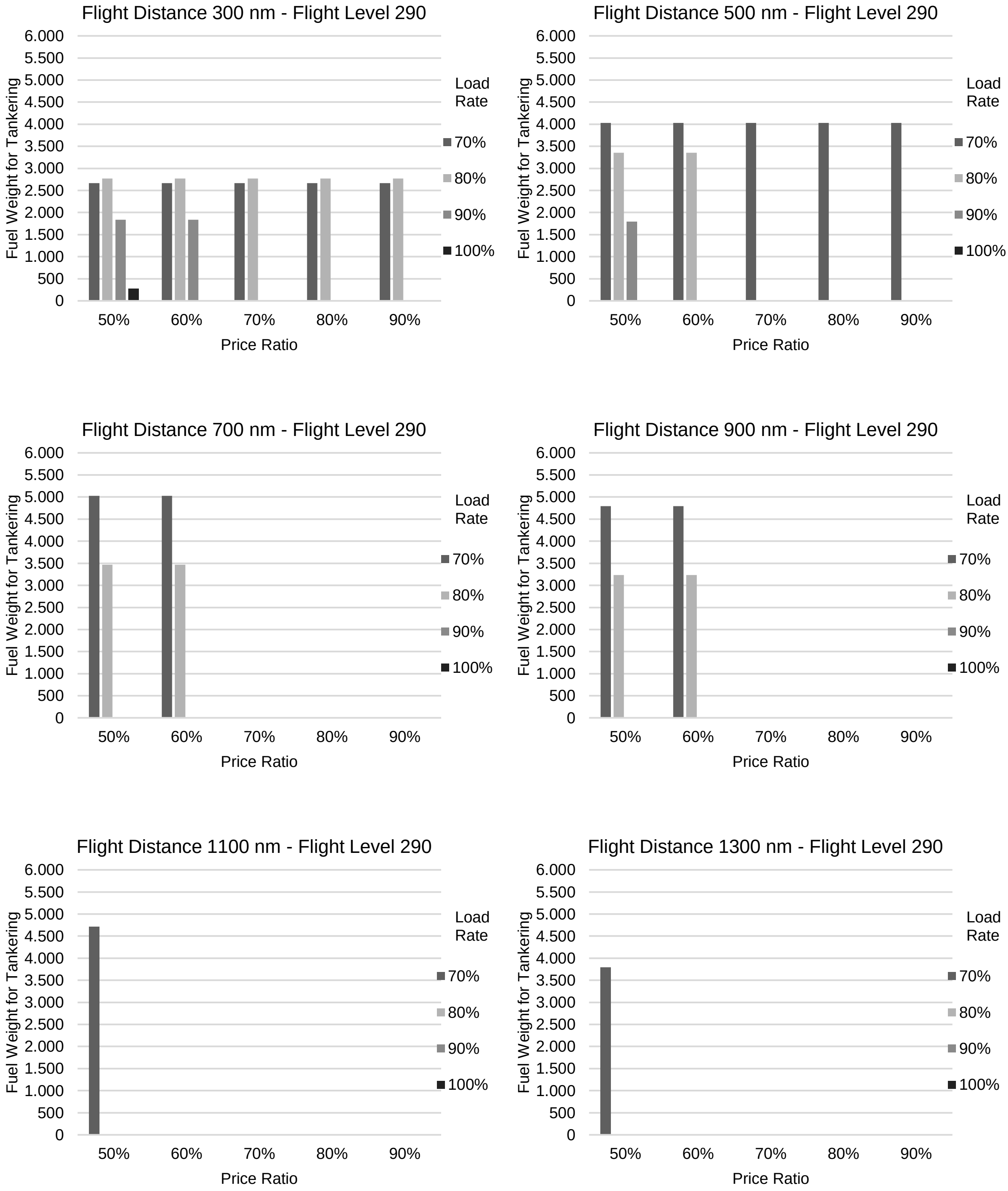


Figure 1. Tankering amount suggested by parametric problem solution for FL290

The maximum tankering capacity was occurred at a distance of 700 nm and the model suggested tankering in some scenarios depending on the appropriate fuel price differences and load factors. Figure 1 shows how much tankering will be done at 29,000 feet flight level, flight distance from 300 nm to 1300 nm at various load rates and different price ratios. In Figure 2, how much savings will be achieved in the specified scenarios is given for 29,000 feet flight level. The amount of tankering and savings realized at other flight levels are specified in the Appendix Table A1 and Table A2.

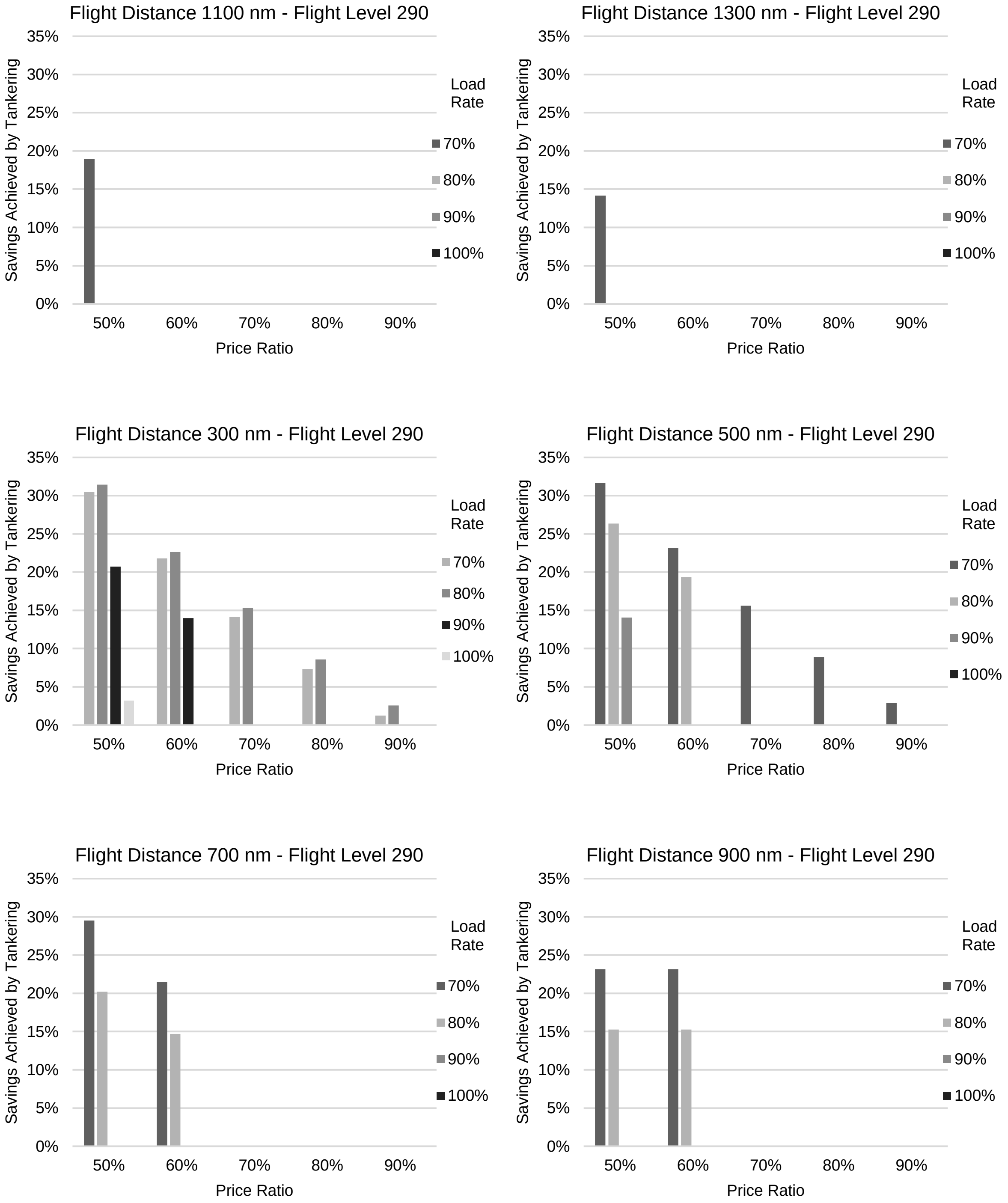
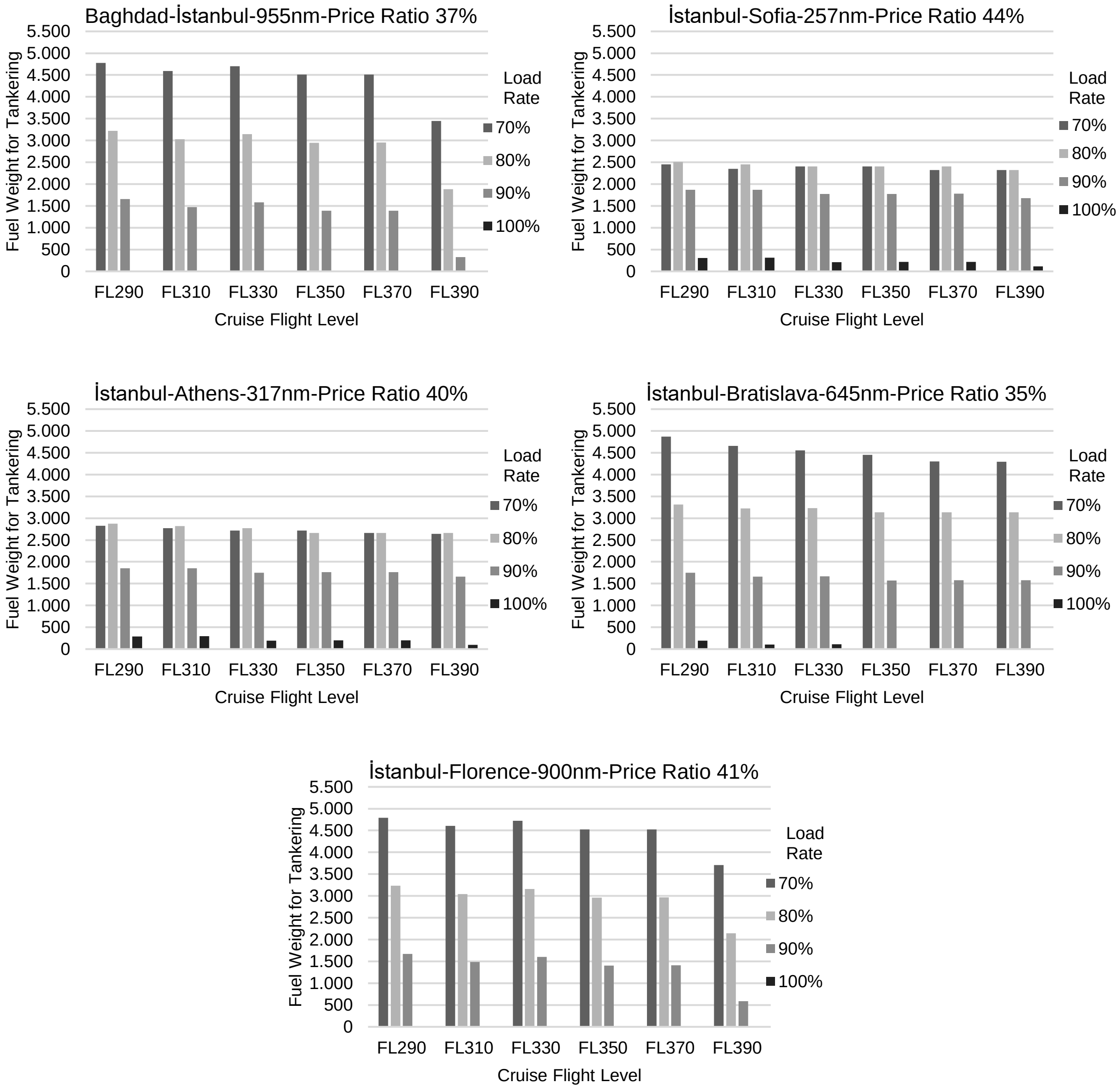


Figure 2. Savings achieved by tankering as a result of parametric problem solution for FL290

It has been observed that fuel consumption increased due to the increased weight resulting from tankering. Increasing fuel consumption caused additional fuel costs. However, in Figure 2, how much the total fuel cost is saved as a result of tankering is calculated by deducting the additional cost caused by the increased weight after fuel transportation. It has been observed that the strategy of reducing the cost by tankering is achieved when the load factor is the lowest and the fuel price difference is the highest. It has been found that the best savings rate with slight differences varies depending on the flight distance when comparing different cruise flight altitudes. It has been determined that the most suitable altitude is 31,000 ft to save with tankering strategy on round trip flights where the flight distance is 300 nm and 500 nm, but when the flight distance reaches 700 nm, the optimum cruise flight altitude is 35,000 ft.

6.3. Analysis of the Problem Solution Results of the Scenario Takes İstanbul as Hub

Since geopolitical position of Turkey is advantageous in accordance with fuel prices compared to many centers and proximity to many centers in Europe and the Middle East where population density is more, tankering application centered on İstanbul has been implemented. Flights from İstanbul are simulated to Bratislava, Florence, Athens, Sofia, and Baghdad airports where the fuel price is more expensive and to Tehran and Tripoli, where fuel prices are cheaper given in Table 3. As in the parametric study, cruise flight levels were tested at 29.000ft, 31.000ft, 33.000ft, 35.000ft, 37.000ft and 39.000ft (Figure 3).


Baghdad-İstanbul-955nm-Price Ratio 37%

İstanbul-Sofia-257nm-Price Ratio 44%

İstanbul-Athens-317nm-Price Ratio 40%

İstanbul-Bratislava-645nm-Price Ratio 35%

İstanbul-Florence-900nm-Price Ratio 41%

Figure 3. Tankering amount suggested by İstanbul based flight network

As can be seen in Figure 3, it is recommended to fly from İstanbul by tankering even if the plane flies with 100% load factor to five centers where the difference in fuel price is not much and the flight distance is short. Tankering to Tehran and Tripoli, where the fuel price difference is less and the flight distance is higher, is not recommended for any load factor. Because of that their chart is not located in the figure. When the amount of extra fuel transport for flights with close flight distances is compared with the different level flight levels flown, it is seen that there is not much difference. Similar to the parametric studies given before, the highest tankering capacity was ocured while flying to Bratislava, whose flight distance was 645nm, and because of the fuel price ratio of 35% in all load factors, extra fuel could be carried. The fact that the price of fuel in İstanbul is more than twice as cheap as Sofia, Athens, Bratislava, Florence, and Baghdad indicates that tankering can be achieved in almost all scenarios. The model returned aircraft from the destination without loading any fuel, thanks to the excess fuel taken in short distances and low load factors.

Figure 4 shows how much savings have been achieved from the total fuel cost for the round trip with tankering on flights based in İstanbul compared to the flight with common fuel. When the table is examined, saving rates around 40% from the total fuel cost is seen on the trips when there are the least load factors and the highest price differences. It is seen that the most savings were achieved in the 645 nm distance Bratislava flight. Although there is no big difference between the levels of cruise flight, it can be said that the most suitable cruise flight altitude for saving with tankering is 33.000ft and 35.000ft. The amount of tankering and savings realized at other flight levels are specified in the Appendix Table A3 and Table A4.

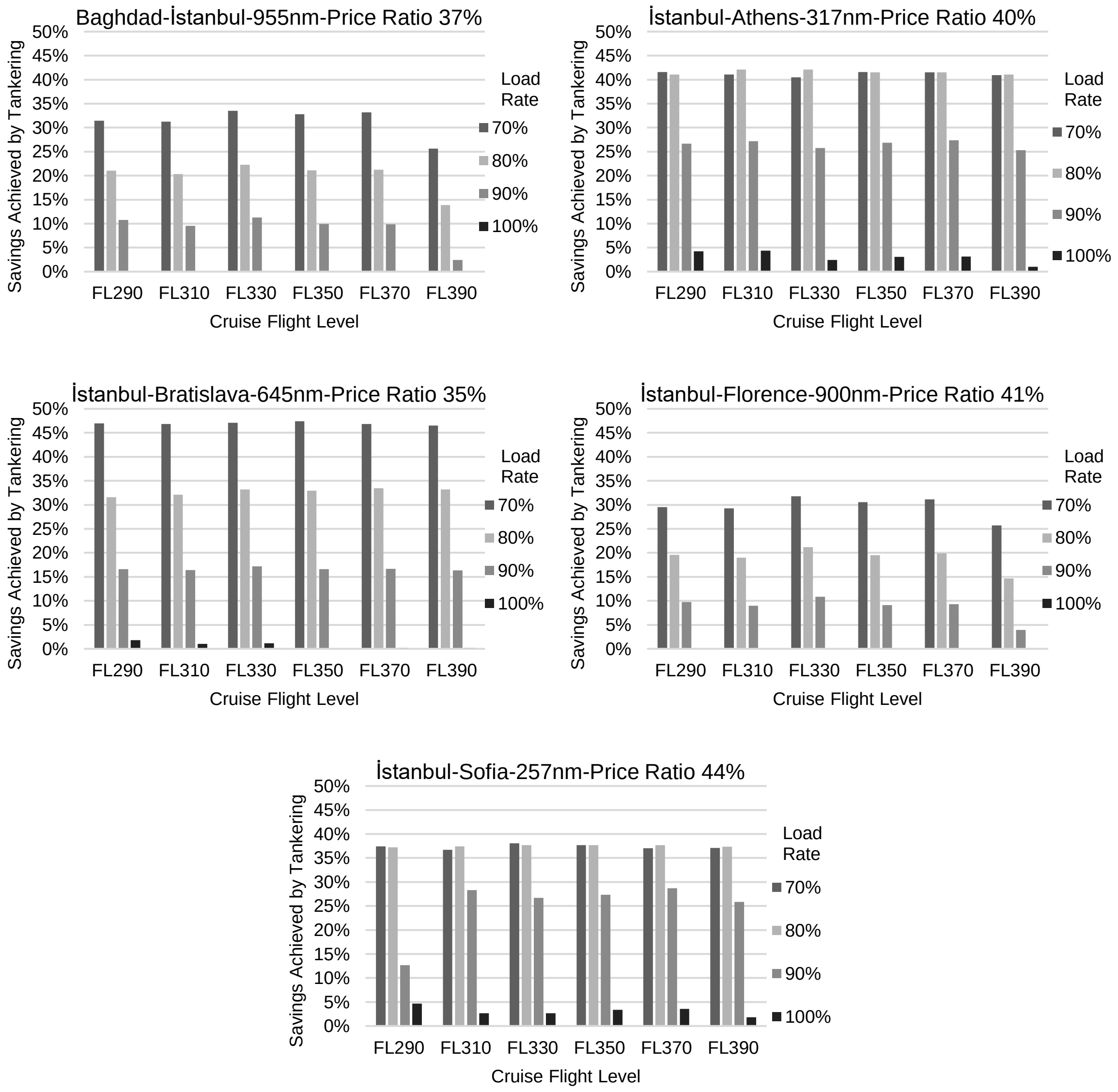


Figure 4. İstanbul based flight network problem solution resulting savings rates

7. CONCLUSION

Fuel cost is one of the major parts of the expenses in airline management. In this study, tankering operational application nonlinear programming problem is modeled and a flight planning application is proposed that will support the determination of optimum fuel purchases from each airport that will optimize the fuel cost for flights between centers with high fuel price differences. The GRG method was used to solve the problems, and a global optimal result was achieved in all of them. MS Excel plug-in was used as a solver.

In the experimental part of the study, scenarios were developed in order to evaluate the most appropriate conditions for tankering and the effect of flight distance, fuel price ratio, cruise flight altitude, and load factor on tankering was shown parametrically. If the fuel price ratio between the flight centers is over 50% and the load factor is over 70%, tankering for 1300nm and above flight distances will increase the total fuel cost. It has been observed that as the flight distance between the centers gets shorter, tankering is allowed even though the fuel price factors are low and the load factors are high. In cases where the flight distance is over 300nm, if the load rate is 80% and above and the fuel price ratio is 70% and above, tankering for these flights will increase the cost. When the problem solution results are examined, it is seen that the cruise flight altitude does not affect the amount of tanker. However, in terms of fuel consumption, it has been observed that the most efficient flight levels increase the amount of extra fuel transport.

Although it is observed that there is an average 3-3.5% increase in fuel consumption with the tankering proposed by the research problem, compared to the flights that will take place without tankering, it has been shown that savings can be made from 2% to 30% which the total fuel cost of the round trip flights is directly proportional to the increase in the fuel price difference and inversely proportional to the increase in the load factor.

The developed model has been applied on an İstanbul-based flight network scenario. This practice is important as there can be large fuel differences between the centers that are close. It has been observed that the model allows tankering at almost all load factors between these centers, as the price of fuel in İstanbul is twice or cheaper than those in cities such as Sofia, Athens, Florence, Bratislava, and Baghdad. With Tankering, it has managed to save between 2% and 47% of the total fuel cost of round trips.

As a result of the interpretation of the right-hand side values of the constraints with the Lagrange multiplier value during the solution of the problem, it has been observed that more savings can be achieved when tankering is applied with efficient and high load carrying capacity aircraft that consume less fuel. Airline companies will be able to save more on fuel costs with tankering, as the new generation aircraft produced have these features.

Airbus A319 aircraft was taken as the basis in the application of this study. FCOM developed by the aircraft manufacturer was used for fuel estimation. In order to reach more accurate results, software that makes more precise fuel estimation calculations from FCOM can be used and airlines can develop their flight planning software themselves without outsourcing.

In the scenario, factors such as various weather conditions, wind, airport elevation, flight level change during cruise flight have been ignored as they increase the complexity of the operation. In addition, it was assumed that flight network is hub&spoke and flights are round trips. Although this default situation is in line with the network structure of traditional business model airline companies, the network structures of low cost, charter, regional airlines and air cargo carriers may differ. In addition to this research, the effect of assumed weather conditions can be emphasized while estimating aircraft fuel consumption for tankering studies. Lastly, since the established model will meet the needs of all network structures, it can be applied to network structures of a low-cost airline, charter airline, regional airline and air cargo carrier for optimization of tankering.

Yazar Katkıları /*Author Contributions*

Niyazi Cem Gürsoy: Literatür taraması, Modelleme, Kavramsallaştırma, Metodoloji, Veri Derleme, Analiz, Makale Yazımı-orijinal taslak *Nesrin Alptekin*: Kavramsallaştırma, Metodoloji, Makale Yazımı-inceleme ve düzenleme

Niyazi Cem Gürsoy: Literature review, Modelling Conceptualization, Methodology, Data Curation, Analysis, Writing-original draft *Nesrin Alptekin*: Conceptualization, Methodology, Writing-review and editing

Çatışma Beyanı /*Conflict of Interest*

Yazarlar tarafından herhangi bir potansiyel çıkar çatışması beyan edilmemiştir.

No potential conflict of interest was declared by the author

Fon Desteği / *Funding*

Bu çalışmada herhangi bir resmi, ticari ya da kâr amacı gütmeyen organizasyondan fon desteği alınmamıştır.

Any specific grant has not been received from funding agencies in the public, commercial, or not-for-profit sectors.

Etik Standartlara Uygunluk / Compliance with Ethical Standards

Yazarlar tarafından, çalışmada kullanılan araç ve yöntemlerin Etik Kurul izni gerektirmediği beyan edilmiştir.
It was declared by the authors that the tools and methods used in the study do not require the permission of the Ethics Committee.

Etik Beyanı / ***Ethical Statement***

Yazarlar tarafından bu çalışmada bilimsel ve etik ilkelere uyulduğu ve yararlanılan tüm çalışmaların kaynakçada belirtildiği beyan edilmiştir.
It was declared by the authors that scientific and ethical principles have been followed in this study and all the sources used have been properly cited.

REFERENCES

Abadie, J. and Carpentier, J. (1969). "Generalization of the Wolfe Reduced Gradient Method to the Case of Nonlinear Constraints", *Optimization*, 37-47.

Abdelghany, K., Abdelghany, A. and Raina, S. (2005). "A Model for the Airlines' Fuel Management Strategies", *Journal of Air Transport Management*, 11(4), 199-206, DOI: 10.1016/j.jairtraman.2004.10.002.

aeroportos.weebly.com. (2020). "Fuel Prices", http://aeroportos.weebly.com/fuel-prices.html#.XfdHkugzbIW (Access Date: 14.02.2020).

Airbus (2004). "Getting to Grips with Fuel Economy", https://ansperformance.eu/library/airbus-fuel-economy.pdf, (Access Date: 06.03.2020).

Airbus (2005). "A319-Airplane Characteristics for Airport Planning", https://www.airbus.com/sites/g/files/jlcbta136/files/2021-11/Airbus-Commercial-Aircraft-AC-A319.pdf, (Access Date: 06.03.2020).

Arora, J.S. (2017). "Introduction to Design Optimization", Elsevier, Waltham. DOI: 10.1016/b978-0-12-800806-5.00001-9.

Darnell, D.W. and Loflin, C. (1977). "National Airlines Fuel Management and Allocation Model", *Interfaces*, 7(2), 1-6, DOI: 10.1287/inte.7.2.1.

De Baudus, L. and Castaigns, P. (2016). "Control Your Speed… in Cruise", *Safety First*, 9-21.

Deo, V.A., Silvestre, F.J. and Morales, M. (2020). "The Benefits of Tankering Considering Cost Index Flying and Optional Refuelling Stops", *Journal of Air Transport Management*, 82, DOI: 10.1016/j.jairtraman.2019.101726.

Erdoğan, N.K. and Alptekin, N. (2006). "Lineer Olmayan Programlama Problemleri". Anadolu Üniversitesi Yayınları, Eskişehir.

EUROCONTROL (2019). "Fuel Tankering in European Skies: Economic Benefits and Environmental Impact | EUROCONTROL", https://www.eurocontrol.int/publication/fuel-tankering-european-skies-economic-benefits-and-environmental-impact (Access Date: 30.12.2019).

Frank, S., Steponavice, I. and Rebennack, S. (2012). "Optimal Power Flow: A Bibliographic Survey I", *Energy Systems*, 3, 221-258, DOI: 10.1007/s12667-012-0056-y.

Gerede, E. and Orhan, G. (2015). "Türk Havayolu Taşımacılığındaki Ekonomik Düzenlemelerin Gelişim Süreci", *Türk Havayolu Taşımacılığındaki Ekonomik Düzenlemelerin Gelişim Süreci*, 169-170, Republic of Turkey Directorate General of Civil Aviation, Ankara.

Guerreiro Fregnani, J.A.T., Müller, C. and Correia, A. R. (2013). "A Fuel Tankering Model Applied to a Domestic Airline Network", *Journal of Advanced Transportation*, 47(4), 386-398, DOI: 10.1002/atr.162.

Hubert, T., Guo, C., Mouton, C.A. and Powers, J. D. (2015). "Tankering Fuel on U.S. Air Force Transport Aircraft: An Assessment of Cost Savings" RAND Corporation, California.

IATA. (2019a). "Fuel Fact Sheet", https://www.iata.org/contentassets/ebdba50e57194019930d72722413edd4/fact-sheet-fuel.pdf (Access Date: 08.07.2019).

IATA. (2019b). "Air Passenger Market Analysis", https://www.iata.org/contentassets/08ea7f485007409d998575257c822ee9/passenger-analysis-jun-2019.pdf (Access Date: 06.03.2020).

ICAO (2018). "Annex 6 - Operation of Aircraft-Part I-International Commercial Air Transport - Aeroplanes" International Civil Aviation Organization, Montréal.

Lasdon, L.S., Waren, A.D., Jain, A. and Ratner, M. (1978). "Design and Testing of a Generalized Reduced Gradient Code for Nonlinear Programming", ACM Transactions on Mathematical Software (TOMS), 4, 34-50, DOI: 10.1145/355769.355773.

Lasdon, L.S., Fox, R.L. and Ratner, M.W. (1974). "Nonlinear Optimization Using The Generalized Reduced Gradient Method", Revue Française D'automatique, İnformatique, Recherche Opérationnelle. *Recherche Opérationnelle*, 3(8), 73-103, DOI: 10.1051/ro/197408V300731.

Nocedal, J. and Wright, S.J. (2006). "Numerical Optimization", 2, Mikosch, T.V., Resnick, S.I., Robinson, S.M., Springer Science and Business Media, New York, DOI: 10.1201/b19115-11.

Rao, S.S. (2009). "Engineering Optimization: Theory and Practice: Fourth Edition", In Engineering Optimization: Theory and Practice (Fourth Edi). John Wiley & Sons, New Jersey, DOI: 10.1002/9780470549124.

Rudd, K., Foderaro, G. and Ferrari, S. (2013). "A Generalized Reduced Gradient Method for the Optimal Control of Multiscale Dynamical Systems", *Proceedings of the IEEE Conference on Decision and Control*, 3857–3863. DOI: 10.1109/CDC.2013.6760478.

Rudd, K., Foderaro, G., Zhu, P. and Ferrari, S. (2017). "A Generalized Reduced Gradient Method for the Optimal Control of Very-Large-Scale Robotic Systems", *IEEE Transactions on Robotics*, 33(2), 1226-1232. DOI: 10.1109/TRO.2017.2686439

Rutner, S. and Brown, J. (1999). "Outsourcing as an Airline Strategy*", Journal of Air Transportation World Wide*, 4(2), 23-31.

Schonland, A. (2019). "Total Ground Time – A Key Operational Metric for Airlines » AirInsight", https://airinsight.com/total-ground-time-a-key-operational-metric-for-airlines/ (Access Date: 13.03.2020).

skyvector.com. (2020). "SkyVector: Flight Planning / Aeronautical Charts", https://skyvector.com/ (Access Date: 16.12.2020).

Smith, S. and Lasdon, L. (1992). "Solving Large Sparse Nonlinear Programs Using GRG", *ORSA Journal on Computing*, 4(1), 2-15, DOI: 10.1287/ijoc.4.1.2.

Stroup, J.S. and Wollmer, R.D. (1992). "Fuel Management Model for the Airline Industry", *Operations Research*, 40(2), 229–237, DOI: 10.1287/opre.40.2.229.

Toplu, M.S. and Körpe, D.S. (2018). "Genel İndirgenmiş Gradyan Metodu İle Eniyileme Çözücüsü Geliştirilmesi", *Sakarya Üniversitesi Fen Bilimleri Enstitüsü Dergisi*, 22(3), 960–969, DOI: 10.16984/saufenbilder.298972.

Tuncer, B. and Aydoğan, K. (2019). "Yakit Maliyetlerinin Taşınan Hava Kargo Miktari Üzerine Etkileri: 2007-2018 Türkiye Örneği", *Uluslararası İktisadi ve İdari İncelemeler Dergisi*, Bor Özel Sayısı, 146–147. DOI: 10.18092/ulikidince.517774

Turkish Airlines (2019). "Turkish Airlines Board Activity Report", https://investor.turkishairlines.com/documents/ceyreklik-sonuclar/2019_yili_yk_faaliyet_raporu_en(2).pdf (Access Date: 03.07.2019).

APPENDIX

Table A1. Tankering amount suggested by parametric problem solution (kg)

	Cruise Flight Altitude	*FL290*					*FL310*					*FL330*					*FL350*					*FL370*					*FL390*				
		Price Ratio (%)																													
Flight Distance	*Load Factor (%)*	*50*	*60*	*70*	*80*	*90*	*50*	*60*	*70*	*80*	*90*	*50*	*60*	*70*	*80*	*90*	*50*	*60*	*70*	*80*	*90*	*50*	*60*	*70*	*80*	*90*	*50*	*60*	*70*	*80*	*90*
300 nm	70	2.665	2.665	2.665	2.665	2.665	2.718	2.718	2.718	2.718	2.718	2.665	2.665	2.665	2.665	2.665	2.665	2.665	2.665	2.665	2.665	2.560	2.560	2.560	2.560	2.560	2.560	2.560	2.560	2.560	2.560
	80	2.770	2.770	2.770	2.770	2.770	2.770	2.770	2.770	2.770	2.770	2.718	2.718	2.718	2.718	2.718	2.665	2.665	2.665	2.665	2.665	2.560	2.560	2.560	2.560	2.560	2.508	2.508	2.508	2.508	2.508
	90	1.834	1.834	0	0	0	1.859	1.859	0	0	0	1.762	1.762	0	0	0	1.762	1.762	0	0	0	1.767	1.767	0	0	0	1.665	1.665	0	0	0
	100	274	0	0	0	0	299	0	0	0	0	202	0	0	0	0	202	0	0	0	0	207	0	0	0	0	105	0	0	0	0
500 nm	70	4.030	4.030	4.030	4.030	4.030	3.925	3.925	3.925	3.925	3.925	3.715	3.715	3.715	3.715	3.715	3.715	3.715	3.715	3.715	3.715	3.610	3.610	3.610	3.610	3.610	3.558	3.558	3.558	3.558	3.558
	80	3.354	3.354	0	0	0	3.261	3.261	0	0	0	3.267	3.267	0	0	0	3.169	3.169	0	0	0	3.174	3.174	0	0	0	3.172	3.172	0	0	0
	90	1.794	0	0	0	0	1.701	0	0	0	0	1.707	0	0	0	0	1.609	0	0	0	0	1.614	0	0	0	0	1.612	0	0	0	0
	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
700 nm	70	5.030	5.030	0	0	0	4.761	4.761	0	0	0	4.772	4.772	4.772	0	0	4.765	4.765	4.765	4.765	4.765	4.660	4.660	4.660	4.660	4.660	4.581	4.581	4.581	4.581	4.581
	80	3.470	3.470	0	0	0	3.201	3.201	0	0	0	3.212	3.212	0	0	0	3.222	3.212	0	0	0	3.119	3.119	0	0	0	3.120	3.120	0	0	0
	90	0	0	0	0	0	1.461	0	0	0	0	1.652	0	0	0	0	1.669	0	0	0	0	1.559	0	0	0	0	1.560	0	0	0	0
	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
900 nm	70	4.794	4.794	0	0	0	4.604	4.604	0	0	0	4.722	4.722	0	0	0	4.522	4.522	0	0	0	4.527	4.527	0	0	0	3.706	3.706	0	0	0
	80	3.234	3.234	0	0	0	3.044	0	0	0	0	3.162	0	0	0	0	2.962	0	0	0	0	2.967	0	0	0	0	2.146	0	0	0	0
	90	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1100 nm	70	4.715	0	0	0	0	4.549	0	0	0	0	4.667	0	0	0	0	4.467	0	0	0	0	4.472	4.472	0	0	0	2.594	0	0	0	0
	80	0	0	0	0	0	2.948	0	0	0	0	3.102	0	0	0	0	2.907	0	0	0	0	2.912	0	0	0	0	0	0	0	0	0
	90	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1300 nm	70	3.794	0	0	0	0	4.092	0	0	0	0	4.509	0	0	0	0	4.417	0	0	0	0	4.417	0	0	0	0	0	0	0	0	0
	80	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	90	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1500 nm	70	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	80	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	90	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table A2. Savings achieved by tankering as a result of parametric problem solution (%)

	Cruise Flight Altitude	*FL290*					*FL310*					*FL330*					*FL350*					*FL370*					*FL390*				
		Price Ratio (%)																													
Flight Distance	*Load Factor (%)*	*50*	*60*	*70*	*80*	*90*	*50*	*60*	*70*	*80*	*90*	*50*	*60*	*70*	*80*	*90*	*50*	*60*	*70*	*80*	*90*	*50*	*60*	*70*	*80*	*90*	*50*	*60*	*70*	*80*	*90*
300 nm	*70*	30.50%	21.81%	14.14%	7.33%	1.23%	31.83%	23.31%	15.79%	9.11%	3.13%	31.80%	23.28%	15.76%	9.07%	5.26%	32.46%	24.01%	16.56%	9.94%	4.02%	31.07%	22.45%	14.85%	8.09%	2.04%	32.43%	23.99%	16.54%	9.91%	3.99%
	80	31.43%	22.61%	15.30%	8.57%	2.56%	32.47%	24.03%	16.58%	9.96%	4.04%	32.68%	24.27%	16.85%	10.25%	4.34%	33.32%	24.99%	17.64%	11.10%	5.25%	31.87%	23.35%	15.84%	9.16%	3.18%	30.80%	22.15%	14.52%	7.74%	1.67%
	90	20.72%	14.01%	0	0	0	21.76%	16.09%	0	0	0	21.03%	15.55%	0	0	0	21.03%	15.39%	0	0	0	21.95%	16.07%	0	0	0	19.82%	14.17%	0	0	0
	100	3.17%	0	0	0	0	3.57%	0	0	0	0	2.46%	0	0	0	0	2.34%	0	0	0	0	2.62%	0	0	0	0	1.30%	0	0	0	0
500 nm	*70*	31.67%	23.13%	15.59%	8.89%	2.90%	32.07%	23.58%	16.09%	9.43%	3.45%	31.31%	22.72%	15.15%	8.41%	2.39%	31.78%	23.25%	15.73%	9.04%	3.05%	31.74%	23.21%	15.68%	8.98%	3.00%	30.99%	22.36%	14.75%	7.98%	1.93%
	80	26.34%	19.36%	0	0	0	26.62%	19.56%	0	0	0	27.32%	19.90%	0	0	0	26.84%	19.53%	0	0	0	27.67%	20.13%	0	0	0	27.12%	19.45%	0	0	0
	90	14.06%	0	0	0	0	13.60%	0	0	0	0	14.03%	0	0	0	0	13.20%	0	0	0	0	14.18%	0	0	0	0	13.96%	0	0	0	0
	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
700 nm	*70*	29.53%	21.46%	0	0	0	29.35%	21.06%	0	0	0	30.74%	22.21%	14.65%	0	0	32.13%	23.65%	16.17%	9.51%	3.56%	31.12%	22.51%	14.91%	8.16%	2.12%	30.52%	21.83%	14.17%	7.35%	1.26%
	80	20.21%	14.70%	0	0	0	19.20%	13.56%	0	0	0	20.54%	14.95%	0	0	0	21.66%	15.91%	0	0	0	20.30%	14.47%	0	0	0	20.30%	14.47%	0	0	0
	90	0	0	0	0	0	9.56%	0	0	0	0	10.48%	0	0	0	0	11.31%	0	0	0	0	10.11%	0	0	0	0	9.99%	0	0	0	0
	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
900 nm	*70*	23.15%	23.15%	0	0	0	22.79%	16.25%	0	0	0	25.14%	18.31%	0	0	0	23.86%	17.12%	0	0	0	24.24%	17.26%	0	0	0	20.05%	14.36%	0	0	0
	80	15.26%	15.26%	0	0	0	14.77%	0	0	0	0	16.79%	0	0	0	0	15.07%	0	0	0	0	15.37%	0	0	0	0	11.42%	0	0	0	0
	90	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1100 nm	*70*	18.89%	0	0	0	0	19.12%	0	0	0	0	20.47%	0	0	0	0	19.43%	0	0	0	0	19.76%	13.96%	0	0	0	11.50%	0	0	0	0
	80	0	0	0	0	0	12.36%	0	0	0	0	13.33%	0	0	0	0	12.47%	0	0	0	0	12.29%	0	0	0	0	0	0	0	0	0
	90	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1300 nm	*70*	14.11%	0	0	0	0	14.47%	0	0	0	0	17.01%	0	0	0	0	16.92%	0	0	0	0	16.48%	0	0	0	0	0	0	0	0	0
	80	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	90	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1500 nm	*70*	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	80	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	90	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	100	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table A3. Tankering amount suggested by İstanbul based flight network problem solution (kg)

	Load Factor	*Cruise Flight Altitude*					
Destination, Flight Distance and Price Ratio	*(%)*	*FL290*	*FL310*	*FL330*	*FL350*	*FL370*	*FL390*
Sofia-257nm-44%	*70*	2.455	2.350	2.402	2.402	2.323	2.324
	80	2.507	2.455	2.402	2.402	2.402	2.324
	90	1.868	1.871	1.771	1.774	1.779	1.675
	100	308	311	211	214	219	115
Athens-317nm-40%	*70*	2.823	2.770	2.717	2.717	2.665	2.639
	80	2.875	2.822	2.770	2.665	2.665	2.665
	90	1.851	1.850	1.750	1.761	1.761	1.660
	100	291	293	194	201	201	100
Bratislava-645nm-35%	*70*	4.870	4.660	4.555	4.450	4.300	4.293
	80	3.310	3.221	3.229	3.134	3.136	3.135
	90	1.751	1.661	1.669	1.574	1.579	1.575
	100	191	101	109	14	16	15
Florence-900nm-41%	*70*	4.793	4.604	4.721	4.522	4.527	3.706
	80	3.233	3.044	3.161	2.962	2.967	2.146
	90	1.673	1.484	1.601	1.402	1.407	586
	100	0	0	0	0	0	0
Baghdad-955nm-37%	*70*	4.778	4.589	4.704	4.509	4.512	3.445
	80	3.218	3.029	3.144	2.949	2.952	1.885
	90	1.658	1.469	1.584	1.389	1.392	325
	100	0	0	0	0	0	0
Tripoli-995nm-59%	*70*	0	0	0	0	0	0
	80	0	0	0	0	0	0
	90	0	0	0	0	0	0
	100	0	0	0	0	0	0
Tehran-1122nm-87%	*70*	0	0	0	0	0	0
	80	0	0	0	0	0	0
	90	0	0	0	0	0	0
	100	0	0	0	0	0	0

Table A4. Istanbul based flight network problem solution resulting savings rates with tinkering (%)

	Load Factor	*Cruise Flight Altitude*					
Destination, Flight Distance and Price Ratio	*(%)*	*FL290*	*FL310*	*FL330*	*FL350*	*FL370*	*FL390*
Sofia-257nm-44%	*70*	37.42	36.68	38.06	37.71	37.01	37.12
	80	37.21	37.40	37.71	37.71	37.69	37.35
	90	12.69	28.33	26.69	27.31	28.68	25.85
	100	4.64	2.65	2.67	3.36	3.59	1.80
Athens-317nm-40%	*70*	41.61	41.05	40.47	41.57	41.55	40.95
	80	41.10	42.13	42.12	41.54	41.54	41.08
	90	26.67	27.20	25.77	26.84	27.36	25.30
	100	4.24	4.36	2.41	3.11	3.17	1.02
Bratislava-645nm-35%	*70*	46.95	46.84	47.11	47.39	46.84	46.52
	80	31.56	32.09	33.19	32.95	33.46	33.22
	90	16.61	16.43	17.17	16.57	16.68	16.35
	100	1.83	1.01	1.13	0.15	0.18	0.16
Florence-900nm-41%	*70*	29.50	29.27	31.81	30.53	31.15	25.68
	80	19.59	18.98	21.21	19.51	19.91	14.64
	90	9.73	8.97	10.83	9.13	9.33	3.91
	100	0	0	0	0	0	0
Baghdad-955nm-37%	*70*	31.42	31.24	33.49	32.80	33.21	25.60
	80	21.03	20.33	22.27	21.12	21.22	13.87
	90	10.80	9.57	11.28	9.91	9.87	2.45
	100	0	0	0	0	0	0
Tripoli-995nm-59%	*70*	0	0	0	0	0	0
	80	0	0	0	0	0	0
	90	0	0	0	0	0	0
	100	0	0	0	0	0	0
Tehran-1122nm-87%	*70*	0	0	0	0	0	0
	80	0	0	0	0	0	0
	90	0	0	0	0	0	0
	100	0	0	0	0	0	0