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TRANSITION FROM STATIC TO DYNAMIC FIELDS IN THE PRESENCE OF DYONS

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Abstract: We assumed the existence of dyons. Since the dyon is a particle with electric and magnetic charges, we simply considered that electric charge and corresponding magnetic charge have the same small velocity. Thus using this proposition and neglecting the interaction between electric and magnetic charges, we constructed symmetric microscopic Maxwell equations in the presence of dyons. Eventually we expanded the theory and obtained macroscopic Maxwell equations in vacuum using averaging process.

Keywords: Dyons, symmetric Maxwell equations, averaging process

Dyonların Varlığında Statik Alanlardan Dinamik Alanlara Geçiş

Öz: Bu çalışmada dyonların var olduğunu farz ettik. Dyon, elektrik ve manyetik yüklerin bulunduğu parçacık olduğundan basit olarak elektrik ve karşı düşen manyetik yükün aynı düşük hıza sahip olduğunu kabul ettik. Böylece bu önermeyi kullanarak ve elektrik yükler ile manyetik yükler arasındaki etkileşimi ihmal ederek dyonların varlığında simetrik mikroskobik Maxwell denklemlerini ortaya çıkarttık. Sonuçta ortalama alma yöntemini kullanarak teoriyi genişlettik ve boşlukta makroskobik Maxwell denklemlerini elde ettik.

Anahtar Kelimeler: Dyonlar, simetrik Maxwell denklemleri, ortalama alma yöntemi

1. INTRODUCTION

Poincaré (1896) conducted works on the dynamics of moving electron in the field of steady magnetic monopole and defined angular momentum. Thomson (1904) later obtained the same results and determined intrinsic angular momentum. However, Dirac (1931, 1948) first developed a quantization condition using the Hamiltonian for an electric charge interacting with the field of a fixed magnetic monopole. Dirac constructed the singularity of electric vector potential in the presence of a magnetic monopole and defined on a line instantaneously extending outward from the monopole to spatial infinity. Dirac was uncertain whether a particle with electric and magnetic charge could exist. Schwinger (1966,1968,1969,1975) generalized this quantization condition to dyons. He attempted to construct field theory of dyons precisely but he failed. However, Dirac, Schwinger and Zwanziger (1971) proved that point-like electric and magnetic charges are defined in electromagnetic theory with the help of the Dirac string or multi-valued potential (Wu and Yang, 1975).

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These microscopic studies require a connection with the macroscopic domain. Lorentz (1902) served electron theory, which supposed that charged particles in material are interacting through electromagnetic waves. He (1909) defined microscopic theory of electrodynamics as the equations of electron theory. Lorentz also obtained the macroscopic Maxwell equations applying time and space averaging over a physically infinitesimal region.

Vleck (1932) and Rosenfeld (1951) expanded Lorentz's averaging theory in the presence of polarization mechanisms. Voisin (1959) conducted averaging theory for arbitrary order multipoles. Mazur and Nijboer (1953) served a statistical ensemble method instead of Lorentz's theory. Jansen (1958) and Schram (1960) improved Mazur and Nijboer's works in a quantum-mechanical basis. De Groot et al. (1965,1965,1969) obtained a general method that includes covariant structure of polarization fields in microscopic scale.

The theory of dyons yielded in the microscopic domain can also be defined in the macroscopic domain through averaging techniques. It should be emphasized that the symmetrization of the Maxwell equations with magnetic charges is an efficient theoretical method that serves solutions of many practical problems.

2. MICROSCOPIC FIELD QUANTITIES

Dyon theory could lead to be adopted dynamics of electric charges to magnetic charges. Thus we can extract physical equalities of magnetic charges using this connection. A static dyon particle with charges (e, g) excites Coulomb-like electric and magnetic fields (Shnir, 2005). If dyon charges are moving with small velocities, we can’t use Coulomb fields to achieve Maxwell equations. Integration process should be conducted to gather the resultant effect of moving charges.

We use three-dimensional delta function (δ) to define the motion of dyons. Since a number of point-like dyons (e_a, g_a) are in motion and they are supposed to be located at points $\vec{r}_a(t)$, $a = 1, 2, ..., n$, the charge densities can be written as

$$\begin{pmatrix} \rho_e(\vec{r},t) \\ \rho_g(\vec{r},t) \end{pmatrix} = \sum_{a=1}^{n} \begin{pmatrix} e_a \\ g_a \end{pmatrix} \delta(\vec{r} - \vec{r}_a(t)) \cdot \tag{1}$$

Electron theory states that all electric currents are purely convective, that is, they are caused by the motion of charged particles. This theory can be expanded through dyons. We can define current densities as moving charge densities in (1)

$$\begin{pmatrix} \vec{j}_e(\vec{r},t) \\ \vec{j}_g(\vec{r},t) \end{pmatrix} = \sum_{a=1}^{n} \begin{pmatrix} e_a \\ g_a \end{pmatrix} \vec{v}_a(t) \delta(\vec{r} - \vec{r}_a(t)), \tag{2}$$

where $\vec{v}_a(t)$ is dyons velocity .

We admitted that electric and magnetic charges on a dyon are net unique charges. Since dyons move in a certain volume, we can obtain total charges as

$$\begin{aligned} \int_{\upsilon} \begin{pmatrix} \rho_e(\vec{r},t) \\ \rho_g(\vec{r},t) \end{pmatrix} d\upsilon &= \int_{\upsilon} \sum_{a=1}^{n(t)} \begin{pmatrix} e_a \\ g_a \end{pmatrix} \delta(\vec{r} - \vec{r}_a(t)) d\upsilon \\ &= n(t) \begin{pmatrix} e \\ g \end{pmatrix} = \begin{pmatrix} q_e(t) \\ q_g(t) \end{pmatrix}. \end{aligned} \tag{3}$$

Surface integration of electric and magnetic fields of some moving dyons in the sphere in R^3 enclosed by S yields

$$\int_S \begin{pmatrix} \vec{e}(\vec{r},t) \\ \vec{b}(\vec{r},t) \end{pmatrix} \cdot d\vec{S} = \int_S \left(n(t) \begin{pmatrix} e \\ g \end{pmatrix} \frac{\vec{r}}{4\pi |\vec{r}|^3} \right) \cdot d\vec{S} \cdot \tag{4}$$

Note that the sphere is large enough that total moving dyons are supposed to be point-like charges. Equation (4) is general field definition of moving charges in low velocity.

If we employ divergence theorem for the left term in (4) and calculate the integration at the right term, we obtain

$$\int_\upsilon \vec{\nabla} \cdot \begin{pmatrix} \vec{e}(\vec{r},t) \\ \vec{b}(\vec{r},t) \end{pmatrix} d\upsilon = \begin{pmatrix} q_e(t) \\ q_g(t) \end{pmatrix} \cdot \tag{5}$$

It can be written these total point charges as volume integral of charge density in (3)

$$\int_\upsilon \vec{\nabla} \cdot \begin{pmatrix} \vec{e}(\vec{r},t) \\ \vec{b}(\vec{r},t) \end{pmatrix} d\upsilon = \int_\upsilon \begin{pmatrix} \rho_e(\vec{r},t) \\ \rho_g(\vec{r},t) \end{pmatrix} d\upsilon \cdot \tag{6}$$

Since this equality must be valid for any volume, Gauss equations can be obtained in microscopic media

$$\vec{\nabla} \cdot \begin{pmatrix} \vec{e}(\vec{r},t) \\ \vec{b}(\vec{r},t) \end{pmatrix} = \begin{pmatrix} \rho_e(\vec{r},t) \\ \rho_g(\vec{r},t) \end{pmatrix} \cdot \tag{7}$$

We supposed all dyons have same velocity $\vec{v}(t)$ which is constant at the same time through the region. Thus, multiply (7) by this velocity yields

$$\vec{v}(t)\vec{\nabla} \cdot \begin{pmatrix} \vec{e}(\vec{r},t) \\ \vec{b}(\vec{r},t) \end{pmatrix} = \vec{v}(t) \begin{pmatrix} \rho_e(\vec{r},t) \\ \rho_g(\vec{r},t) \end{pmatrix} \cdot \tag{8}$$

If we take into account the equality

$$\begin{pmatrix} \vec{j}_e(\vec{r},t) \\ \vec{j}_g(\vec{r},t) \end{pmatrix} = \vec{v}(t) \begin{pmatrix} \rho_e(\vec{r},t) \\ \rho_g(\vec{r},t) \end{pmatrix} \tag{9}$$

and using simple vector identity

$$\vec{v}(t) \left[\vec{\nabla} \cdot \begin{pmatrix} \vec{e}(\vec{r},t) \\ \vec{b}(\vec{r},t) \end{pmatrix} \right] = \left(\vec{v}(t) \cdot \vec{\nabla} \right) \begin{pmatrix} \vec{e}(\vec{r},t) \\ \vec{b}(\vec{r},t) \end{pmatrix} + \vec{\nabla} \times \left[\vec{v}(t) \times \begin{pmatrix} \vec{e}(\vec{r},t) \\ \vec{b}(\vec{r},t) \end{pmatrix} \right], \tag{10}$$

we obtain

$$\begin{pmatrix} \vec{j}_e(\vec{r},t) \\ \vec{j}_g(\vec{r},t) \end{pmatrix} = \left(\vec{v}(t) \cdot \vec{\nabla} \right) \begin{pmatrix} \vec{e}(\vec{r},t) \\ \vec{b}(\vec{r},t) \end{pmatrix} + \vec{\nabla} \times \left[\vec{v}(t) \times \begin{pmatrix} \vec{e}(\vec{r},t) \\ \vec{b}(\vec{r},t) \end{pmatrix} \right] \cdot \tag{11}$$

If we take divergence of (11), we obtain

$$\vec{\nabla}\cdot\begin{pmatrix}\vec{j}_e(\vec{r},t)\\ \vec{j}_g(\vec{r},t)\end{pmatrix}=\vec{\nabla}\cdot\left[\left(\vec{v}(t)\cdot\vec{\nabla}\right)\begin{pmatrix}\vec{e}(\vec{r},t)\\ \vec{b}(\vec{r},t)\end{pmatrix}\right]. \tag{12}$$

The charge distributions of dyons (ρ_e,ρ_g) in motion generate currents coupled as $(\vec{j}_e,\vec{j}_g)$. Therefore, it can be postulated that the equation of electric charge conservation is also coupled to the equation of magnetic charge conservation,

$$\frac{\partial}{\partial t}\begin{pmatrix}\rho_e(\vec{r},t)\\ \rho_g(\vec{r},t)\end{pmatrix}=-\vec{\nabla}\cdot\begin{pmatrix}\vec{j}_e(\vec{r},t)\\ \vec{j}_g(\vec{r},t)\end{pmatrix}. \tag{13}$$

Thus, combining (7), (12) and (13) yields

$$\frac{\partial}{\partial t}\begin{pmatrix}\vec{\nabla}\cdot\vec{e}(\vec{r},t)\\ \vec{\nabla}\cdot\vec{b}(\vec{r},t)\end{pmatrix}=-\vec{\nabla}\cdot\left[\left(\vec{v}(t)\cdot\vec{\nabla}\right)\begin{pmatrix}\vec{e}(\vec{r},t)\\ \vec{b}(\vec{r},t)\end{pmatrix}\right]. \tag{14}$$

And using the vector identity (11) again, we obtain

$$\frac{\partial}{\partial t}\begin{pmatrix}\vec{\nabla}\cdot\vec{e}(\vec{r},t)\\ \vec{\nabla}\cdot\vec{b}(\vec{r},t)\end{pmatrix}=-\vec{\nabla}\cdot\left\{\begin{pmatrix}\vec{j}_e(\vec{r},t)\\ \vec{j}_g(\vec{r},t)\end{pmatrix}-\vec{\nabla}\times\left[\vec{v}(t)\times\begin{pmatrix}\vec{e}(\vec{r},t)\\ \vec{b}(\vec{r},t)\end{pmatrix}\right]\right\}. \tag{15}$$

We can change the cross partial operation in (15) and then write

$$\vec{\nabla}\cdot\left[\frac{\partial}{\partial t}\begin{pmatrix}\vec{e}(\vec{r},t)\\ \vec{b}(\vec{r},t)\end{pmatrix}\right]=-\vec{\nabla}\cdot\left\{\begin{pmatrix}\vec{j}_e(\vec{r},t)\\ \vec{j}_g(\vec{r},t)\end{pmatrix}-\vec{\nabla}\times\left[\vec{v}(t)\times\begin{pmatrix}\vec{e}(\vec{r},t)\\ \vec{b}(\vec{r},t)\end{pmatrix}\right]\right\}. \tag{16}$$

We can use the equalities for convenience (Schwinger et al. 1998)

$$\vec{v}(t)\times\begin{pmatrix}\vec{e}(\vec{r},t)\\ \vec{b}(\vec{r},t)\end{pmatrix}=c\begin{pmatrix}\vec{b}(\vec{r},t)\\ -\vec{e}(\vec{r},t)\end{pmatrix}, \tag{17}$$

(c is speed of light) in (16). By adopting a gauge condition, we can obtain microscopic Maxwell Ampere and Faraday equations

$$\frac{\partial}{\partial t}\begin{pmatrix}\vec{e}(\vec{r},t)\\ \vec{b}(\vec{r},t)\end{pmatrix}=-\begin{pmatrix}\vec{j}_e(\vec{r},t)\\ \vec{j}_g(\vec{r},t)\end{pmatrix}+c\vec{\nabla}\times\begin{pmatrix}\vec{b}(\vec{r},t)\\ -\vec{e}(\vec{r},t)\end{pmatrix}. \tag{18}$$

In the presence of dyons, we obtained symmetric Maxwell equations which have dual quantities.

We need to define Lorentz force equations to complete dynamical structure. In co-moving frame, the generalized force on charged dyon particle can be defined as

$$\vec{F}=e\left(\vec{e}(\vec{r},t)+\frac{\vec{v}(t)}{c}\times\vec{b}(\vec{r},t)\right)+g\left(\vec{b}(\vec{r},t)-\frac{\vec{v}(t)}{c}\times\vec{e}(\vec{r},t)\right). \tag{19}$$

This is a force on moving dyon in the fields of $\vec{e}(\vec{r},t)$ and $\vec{b}(\vec{r},t)$ at microscopic scale.

3. MACROSCOPIC FIELD QUANTITIES

In the medium, there can be a large number of particles exhibiting motion over a microscopic time period and positions at the atomic scale. Hence, it is impossible to calculate total fields excited by all these particles. Even if such a calculation could be performed, it could not be verified by macroscopic measurement instruments. Therefore, different approaches can be served.

Since a statistical method models the structure of medium, we can define a relation between microscopic and macroscopic theory. Therefore, the averaged values of the charge-current densities are introduced, which then give rise to averaged electromagnetic fields.

The averaged values can be obtained by averaging over a temporal interval and a spatial region. Thus, the linear differential equations for the microscopic field variables hold for the macroscopic (averaged) fields. Eventually we choose averaged sources and fields to apply Maxwell equations as

$$\bar{\rho}_e(\vec{r},t) = \frac{1}{\sqrt{\varepsilon_0}}\rho_{fe}(\vec{r},t), \tag{20}$$

$$\bar{\vec{j}}_e(\vec{r},t) = \frac{1}{\sqrt{\varepsilon_0}}\vec{J}_{fe}(\vec{r},t), \tag{21}$$

$$\bar{\vec{e}}(\vec{r},t) = \sqrt{\varepsilon_0}\vec{E}(\vec{r},t), \tag{22}$$

$$\bar{\rho}_g(\vec{r},t) = \sqrt{\mu_0}\rho_{fg}(\vec{r},t), \tag{23}$$

$$\bar{\vec{j}}_g(\vec{r},t) = \sqrt{\mu_0}\vec{J}_{fg}(\vec{r},t), \tag{24}$$

$$\bar{\vec{b}}(\vec{r},t) = \frac{\vec{B}(\vec{r},t)}{\sqrt{\mu_0}}, \tag{25}$$

where (ε_0, μ_0) are the permittivity and permeability of vacuum. Thus, if we directly substitute (20)-(25) in (7) and (18), the macroscopic Maxwell equations in vacuum are obtained

$$\vec{\nabla}\times\vec{E}(\vec{r},t) = -\frac{\partial\vec{B}(\vec{r},t)}{\partial t} - \mu_0\vec{J}_{fg}(\vec{r},t), \tag{26}$$

$$\vec{\nabla}\times\vec{B}(\vec{r},t) = \frac{1}{c^2}\frac{\partial\vec{E}(\vec{r},t)}{\partial t} + \mu_0\vec{J}_{fe}(\vec{r},t), \tag{27}$$

$$\vec{\nabla}\cdot\vec{E}(\vec{r},t) = \frac{1}{\varepsilon_0}\rho_{fe}(\vec{r},t), \tag{28}$$

$$\vec{\nabla}\cdot\vec{B}(\vec{r},t) = \mu_0\rho_{fg}(\vec{r},t). \tag{29}$$

And macroscopic Lorentz force can be written similarly as

$$\vec{F} = q_{fe}\left(\vec{E}(\vec{r},t) + \vec{v}(t)\times\vec{B}(\vec{r},t)\right) + q_{fg}\left(\vec{B}(\vec{r},t) - \frac{\vec{v}(t)}{c^2}\times\vec{E}(\vec{r},t)\right). \tag{30}$$

These expressions define the dynamics of charged particles in vacuum.

4. CONCLUSION

Postulated microscopic domain field quantities are compatible with Maxwell equations. We assumed the existence of dyons, thus microscopic theory leads to new formulations that can be adapted to the macroscopic domain.

When the charges are dual (dyons) and are assumed to give rise to Coulomb-like fields, generalized Maxwell equations are obtained considering the dynamics of dyon particles. Thus, macroscopic Maxwell equations can be constructed using averaging process.

CONFLICT OF INTEREST

The author confirms that there is no known conflict of interest or common interest with any institution/organization or person.

AUTHOR CONTRIBUTION

Ömer Zor takes all responsibility of the manuscript.

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