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A Case Study on Mathematical Classroom Discourse in a Fifth Grade Classroom

Beşinci Sınıf Matematiksel Söylem Üzerine Bir Durum Çalışması

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Abstract

The purpose of this study was to analyze and interpret characteristics of classroom discourse of an elementary mathematics classroom. To examine the classroom discourse, a fifth grade mathematics classroom was observed during sixteen weeks, and twenty lesson hours in total. The analysis was based on Student Learning as the main category, which was further divided into two sub-categories, including content and learning. Results showed that despite the recent reform efforts in school mathematics in Turkey; still teacher-centered instruction continues to be the dominating instructional method. Although the results did not meet the assumptions of discursive classroom at all; based on the results, it could be said that in classroom practices, mathematics teachers try to make connections between mathematical content and other disciplines where they tried to give examples from real-world situations and also encourage students in that way; as pointed out in the school mathematics curriculum.

Keywords: Mathematical classroom discourse, middle school mathematics, teacher-student interaction, student learning.

Özet

Bu çalışmanın amacı bir ilköğretim matematik sınıfındaki matematiksel söylemin analiz edilmesi ve yorumlanmasıdır. Sınıf söyleminin incelenmesi için bir beşinci sınıfın yirmi matematik dersi on altı hafta boyunca incelenmiştir. Analizler Öğrenci Öğrenmesini temel kategori olarak merkeze almıştır. Bu kategori İçerik ve Öğrenme olmak üzere iki alt kategoriye ayrılmıştır. Türkiye'de son yıllarda sürmekte olan reform hareketlerinin aksine,

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bulgular öğretmen merkezli öğretimin yoğun olarak devam ettiğini göstermektedir. Her ne kadar gözlem yapılan dersler matematiksel söylemin etkili özelliklerine sahip olmasa da sınıf içi uygulamalarda öğretmenlerin matematiksel içerik ile diğer disiplinler ve günlük hayat ile ilişki kurdukları ve öğrencileri de bu tür ilişkileri kurma konusunda teşvik ettikleri görülmüştür. Bu tür uygulamalar ilköğretim matematik dersi programı ile uyumludur.

Anahtar Kelimeler: Matematiksel sınıf söylemi, ortaokul matematik, öğretmen-öğrenci etkileşimi, öğrenci öğrenmesi

Introduction

Research studies on mathematics education have gained more importance due to continuous student learning problems in mathematics. Since this seems to be a general issue all over the world, educators have developed a great deal of approaches and implemented various educational reform efforts in school mathematics (Hiebert & Wearne, 1993). From many aspects of these educational reforms, process of learning mathematics is generally perceived as a social enterprise, taking place during the interactions in a classroom community, which provide opportunities and chances for students to learn by thinking, talking, agreeing, and disagreeing about mathematics (Cobb, Yackel, & Wood, 1992; Lampert, 1990). The US National Council of Teachers of Mathematics [NCTM] (2000) has a supportive effect on increasing popularity of usage of communication and writing (Green & Johnson, 2007). Related reform movements have also highlighted that communication is a necessary tool for teaching and learning of mathematics (NCTM, 2000).

In general, mathematical discourse has been defined as a "purposeful talk on a mathematics subject" (Pirie & Schwerzenberger, 1988, p.460). In addition, Gee (1996) defines discourse as: "…ways of talking, listening, acting, interacting, believing, valuing, and using tools and objects in particular settings…" (p.128). However, it would not be right to see discourse just as a way of talking (Hiebert & Wearne, 1993). Mathematical classroom discourse can be described as whole-class discussions in which students talk about mathematics to get deeper understanding of mathematical concepts. Students learn to engage in mathematical ways of thinking and self-perceiving which would be described as understanding of concepts (McCrone, 1997). Research studies have shown that to provide a conceptual understanding, quality and type of discourse is also important (Kazemi & Stipek, 2001). Zolkower and Shreyar (2007) believed that for a meaningful classroom discourse, students should be involved in a "thinking aloud" (p.178) discussion and they should have chance to share their own mathematical ideas and solutions with classmates under the leadership of the teacher.

In an earlier study on classroom discourse, Clement (1997) worked with a mathematics instructor who believed in the importance of discussions and communication in mathematics classes. Results of this study showed that only engaging in conversations, questioning students, probing them for alternative solution strategies, making them to work in groups or in pairs, using manipulatives, do not mean that the teacher effectively facilitate student learning. McClain, McGatha and Hodge (2000) mentioned that in discourse-

based classroom settings, the teachers are expected to lead and orchestrate the classroom norms to make students involve in meaningful mathematical discussions. These discussions are expected to include asking questions and providing answers, problem solving activities, drawing inferences and evaluating mathematical interactions. McClain, McGatha and Hodge also (2000) pointed out that an increasing emphasis on discourse and communication in mathematics classes enables students to talk about mathematical ideas and strategies.

The nature and requirements of classroom discourse is also mentioned below:

> Its nature is reflected in what makes an answer right and what counts as legitimate mathematical activity, argument, and thinking. Teachers, through the ways they orchestrate discourse, convey messages about whose knowledge and ways of thinking and knowing are valued, who is considered able to contribute, and who has status in the group (NCTM, 1991, p.20).

According to Brophy (1999), some teaching features have the potential to create an effective discursive environment. For instance, a teacher is expected to ask questions to students whether they are able to find different solution strategies for problems. In addition, classroom discussions should elicit the meaning of procedures rather than the procedure or rule itself. Classroom discourse should also help teachers identify problems and misconceptions that students have. Discourse is not only crucial in developing interaction between students and their mental skills, but also in enhancing the teacher's evaluation of analyzing their own effectiveness and deficiencies in the classroom. Furthermore, classroom discourse should allow students to experience the mathematical understanding process and increasing their mathematical empowerment (Moore, 2002).

In this study, characteristics of classroom discourse are analyzed in terms of process and content of discourse. More specifically, under the content topic student thinking and their way of making connections in and outside mathematics are investigated; and in the process part, students' engagement in activities, discussions and usage of variety of demonstration models are evaluated.

Looking at the reform movements in mathematics all around the world and issues in the quality of mathematics education, there have been some efforts to increase the quality of teaching mathematics in Turkey. One of these efforts is the continuous revision of the School Mathematics Curricula in Turkey. In the elementary and middle school mathematics curriculum, it was aimed to increase the quality of mathematics teaching. With this goal in mind, the importance of quality of classroom discourse is stressed in the curriculum (MoNE, 2005; 2013). Bulut and Koç (2006) stated that teacher and student roles were redefined in the curriculum. To be more specific, students are expected to participate, inquire, response, discuss, understand, involve in problem solving, think, and decide independently. Additionally, there is a strong emphasis on the quality of classroom discourse in mathematics classrooms as follows:

> Students can be required to write about the way of their solving a problem or want them to explain what a rule means. To talk and write about/on mathematics will help them to construct the mathematical concepts easier. Teacher should

> provide appropriate classroom discourse in which students have opportunities to explain their ideas, discuss, and explain by writing (MoNE, 2005, p.13).

In this context, this study aims to provide a general picture on classroom discourse by observing a classroom. This study also wants to fill one of the gaps by trying to give detailed information about kind of practices that are being conducted in fifth grade elementary classroom. Thus, the purpose of this study is to observe and interpret a mathematical classroom discourse of fifth grade students. In this regard the main and sub-questions could be stated as follows:

In what ways mathematical classroom discourse is practiced in a fifth grade mathematics class in a regional boarding elementary school?

- What are the general characteristics of student learning during classroom discourse?
- What are the features of students' behaviors related to content of the lesson with respect to classroom discourse?
- What are the features of students' behaviors related to learning case of the classroom discourse?

Method

Research Design

Since the aim was to focus on discourse practices of a classroom with all class members, a qualitative approach was more appropriate for this study. The design of the project is observational, exploratory, and interpretive. This framework calls for an observational approach for data collection that involves description of everyday practices in the natural settings of related field and an effort for discovery of the significance of actions in those events. Merriam (1998) defined the qualitative research as an umbrella, which covers different aspects of inquiry; by this way, it helps us understand and explain the phenomena in its natural settings. Similarly, in this study, a fifth grade mathematics classroom was observed for a relatively prolonged amount of time to deeply understand the nature of classroom discourse occurred as part of the teacher-student and student-student interactions.

Participants

The study was conducted in a public elementary school in Kızılırmak, a district of Çankırı. The school was mainly serving students from low socioeconomic backgrounds. The participating classroom had 38 students with 23 females and 15 males. The school was chosen to participate in this study due to its convenience. The classroom teacher was 33 years old and had been teaching from first thru fifth grades for 5 years. She was trained as a chemistry teacher; but, she once had the chance to change her field of teaching; and she preferred to work as an elementary teacher. When the researcher asked the reason for her changing teaching area, she stated that she liked children very much. In her first three years, she worked in a village and for two years she had been working in the participating school.

Data Collection

Data was collected by visiting a fifth grade classroom of a public school during twenty lessons, to observe students and teacher in their natural settings and take observational field notes of classroom experiences. The first researcher took part in the observation process as a non-participant observer. During the observations, she was sitting at the back of the classroom; so, she could have an overall picture of the classroom with all students and the teacher.

Procedure

For data collection, the first author, as a non-participant observer, observed the classroom while she was sitting at the back of the classroom to have an overall view of the classroom, including all students and the teacher. During the observations, the researcher took thick descriptions of the discursive activities of classroom. An observation instrument was used as a guide for data collection. The guide was adapted from a previous form, which was developed by Chicago Mathematics and Science Initiative (CMSI, 2007). Necessary permissions to use an adapted version in Turkish were received from the Chicago Public Schools. The guide was designed to support observation and conversation about learning in a mathematics classroom. It was expected to support an observation that focuses on the key mathematical ideas, student experiences designed to address those ideas, and evidence of student learning. Moreover, the observer is not expected to see all of the components in a single lesson; but, over time, evidence related to all questions should emerge (CMSI, 2007). The adapted version of the observation guide is presented in Appendix A. The Mathematics Classroom Observation Guide was mainly focused on Student Learning; including the content and learning aspects; thus the form was formed into two sections: Content and Learning. The Content section contained guiding questions to observe and record the mental activities that student engaged in during lessons such as problem solving, justification and explanation. The Learning section was used to record student engagement. In the following paragraphs, these two sections will be explained.

Content

This section is interested in mental activities that student engaged in during lessons. Their problem solving process, justifying their answers, explaining ideas, and similar cognitive activities were considered in this section. In order to understand the kinds of mathematical thinking students are engaged in the researchers used the observation guide (CMSI, 2007).

Students' engagement in procedural thinking is explained as solving problems involving procedures or standard algorithms. An example is given as "the standard procedure for comparing fractions by first getting a common denominator, and then comparing the numerators." (CMSI, 2007, p.7) Another example would be suggested as conducting operations after learning addition, subtraction, multiplication, or division.

Students' engagement in conceptual thinking relates with students' developing conceptual understanding of the mathematical ideas. As an example,

they can use equally divided bread to understand fractions as visual models, or they can learn equations by using scales.

Problem solving practices should be away from being non-routine processes. Samples may include word problems or experiments; rather than students working on problems demanding low=level cognitive skills. As an example, they can act in a small scenario, which is based on shopping process to learn four-basic operations.

In the justification process, students are expected to justify their solutions. As an example, they can "prove that a number trick works by using variables to show that it is true for all cases." (CMSI, 2007, p.7)

In order to determine how connections made to other disciplines and real-world situations promote understanding of the mathematical ideas, the students were expected to find and make connections to other disciplines or real-life situations. For example, after understanding the proportion, they can use similar triangles to find the height of a building (CMSI, 2007). Another example would be the usage of ratio to make a model of a building, or to draw a sketch of a room.

Regarding the connections made to prior work in the mathematics class, there should be demonstration of familiarity between procedures and concepts, which developed in their prior work. For instance, they can solve a new problem by connecting the ideas to prior problems they have solved. (CMSI, 2007) Another example can be proving ones idea with a connection to the knowledge from previous years.

Learning

The section is interested in students' physical activities practiced during lessons. Their participation to the discussions, solving problems, and usage of representations are discussed in this topic.

For *active student engagement,* the researchers conducted observations to determine whether all students focus on the work of exploring, understanding, and solving mathematics problems. Student engagement means that they actively participate in classroom discourse. Their attention should focus on the mathematics of the lesson. In addition, they may participate in a whole class discussion or in a group work. They can work together to find and explain alternative solution strategies would be given as an example (CMSI, 2007).

Regarding how students were justifying their answers, offering alternative solution strategies, or demonstrating that their strategies work, the researchers looked for justification of students' answers or demonstration of their strategies work. Students are required to prove these strategies by operating the found reasoning in solutions. They may notice patterns while solving problems, and use this reasoning to justify their thinking; and it is possible that they may recognize connections between mathematics problems. As an example, "One can use reasoning to solve 99 + 76 by creating a new problem: 100 + 75 = 175. This demonstrates that student's understanding of an

equivalent addition expression can be formed by increasing one addend by 1 and decreasing the other addend by 1.

$$x + y = (x+ 1) + (y – 1)$$

Students' demonstration of their strategies may be operated in variety of ways such as using drawings, diagrams, models, graphs, equations, written explanations, examples" (CMSI, 2007, p. 9). The observation guide also aimed to understand how students use a variety of representations – models, graphs, drawings, manipulatives, and writing – to demonstrate their understanding of mathematics. The major purpose was to observe whether students are comfortable using a variety of representations depending upon the problem or situation. As an example, if students could easily access calculators or physical models (CMSI, 2007).

Regarding classroom interactions, the observation guide was designed to understand if the interactions reflect collaborative relationships and peer support, and promote understanding of the mathematical ideas. In group work students collaborate with others to solve problems and share ideas. They build on each other's ideas and share responsibility for solving problems. It is important that each member of the group should be willing to help other members to understand the solution, and each of them should be able to demonstrate understanding of the problem (CMSI, 2007).

Data Analysis

Four mathematics teachers, including the first author, coded the observational field notes. At the first meeting, the field notes from one of the observed lessons were read and coded by each coder individually (Appendix B). The coding framework was a slightly modified version of the classroom observation form that the first author used for observing the classrooms. While coding each case, the teachers wrote down their own interpretations and examples to accurately describe the lesson. For example, after reading the lesson of October 13, the coders read the questions given in the coding table. For instance for the question, "In what kinds of mathematical thinking are students engaged?", each coder looked for whether there were any of sub-categories of procedural, conceptual, problem solving, justification. The decision of coding the lessons in one of the categories was made according to definitions and requirements defined in the CMSI Guide (2007). As an example; the following interaction was chosen from the lesson of 'addition with five digits natural numbers'.

The teacher wrote the following addition on the board:

$$\begin{array}{r} 3\,6\,8\,4\,\_ \\ 2\,\_\,7\,7\,3 \\ +\ 1\,4\,\_\,4\,9 \\ \hline \_\,10\,\_\,8 \end{array}$$

Teacher: Let's do it altogether. Watch me carefully. What if we add 9 to 3?
Class: 12
Teacher: Ok. Which number we need to add to get 8?
Class: 6

The session continued by following same procedure. The teacher asked and students gave responses. The teacher executed the operation. After this example, they solved a similar question by following the same procedure. According to the definition of CMSI (2007), procedural thinking refers to the traditional, teacher dominated classroom practices and experiences. Thus, the coding team decided that the above interaction was an example of procedural thinking practice of students.

Similar procedures were followed by the team during the analyses sessions. Each coder looked for whether there was any practice related to the question. If an example was found, the coder put a mark on the coding table, and wrote down the example to illustrate the case. If there was no example for the issue, a cross was put on the table and passed to next case. After completing the analysis of each lesson independently, they compared their coding and comments. They looked for whether there were any different disagreements across the coders; if there was; they discussed to reach a consensus. At the end of the coding process, the coders reached 100% agreement, which indicates high inter-coder reliability.

Results

The findings are presented under the main theme of Student Learning, which is formed into two categories: content and learning. As mentioned in the CMSI Guide (2007); the student learning theme refers to the activities that students are engaged in during mathematics classes. Moreover, their having of chances to express their opinions about subjects, abilities of finding alternative solution strategies, proving those strategies works, using various representations as solutions of problems or as proofs for demonstration of their understanding issues were determined with the theme. Additionally, student participation and abilities of involving in classroom activities or group work were investigated under the same theme.

Content

According to the Mathematics Classroom Observation Guide (2007), the content category specifically focused on the mental activities of students. Their thinking, understanding, and knowledge construction were evaluated under the content category. Based on the data analysis, there were five sub-categories of the content category: procedural thinking, conceptual thinking, routine problem solving, justification and real life connections. Table 1 presents the frequencies and percentages of the number of lessons coded under the subcategories.

Table 1. Frequencies and percentages of the lessons coded under the subcategories of Content

Subcategories of Content	Frequency (%)
Procedural thinking	20 (100%)

Conceptual thinking	3 (15%)
Routine Problem solving	9 (45%)
Justification	5 (25%)
Real life connections	3 (15%)

From Table 1, it is seen that students engaged in procedural thinking in all 20 lessons. Procedural thinking refers to the traditional, teacher dominated classroom practices and experiences. While procedural thinking was promoted in all the lessons, conceptual thinking was fostered in only three lessons. Conceptual thinking refers to students' development of relational understanding of the mathematical ideas (CMSI, 2007; Hiebert & Lafarve, 1986). As seen in Table 1, the students were given the opportunity to solve routine mathematics problems in nine lessons. Such problems were embedded into relatively more teacher-centered instructional environments. Also, in general, the students were solving the problems that were requiring procedural thinking and low-level cognitive demand (Smith & Stein, 1994). In three of the lessons, there were real life connections. The findings also indicate that the students were engaged in justifying their solutions in five lessons. In the justification process, students were expected to justify their solutions.

Procedural Thinking

The following excerpt illustrates how the teacher promoted procedural thinking in a lesson on polygons. The lesson started with an introduction of the teacher to the topic of triangles:

> Today we will learn the kinds of triangles. We have three types of triangles. First one is equilateral triangle, which has three equal sides. We find the perimeter of it by multiplying one side by 3. (She drew the picture and wrote the formula on the board)
> Second one is isosceles triangle with only two equal sides. We find the perimeter by multiplying one of equal sides with two and adding the third side to it. (She drew the picture and wrote the formula on the board)
> And the last one is scalene triangles with no equal sides. We find the perimeter by adding up all sides. (She drew the picture and wrote the formula on the board)

Students were familiar with the subject from the previous lesson and from fourth grade. During the instruction, the teacher presented the subject directly without asking any questions to the students. Furthermore, students rarely met experiences that provided them with a conceptual understanding of mathematics and they did not have any opportunity to justify their arguments. Thus, it was unlikely for them to transfer what they have learned into real world situations.

The following dialogue is another example from the same lesson. The students were working on a geometry problem. The teacher called a student to the board:

Teacher: Ok. First read the question. What do you understand?
Student: We are given two equilateral triangles with their perimeters and are asked to find the perimeter of rectangle.
Teacher: That is good. Now look at the picture. These two triangles will help us find the perimeter of rectangle. What feature of the equilateral triangle will help us here?
Student A: It has two equal edges.
Teacher: No, no, no! Be careful! It has three equal edges.

After this interaction, the teacher took the board marker and solved the question by explaining it to the whole class. The session continued via the same interaction pattern: The teacher asks and students respond. The teacher completed the operation on her own. After this example, they solved a very similar question by following the same procedure.

Conceptual Thinking

The following example demonstrates conceptual thinking as illustrated in one of the lessons observed:

The teacher brought the class a small cloth bag with marbles in it. The lesson started with a recall from the previous lesson.

Teacher: We have eight red, four orange, and two yellow marbles in this bag. Now, I want to make a random selection from it. Which marble do you think have the highest probability of coming out?
Class: Red
Teacher: What is the reason for this? Yes, Batuhan.
Student A: The number of red marbles is more than others.
Teacher: All right. Whose probability is less than others?
Class: Yellow.
Teacher: Reason? Yes, Berna.
Student B: Because, it is fewer than others.
Teacher: Now, let's try and see if we are right.

After this dialogue, she made 20 random selections from the bag; and drew a tally table on the board:

Teacher: This practice helped us prove our claims. Red marbles were drawn more than other colors and yellow marbles were the least drawn. What did we do here? How do you define our activity?
Student C: You took marbles out of the bag.
Teacher: Yes, we call this situation as an "experiment" in probability.

The above probability experiment might have provided the students with the opportunity to understand basic probability. The experiment made the subject concrete and promoted conceptual learning.

Routine Problem Solving

Third sub-category is about the problem solving activities. As it was mentioned in the previous section, problem solving should be away from being non-routine processes. Samples may include word problems or experiments; rather than being traditional in which students working on low-level problems on the board (CMSI, 2007). As an example, they can have small roles in a small scenario, which is based on shopping process to learn four-basic operations. It was found out that solving problems by following traditional methods occurred 9 times. Examples were presented below about the issue.

The following lesson was based on solving problems since they had already learned the subject in the previous lesson. Before getting started, the teacher reviewed the subject briefly. The following is a sample of the lesson.

Teacher: Yes, all of you remembered the subject, now we will solve questions to provide a better understanding. Listen to me carefully.
If we need 2 kg butter for 5 liters of milk, with 15 liter milk how much butter do we need?

Teacher: Have you all understood the question? First, think about the amount of the butter. In the new situation, do you think the amount of butter will increase or decrease?

Class: It will increase.

Teacher: Canan. Tell me the reason for the increase of butter.

Student A: Because in the second situation, we use much more milk compared to the first situation.

Teacher: Ok. I will solve the first question to help you understand better. You will solve these kind of questions in three ways. First, you can organize the given data like this *(explained by writing on the board):*

5 liters of milk		2 kg butter
15 liters of milk		? kg butter

Teacher: Here, you will multiply the two known number and divide it into another. In the second way, you will write a proportion as follows:

$$\frac{5}{15} = \frac{2}{?}$$

Can you see the ratio between 5 and 15?

Class: Yes. It is 1/ 3

Teacher: So the same ratio will be between 2 and which number?
Class: 6
Teacher: Ok. The third way is using the "multiplication of inner and outer terms."

$$\frac{5}{15}=\frac{2}{?}$$

15x2 = 5x? ⟶ ?= 6 kg
Teacher: Is it okay? You will use one of these ways.

In this example, the teacher followed a way of questioning method, which was followed by direct teaching. They solved four problems throughout the lesson. Students came to the board to solve them. They were required to use the ways that the teacher wanted them to use. At the beginning of the problem solving process, the teacher asked students about the amount of butter in new situation. So, students first had a chance to see in what ways they need think to solve it.

Justification

Results indicated that students were involved in justification for five times during the observation process. However, in only one of them, a student justified his solution. At other times, the teacher developed the justification as a process of teaching session; students were only involved in those instances. In the justification process, students were expected to justify their solutions. As an example, they could "prove that a number trick works by using variables to show that it is true for all cases." (CMSI, 2007, p.7) In the following dialogue, an example was presented for the issue. The sample was chosen from the lesson of 'demonstration of exponential number'.

After a summary of previous lesson and introduction of the subject, the teacher drew a house on the board. The house had three windows and three small windows on each big one. By asking questions to the students, the teacher helped them see the total number of windows of the house. She demonstrated that:

$$3\times3=9 \quad \text{or} \quad 3\times3=3^2$$

Then the teacher added two more of the same house.

Teacher: How many big windows do these houses have?
Class: Three
Teacher: How many small windows do the each big window has?
Class: Three
Teacher: So, how many windows do these houses have in total?
Some Students: 27
Teacher: Why, do you think that? Or How did you get that answer?
Student A: In this example, we have three houses. Other cases are the same as with the previous one. It is enough to multiply the

previous result with the number of houses. So if we multiply 9 and 3, we get the total number of windows.

Real life connections

As the final category of the Content, the issue of giving examples from daily life was indicated. Before presenting the samples from this context, it would be significant to mention about making connections to real world situations, connections to other disciplines and connections to prior work. The students are expected to find and make connections to other disciplines or real-life situations. For example, after understanding the proportion, they can use similar triangles to find the height of a building (CMSI, 2007). Another example would be the use of ratio to make a model of a building, or to draw a sketch of a room. To sum up, these features of the Content expect students to transfer the knowledge learned in mathematics lessons, to other situations and find practice areas for them. Table 1 indicated that students did not engage in these kinds of connections. Rather, they found examples from daily life for three times. Following a sample was presented for related issue.

The teacher made an introduction by asking students if they heard the term "ratio" before.

Student A: I have heard from my sister. She mentioned about it several times while she was working.

Student B: I have heard from my father. He is a carpenter and he uses this term regularly while doing his work.

The teacher asked girls whether they had ever observed their mother while they were cooking. If yes, how they were doing it. She was willing to hear students using the term "ratio"; or she wanted to obtain their prior knowledge on the issue.

Student C: I always watch my mother while she is baking cake. She uses ingredients according to some ratio. For example, I know that she adds three glasses of flour for one glass of milk.

Student D: I also know that my mother cooks rice with a ratio of two glasses of water for one glass of rice.

Teacher: All of your examples were very good and true. Ok, now. What about maps? Who has an idea about them? Do you think that the areas of the countries or cities are the same as you see in the map also in reality?

Student E: No, map designers make them smaller.

Teacher: Do you think that they do this job randomly?

Student E: I don't think so. They should use a particular ratio.

Teacher: Ok. They use ratio; for example, when we look at our map on the wall, we see a ratio of 1/ 100 000. Ok, now I will write the descriptions and then the questions on the board. Just watch and listen to me carefully.

Looking from a general view to the case of Content, Table 1 indicated that students were generally engaged in procedural thinking that indicates traditional methods of teaching according to the CMSI (2007). Furthermore, students rarely met experiences that provided them with a conceptual understanding of mathematics. Additionally, solving problems were practiced by following traditional methods, which did not have the features defined in the CMSI (2007). Moreover, students did not have chance or opportunities to make justification. As a final point, they did not transfer the gained knowledge by making connections to real world situations, to other disciplines or to the prior work. They only found examples from daily life in limited number of lessons.

Learning

Since the aim of the study was to observe students' practices in the classroom during the mathematics courses; the learning category mainly focused on the physical activities of students in learning mathematics. According to the CMSI (2007), the level of student participation, offering alternative solution strategies to the problems and proving whether those strategies work, and using various representations for demonstrating their understanding of mathematical content were considered as essential evidence of student learning. Moreover, students' relationship with each other from aspects of sharing ideas and working collaboratively were evaluated under the Learning category (CMSI, 2007). Table 2 presents the frequencies and percentages of the lessons coded under the subcategories of Learning, including active engagement, justification, alternative solution strategies, representations.

Table 2. Frequencies and percentages of the lessons coded under the subcategories of Learning

Categories of Learning	Frequency (%)
Active engagement	10 (50%)
Justification	1 (5%)
Alternative solution strategies	1 (5%)
Representations	1(5%)

Table 2 indicates that active engagement was the most apparent subcategory of the Learning category (10 lessons). As it is defined in the CMSI guide (2007), active engagement refers to whether all students explore and understand mathematical ideas and procedures, and solve mathematics problems. Student engagement also means that students actively participate into classroom discourse. Their attention should focus on the mathematics itself rather any other issue. In addition, they are expected to participate in whole class discussion or small group work. Working together to find and explain alternative solution strategies are also distinctive feature of active student engagement (CMSI, 2007). As result, in half of the lessons, it was observed that the classroom environment showed various evidence of active engagement. As it was mentioned in the previous section, students are expected to justify their answers or solutions in meaningful way. Results indicated that students

produced justifications only once during the study. From Table 2, it is also seen that only one time a student offered an alternative method to solve a problem.

Active engagement

In order to illustrate occurrence of active engagement, the following exchange between the teacher and a student is provided below:

The probability lesson was started with the teacher's introduction the subject. She gave papers to students and mentioned that the lesson would be activity-based. She wanted students to draw squares on their papers.

Teacher: I want you to draw a diagonal of your squares. First, tell me the meaning of the term "diagonal". Yes, Burak. (There were only a few raising hands).

Student A: A line, which is drawn from the corner of our figure.

Teacher: Remember from last year. Did we make such a definition? (Silence for a while. They knew the meaning, but cannot define it in mathematical terms.) Ok. Who wants to show what a diagonal is? Berna, come to the board. (Student came) Now, draw a square and one of its diagonal.

Student drew what she wanted; and then the teacher told the definition of diagonal for students. After students had written the definition, all students draw one diagonal of their squares.

Teacher: All right, now fold your squares from these diagonals and tell me what happened to them? Who wants to answer?

Student B: The pieces are the same.

Teacher: That's right. The pieces are all the same. We define these "symmetry lines". Now write the definition. (She told and student wrote). Now, look at your squares whether it is possible to find another symmetry lines. Yes, what do you think? Is there anyone who found other symmetry lines?

Student C: Another diagonal is one of the symmetry lines.

Teacher: Good. What else? (A few students raised their hands.)

Student D: If we fold from middle of the square straightly, not diagonally, we get two equal pieces again.

Teacher: Perfect. That's right. Now, I want you to draw all symmetry lines of your squares with colored pencils; then you will tell me the total number. How many diagonals did you find?

Class: Four

Teacher: Yes, a square has four symmetry lines in total. Now, I will draw an equilateral triangle on the board; write it in your notebooks.

Student engagement was at high levels in this lesson. Students were involved in different activities in addition to procedural ones. They reached the

rule with an activity. This was a task requiring justification and proof. An increased interest of students was observed. This was the first classroom activity that required the participation of whole class. Except for a few students, most of them tried to do and understand what the teacher expected of them. Furthermore, this was a lesson with more student-student interaction. The activity was the first that made students ask questions and communicate with each other.

Alternative solution strategies

Another example is about alternative solution strategies.

> Teacher: Is there anyone with an alternative solution to make our operation easier? Remember, what we use in these kinds of problems.
>
> Student: We can draw a figure to make our operations easier. (Student drew the figure on the board. The figure was a simple rectangle divided into rows and columns. He wrote the given data in these rows and columns.)

The above quotations provide clear evidence of using *representations also*. In particular, upon teacher's request the student formed an alternative solution to a problem on subtraction. The above example, additionally, constitutes a case of using representations. The student drew a figure to approach the problem from a geometric perspective.

It can be concluded from the findings that students did not always actively participate in classroom discourse practices. They rarely found or offered alternative solutions to problems, and justified that those strategies work. Additionally, it was concluded that students' involvement in procedural teaching-learning practices had also significant effect on their realization of other aspects of discursive experiences. When the literature is considered, it is clear that active participation has an important effect on shaping the nature of classroom discourse. In this study, classroom discourse was mainly based on traditional dialogues between teacher and students. More specifically, a pattern was determined for this classroom as first teacher taught the subject (with its descriptions, mathematical concepts, formulas; wrote a question about the subject and solved it for children to make them understand better; the teacher emphasized the procedure for how they would solve other questions; and finally other questions were written on the board and students came to board to solve them. Generally student who came to solve the problem used the way which teacher had told him or her to use.

Discussion and Implications

In the present study the findings indicated that students in general were exposed to instructional activities that may potentially foster their procedural thinking; but, conceptual thinking was not the focus of instruction in most of the lessons. Also, students were rarely given the opportunity to solve non-routine

problems demanding high level cognitive thinking skills, develop multiple solution methods and justify their solutions. Therefore, the analysis of the data based on the observation of twenty fifth-grade mathematics lessons shows that teaching mathematics for understanding was not given a prominent importance.

The mathematics classroom is expected to be a community where the teacher fosters critical thinking, sharing, agreeing, and disagreeing (NCTM, 1991, 2000). These essential features are expected to enhance the quality of mathematical discourse in the classroom. However, in the current study, it was found out that students could not experience these practices and they were rarely encouraged to justify or explain their own solutions. For instance results indicated that in only five of twenty lessons –that correspond to 25%- students practiced justification. There were only in three lessons which correspond to 15% they were experienced conceptual thinking. More specifically, the classroom environment was not supporting sharing and discussing student solutions and ideas. This certainly is a reflection of the teacher's teaching philosophy and method. It can be argued that the nature of the questions she used might have been the reason

Another reason for the procedural nature of the mathematics noted in this study can be the characteristics of teacher questions. It was observed that teacher questions were not motivating the students to think in alternative ways and develop different solution strategies. Moreover, the data was collected from a relatively large classroom with 38 students; hence, the class size might have prevented the teacher from using activities that promote mathematical and so conceptual understanding. Additionally teacher's level of pedagogical knowledge about creating and leading this kind of environment might be the reason for not having effective discursive practices.

Classroom characteristics as illustrated by the findings of the present are reflections of traditional classrooms (Yackel & Cobb, 1996; Yackel, 2002). Based on the data collected in this study, it can be concluded that traditional teaching methods still dominate mathematics classrooms. Thus, it can be concluded that the participated classroom shows characteristics of a traditional classroom. Additionally, such findings obtained observational data were parallel with Doğan's (2006) study where he found out that teacher-dominated classroom was prevalent in Turkish mathematics classrooms.

In the present study during the observation process all the lessons were mainly based on this teacher-centered classroom culture where the classroom discourse was mainly dominated by the teacher talk. For instance, in all observations, students engaged in procedural thinking practices and in 45% of the lessons traditional routine problem solving practices occurred. In all these cases, the teacher was the most dominant of all in class. Sometimes there were small discussions when the examples were given about the subject or a student could not understand any particular issue or question. By asking a simple question, a classroom discussion can be started and this would make students see their thinking abilities and develop their skills of sharing ideas, agreeing and disagreeing with peers and mainly communicating in mathematical language (Clements, 1997). In order to take place in a discussion, classroom (both social

and mathematical) norms need to be established so students can feel comfortable in explaining and justifying their responses. Establishing this classroom culture can be done by expecting students to explain and justify their answers, whether they are correct or not; emphasizing the importance of contributing to the discussion by explaining their strategy rather than producing correct answers and expecting students to listen to others' explanations (McGraw, 2002; NCTM, 1991; Peng, 2009; Rojas- Drummond & Mercer, 2003)

The findings indicated that the participants were not exposed to high level questions in mathematics lessons as suggested in the Turkish School Mathematics Curricula (MONE, 2005; 2013). Rather, the teacher questions were mostly focusing on practicing mathematical procedures. They rarely discussed and shared their ideas about a mathematics problem or concept. Additionally, justification and problem solving processes were hardly observed. Hence, the results were not consistent with that of a discursive classroom environment (CMSI, 2007).

Considering the literature, Lampert's (1990) study examined the kind of reasoning abilities that occurred during mathematical classroom discourse when students and teacher engaged in. That study was an example of importance of making connections to real world situations and other disciplines during teaching-learning process. In that study, students and teachers worked on some problems that were generally related to real world situations. With these changes in classroom discourse, students' performances clearly improved on tests. However, in the current study the situation was reverse as mentioned above. Practices which included examples of making connections to other disciplines and giving examples from real world situations were in very limited numbers.

Additionally, observational data from the current study showed that the same teaching and practicing procedure was followed during the observation process. Parallel to Doğan's (2006) study; the teacher first talked about the subject and then solved questions about the subject. The classroom had a characteristic of teacher-dominant environment. The teacher generally did not create a classroom environment with the participation of all the students. Although they had chances to talk and they were flexible about explaining the ideas, they gave answers to the questions only when asked by the teacher instead of constructing their own process or finding different strategies to solve the questions. However, when the literature was considered, Ping's (2001) study indicated that classroom discourse would be accepted as a tool for learning and as an indicator of identity. Furthermore, teachers have a critical effect as models on children's attitudes toward communication in the usage of mathematical language. Additionally, Casa's (2004) doctoral dissertation focused on teachers' decision-making in discourse practices in elementary level mathematics classrooms. Results of that study indicated that teachers should examine and understand the purpose, the nature, and the requirement of discourse. To provide an environment for students in which they would discuss and prove mathematical solutions, strategies and ideas, teachers expected to learn how to

question them in a way of engaging in discussions; and how to guide the classroom discourse.

Based on the findings, it can be suggested that mathematics teachers should be more careful while they are planning their lessons by considering the requirements of the mathematics curriculum; thus, they should provide students with more opportunities for effective classroom discourse and they should also orchestrate the classroom discussion in a way to help student improve their mathematical understanding. More activities should be organized for providing classroom activities that will encourage students to participate in classroom discussions. Teachers should also be careful about the language they use in classroom as students need to start using the mathematical language correctly from their early years for better usage of mathematical language in later years while sharing their ideas, and discussing, agreeing and disagreeing with their peers. Because, teachers are responsible for leading classroom activities to support mathematical understanding; and they need to be careful not to lose the control in class. They should be aware of the language used in classroom, and also should be able to guide the classroom discourse effectively. In addition, the quality of teacher questions should be determined by their levels of cognitive demand. Also, teachers should pose tasks and problems that can be solved via different approaches. The questions of a discursive classroom environment should make students think deeper about the mathematical idea, and see the connections with prior knowledge and use it (Forman, 1996).

Additionally, mathematics teachers play an important role in process and content of classroom discourse. Observational data from the study showed that teachers should be active as well as students to orchestrate the class effectively, to encourage children to involve in discourse community and to create a learning environment parallels with curriculum goals. They need to be aware of the necessity of improving their own content knowledge and pedagogy. To support this improvement, it is inevitable to follow and learn changes and new approaches that have developed in the field of education, specifically about the teacher role in classroom. Related to this, they need to change their teaching and participation methods. In this context, McCrone (1997) states teachers are responsible for constructing classroom environments, which enhance mathematical discussion. Moreover, they need to choose appropriate tasks for classroom discourse. Furthermore, teachers should be aware of the usefulness of their questions, whether they facilitate students' mathematical understanding. As an essential role, they also need to listen to the students' ideas to make them work with each other; and participate in classroom activities and discussions. Similarly, Forman and Cazden (1985) mention that students can only learn by communicating and interacting and by using mathematical language in a meaningful way under the appropriate guidance of their teacher.

This study focused on the nature of mathematical classroom discourse with one-fifth grade classroom. Although, the study is important in providing important information about some aspects of classroom discourse, and demonstrating the deficiencies of these practices to help teachers improve their teaching effectiveness, it can be improved in a few aspects. First, one may want

to conduct a future study comparing two classrooms with respect to the nature of the mathematical classroom discourse. For example, while an experienced teacher teaches one group, an inexperienced teacher can teach another group. This research design will be useful to understand differences and similarities between experienced and inexperienced mathematics teachers regarding their orchestration of mathematics classrooms. Another future study may investigate classroom discourse at different grade levels; for example, 6th, 7th or 8th grade classrooms can be studied and compared for a better understanding of how classroom discourse takes place in classrooms with students at different developmental levels.

The aim of the present study was to draw a general picture of classroom discourse in mathematics lessons. Findings indicated that students in general are exposed to instructional activities that may potentially foster their procedural thinking; but, conceptual thinking was not the focus of instruction in most of the lessons. Also, students were rarely given the opportunity to solve non-routine problems demanding high level cognitive thinking skills, develop multiple solution methods and justify their solutions. Therefore, the analysis of the data based on the observation of twenty fifth-grade mathematics lessons shows that teaching mathematics for understanding was not given a prominent importance. This meant that more traditional classroom practices continue to be dominant in general.

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APPENDIX A

Reformed Mathematics Classroom Observation Guide

Focus on Student Learning

Content

1. In what kinds of mathematical thinking are students engaged? (Examples – procedural, conceptual, problem solving, justification)
2. Do connections made to other disciplines and real-world situations promote understanding of the mathematical ideas?
3. How are connections made to prior work in the mathematics classroom?

Learning

4. Are students actively engaged?
5. How are students justifying their answers, offering alternative solution strategies, and demonstrating that their strategies work?
6. Do students use a variety of representations – models, graphs, drawings, manipulatives, and writing – to demonstrate their understanding of the mathematics?
7. Do the interactions reflect collaborative relationships and peer support?

APPENDIX B

Teachers' Coding Tables

Focus on Student Learning	Lesson #1	Lesson #2	Lesson #3
Content			
1. In what kinds of mathematical thinking are students engaged? (Procedural, Conceptual, problem solving, justification)			
2. How do connections made to other disciplines and real world situations promote understanding of the mathematical ideas?			
3. How are connections made to prior work in the mathematics classroom?			
Learning			
4. Are students actively engaged? How?			
5. How are students justifying their answers, offering alternative solution strategies, and demonstrating that their strategies work?			
6. Do students use a variety of representations – models, graphs, drawings, manipulative, word problems, and writing – to demonstrate their understanding of the mathematics? Give examples.			
7. Do the interactions reflect collaborative relationships and peer support How?			