Incorrect Strategies Developed by Seventh-Grade Students to Solve Proportional Reasoning Problems[*]

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Abstract: This study was conducted to determine the incorrect strategies developed by seventh-grade students to solve problems that require proportional reasoning and evaluate their solutions according to SOLO Taxonomy. This two-stage case study was conducted in a public school in the Central Black Sea region. The Proportional Reasoning Skill Test was administered to 33 seventh-grade students in the first stage. In the second stage, semi-structured interviews were held with 10 students by determining the 5 problems that students made mistakes in most. The students developed 5 different incorrect strategies when solving problems requiring proportional reasoning: additive relationships, data neglect, using numbers and no content, giving an emotional response, and failure to identify non-proportional situations. Students had difficulty identifying non-proportional situations and established additive relationships in problems requiring multiplicative relationships. Also, the levels of the participants were examined with the SOLO Taxonomy Rubric developed for proportional reasoning skills. Accordingly, the students with a high level of proportional reasoning were at abstract and relational structure levels, whereas those with moderate scores were at uni-structural and multi-structural levels. The students with low scores had low-level skills according to SOLO Taxonomy criteria.

Keywords: proportional reasoning, SOLO taxonomy, ratio, proportion, solution strategy

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Introduction

Mathematical thinking skill is defined as a necessary feature for an individual to reach new knowledge or concepts by using mathematical knowledge and concepts based on their prior learning through reasoning, estimating, generalising, abstract thinking, forming hypotheses, testing, reasoning, and proving (Bukova, 2006, 2008). Logical thinking is a complex process that includes exploring, using information effectively, creating formulas, and discovering unusual methods (Ersoy & Baser, 2013; Langrall & Swafford, 2000). Mathematical thinking skills include thinking processes related to life skills, such as producing practical solutions to daily living problems and establishing cause-effect relationships (Aslan & İlkorucu, 2017), as well as direct mathematical skills, such as developing a number sense and using abstract mathematical concepts skillfully (Yesildere, 2006). Mathematical thinking skills need to be developed from an early age as a necessary skill for problem-solving skills to meet daily living needs and for the effective use of methods of accessing, analysing, and producing information (Yildirim, 2015). Beyond developing a solution for the problem, by handling the different dimensions of the problem, it is possible to evaluate the source of the problem, the aspects affecting its emergence, the factors that will contribute to the solution, and the use of auxiliary skills as situations that require mathematical thinking (Karakoca, 2011).

Proportional thinking, a skill that can be evaluated within the scope of mathematical thinking (Akkus & Duatepe-Paksu, 2006), is necessary for students to use the concepts related to proportion and ratio correctly, understand mathematical relationships in proportionality problems based on multiplication, and discriminate between proportional and non-proportional situations (Cramer, Post, & Currier, 1993; Lesh, Post, & Behr, 1988; Pittalis, Christou, & Papageorgiou, 2003). According to the principles and standards determined by the Council of Teachers of Mathematics [National Council of Teachers of Mathematics (NCTM)] (2020), students' proportional thinking skills should be developed so that they can learn mathematical concepts including ratio and proportion, slope, percentage, similarity, linear equations, histogram, and probability, with a holistic approach.

For this reason, it is thought that it is important to conduct studies on proportional reasoning skills to examine complex processes that require mathematical thinking in mathematics education. Many studies in the literature have examined proportional reasoning skills. These studies have been conducted with middle school students to determine the proportional thinking skill levels (Akkus Cıkla, & Duatepe, 2002; Akkus & Duatepe-Paksu, 2006; Ben-Chaim, Fitzgerald, Benedetto, & Miller, 1998; Duatepe, Akkus, Cıkla, & Kayhan, 2005; Musan, 2012; Umay, 2003), qualitative and quantitative analysis of answers to problems requiring proportional reasoning skills (Aladag, 2009; Aladag, & Artut, 2012; Altayli, 2012; Atabas, 2014; Bayazit, & Donmez, 2017; Celik, 2010; Celik & Ozdemir, 2011; Debreli, 2011; H. Cetin, 2009; Kupcu, 2008; Kupcu & Ozdemir, 2011; Unsal, 2009; Wells, Dole, & Makar, 2014), and strategies used and mistakes made while solving ratio and proportion problems

(Avcu, 2010; Cramer & Post, 1993; İ. Cetin, 2009; Kayhan, 2005; Kaplan, İslenen, & Ozturk, 2011; Pakmak, 2014; Pelen, 2014; Sen & Guler, 2018; Umay & Kaf, 2005).

Students must first recognise the multiplicative relationships between quantities to develop proportional reasoning skills (Pelen, 2014; Toluk-Ucar & Bozkus, 2016). It is thought that a student who can distinguish multiplicative relationships between objects can easily notice whether the problem situation is proportional or not. For example, when a student is asked to enlarge a rectangle with side lengths of 4 cm and 8 cm by photocopying at a certain rate and make its breadth 6 cm, they are expected to calculate the length as 12 cm, which is 1.5 times 8 cm, considering the proportional, that is, the multiplicative relationship between 4 and 6. Instead, if the student considers the difference as 2 cm for the length, thinking that there is a 2-cm increment between 4 and 6 for the breadth, this can be interpreted as establishing an additive relationship.

It is possible to say that the proportional reasoning skill is integrated into almost every subject in mathematics. For example, subjects such as similar triangles, slope, probability, equations, fractions, logarithms, and trigonometry are related to proportional reasoning skills (Van de Walle, Karp, and Bay-Williams, 2013). In addition, it is thought that the process of expressing the ratios between the sides of geometric shapes symbolically will contribute to the development of geometric and algebraic thinking (Langrall & Swafford, 2000). For this reason, since the development of proportional reasoning skills will support the development of mathematical skills in many related subjects, studies on this skill gain importance (Musan, 2012).

Strategies Used in Solving Proportional Reasoning Problems

It is known that students develop incorrect and correct strategies while solving problems that require proportional thinking skills. Solution strategies used in solving problems related to proportional reasoning are classified as the unit ratio, change multiplier, cross multiplication algorithm, equivalent fraction strategy (Cramer and Post, 1993), and equivalence class strategy (Bart, Post, Behr, and Lesh, 1994). Incorrect solution strategies include giving an emotional response, additive relationships, data neglect, and using numbers and no content (Ben-Chaim, Fey, Fitzgerald, Benedetto, & Miller, 1998; Kayhan, 2005).

In the "additive relationship" strategy, an incorrect solution strategy, it has been seen that students cannot notice the multiplicative relationship and that they try to find the other ratio in the proportion by adding a certain value to the ratio by establishing an additive relationship between the variables. For example, in the problem, "If two glasses of sugar are used to make cookies for eight people, for how many people can you make cookies with five glasses of sugar?", designed to find the unknown value, students can make the following mistake:

5 cups of sugar – 2 cups of sugar = 3 cups of sugar

8 people + 3 people = cookies for 11 people

Here, instead of using the proportional relationship between the number of cups of sugar, the student considers the difference between the number of cups of sugar and says that it will make cookies for 11 people. This calculation suggests that the student establishes an additive relationship between the data.

In the "emotional response" strategy, students make mistakes by giving subjective answers when trying to relate the data in the problem with real-life situations. For example, consider the following comparison question: "A boy goes to the grocery and sees that 6 bottles of mineral water are 12 liras, and 4 bottles of soda are 10 liras. Which product is more economical?". Examples of their answer to this question include, "Soda is always more expensive, so mineral water is cheaper." or "I don't like mineral water at all; it is flavourless. I'd choose soda because it is better." These are subjective answers that are away from mathematical solutions. In this case, it can be said that students make an emotional response mistake.

In the "data neglect" strategy, students only focus on one situation or variable. For example, the answers to the grocery question above can be as follows: "Since soda is 12 TL, its price is higher, so soda is more expensive" or "4 bottles are less than 6 bottles. We pay less for fewer products." It can be said that students giving these answers focus only on the price or only the count of bottles and neglect other data.

In the last incorrect strategy, "using numbers and no content", students are aware that they need to use addition, subtraction, multiplication, and division operations with the numbers in the problem, but they use operations unrelated to the problem's solution.

When the literature is examined, it can be seen that the proportional reasoning skill is an important subject that can be developed from the primary education level and should not be neglected as a subject that supports the transition from arithmetic to algebra (Akkus-Cıkla & Duatepe, 2002; Akkus & Duatepe-Paksu, 2006; Aladag, 2009; Aladag & Artut, 2012; Altayli, 2012; Atabas, 2014; Avcu, 2010; Bayazit and Donmez, 2017; Ben-Chaim et al., 1998; Cramer & Post, 1993; Celik, 2010; Celik & Ozdemir, 2011; Fielding-Wells, Dole, & Makar, 2014; H. Cetin, 2009; İ. Cetin, 2009; Debreli, 2011; Duatepe et al., 2005; Kaplan, İsleyen, & Ozturk, 2011; Kayhan, 2005; Kupcu, 2008; Kupcu & Ozdemir, 2012; Martinez Ortiz, 2015; Ozdemir & Celik, 2011; Pakmak, 2014; Pelen, 2014; Sen & Guler, 2018; Umay, 2003; Umay & Kaf, 2005; Unsal, 2009). In this context, it is thought that proportional reasoning is important in the teaching of ratio and proportion and subjects related to ratios, such as rational numbers, fractions, percentage calculations, decimal representations, and similarity in triangles. In the current study, we tried to reveal the participants' thoughts when they needed to think proportionally and their mistakes (e.g., establishing additive relationships, establishing multiplicative relationships, failure to identify non-proportional situations). We focused on where these mistakes came from. It is thought that the study is important in giving teachers ideas about this subject and raising awareness about the importance of ratio and proportion, which is the most basic mathematics subject.

In the current study, the answers given by the students to the problems that require proportional thinking skills were evaluated with the help of SOLO Taxonomy. There are a lot of taxonomies in the literature, such as Bloom, Anderson, MATH, Fink, and Dettmer Taxonomy (Ari, 2013; Kocyigit & Moralı, 2020). In this study, we used the SOLO taxonomy, which is thought to be the most useful in determining the solution levels of the participants and categorizing them according to the indicative verbs.

SOLO Taxonomy

SOLO Taxonomy was developed by Biggs and Collis (1982) to analyse students' solutions following each grade level. It consists of 5 different hierarchical levels (Wadhwa, 2008). These levels are the "pre-structural level", where the student shows no signs of learning; "uni-structural level", where the student tries to learn by dealing with one single aspect of the subject; "multi-structural level", where the student can deal with several aspects of the subject but cannot make the connections between the aspects; "relational level", where the student can establish reasonable connections between the several aspects learned; and the "abstract level", where the student can make generalizations by reasoning beyond all the things learned. Due to the hierarchical structure of the SOLO Taxonomy, students' answers show improvement in terms of associations, consistency, and higher-order thinking skills as students move towards higher levels (Biggs & Collis, 1982).

At the first three levels of taxonomy, namely, pre-structural, uni-structural, and multi-structural levels, students progress quantitatively in their solutions and achieve surface learning (Ozdemir & Goktepe-Yıldız, 2015). At the relational and abstract levels, which represent more advanced levels, students grasp the question qualitatively and quantitatively and have deeper learning. Aiming to describe observable learning outcomes, SOLO Taxonomy is used in Mathematics, History, Geography, English, modern languages, and similar fields. Studies have generally used SOLO Taxonomy to do a detailed analysis of student answers, determine the thinking level of students at different grade levels according to SOLO Taxonomy, reveal how advanced mathematical structures are perceived in the minds of students in studies conducted with university students, and learn the details about these solutions.

In the mathematics education literature, there are many studies examining students' knowledge levels by using SOLO Taxonomy (Akbas, 2016; Akkas, 2009; Ardic, Yılmaz, & Demir, 2012; Ari, 2013; Bagdat, 2013; Bagdat & Anapa Saban, 2013; Celik, 2007; Goktepe & Ozdemir, 2013; Groth & Bergner, 2006; Kıanı, 2004; Kanyalihatipoglu, 2016; Lian & Idris, 2006; Rider, 2004; SArihan-Musan, 2012; Tuna, 2011). When the literature is evaluated in general, it can be seen that most of these studies have a qualitative or mixed design and that the participants consist of students at every grade level from secondary school to higher education. SOLO Taxonomy has also been used in studies as a tool in determining and categorizing the degree of accuracy of students' solutions in various mathematics subjects, such as algebraic expressions (Akbas, 2016; Bagdat, 2013; Bagdat & Anapa-Saban, 2014; Celik, 2007;

Rider, 2004), statistical thinking processes (Akkas, 2009; Groth and Bergner, 2006), data analysis (Ardic et al., 2012; Kiani, 2014), polygons (Kanyalihatipoglu 2016), and equations (Lian & Idris, 2006; SArihan-Musan, 2012). Similarly, in the current study, SOLO Taxonomy was used to examine the mistakes made by seventh-grade students in the process of solving problems that require proportional reasoning skills and to categorise and evaluate their solutions.

Method

Study Design

This study used qualitative case study methodology. According to Creswell (2007), a case study is a type of qualitative research in which the researcher examines one or more situations in depth that they have limited over time with the help of observations, interviews, documents, and reports. This is a design in which an event or situation is examined longitudinally, and there is a systematic data collection process (Buyukozturk et al., 2010; Subasi and Okumus, 2017). A special case was analysed in-depth using more than one data collection method in the current study.

Study Group

The study was carried out in a public school in a city centre in the Central Black Sea region in the 2016-2017 academic year. The research participants consisted of 10 students selected following the purpose of the study among 33 seventh-grade students. The criterion sampling method, one of the purposive sampling methods, was employed to determine the study group. The sample is selected among people, situations, or events related to the predetermined criteria and the problem is investigated in-depth to enrich the data in criterion sampling (Patton, 2014). According to the study's inclusion criteria, students with middle or high test scores were included in the study sample. Ten students who got the highest score among 33 students who participated in the proportional thinking skill test constituted the study sample.

Data Collection and Analysis

The proportional reasoning skill test and a semi-structured interview form were used as data collection tools in the study. The study data were analysed, and the student levels were classified into level groups with the SOLO Taxonomy rubric adapted by the researchers to reveal the mistakes made by the participants in the questions requiring proportional thinking skills. Individual interviews were conducted with the participants to analyse their answers and the solutions they developed in more detail. The next section presents more detailed information about data collection tools and analysis methods.

The Proportional Reasoning Skill Test and Analysis

The data were collected using the Proportional Reasoning Skill Test, which was developed by Akkus and Duatepe-Paksu (2006) to measure the students' proportional reasoning skills. Cronbach's Alpha value, which is the internal consistency coefficient of the 15-item test, was calculated as 0.86. The discriminating power indices of the test items varied between 0.50 and 0.71 (Akkus & Duatepe-Paksu, 2006).

The test was administered to 33 students in the study. Then, the researcher scored the students' solutions, and 5 problems numbered 2, 7, 9, 10, and 15 on the test and had the lowest correct solution percentage by the students were selected to be analysed to reveal students' mistakes. The correct solution rate was 33.33% for the 2nd problem, 12.12% for the 7th problem, 24.24% for the 9th problem, 18.18% for the 10th problem, and 6.6% for the 15th problem. According to the scoring key of the test, the minimum and maximum scores that students can get from these 5 problems ranged between 0 and 15. The students in the study group were divided into 3 score groups as low, middle, and high. Accordingly, students in the 0-4 score range (S2, S4, S5, S10) were classified as low score group, those in the 5-9 score range (S1, S3, S6, S7) were classified into middle score group, and those with scores between 10 and 15 (T8, T9) were assigned to the high score group.

The mistakes in the solutions of the 10 students were categorised and grouped according to the incorrect solution strategies in the literature, taking into account their similarities. The accuracy of the identified erroneous strategies was questioned in one-on-one interviews with the students. After the interviews, the strategies matched with the students' solutions were submitted for the approval of three academicians, experts in the field. It was difficult to classify the student's solution coded S10 for the 9th problem. Initially, the researchers considered this solution suitable for both the "using numbers and no content" and "data neglect" strategies. In this solution, which was submitted to expert opinion, the mistake was eventually coded as "using numbers and no content" strategy, taking into account the student's voice recordings and the interview process. In another example, students' incorrect solutions that involved setting up proportions in situations that did not contain proportions were classified as "multiplicative relationship strategy". However, as this mistake has not been included in the relevant literature with this title and in line with the experts' suggestions, the name of this strategy has been changed to "failure to identify non-proportional situations". The incorrect solution strategies determined due to the data analysis carried out under the control of experts consisted of five categories: additive relationships, data neglect, using numbers and no content strategy, emotional response, and failure to identify non-proportional situations.

The Semi-Structured Interview Form and its Analysis

An interview form is a tool used to collect similar types of data from a sample of different participants to obtain detailed information about a subject (Yildirim & Simsek, 2008). To analyse the data obtained from the test in detail and to make the students'

solutions clearer, a "semi-structured interview form" developed by the researcher was used.

The form was submitted to the opinions of experts to increase the validity of the interview form developed and determine the appropriateness of the proportional reasoning process for sub-skills; the experts consisted of one faculty member from the department of measurement and evaluation in education, one faculty member from the mathematics education department, one mathematics teacher, and one Turkish teacher to determine the appropriateness and intelligibility of the language of the test.

In the interview form, there were 8 questions that would help students explain problems 2, 7, 9, 10, and 15 selected for the study in detail. The first 3 questions on the form were designed to get general ideas of the students about the test administered and to reduce their excitement during the interview. In the next 5 questions, students were asked to summarise the 5 problems in question and explain their solutions. The audio recordings from the interviews were deciphered and analysed descriptively by grouping them according to predetermined incorrect strategies.

The SOLO Taxonomy Rubric and Its Analysis

Taxonomy is a tool that helps analyse an answer given by a student to a question in-depth and classify the students' thinking levels (Lung, 2000). The current study used SOLO Taxonomy to classify student solutions according to levels. The researcher conducted an adaptation study, and the adaptation was used as a rubric in the study to use the SOLO taxonomy to determine the students' proportional reasoning skills.

The levels of the SOLO Taxonomy are called pre-structural level (PS), uni-structural level (US), multi-structural level (MS), relational level (RL), and abstract level (AL). SOLO levels give the researcher information about the depth of learning by allowing the analysis of the answers given by the student to a question at five different levels. The properties of taxonomy levels are given below.

Pre-Structural Level: This is the lowest level of the SOLO Taxonomy. Students at this level generally do not understand or have little knowledge of the subject they are studying (Biggs, 1995). The students answer to the question, and the problem does not match. The student's attention is easily distracted by situations unrelated to the solution of the problem (Cetin & İlhan, 2016). Therefore, the student cannot perform the expected task properly.

Uni-Structural Level: At this level, students deal with the problem from a narrow perspective. Although the student has the necessary theoretical knowledge for the solution, they cannot successfully apply it (Biggs & Collis, 1991). At this level, students can explain the subject and perform simple operations. However, they cannot establish a relationship between the part they focus on for the solution and other parts. Therefore, there are inconsistencies in their answers.

Multi-Structural Level: At this level, students can recognise more than one aspect of the subject in the problem, but they cannot connect these different aspects (Biggs & Collis, 1991). They use a lot of theoretical information in their plans for the solution. However, they cannot develop a logical and consistent solution by bringing together different ideas. They try to explain their solutions at this level but fail to establish a cause-effect relationship.

Relational Level: At this level, students can deal with the subject or problem situation they are working on in a way that creates a logical and consistent whole (McGill, 2013). They can generalize their results to a similar problem situation. However, since they are limited to their knowledge, they cannot reach a conclusion beyond their knowledge and cannot make generalisations.

Abstract Level: At this level, the peak of SOLO Taxonomy, students can reason much more than their knowledge and reach generalizations by making connections between relationships. They can show examples of abstract thinking. They systematically reconstruct their knowledge by combining it with the power of interpretation and logic (Biggs & Collis, 1991). They can present new and different ideas, produce hypotheses and theories by making references, and follow more deductive ways to reach generalizations.

It was updated using the SOLO taxonomy levels and the indicative verbs prepared by Cetin and İlhan (2006), presented in Table 1, to use the SOLO Taxonomy Rubric for data analysis of the current study.

Table 1.

SOLO Taxonomy Levels and Indicative Verbs

	Quantitative Increase and Surface Learning			Qualitative Increase and Deep Learning	
	Pre-Structural	Uni-Structural	Multi-Structural	Relational	Abstract
Key Features	Things learned about the subject studied are incorrect, and no learning has occurred.	Focuses on a single aspect of the subject being studied.	Two or more aspects of the subject studied are understood. But no relationship between the parts can be established.	Different aspects of the subject studied are associated with each other.	Reasoning and generalisations beyond the available knowledge can take place.
Indicative Verbs	- Repeating what is given in the problem - Saying, "I don't know." - Failing to give an answer	- Explain - Describe - Memorize - Apply a simple operation - Name - Sort - Count	- Combine - List - Describe - Talk metaphorically - Plan - Applying algorithms and methods	- Analyse - Compare - Combine - Associate - Establish relationships between unknowns - Explain cause and effect - Apply a given theory to the relevant field	- Build a theory - Make generalizations - Guess - Build a hypothesis - Reflect - Apply the theory to a new field - Discuss - Examine in-depth

To carry out the adaptation study, first of all, the solutions created by the students for the problems 2, 7, 9, 10, and 15 were listed starting from the lowest level, namely, the pre-structural level, to the highest level, that is, the abstract level, and then the indicative verbs suitable for each level were selected. Thus, a rubric that is compatible with both the verbs and the information in the scoring key of the test was developed. The match between these five levels and students' solutions was determined by considering the accuracy of the solution and the inclusion of the verbs presented in Table 1. For example, phrases such as "I don't understand; makes no sense" in the student's solution were evaluated as pre-structural. If the student tried to reach a solution based on only one data in the question, the solution was evaluated as the uni-structural level. If the student reached the correct result through an unusual solution or reasoning, this solution was evaluated as the abstract level. Before using the rubric in the study, it was piloted to a small sample group. After necessary corrections were made, it was used in the main study to determine the proportional reasoning levels of the students.

Credibility and Ethics

Reliability and validity are the two most important criteria for qualitative research's credibility. The degree to which the measurement results obtained in a study are free from random errors is defined as the reliability of the research. The reliability of the research increases depending on the increase in the rate of the accurate measurement of the feature studied (Buyukozturk et al., 2010; Tanriogen et al., 2012). we used the rubric of the test and the SOLO Taxonomy rubric developed for the current study to progress with an objective and reliable approach in scoring the proportional reasoning skill test administered to the students. The inter-rater reliability was calculated to ensure scoring reliability during the data analysis phase. In this method, the reliability of the scoring of two or more observers for the same data is measured by the agreement between the score groups. It is stated in the relevant literature that as the scores given by the observers for the existing data and solutions get closer to each other, the reliability will increase (Bilgen & Dogan, 2017). A researcher who is an expert in primary school mathematics education was provided information about the SOLO Taxonomy. The rubric developed was chosen as the second-rater to determine the reliability. This rater scored the solutions of the 10 students for problems 2, 7, 9, 10, and 15 in the test in an isolated environment. Predetermined categories and the SOLO Taxonomy rubric were used as criteria in scoring. The researcher and the expert rater categorised the students' incorrect strategies and placed them at appropriate levels in the rubric. The solutions created by the 10 students for the 5 questions were scored and recorded in the table. Scoring was based on the levels in the rubric that correspond to 0 points for the lowest level and 5 points for the highest level and their explanations. When the score tables of the rater and the researcher were compared at the end of the process, it was found that 7 questions were coded differently: problem 2, S2 (researcher: 2, rater: 3) and S7 (researcher: 3, rater: 4); problem 7, S6 (researcher: 3, rater: 2) and S4 (researcher: 4, rater: 3); problem 9, S5 (researcher: 3,

rater: 4); and problem 10, S1 (researcher: 3, rater:4) and S3 (researcher: 3, rater: 2). In the scoring of problem 15, it was seen that the coding of both raters was consistent. It was observed that the differences between the scores were 1 point for all students. In cases where the raters disagreed about the level of the students, a consensus was reached by discussing the student's answers in detail. The inter-rater reliability of the rubric, which was finalised after the scoring process, was scored according to Miles and Huberman (1994).

$$\text{Reliability} = \frac{\text{Agreement}}{\text{Agreement} + \text{Disagreement}}$$

According to Miles and Huberman (1994), a reliability percentage of 70% or above in scoring is considered enough for inter-rater reliability. The reliability percentage was calculated as 86% using the formula above in the current study. It is possible to say based on this calculation that the rubric developed is a reliable tool for placing students' levels of proportional reasoning skills at the appropriate level of the SOLO Taxonomy.

Validity is defined as the degree to which a feature that is intended to be measured in research can be measured without any interference of different factors in the measurement process (Merriam, 2015). Interviews were held with the students in the study group using the data obtained in the present study. During the semi-structured interviews, the data were collected by the researcher personally. All interviews were audio-recorded. The researcher tried to proceed without bias until the data collection stage was completed and to be objective in her analysis by using the data of the voice recordings, the rubric of the administered test, and the developed rubric. After the voice recordings were transcribed, they were validated through member-checking. The recordings transcriptions were given to the 10 interviewed students. After they confirmed the accuracy of the content, the data were analysed. In the implementation phase of the research, necessary steps were taken to fulfil all ethical responsibilities, such as protecting the participants from all kinds of harm and keeping confidentiality. At the outset, written permission of the Provincial Directorate of National Education (Date: May 31, 2017, Issue: 27001677-44-E.7951860) and students' written consent, showing their voluntary participation, was obtained. Participants were informed that they could leave the study at any time, and it was ensured that their personal data would not be shared with anyone other than the research team.

Findings

In this section, the answers given by the students to the selected problems and the transcriptions of the interview recordings about the solution strategies they developed are presented. For the analysis, the solutions of problems 2, 7, 9, 10, and 15, in which the students made the most mistakes, were used. The five different incorrect solution strategies used by the 10 students in the study group were examined under separate headings. These incorrect strategies, presented under sub-headings in the chapter,

include establishing additive relationships, data neglect, giving emotional responses, using numbers and no content, and failure to identify non-proportional situations.

Findings Regarding Additive Relationship Strategy

This incorrect strategy is used by students in situations where ratios need to be compared. The solution requires adding a value to one of the ratios in proportion to find the other ratio. Students establish an additive relationship mistakenly instead of the multiplicative relationship that should be used due to the nature of the proportional relationships. The problems that the 10 students in the study group made mistakes by using the additive relationship strategy and the evaluation of these problems according to the SOLO Taxonomy Rubric are given in Table 2.

Table 2.

Use of Additive Relationship Strategy by the Participants

| Strategy	Student Code	Problem No.	Level of SOLO
Additive relationship strategy	S1	10	Multi-structural level
	S2	2, 10	Uni-structural level
	S3	10	Uni-structural level
	S4	10	Uni-structural level
	S5	2	Uni-structural level
	S6	2	Uni-structural level
	S7	10	Multi-structural level
	S8	-	-
	S9	2	Multi-structural level
	S10	7	Uni-structural level

According to Table 2, 9 out of the 10 students in the study group used the incorrect additive relationship strategy in their solution. The student coded S8 did not use this incorrect strategy for solving any problem. In this respect, it is possible to say that the additive relationship strategy was the incorrect strategy that the majority of the students used. For example, the solution of problem 10 by the student coded S2, who used the additive relationship strategy in the solution of the 10th problem given below, is presented in Figure 1.

Problem 10: "Orange juice is made in pitchers A and B in the figure. There is concentrated orange juice in dark-coloured glasses and water in light-coloured glasses. As shown in the figure, 2 glasses of concentrated orange juice and 3 glasses of water are put into pitcher A, and 3 glasses of concentrated orange juice and 4 glasses of water are put into pitcher B. Accordingly, which pitcher has sweeter orange juice? Please explain."

This problem requires quantitative comparison; accordingly, students are expected to conclude that pitcher B with a greater ratio of orange juice has sweeter content by setting up 2/5 and 3/7 or 2/3 and 3/5 proportions.

Figure 1.

Solution of S2 for problem 10
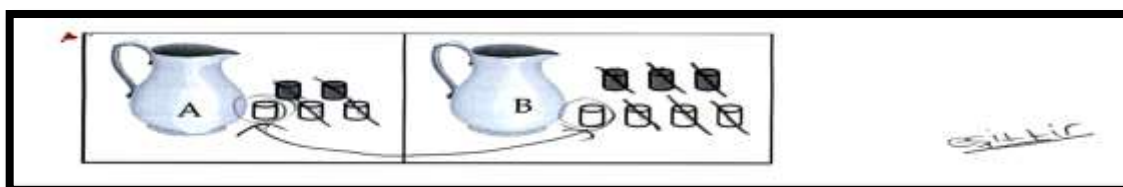
Quotations from the interview for the solution in Figure 1 are given below.

R　: Which pitcher do you think has sweeter orange juice?

S2　: I think both of them will be equal.

R　: Why will they be equal?

S2　: The pitchers are the same size. For example, this is paired with this, and this is paired with this, and 1 glass of water remains (the student wants to say that he has matched 1 glass of water with 1 glass of orange juice in pitcher A. According to this pairing, only 1 glass of water remains in pitcher A). Again, this is paired with this, and this is paired with this, and we have 1 glass of water remaining (the student matches concentrated orange and water for pitcher B, too). However, the taste does not change. There is no change in ratios. The liquid is more only in one of them.

R　: Well, you have just said they are equal, and the ratios are the same. What ratio are you talking about?

S2　: Teacher, the difference is 1 here (pitcher A) and 1 here (pitcher B). It has the same ratio.

R　: How did you use the ratio here?

S2　: I have just shown it, teacher. I matched orange juice glasses and water glasses exactly, so I proportioned them; only 1 glass of water was left. So, they taste similar.

Based on the student's statement, "… I matched them one-to-one, that is, I proportionated them. So, they taste equal", it is thought that while he was solving the problem, he focused on the difference between the number of glasses, neglected the multiplicative relationship between the number of glasses for each pitcher, and made a mistake by using the additive relationship strategy. The student thought he set up a proportional relationship by making one-to-one matching. In terms of taxonomy, student S2 misinterpreted the number of glasses in the problem and could not determine which pitcher had sweeter juice. Instead of setting up a proportional relationship between the numbers of glasses, he tried to establish an additive relationship. Since the difference between the numbers of glasses in both pitchers was

1, he thought that their taste was the same and gave an incorrect answer. According to the rubric, this incorrect solution corresponds to the uni- structural level.

Similarly, the solution of the student coded S2 for the 2nd problem is presented in Figure 2.

Problem 2: "Mr Short has a friend named Mr Tall. When the height of Mr Short was measured with a paper clip, it was found that he was 6 paper clips tall. When the height of Mr Tall and Mr Short was measured using a button, it was found that Mr Tall was 6 buttons tall, and Mr Short was 4 buttons tall. Accordingly, how tall is Mr Tall in paper clips?".

In this problem, which requires finding the missing value, students are expected to find the proportion between a paper clip and button length, that is, the equation of 1 button = 1.5 paper clips, and reach this result: "6 buttons = 6 x 1.5 = 9 paper clips."

Figure 2.

Solution of S2 for problem 2
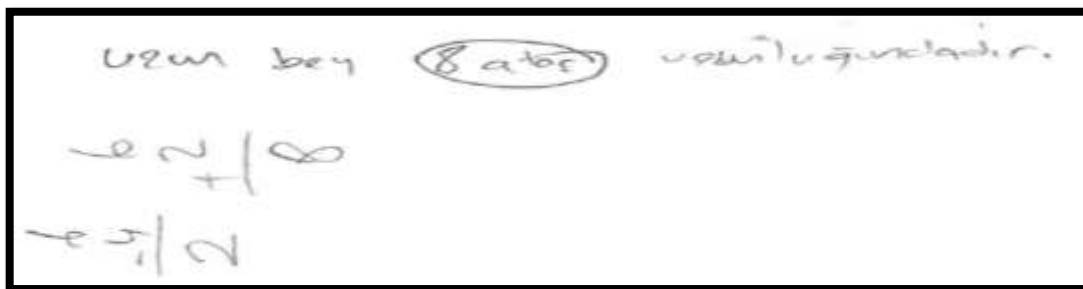
Quotations from the interview for the solution in Figure 2 are given below.

R   : How did you solve the problem?

S2   : Teacher, there is only one ratio for the buttons. Mr Tall is 6 buttons, and Mr Short is 4 buttons tall (he is talking about the heights of Mr Tall and Mr Short in buttons).

R   : What ratio are you talking about?

S2   : Teacher, the buttons have a certain proportional increment here.

R   : What kind of increment is this?

S2   : Mr Short is 4 buttons tall, and Mr Tall is 6 buttons tall. So, the increment is 2 units here.

R   : How did you determine the ratio here?

S2   : The ratio is 2. If the difference between the buttons is 2, then the same will apply to the paper clips.

R   : What does ratio mean? Can you explain to me the way you think of it?

S2   : The division of two things by each other.

R   : What does the number 2 that you have found mean then?

S2 : I found the difference between them. Isn't that the ratio? The same will be true for paper clips, too.

In this incorrect solution, it is possible to say that the student mistakenly established an additive relationship. The student's explanation, "The ratio is 2. If the difference between the buttons is 2, then the same will apply to the paper clips", supports the finding that he established an additive relationship in the solution. In the interview, when the student was asked to explain a ratio, he described it as the division of two things by each other. Although the explanation has a multiplicative meaning, the student used the difference between button lengths in his solution and tried to establish an additive relationship using this information. Based on his interpretation of the ratio as 2, when the student was asked what "2" meant, he said, "I calculated the difference. Wouldn't that be the ratio?" He identified the concept of ratio with multiplication and division operations. Still, while solving the problem, he made a mistake by thinking that he would obtain a ratio by calculating the difference between the variables. In terms of taxonomy, the student coded S2 realised that he needed to obtain a ratio by using the information given in the problem. However, he tried to find the measurements of Mr Tall by making use of the difference of the measurements of Mr Short. It can be said that the student who defined this process as a ratio could not make enough explanations for the solution. These indicative verbs correspond to the 'uni-structural level' in the rubric.

Another student who is thought to have used the additive relationship strategy was the student coded S1. His solution to problem 10 is given in Figure 3 (See Appendix 1- Question 10).

Figure 3.

Solution of S1 for problem 10
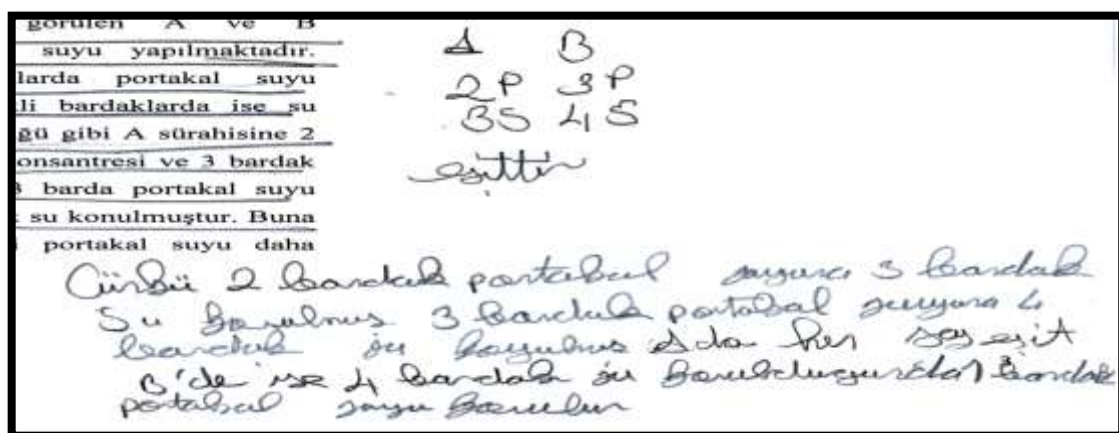
Quotations from the interview for the solution in Figure 3 are given below.

R   : How do we decide which is sweeter?

S1   : Teacher, they are all equal here.

R   : Why are they equal?

S1   : Because, for example, there are 2 glasses of orange juice here (he is talking about pitcher A) and 3 glasses of water here. The difference between them is 1. Here, too (he is talking about pitcher B), there are 3 glasses of orange juice and 4 glasses of water. So I said, "they are equal" because the difference between them is 1.

Based on the interview, it is thought that the student used the 'additive relationship' strategy in his solution. When the student was asked how he decided that the taste of the juice in the pitchers was equal, he replied, "It is equal because the difference is 1." According to this answer, he focused only on the quantities of the glasses and therefore could not see the multiplicative relationship between the glasses. As a result, he reached an incorrect solution by establishing an additive relationship. In terms of taxonomy, student S1 neglected the number of glasses, which has a multiplicative relationship, and tried to establish an additive relationship between these numbers and solved the problem incorrectly. These indicative verbs correspond to the "multi-structural level" in the rubric.

Findings on the Data Neglect Strategy

The type of mistake in which two ratios or only one of the proportional relationships is considered and the other is neglected is defined as the 'data neglect strategy'. Students who make this mistake focus on a single situation, relationship, or variable. The problems that the 10 students in the study group made mistakes by using the data neglect strategy and the evaluation of these problems according to the SOLO Taxonomy Rubric are given in Table 3.

Table 3.

Use of Data Neglect Strategy by the Participants

Strategy	Student Code	Problem No.	Level of SOLO
Data neglect strategy	S1	9	Pre-structural
	S2	-	-
	S3	-	-
	S4	10	Uni-structural
	S5	9, 10	Multi-structural, Pre-structural
	S6	-	-
	S7	2, 7	Relational, Uni-structural
	S8	-	-
	S9	7	Relational
	S10	-	-

As seen in Table 3, students coded S1, S4, S5, S7, and S9 used this incorrect strategy in their solutions. Among the 5 incorrect strategies included in the study, the data neglect strategy was determined as the most frequently used incorrect strategy after the additive relationship strategy.

The solution of the student coded S5, who is thought to have used the data neglect strategy in his solution for problem 9, is shown in Figure 4.

Problem 9: "Pita bread of the same size is produced in a restaurant. 7 girls eating at this restaurant share 3 loaves of pita, and 3 boys share 1 loaf of pita. Is the amount of pita bread per girl or boy more in this restaurant? Please explain." In this problem, which requires quantitative comparison, students must compare the ratios between pita bread. The ratios are 3/7 for girls and 1/3 for boys. Students are expected to find that the girl group with a larger ratio eats more pita bread.

Quotations from the interview for the solution in Figure 4 are given below.

S5 : Teacher, each girl ate half a pita.

R   : You say the girls ate half pita each. How much did the boys eat?

S5 : I think it is 1/3, that is, a quarter for boys.

R   : Does your drawing show that each girl ate half and each boy a quarter of a pita?

S5 : It says, "3 boys eat 1 loaf of pita". Do I not need to find pita per person?

R   : Well, you said boys ate 1/3 of a pita. How did you find that the girls ate half a pita?

S5 : 7 girls were eating 3 loaves of pita bread. I divided half of that pita and shared 1 piece for everyone.

R   : If you explain your solution according to your drawing, you haven't cut the girls' pita in half.

S5 : If I divide 7 by 3, they almost eat half a pita.

R   : Did you find ½, that is, half a pita answer by dividing 7 by 3?

S5 : If it were 6, it would be exactly half, but since it is 7, it is very close to half.

Figure 4.

Solution of S5 for problem 9
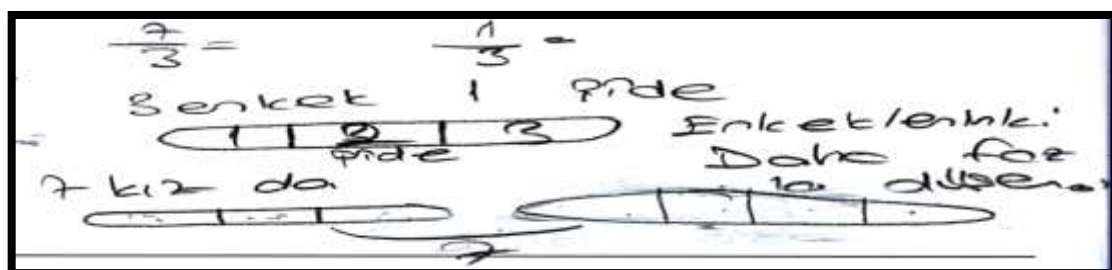
It was observed that S5 initially tried to solve the problem by drawing. It can be said that he used the 'data neglect strategy' in his solution, looking at the student's drawings. In the drawing seen in Figure 4, the fact that the student drew 1 pita incomplete, unlike the information given in the problem for girls, and focused only on the number of people, supports the finding that he used the data neglect strategy. While explaining

why the girls ate half a pita, the student said, "If it were 6, it would be exactly half, but since it is 7, it is very close to half". It can be said that the student can use reasoning about the concept of ratio, albeit superficially, looking at this explanation. However, it was observed that the student had difficulty applying his limited knowledge to solving the problem. He thinks that when he divides 6 by 3, he will get half, which supports the finding that he had difficulty solving the problem. In taxonomy, the student coded S5 drew a figure using the information given in the problem, randomly dividing 3 loaves of pita into 7 pieces and 1 pita into 3. S5 tried to determine the group that ate more pita bread according to his drawing. It was observed that the student had difficulty in understanding the concept of the unit ratio and applying it to the problem, so by setting up incorrect proportions (and incorrect ratio drawings), he found an incorrect answer. These indicative verbs correspond to the 'multi-structural level' in the rubric.

The solution of the student coded S9, who was thought to have used the data neglect strategy for problem 7 is shown in Figure 5.

Problem 7: "Mert and Mine work at the same speed and paint a wall in 10 days. How many days will it take to paint the same wall when 3 more people working at the same speed join them?" In this problem, students are expected to find the missing value by setting up an inverse proportion.

Figure 5.

Solution of S9 for problem 7

Quotations from the interview for the solution in Figure 5 are given below.

S9   : 2 people paint a wall in 10 days. Now, 3 more people who work at the same speed will join them. Considering the direct proportion, if 2 people do it in 10 days, it makes 5 people when 3 more people join them. We can find it with a direct proportion considering 5 people. That is, 2 people do it in 10 days. When 3 more people come, it makes 5 people. Let's call it x (talking about the number of days in the proportion that will take 5 people to paint the wall). It turns out that 2x = 50. When we divide the result by 2, x is equal to 25.

R   : Why do we need to use direct proportion to find it?

S9   : Because, teacher, the number of people increases here. As the number of people increases, the number of days will necessarily increase. For example, the more people eat at a table, the more bread should be on that table.

R   : But in our question, our bread count is fixed.

S9 : Wrong ... (thinking). Well then, it would be correct if we solved it using inverse proportion instead of direct proportion. I am confused.

R : So why do you think you should use inverse proportion?

S9 : I was confused about the proportion. I thought that if the number of people increases, the number of days should also increase. But since work is done here, the more people join, the sooner the work is done.

It is thought that the student found an incorrect solution because he used the data neglect strategy. He only focused on the increase in the number of people and neglected how the time would change according to the number of people. The student's interpretation, "Because, teacher, the number of people increases here. As the number of people increases, the number of days will necessarily increase", supports the finding that he only focused on the increase in the number of people. In addition, it was observed that the student had difficulty choosing the appropriate type of proportion for the problem. When the student was asked why he preferred the direct proportion in his solution, he admitted to using the incorrect proportion type. Based on this, it is thought that the student knows that directly proportional multiplicities require a proportional increment. Still, he made a mistake while transferring this knowledge to the problem. In taxonomy, student S9 correctly determined the person information in the problem but chose the type of proportion incorrectly.

The increase in the number of people in the problem was interpreted by the student as the need for more time to do the work. It was observed that the student's incorrect solution was that he thought he needed to use the direct proportion. These indicative verbs correspond to the "relational structure level" in the rubric.

Findings Regarding Emotional Response Strategy

Students' solutions based on their personal and real-life experiences without mathematical knowledge and solution processes are classified as the 'emotional response' strategy. In this type of mistake, the students' thoughts outweigh the requirements of the problem situation.

The problems that the 10 students in the study group made mistakes by using the emotional response strategy and the evaluation of these problems according to the SOLO Taxonomy Rubric are presented in Table 4.

Table 4.

Use of Emotional Response Strategy by the Participants

Strategy	Student Code	Problem No.	Level of SOLO
Giving emotional response strategy	S1	-	-
	S3		
	S4	10	Uni-structural
	S5		

S6	-	-
S7		
S8	10	Relational
S9	10	Relational
S10	-	-

According to Table 4, it was observed that only students coded S4, S8, and S9 used this incorrect strategy in their solutions. The emotional response strategy was one of the least used strategies in the study. The solution of student S8 for the 10th problem is given in Figure 6 (See Appendix-1, Question 10).

Figure 6.

Solution of S8 for problem 10

Quotations from the interview for the solution in Figure 6 are given below.

R : How did you decide which pitcher would have sweeter juice?

S8 : Teacher, 3 glasses of water are added to 2 glasses of concentrated orange juice. 4 glasses of water are added to 3 glasses of concentrated orange juice. I set up a proportion here. Teacher, there are 2 different pitchers in this question. There are two different mixtures in these pitchers with different numbers of glasses. I know I have to compare these mixtures, so I decided to find the number of glasses of water corresponding to one glass of orange juice to make the comparison. I had to set up a proportion while doing this operation because I had to calculate it for 1 glass out of many glasses. This decrease should be at the same ratio to not spoil the taste. I needed to use the direct proportion for the number of glasses to decrease at the same ratio.

R : Go ahead.

S8 : 3 glasses of water (pitcher A) for 2 glasses of orange juice, 4 glasses of water for 3 glasses of orange juice (describes pitcher B). 4 glasses are added. Teacher, I found 1 glass from here.

A : 1 glass of what?

S8 : Water. I thought if it was 3 for 2 glasses of orange juice, I took 1 glass of orange juice as x. I set it up as 2x=3. I divided it by 2. It was 3/2 (he found the amount of water in pitcher A corresponding to 1 glass of concentrated orange juice). Then I thought if it is 4 glasses of water for 3 glasses of orange juice (he is talking about

water), I took 1 glass of orange juice as x. Direct proportion: If 3x=4, x is 3/4. No, it is 4/3.

R : OK, which pitcher do you think will have a sweeter mixture after these operations?

S8 : Teacher, let's equalise the denominators. Let's use 3 for this (for A) and 2 for this (for B). 9/6 = 1.5 (pitcher A), and 8/6 = 1.3 (pitcher B), which has a repeating decimal. So, this (pitcher A) gets sweeter.

R : So what gives the content of the pitchers a sweet, sugary taste here?

S8 : Orange juice.

R : Well, here we found how much water is added to 1 glass of orange juice. Will adding more water or less water make it sweeter?

S8 : Of course, it will be B. Yes, I did not realise it, teacher. As it has a greater number, I thought pitcher A would have sweeter content. The one with less addition will be sweeter. The answer is B.

It is thought that the student found an incorrect solution because he used the emotional response strategy. During the interview, he successfully reasoned and did the necessary operations. However, when he made the right choice by interpreting the quantitative data available, he did not think that the glasses he was calculating contained water but only considered the large number and turned to an emotional response that the larger number would make it sweeter. The student's interpretation, "As it has a greater number, I thought the pitcher A would have sweeter content", supports the finding that he used the emotional response strategy. In terms of taxonomy, although the result of S8 was incorrect, it was observed that he had skills that required proportional reasoning in the solution process. The student, who had already grasped the concept of proportion correctly and explained why he preferred the direct proportion in the solution successfully, made the mistake of giving an emotional response in the final decision stage and therefore gave an incorrect answer. These indicative verbs correspond to the rubric's 'relational structure' level.

Figure 7 shows the solution of the student coded S4, who is thought to have used the emotional response strategy for problem 9 (See Appendix-1, Question 9).

Figure 7.

Solution of S4 for problem 9

Quotations from the interview for the solution in Figure 7 are given below.

R : What method should we follow to solve this problem?

S4 : Teacher, the pita bread of the same size is consumed in a restaurant. While 7 girls share 3 loaves of pita, 3 boys share 1 loaf of pita. Teacher, I perceived it as bigger. Boys will have more.

R : Why will boys have more?

S4 : Teacher, the number of girls is greater: 7 persons. But because there are fewer boys, the pita bread will be divided into fewer pieces.

It is thought that the student found an incorrect solution by using the emotional response strategy. It was observed that he only compared the number of people by using his personal experiences and without going through a mathematical solution process. He had a belief that the decrease in the number of people would increase the amount of pita bread per person. He was not interested in the fact that the number of pita bread that the girls had was greater and stated that the boys would eat more pita by considering the low number of boys. The student's interpretation, "Teacher, I perceived it as bigger. Boys will have more", supports the finding that he used the emotional response strategy. In terms of taxonomy, student S4 wrote his subjective evaluation without doing operations. He did not realise that he had to set up a proportion and had difficulty setting it up. Therefore, he made a mistake in his solution. These indicative verbs correspond to the 'uni-structural level' in the rubric.

Findings on Using Numbers and No Content Strategy

Using numbers and no content is a method that students prefer when they realise that they need to use the numbers in the problem but cannot establish a logical relationship between them. It was observed that students who used this strategy did different operations randomly instead of multiplication and division, which are especially used in proportional relationships.

The problems that the 10 students in the study group made mistakes by turning to the "using numbers and no content" strategy and the evaluation of these problems according to the SOLO Taxonomy Rubric are presented in Table 5.

Table 5.

Use of Using Numbers and No Content Strategy by the Participants

Strategy	Student Code	Problem No.	Level of SOLO
Using numbers and no content strategy	S1		
	S2	-	-
	S3	7	Uni-structural
	S4	7	Uni-structural
	S5	15	Uni-structural
	S6	-	-
	S7		
	S8	-	-
	S9		
	S10	9	Uni-structural

As seen in Table 5, only the students coded S3, S4, S5, and S10 used this strategy. It was observed that using numbers and no content strategy was used by 4 students. The solution of the student coded S3, who is thought to have used the strategy of using numbers and no content for problem 7 is given in Figure 8 (See Appendix-1, Question 7).

Figure 8.

Solution of S3 for problem 7
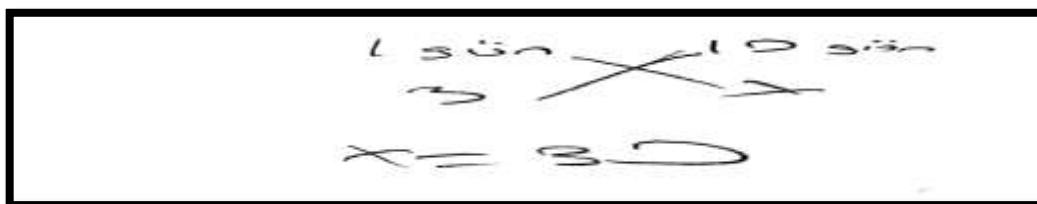
Quotations from the interview for the solution in Figure 8 are given below.

R : Can you explain how you have come up with the solution?

S3 : Mert and Mine finish it in 10 days, and 3 more people join them. When there are 3 more people, there many and finish it in a day. For example, 3, 5; they finish it on those days.

R : How did you interpret this information?

S3 : As the number of people increases, the number of days should also increase. 2 people in 10 days, 3 people in x days. Is that so?

R : Show me the way you think.

S3 : How many people paint the wall? 2? No, they are 3 people.

R : Is the information about the number of people given in the question?

S3 : Yes, teacher. 3 people paint it, which is the direct proportion. That's equal to 30.

It is thought that the student used the using numbers and no content strategy in his solution. When the student's comments were examined, it was seen that he noticed the numbers in the problem. However, he could not grasp the change in the number of people correctly. When 3 more people join Mert and Mine, the total number of people becomes 5, but the student said, "How many people paint the wall? 2? No, they are 3 people." He stated that he found the total number of people as 3. This student's interpretation supports the view that he could not understand how to use the quantitative data in the problem. The student's statement, "… is the direct proportion. That is equal to 30", shows that he uses direct proportion in a situation where he should use inverse proportion. The student could not grasp the problem quantitatively and did incorrect operations, which supports the finding that he used numbers and no content to solve the problem. In terms of taxonomy, student S3 misinterpreted the information about the number of people given in the problem and evaluated the total number of people as 3. The student who used the information about the number of

days and people given in the problem without considering their relationship made a mistake in his solution. These indicative verbs correspond to the 'uni-structural level' in the rubric.

Another student who was thought to have used numbers and no content in his solution was the student coded S10. This student's solution to problem 9 is given in Figure 9 (See Appendix-1, Question 9).

Quotations from the interview for the solution in Figure 9 are given below.

> R : You have written your answer in the solution, but you haven't given a satisfactory explanation. How should we go about solving this problem?

> S10 : Teacher, in this question, girls have more pita bread. When shared equally, boys eat less.

> R : How did you find it?

> S10 : Now, there are 3 boys and 7 girls. If we increase men to 7, we add 4. We increase these by 4, too (he wants to describe the quantity of pita that boys eat), and it becomes 5.
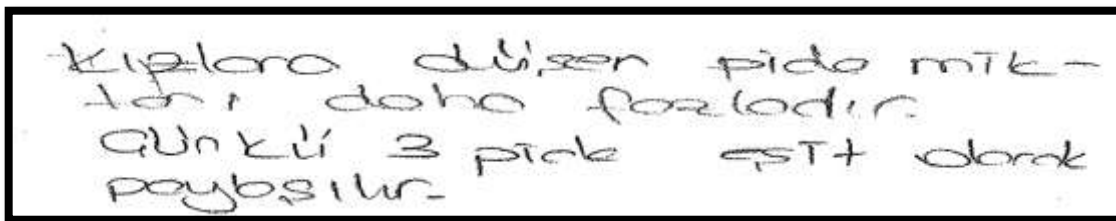
> R : So, how did you do these operations?

> S10 : I increased the number of boys at the same ratio, teacher. Now girls and boys are 7 each.

> R : Well, you increased the boys by 4, and they became 7. Why did you increase the quantity of pita by 4?

> S10 : I need to increase it at the same ratio, that's why.

Figure 9.

Solution of S10 for problem 9

It is thought that the student used the strategy of using numbers and no content in his solution. Although S10 wrote the correct answer on the answer sheet, it was observed that he could not make the necessary explanation. The student understood that he had to use the number of people and loaves of pita in the problem, but he could not realise how to use these numbers correctly in his solution. The student, who tried to reach the solution by equalising the number of persons given in the problem, stated that he needed to use a ratio while doing this operation. This was stated in his comment as follows: "I increased the number of boys at the same ratio, teacher. Now girls and boys are 7 each." It was observed that the student could not understand the concept of the ratio and thought that he calculated ratios by performing four operations with

numbers. These data support the finding that the student used numbers and no content in his solution. In terms of taxonomy, student S10 wrote the correct answer in the solution but could not make a sufficient and appropriate explanation. It was observed that he thought increasing values and proportioning were the same things. During the interview, the student thought that the group with less pita bread might have eaten less pita because he evaluated the given values quantitatively by considering their largeness or smallness. These indicative verbs correspond to the uni-structural level' in the rubric.

Findings Regarding the Failure to Identify Non-proportional Situations

It was determined that the students who made a mistake in problem 15, which did not require establishing a proportional relationship, used direct or inverse proportion in their solutions. The students' solutions, as if there was a proportion, without examining whether there was a proportional relationship, were evaluated under this heading.

Table 6.

Participants Making a Mistake by Failing to Identify Non-Proportional Situations

Method	Code	Problem No.	Level of SOLO
Failure to Identify Non-Proportional Situations	S1	-	-
	S2	15	Uni-structural
	S3	15	Uni-structural
	S4	15	Uni-structural
	S5	-	-
	S6	15	Multi-structural
	S7	15	Multi-structural
	S8	-	-
	S9	15	Uni-structural
	S10	15	Multi-structural

Table 6 presents problem 15, in which the 10 students in the study group made a mistake by failing to identify the non-proportional situation and the evaluation of this problem according to the SOLO Taxonomy Rubric.

As seen in Table 6, the students coded S1, S5, and S8 did not use this strategy in their solutions. It was observed that the strategy of using numbers and no content was used by 7 students, especially in the 15th problem, and together with the additive relationship strategy, it was one of the most frequently used incorrect solution strategies in the study.

The solution of the student coded S6, who was thought to have been unable to determine the non-proportional situation in his solution for problem 15 is given in Figure 10.

Problem 15: "Nevzatcan and Nergis have the same walking speed on a track. Nevzatcan starts walking first. If Nergis has completed 3 laps when Nevzatcan has completed 9 laps, how many laps will Nevzatcan complete when Nergis has completed 15 laps? Also, write an explanation of your solution." In this problem, which requires qualitative comparison, students are expected to notice the non-proportional

relationship between the number of laps. They are expected to find the answer as 15+6=21 by noticing that there are 6 laps between two people.

Figure 10.

Solution of S6 for problem 15
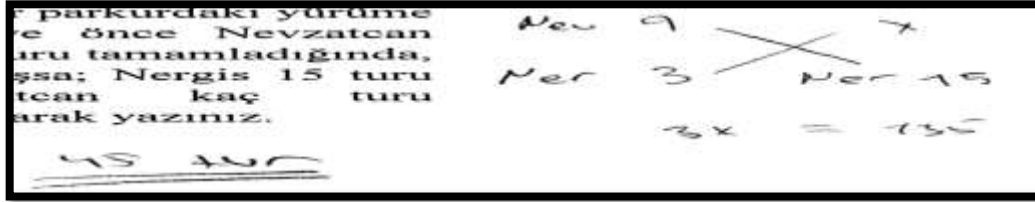
Quotations from the interview for the solution in Figure 10 are given below.

    R   : Can you explain what this question asks? How did you find the answer as 45 laps?

    S6 : Teacher, here is the solution: it increases from 3 to 15 laps, which increases by 5 times. 9 laps will also increase by 5.

    R   : How did you find that it would increase 5 times?

    S6 : Direct proportion. If one increases, the other will also increase. If it were otherwise, that is, if it decreased, it would not be a direct ratio; it would be an inverse ratio.

It is thought that the student could not identify the non-proportional situation in his solution. When asked to explain how he found the answer as 45 laps, he said, "…it will increase by 5 times." This comment supports the research finding that the student believed that although the problem required an additive relationship, he believed that he needed to use proportion in his solution. The student's comment, "Direct proportion. If one increases, the other will also increase. If it were otherwise, that is, it decreased, it would not be a direct ratio; it would be an inverse ratio", indicated that he did not have any difficulty in defining the direct and inverse proportion. However, it can be said that the student who could not realise the additive relationship required for the solution made a mistake by failing to determine the non-proportional situation. In terms of taxonomy, student S6 tried to compare the number of laps. However, instead of finding the difference between the number of laps, he thought there was a direct proportion between them and tried to compare the number of laps. These indicative verbs correspond to the 'multi-structural level' in the rubric.

Another student who was thought to have made a similar mistake in his solution was the student coded S7. This student's solution to problem 15 is given in Figure 11 (See Appendix-1, Question 15).

Figure 11.

Solution of S7 for problem 15

Quotations from the interview for the solution in Figure 11 are given below.

R : Can you explain this question?

S7 : Nevzatcan walks 9 laps, and Nergis walks 15 laps. But the question is asking us what Nevzatcan did last. I used inverse proportion here.

R : Why did you use inverse proportion?

S7 : I did it to find x.

R : Why did you use inverse proportion to find x?

S7 : Because we would have a small value. The number of laps would be very small.

R : Why does it matter to you whether the answer is a small or big value?

S7 : If x is small… Nevzat has already walked 9 laps at the beginning. If I set up a direct proportion, the result would be 5 laps. This guy has already walked 9 laps; how can the answer be 5 laps when he has already walked more than this?

It is thought that the student's reasoning was incorrect because he established a multiplicative (proportional) relationship instead of an additive relationship in his solution. The student stated that he used inverse proportion in his solution during the interview. This explanation clearly shows that he could not notice the non-proportional situation. When the student was asked how he got the answer as 45 laps, he said, "I set up an inverse proportion." When we look at the comment, it can be seen that he did not consider whether a proportion was necessary and chose an inverse proportion to avoid a mathematical contradiction. It is thought that the student, who focused on setting up the proportion, found an incorrect solution because he could not notice the additive relationship between the numbers of laps. In terms of taxonomy, it is thought that the reason for the incorrect solution found by the student coded S7 was that he believed that there should be a direct or inverse proportion between the numbers of laps. These indicative verbs correspond to the 'multi-structural level' in the rubric.

The frequency table of the strategies used by the 10 students is presented in Table 7.

Table 7.

Frequency Table of the Strategy Use

Strategies used	Number of students (out of 10)
Additive relationship	9
Failure to identify non-proportional situations	7
Data Neglect	5
Using numbers and no content	4
Emotional Response	3

When Table 7 is examined, it can be seen that the additive relationship strategy was used the most throughout the study, and it was followed by the failure to identify non-proportional situations mistake with a close number. It was observed that approximately half of the students made a mistake due to using the strategies of "using numbers and no content" and "data neglect". It was determined that the least common incorrect solution strategy was the emotional response.

Conclusion and Discussion

This study aimed to identify incorrect solution strategies that students created while solving problems requiring proportional reasoning skills and categorise these incorrect solutions according to SOLO Taxonomy. The results obtained from the study indicated that the students had difficulty in setting up proportions and made mistakes in their solutions. In the study, it was observed that the students used 5 different incorrect strategies. Among these strategies, the incorrect solution strategy that was most frequently used was the additive relationship. This strategy has also been seen in the literature as one of the incorrect strategies that students frequently use (Atabas, 2014; Bart et al., 1994; Ben-Chaim et al., 1998; Duatepe et al., 2005; Misailidou & Williams, 2003; Pakmak, 2014; Pelen, 2014; Wells et al., 2014).

It was observed that the students got 1 or 2 points in terms of proportional thinking skills in the 2nd and 7th problems, which required the use of inverse proportion and finding the missing value in the test administered to them. They got very low scores in these question types, which required finding the missing value by setting up a proportion and had difficulties setting up the proportion. Based on the interviews, it is thought that the students got these low scores because they were confused about which variables to set up a proportion between; they could not understand the mathematical requirements of the question. They did not realise that the information in the 7th question required inverse proportion. Similarly, it was observed that the students got 1 or 2 points in problems 9 and 10 of the test that required quantitative comparison. It is thought that they had difficulty in perceiving the concept of unit ratio in these problems, which involved a very close mathematical solution process. For example, in question 9, which required comparing the quantity of pita bread between girls and boys, the students often tried to do operations based on the total number of persons. It was found that they did not make this calculation over the unit number of persons, or those who made it failed because they made mistakes in comparing the ratios. Similar

mistakes were made in the 10th question, in which orange juice is prepared. Most of the students focused on the quantitative characteristics of the glasses, ignoring the relationship between them. In the 15th question of the test, which required additive comparisons, the students mostly got 0 points. Although the question did not contain a proportional comparison in terms of content, most of the students tried to solve the question by setting up a direct proportion. The increase in the quantitative data in the question might have caused this mistake. In the secondary school curriculum, the concept of direct proportion is explained as "a situation where one of the variables increases, the other must increase at the same rate". The fact that the number of laps that Nergis takes increases and that the number of laps that Nevzatcan takes increases because they continue to walk with the same speed may have indirectly led the students to the definition of direct proportion. For this reason, most of the participants in the study tried to solve this question by using a direct proportion, but they came up with a wrong solution for this non-proportional situation. These results show that, in the most general sense, students may have difficulty understanding proportion and identifying whether there is a proportional situation.

When the students' proportional reasoning scores and levels were compared with the SOLO Taxonomy levels, it was observed that consistent results were obtained. The students with high reasoning scores (S8 and S9) were at relational and abstract structure levels, the two highest levels in taxonomy criteria. When the answers of these two students at this level were examined, it was determined that these students mostly had difficulty discriminating the non-proportional situations. Although they found the correct answer, they made a mistake because they could not interpret the real-life situation required by the question. It was observed that these students, who did not have problems discriminating the types of proportions, were more successful in setting up proportions than other students.

The students with medium reasoning scores (S1, S3, S6, S7) were in the uni-structural and multi-structural levels, which correspond to the middle level of the taxonomy. These 4 students got the lowest score from the 9th, 10th, and 15th questions, which required quantitative-qualitative comparison. It was observed that the students focused on only one variable in these problem situations where they had to make comparisons. These variables showed themselves in the solution as focusing on the number that was greater or paying attention to the number of laps that only one person took.

The students who got low reasoning scores (S2, S4, S5, and S10) were placed in the low levels of taxonomy criteria. The thinking score of all these four students was 0 points for making a qualitative comparison in the 15th problem and was placed in the uni-structural level of the taxonomy that corresponded to 1 point. Based on the general scores and the interviews, it can be said that these students could not notice the direct and inverse proportional situations, they could not notice the non-proportional situations, and they could not understand the question well. It was observed that as the score level decreased, students' mistakes due to the lack of proportional reasoning increased. These findings are similar to the proportional reasoning levels defined by Akkus and Duatepe (2002), Langrall and Swafford (2000), and Pittalis, Christou, and

Papageorgiou (2003). Similar to Sen and Guler (2018), students with low proportional reasoning levels could perceive proportional situations in general but often made calculation errors. Students who found the correct answer could not explain their reasons satisfactorily. The results obtained from studies using SOLO taxonomy and conducted on different subjects of mathematics showed that the majority of the participants were below the relational structure level (Akkas, 2009; Ardic et al., 2012; Bagdat, 2013; Celik, 2007; Goktepe, 2013; Groth & Bergner, 2006; Lian & Idris, 2006). In this respect, it is possible to say that the present study's findings are consistent with the literature.

According to the results of this study, the students had difficulty understanding terms and concepts, such as ratio, proportion, direct proportion, and inverse proportion, and using them appropriately in the problem-solving process. It is thought that students make mistakes in learning ratio and proportion due to their difficulties in questioning situations requiring proportional relationships and doing the necessary mathematical reasoning. There are studies with similar findings in the literature (Altayli, 2012; Debreli, 2011; Gozkaya, 2015; İ. Cetin, 2009; Kurdal, 2016; Kocyigit-Gurbuz, 2018; Ozturk, 2011).

In the present study, it was observed that students tended to use additive relationships instead of multiplicative relationships while setting up a proportional relationship between the variables. For this reason, it is thought that it is necessary to focus on conceptual explanations instead of algorithmic operations in teaching the concepts of direct and inverse proportion. Using daily living problems that students can reason with, make judgments, and verify to develop proportional thinking skills is recommended. In addition to students' answers to the questions, the thinking processes are also very important in teaching. Especially during the lesson, it is thought that asking students to express their opinions about the subject or questions aloud is necessary to determine what kind of thinking process the students are in and prevent possible learning errors. In this process, it is thought that it will be useful to ask additional questions that will reveal why the student thinks that way and how they have reached the result and question the answers together. In addition, problem-solving processes can be enriched with daily living activities. For example, in the 15th problem, the student who found the answer as 45 laps may be asked to go out to the school garden with his friends and check his answer. In this way, it is thought that students will be provided with the opportunity to understand mathematically why their answer is wrong and check whether their answer fits the reality of daily living in problems involving daily life situations. In the study, SOLO Taxonomy and indicative verbs were utilised to analyse the thinking processes about ratio and proportionality problems in detail. Studies can also be conducted using SOLO Taxonomy for different mathematical skills, such as algebraic thinking, statistical thinking, and geometric thinking. Studies can also be planned using the revised Bloom Taxonomy and Fink and Dettmer Taxonomies instead of SOLO Taxonomy. In the study, it was observed that there were very few student solutions placed at high levels of SOLO Taxonomy, especially at the relational structure and multi-structural levels. For this reason, it is recommended to conduct studies with

students from higher grade levels to see detailed analyses of the specified taxonomy levels in future studies using the SOLO Taxonomy.

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