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From physical reality to the Metaverse: a Multilayer Network Valuation

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***Abstract*— The physical reality can be partially mapped with network theory, showing the edging links between connected nodes, and their spatial and intertemporal dynamic interaction. The Internet is a network of networks representing a global system of interconnected computer networks. The metaverse is a network of 3D virtual worlds focused on social connection. There is so an evident Ariadne's thread between these ecosystems, interpreted with multilayer network theory that examines the connectivity and interdependency between nodes positioned in the physical world, the web, or the metaverse.**

This pioneering study illustrates a new research avenue, analyzing the application of some of the most evident properties of network theory to the case, showing for instance how replica nodes can link through an avatar the physical world with the metaverse. A valuation methodology of the metaverse ecosystems will be proposed, using a with-and-without approach or multilayer network metrics.

Keywords*— *Avatar, MetaEconomics, connectivity, digital platform, scale-free network, scalability.

I. Introduction

The three-step supply and value chain patterns from the physical world to the metaverse pass through the Internet and follow a technological upgrade whose eventual outcome is still uncertain. In the future, the metaverse [1; 2; 3] is likely to fully incorporate the earlier-stage Internet dimension.

This study starts from the evidence that physical reality can be partially mapped with network theory, showing the edging links between connected nodes, and their spatial and intertemporal dynamic interaction. The Internet is a network of networks representing a global system of interconnected computer networks. The metaverse is a network of 3D virtual worlds focused on social connection. There is so an evident Ariadne's thread between these ecosystems, interpreted with multilayer network theory that examines the connectivity and interdependency between nodes positioned in the physical world, the web, or the metaverse.

The metaverse is an Internet evolution that is oriented towards shared activities (mainly through social networking) with an exponential rise in creativeness, unleashed by a decentralized ecosystem, and integrated technologies.

This innovative study investigates a new research field, analyzing the application of some of the most evident properties of network theory to the case.

Consistently with this framework, *the main research question of this paper is to investigate the potential market value of metaverse ecosystems, using a with-and-without approach or multilayer network metrics.*

II. Scale-Free Networks and the Metaverse Topology

Network theory [4; 5; 6; 7; 8], is the study of graphs as a representation of either symmetric or asymmetric relations between discrete objects.

The World Wide Web is a network whose nodes are documents, and the links are the uniform resource locators (URLs) that allow to "surf" with a click from one web document to the other [4].

A scale-free network is a decentralized network whose degree distribution follows a power law and is characterized by the presence of large hubs (pivoting nodes). Decentralization is consistent with the web, blockchains, and the metaverse.

A scale-free network looks like the air-traffic network, whose nodes are airports and links are the direct flights between them.

Scale-free networks have a very heterogeneous distribution of degrees, and their dynamical behavior is dominated by the hub nodes having a degree order of magnitude larger than the average.

The topology of the metaverse influences its dynamics, workings, and wealth distribution. According to [9], the metaverse, consistently with the Internet, is more like a scale-free network than to a "hub and spoke" architecture, where every network node connects to a central authority that is responsible for controlling access and managing any of the exchanges.

In scale-free networks (e.g., the Internet, open-source software, smart contract blockchains, etc.), the central node acts more like a facilitator than an authority, and where nodes are then free to connect.

III. Scalability and the Network Effect

Scalability indicates the ability of a process, network, or system to handle a growing amount of work. Scalability fosters economic marginality, especially in intangible-driven businesses – such as the Internet or the metaverse – where variable costs are typically negligible. Massive volumes may offset low margins, producing economic gains. Digitalization is defined as the concept of "going paperless", the technical process of transforming analog information or physical products into digital form. Digital scalability operates in a web context, where networked agents interact to generate co-created value.

Economic and financial margins that represent a primary parameter for valuation are boosted by cost savings and scalable increases of expected revenues. Digitalized intangibles synergistically interact through networked platforms that reshape traditional supply chains. Link (edge) overlaps and replica nodes foster these synergies.

Table I. synthesizes the main properties of the network laws [10].

TABLE I. – NETWORK VALUE

Scalability law	formula	Features / properties
Sarnoff's law	Network Value=n	The value of a network seemed to increase in direct proportion to the size of the network - proportional to *N*, where *N* is the total number of users on the network.
Metcalfe's law	Network Value=n^2	Network value increases exponentially with an increasing number of devices on the network
Reed's law	Network Value=2^n	Network value increases even more than Metcalfe's as subgroups (social networks; messaging apps, etc.) become easier to form. Reed's law is consistent with multilayer network extensions.
Beckström's law		The value of a network equals the net value added to each user's transactions conducted through that network, summed over all users. This model values the network by looking from the edge of the network at all of the transactions conducted and the value added to each. It states that one way to contemplate the value the network adds to each transaction is to imagine the network being shut off and what the additional transactions costs or loss would be.
Radoff's law	[qualitative proposition]	The degree to which a network facilitates interconnections determines the extent of its emergent creativity, innovation, and wealth.
Metaverse extension		3D dimension, increased networking, technological upgrade, etc. improve scalability and so value

Reed's Law is often mentioned when explaining the competitive dynamics of internet platforms. As the law states that a network becomes more valuable when people can easily form subgroups to collaborate, while this value increases exponentially with the number of connections, a business platform that reaches enough members can generate network effects that dominate the overall economics of the system.

Other analysts of network value functions [11] have argued that both Reed's Law and Metcalfe's Law overstate network value because they fail to account for the restrictive impact of human cognitive limits on network formation.

IV. MULTILAYER NETWORKS

Multilayer networks are networks with multiple kinds of relations with multiplex or multidimensional configurations [12]. In a multiplex network, the same set of nodes is connected via more than one type of link, enhancing scalability.

The world is more complex than conventional economic models traditionally assume. Many real-world complex systems are accordingly best modeled by multiplex (multidimensional) networks of interacting layers [13]. These interconnected systems are very sophisticated and may explain better the applications in the field of social network analysis, economics, operations management, finance, etc., being consistent with corporate governance concerns.

Multilayer networks are an extension of the traditional networks and are fully consistent with the framing and research aim of this study. Multilayer networks are intrinsically fit for leveraging the scalability features already examined, since they host bridging (replica) nodes, digital networks, or firms that are simultaneously present in several layers. These properties have deep, albeit non investigated, governance consequences.

Complex multidimensional networks host multiple kinds of relations (multiplex, multilayer, multilevel, multi-relational, interconnected, interdependent, etc.), and may yield valuable insight in many interdisciplinary fields. These networks of networks may affect social networks that involve different types of connections, networks of airports connected by different air carriers, multiple infrastructures of a country that are mutually connected, etc.

Nodes that simultaneously belong to different layers (networks) can be represented mathematically by adjacency tensors with inter-layer edges that connect each network to the other. These links enhance the overall value of the network of networks, boosting Metcalfe's formulation.

Whereas the sophisticated mathematics that explains these relations [12] goes far beyond this preliminary study, some economic implications may be worth considering.

A multilayer network - of which multiplex and interdependent networks are peculiar cases - is a network made up of multiple layers, each of which represents a given operation mode, social circle, or temporal instance.

Multilayer networks show connectivity links between the nodes of each layer that bring interdependency links.

A key feature of any network is represented by its dynamic properties: a network is hardly ever static, and this is never the case on the Internet- metaverse dimension. Many networks expand and grow by increasing their number of nodes and links over time. examples include the Internet and social online networks this evidence is described by non-equilibrium dynamics important information can be gained by studying non-equilibrium models of growing networks that show scale-free properties that can emerge from simple dynamical rules of network growth.

Dynamical processes include:

a. Percolation (the behavior of a network when nodes or links are added);

b. Diffusion/propagation; (the ability to amplify association between nodes that lie in network proximity).

c. Spreading processes on complex networks, showing the transmission probability within and outside a network.

The interplay between the “random versus orderly” structure and the dynamics of multilayer networks contributes to explaining their essential features.

Relevant information from multilayer network data sets cannot be found by considering networks in isolation. Connectedness allows for the possibility of diffusing information and hyper-navigating the network. A further characteristic is represented by communicability - the number of paths that connect the node to the rest of the nodes within the network.

Precious informative sets are represented by big data that are gathered in cloud databases. The digital nature of data softens interoperability concerns, easing information dissemination and exploitation.

This contributes to explaining why interconnected (multilayer) networks are worth more than isolated networks. The implications for metaverse networks that are ontologically connected are evident, albeit hardly investigated.

A further feature is represented by navigability - the possibility of exploring large parts of the network by following its paths through connectedness. This concept can be associated with supply chains and shows which are the iterative patterns from the real world to the metaverse, passing through the internet.

V. Multilayer networks

A metaverse is a network of 3D virtual worlds focused on social connection; a set of virtual spaces where an individual can create and explore with other people who are not in the same physical space.

The term "metaverse" has its origins in the 1992 science fiction novel Snow Crash as a portmanteau of "meta" (in Greek μετα = beyond) + "universe".

The metaverse building features are represented by:

- Social media
- Online gaming
- Digital identity/avatar
- Decentralization/blockchains
- Cryptocurrencies
- Virtual reality
- Augmented reality
- Creator economy (value co-creation patterns)

Complex and interdependent technologies are the core constituents of the metaverse ecosystems:

- Multilayer networks
- Digital platforms
- Interactive technologies
- 5G/6G
- Computer vision
- IoT / robotics
- 3D print
- Distributed computing / Blockchains
- Distributed storage
- Quantum computing
- Cloud/edge computing
- Hardware infrastructure
- Artificial intelligence

VI. Synchronizing the Physical and Virtual: The Avatar Bridging Node

In computing, an avatar is a graphical representation of a user or the user's character or persona. It may take either a two-dimensional form as an icon in Internet forums and other online communities (where it is also known as a profile picture) or a three-dimensional form, as in games or virtual worlds. Another use of the avatar has emerged with the widespread use of social media platforms. There is a practice in social media sites: uploading avatars in place of the real profile image.

Avatars – a sort of virtual second life – can be considered the main bridging node connecting the real world to the metaverse (through the web).

Even if avatars are traditionally linked to “light” applications (e.g., video games or social entertainment), they are increasingly used in more significant practices. For instance, digital twins are used in medicine. A digital twin is a digital representation of real-world entities — an object, system, or process — that is synchronized with the real world.

Even if the original node in the real world (represented by a physical person) is different from Her digital avatar, they may tentatively be considered, as a necessary simplification in this study, substantially coincident.

Thanks to augmented (and virtual) reality, an avatar can be identified, copied, measured, increasing Her value, if compared to the original.

According to [14], “In Multiplex Networks a set of agents might interact in different ways, i.e., through different means. Since a subset of agents is present at the same time in different networks of interactions (layers), these layers become interconnected”. These agents are represented, in our case, by bridging avatars and other players (digital platforms, etc.).

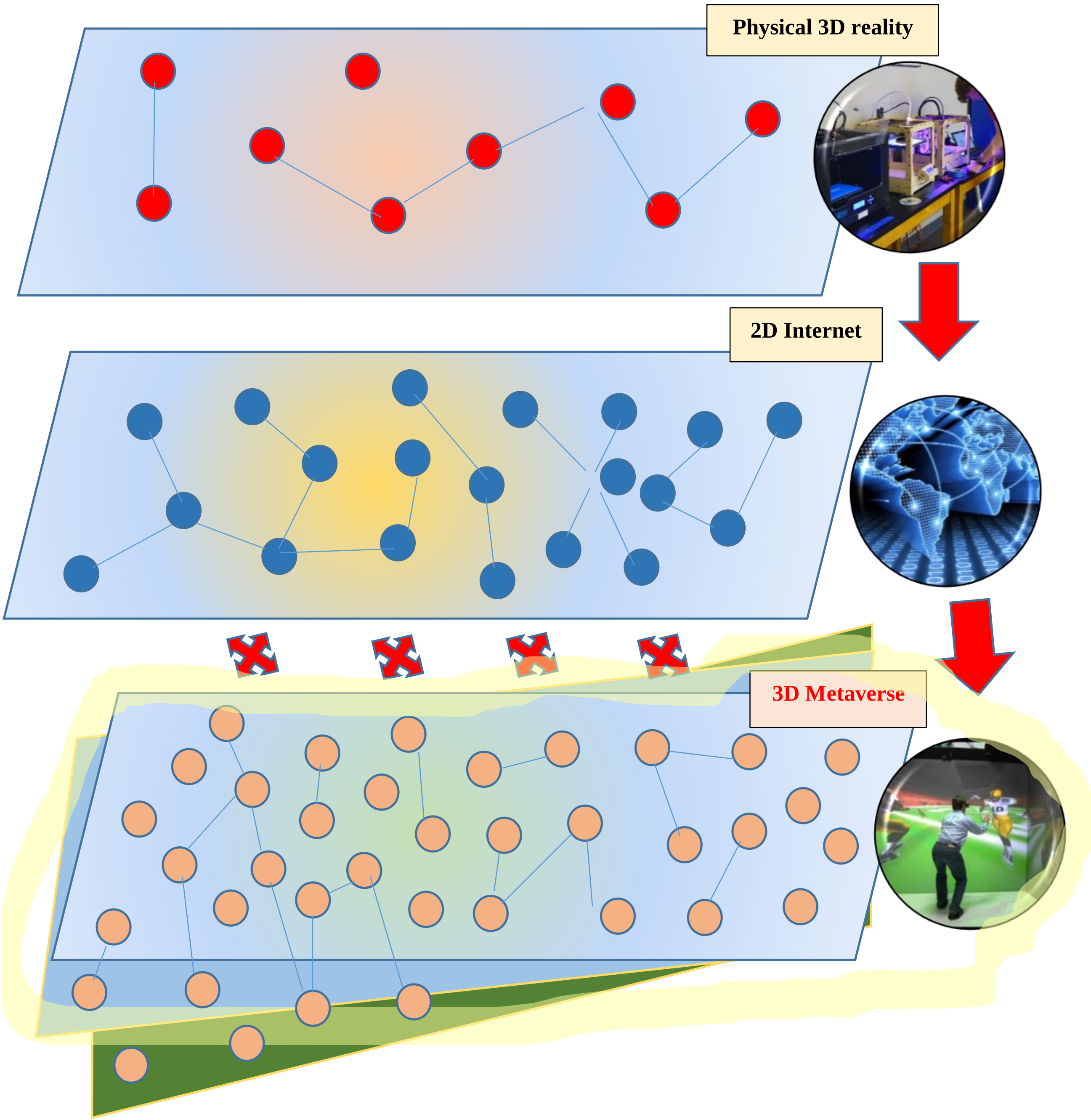
VII. A Holistic Ecosystem: From physical reality to the Internet and the Metaverse

The metaverse and the physical world interact in both directions, generating value-enhancing synergies.

According to [15], “MetaEnterprises and MetaCities can be regarded as the mapping of real enterprises and cities in the virtual cyberspace. They are virtual enterprises and cities running parallel to real enterprises and cities, which can realize the description of real enterprises and cities. Corresponding to the human, material, organizations, scenarios, and other elements in real enterprises and cities, there are various virtual elements such as virtual human, virtual objects, virtual organizations, and virtual scenarios in MetaEnterprises and MetaCities. These virtual elements in MetaEnterprises and MetaCities can be used to analyze and evaluate the decision-making scenarios with computational experiments approach so as to realize the prediction of real enterprises and cities. Through the interaction and feedback between MetaEnterprises/MetaCities and real enterprises/cities, we can realize the prescription of decision-making in real enterprises and cities, so as to effectively improve the efficiency and effect of various decisions in real enterprises and cities".

The three-step pattern from the physical reality to the metaverse can be illustrated in Figure I.

FIGURE. I. - FROM PHYSICAL REALITY TO THE INTERNET AND THE METAVERSE



Any consideration about the potential market value of the metaverse would be considered as science fiction since the underlying concept is uneasy to define and impossible to measure.

An estimate of the value of the metaverse is, however, important because investors need market traction to drive their efforts, envisaging potential returns out of their expenditures.

The metaverses are spaces where you can not only admire places, monuments, and works of art but also conclude business. The metaverses represent worlds in which commercial exchanges are becoming increasingly important together with the market value of the companies that produce supporting technology. According to a Bloomberg Intelligence report, the market value of companies operating in metaverses is expected to reach $ 800 billion by the middle of this decade and $ 2.5 trillion by 2030.

Inside the metaverse, on the other hand, commercial exchanges take place on two floors. On the one hand, there is the presence of shops that act as a showcase and traditional e-commerce [16], while allowing a more immersive experience in which the customer chooses a product that exists in the real world and pays for it with fiat currency. At the same time, in the metaverse, there are properties, goods, and other values that are represented by NFTs or, more precisely, they are NFTs whose exchanges take place through digital currencies.

The metaverse, through the interactions between avatars, makes possible strategies of value co-creation in which the single virtual nodes actively participate in this creation, receiving remuneration in tokens/cryptocurrencies that are usually lacking in many traditional business models. Even the feedback on the Internet (such as, for example, the reviews on Tripadvisor) does not involve any direct remuneration for the user, who for her part provides valuable big data that feeds increasingly advanced profiled marketing strategies. With the metaverse, there is a customer-centric qualitative leap, which places the user at the center of value co-creation and sharing of new value.

The currency of the metaverse is currently represented by controversial cryptocurrencies, linked to blockchain technologies potentially harbingers of tax evasion and money laundering. The boundaries and exchanges between fiat money and cryptocurrencies are still confused, although full convertibility could, in perspective, represent an important milestone in the convergence of segregated ecosystems.

The value of the metaverse can be direct, if it concerns this integrated ecosystem (declined, as it has been shown, in many interrelated dimensions), or transferred to the real world and the Internet, which expand the range of goods and services exchanged, also in terms of usability, generating a differential / incremental surplus value.

The new paradigms of value co-creation rely, in many cases, on social networks and behavioral models inspired by the sharing economy, facilitated by the plasticity and resilience of the digital and virtual world.

The growing sensitivity towards development paths inspired by sustainability and ESG metrics must be confronted, first, with the energy-intensive trends of blockchains.

In the logic of the metaverse, the experiential experience and the contents shared between the virtual players will assume preponderant contents in the co-creation of shared value. In perspective, the content will be able to count more than the technological infrastructure, destined to become a commodity (this trend is already visible in the world of digital media).

Metaverse business models fuel value creation and monetization strategies. A summary is shown in Table II.

TABLE II. - THE METAVERSE: BUSINESS MODELS AND VALUE CREATION

Business model	Value leverage and monetization strategies
3D clothing e-commerce	Presentation of the product in the metaverse with offline delivery to the customer. Sale of virtual products (see the "Gucci Garden Experience" event on the Roblox platform for all).
Metaverse platforms	Sale of cryptocurrency that can be spent on the platform, advertising agreements, license agreements, and royalties on the sales of products on the platform.
Real Estate	Sale of "finished" properties and both virtual plots of land. The user, in turn, can monetize the investment through renting and other forms of management. Sale of professional services (architecture, design, etc.) necessary or useful for the construction of the virtual property. Sale of waste materials following the demolition of virtual properties.
NFT	Direct or auction-based sale of unique "computer codes" linked to digital-only works or digital representations of physical works with or without transfer of the relative copyright.
Digital / profiled advertising	Sale of experiential marketing services in which the customer can experiment with a product and submit indications for its further development.
Gaming	Sale of the cryptocurrency needed to purchase the characters and accessories to participate in the game. Play-to-earn model that awards rewards to users based on their participation in the game. These rewards are in cryptocurrency that users reinvest in the game itself, guaranteeing liquidity to the system. Sale of game characters in the form of NFT.
Immersive experiences	Sale of the devices necessary to take advantage of the gaming experience or participation in virtual events (3D viewers, headphones, gloves, etc.).

The pioneering investments in the metaverse are based on prospects of economic returns in the medium-long term, which in turn depend on the revenue model incorporated in the business models and disruptive strategies, with a highly innovative and discontinuous scope. These investments are made above all by Big Tech (starting with FAANG - Facebook, Amazon, Apple, Netflix, and Google), intent on creating new lifestyles and entertainment, in the hope that they will become market standards, guaranteeing promoters the role of first movers and standard settlers, from which oligopolistic rents can derive (where barriers to entry are created for new competitors).

Consistently with the methodology of this study, the potential value of each metaverse ecosystem may be tentatively appraised with complementary approaches that are based on the mentioned monetization strategies:

1. With-and-without approach (comparing a hypothetical value with or without the metaverse, so showing its incremental/differential contribution in value creation);
2. Mathematical modeling consistent with multilayer networks;
3. Network effect laws.

The with-and-without methodology is traditionally used in the evaluation of intangibles and is so consistent with the intrinsic features of the metaverse. For instance, the International Valuation Standard (IVS) 210 considers this method within the income approach methods (§ 60.5).

The incremental (marginal) value of an added network, represented in this case by a metaverse ecosystem, can be economically appraised by the with-and-without approach, and mathematically evaluated using (multilayer) network theory. The Network Effect Laws illustrated in section III. are complementary to these approaches.

To the author's best knowledge, this joint approach has never been used in network valuation.

Every network can be expressed mathematically in the form of an adjacency matrix. In these matrices, the rows and columns are assigned to the nodes in the network and the presence of an edge is symbolized by a numerical value.

The matrix representation is fully consistent with:

a) The with-and-without approach (each "with" is represented by the introduction of a new node and edging link; "without" represents node absence, deletion, or isolation);

b) Network effect laws that are enhanced by new nodes and edges;

c) Multilayer networks, where interconnections are mapped with adjacency matrices.

The with-and-without approach is also consistent with the differential analysis (confrontation) between networks without and then with interlinks (that depend on avatars or other linking nodes or edges). Digital links are intrinsically flexible and favor immediate linkings through the web or the metaverse.

VIII. Metaverse Evaluation with Multilayer Network Analysis

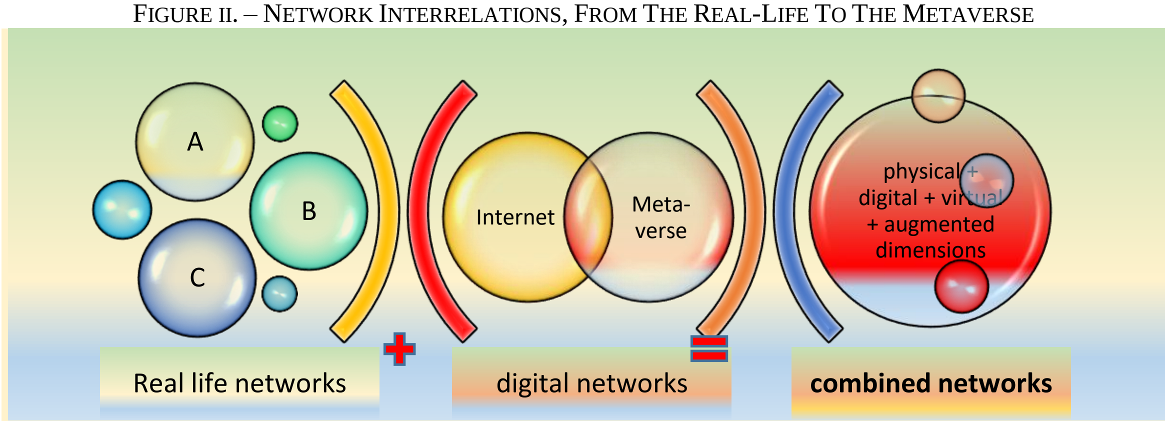
Metaverse evaluation is still a frontier research issue. Each single metaverse ecosystem, ideally corresponding to a closed network (with well-defined boundaries) may be evaluated, even in economic terms, considering it as a static single entity. This is just the first step for appraisal that needs to be complemented (and complicated) introducing dynamic interactions, and interrelations with other networks (both within and outside the metaverse).

Interrelations (exemplified in figure II) incorporate economic synergies, for instance, if a node (individual) in the physical world is complemented by her avatar in the metaverse – their joint value, albeit difficult to assess, is certainly higher than their straightforward sum.

Porous borders make networks intrinsically unstable and more difficult to appraise, but also increasingly valuable.

Scalability is a further feature that encompasses the physical world – internet – metaverse value chain, as shown in Figure II.

Figure II. – Network Interrelations, From The Real-Life To The Metaverse



Multilayer network analysis is a powerful tool for the evaluation of a metaverse (considering every single layer-finite ecosystem, wherever possible).

The mathematical properties of the network and its multilayer extensions preside over the evaluation patterns that should conveniently consider:

a) The architecture of the metaverse network (scale-free, etc.);

b) The number of its nodes and edges;

c) The intensity of the edging relationships;

d) The hyperlink with other (multilayer) networks (real life, internet, other metaverses);

e) The dynamic evolution of both the networks and their interactions.

The monetization of the edging relationships within and outside the metaverse stands out as the final valuation target.

IX. Discussion

The paper's research question is to investigate the potential market value of metaverse ecosystems, using a with-and-without approach or multilayer network metrics. The metaverse can be roughly interpreted as a digital (virtual/augmented) social network, confirming its consistency with network theory – by now a well-established discipline, albeit still underexploited in its economic applications.

Network theory, with its multilayer extensions, is well suited to represent a cornerstone in the interpretation of the evolutionary path from real-life to the metaverse. This explanation is also consistent with the attempt to give a rough estimate of the potential economic value of metaverse layers.

The metaverse can be roughly interpreted as a digital (virtual/augmented) social network, confirming its consistency with network theory – by now a well-established discipline, albeit still underexplored in its economic applications.

The segregation of the metaverse from the real-life and the bridging internet is far from representing a real threat in this still pioneering phase, even if it may represent a critical issue in the future. Node or edging deletion is the primary cause of isolation, a state of the world that typically destroys value (erasing the synergies between the real-life and the metaverse). This occurrence is well known in network theory [17] and should be carefully examined, to anticipate and possibly fix unwanted consequences (e.g., digital identity theft, when the avatar is detached from the originating individual). In most networks, some nodes are likely to disappear. As long as the network continues to grow, its scale-free nature can persist [4].

X. Conclusion

This preliminary study considers the value chain relationship that starts from real-life networks and arrives at metaverse networks, passing through the internet (that, sooner or later, is expected to "merge" with the metaverse).

These findings are consistent with the following definition: "The Metaverse is the post-reality universe, a perpetual and persistent multiuser environment merging physical reality with digital virtuality. It is based on the convergence of technologies that enable multisensory interactions with virtual environments, digital objects, and people such as virtual reality (VR) and augmented reality (AR). Hence, the Metaverse is an interconnected web of social, networked immersive environments in persistent multiuser platforms" [18].

As most complex ecosystems are composed of interacting elements, networks are ubiquitous and are linked among themselves through replica nodes. Multi-slice networks describe time-dependent interactions that represent an intrinsic feature of naturally evolving temporal social networks. Metaverse interactions are continuously updated in real-time, being consistent with this dynamic framework that instantaneously incorporates socio-pattern data.

Networks are a powerful common denominator of all these states of the real or digital world, so representing a clue for joint interpretation. Since these networks operate on different layers, multilayer networking applications need to be considered. While the investigation area is fascinating, the evaluation issues are complicated by several factors, starting from the novelty of still unstable metaverses, up to the mathematical intricacies of multilayer networks.

Adjacency matrices represent a powerful tool to estimate the potential value of each metaverse subset, being consistent with the with-and-without approach, the network effect laws, and the multilayer network interpretation.

Further scrutiny is needed for a more comprehensive dynamic evaluation that inspires and justifies the huge investments that Big Techs are pouring into the metaverse.

A better comprehension of the mathematical and appraisal properties of these extended networks, from the real-life to the metaverse, can ease the understanding of value co-creation patterns, systemic failures, and shock resilience (even to targeted cyber-attacks), improving the overall efficiency, to the benefit of all the real and virtual stakeholders.

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