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Similarity measures for interval-valued intuitionistic fuzzy soft sets and its application in medical diagnosis problem

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Abstract: Similarity measure is an important topic in fuzzy set theory (L. A. Zadeh, 1965). Similarity measure of fuzzy sets is now being extensively applied in many research fields such as fuzzy clustering, image processing, fuzzy reasoning, fuzzy neural network, pattern recognition, medical diagnosis, game theory, coding theory and several problems that contain uncertainties. The aim of this paper is to introduce the concept of similarity measure for interval-valued intuitionistic fuzzy soft sets based on set theoretic approach, some examples and basic properties are also studied. Lastly an application in a medical diagnosis problem is illustrated.

Keywords: **:** Soft set, intuitionistic fuzzy set, interval-valued fuzzy soft set, interval-valued intuitionistic fuzzy soft set, Similarity Measure.

1. Introduction

After the introduction of fuzzy set (L. A. Zadeh, 1965) several researchers have extended this concept in many directions. Similarity measures of fuzzy sets or fuzzy soft sets or generalized fuzzy soft sets has wide applications in many problems which contains uncertainty such as fuzzy clustering, image processing, fuzzy reasoning, fuzzy neural network, pattern recognition, medical diagnosis, game theory, coding theory etc. Similarity measure between vague sets (S. M. Chen, 1995), similarity measures on intuitionistic fuzzy sets (Zhizhen Liang and Pengfei Shi, 2003), similarity measure of soft sets (P. Majumdar and S. K. Samanta, 2008), similarity measures of fuzzy soft sets (P. Majumdar and S. K. Samanta,2011), similarity measure for interval-valued fuzzy sets (Hong-mei Ju and Feng-Ying Wang, 2011), similarity measure of intuitionistic fuzzy soft sets(Naim Cagman and Irfan Deli, 2013), similarity measure of interval valued intuitionistic fuzzy sets(Kai Hu and Jinquan Li, 2013) , similarity measures of interval-valued fuzzy soft sets (A. Mukherjee and S. Sarkar, 2014) have been studied by many researchers. There are several techniques for defining similarity measure in such cases. Some of them are based on distances and some others are based on matching function. There are techniques based on set-theoretic approach also. Some properties are common to these measures and some are not, which influence the choice of the measure to be used in several applications. One of the significant differences between similarity measure based on matching function S and similarity measure S' based on distance is that if $A \cap B = \varphi$, then $S(A,B) = 0$ but $S'(A,B)$ may not be equal to zero, where A and B are two fuzzy sets. But it is easier to calculate the intermediate distance between two fuzzy sets or soft sets. Therefore, distance-based measures are also popular. Let $U = \{x_1, x_2, x_3, ..., x_n\}$ be the universe and A, B be two intuitionistic fuzzy sets (IFS) over U with their membership function μ_A, μ_B and non membership function ν_A, ν_B respectively. Then using the distances between A and B (E. Szmidt and J. Kacprzyk 2000), similarity measure between A and B can be calculated. Again in several problems it is often needed to compare two sets. The sets may be fuzzy, may be vague etc. We are often interested to know whether two patters or images are identical or approximately identical or at least to what degree they are identical.

We have extended these concepts of similarity measure in interval-valued intuitionistic fuzzy soft sets. The aim of this paper is to introduce similarity measure of interval-valued intuitionistic fuzzy soft sets based on set theoretic

approach. A decision making method based on proposed similarity measure is established. An illustrative example demonstrates the application of proposed decision making method in a medical diagnosis problem.
The rest of the paper is organized as follow: section 2: some preliminary basic definitions are given in this section. In section 3 similarity measures between two IVIFS sets is defined with example and some basic properties are studied. In section 4 a decision making method based on similarity measure is established with an example in a medical diagnosis problem. Finally in section 5 some remarks of the similarity measures between IVIFS sets and the proposed decision making method are given.

2. Preliminaries
In this section we briefly review some basic definitions and examples related to interval-valued intuitionistic fuzzy soft sets which will be used in the rest of the paper.

Definition 2.1 [16] (L. A. Zadeh, 1965) Let X be a non empty collection of objects denoted by x. Then a fuzzy set *(FS for short)* α in X is a set of ordered pairs having the form $\alpha = \{(x, \mu_\alpha(x)) : x \in X\}$,
Where the function $\mu_\alpha : X \to [0,1]$ is called the membership function or grade of membership (also degree of compatibility or degree of truth) of x in α .The interval M = $[0,1]$ is called membership space.

Definition 2.2 [17] (L.A. Zadeh 1975) Let X be a non empty set and D be the set of closed subintervals of the interval [0, 1]. Then an *interval-valued fuzzy set A* in X is an expression A given by
$A = \{(x, M_A(x)) : x \in X\}$, where M_A:X → D.

Definition 2.3 [9,13] (D. Molodtsov 1999 and P. K. Maji et al 2003) Let U be an initial universe and E be a set of parameters. Let $P(U)$ denotes the power set of U and $A \subseteq E$. Then the pair (F, A) is called a *soft set* over U, where F is a mapping given by $F : A \to P(U)$.

Definition 2.4 [15](X. B. Yang et al 2009) Let U be an initial universe and E be a set of parameters, a pair(*F,E*) is called an *interval valued- fuzzy soft set* over *F*(U), where F is a mapping given by F:E→ *F*(U).

An interval-valued fuzzy soft set is a parameterized family of interval-valued fuzzy subsets of U, thus, its universe is the set of all interval-valued fuzzy sets of U, i.e. *F*(U). An interval-valued fuzzy soft set is also a special case of a soft set because it is still a mapping from parameters to *F*(U),∀ e ∈ *E* , *F*(U) is referred as the interval fuzzy value set of parameters e , it is actually an interval-valued fuzzy set of U where $x \in U$ and $e \in E$, it can be written as:

$$F(e) = \{(x, \mu_{F(e)}(x)) : x \in U\}$$

where, *F*(U) is the interval-valued fuzzy membership degree that object *x* holds on parameter.

Definition 2.5[1] **(**K. Atanassov 1986**)** Let X be a non empty set. An *intuitionistic fuzzy set* (*IFS* in short) α in X is a set of ordered triples given by,

$$\alpha = \{(x, \mu_\alpha(x), \gamma_\alpha(x)) : x \in X\}$$

where, the functions $\mu_\alpha : X \to [0,1]$ and $\gamma_\alpha : X \to [0,1]$ called degree of membership and the degree of non-membership of each element $x \in X$ to the set α respectively and $0 \le \mu_\alpha(x) + \gamma_\alpha(x) \le 1$ for each $x \in X$.

Definition 2.6 [2] **(**K. Atanassov and G. Gargov 1989**)** An *interval valued intuitionistic fuzzy set* (*IVIFS* in short) α over a universe X is defined as the object of the form $\alpha = \{\langle x, \mu_\alpha(x), \gamma_\alpha(x)\rangle : x \in X\}$, where $\mu_\alpha(x): X \to D([0,1])$ and $\gamma_\alpha(x): X \to D([0,1])$ are functions such that the condition: $\forall x \in X \quad 0 \le sup\mu_\alpha(x) + sup\gamma_\alpha(x) \le 1$ is satisfied (where $D([0,1])$ is the set of all closed intervals of $[0,1]$).

Definition 2.7 [6] (Y. Jiang et al 2010) Let U be an initial universe and E be a set of parameters. Let $IVIFS^U$ be the set of all interval valued intuitionistic fuzzy sets on U and $A \subseteq E$. Then the pair (F, A) is called an *interval valued intuitionistic fuzzy soft set (IVIFSS for short)* over U, where F is a mapping given by $F : A \to IVIFS^U$.

3. Similarity measures for interval-valued intuitionistic fuzzy soft sets

In this section we give the definition of similarity measures for interval-valued intuitionistic fuzzy soft sets (IVIFS-sets) based on set theoretic approach with examples. Some properties are also studied.

Definition 3.1 Let U = **{** h_i : i = 1,2,3,4, ….. , m **}** be the universal set of elements h_i and E = {e_j : j= 1,2,3, …. , n } be the set of parameters e_j . Let $\hat{F}$ = (F,E) and $\hat{G}$ = (G,E) be two interval-valued intuitionistic fuzzy soft sets over U . Then $\hat{F}$ = { F(e_j) ∈ $IVIFS^U$, e_j ∈ E }and $\hat{G}$ = { G(ej) ∈ $IVIFSs^U$, e_j ∈ E } , where F(e_j) is called the e_j th approximation of $\hat{F}$ and G(e_j) is called the e_j th approximation of $\hat{G}$ and $IVIFS^U$ is the set of all interval-valued intuitionistic fuzzy sets on U.

Let M($\hat{F}$, $\hat{G}$) indicates the similarity measure between the IVIFS-sets $\hat{F}$ and $\hat{G}$. To find the similarity measure $\hat{F}$ and $\hat{G}$ we first have to find similarity between their e_jth approximations. Let M_j($\hat{F}$, $\hat{G}$) denotes the similarity measure between the two e_jth approximations of F(e_j) and G(ej). Then M_j($\hat{F}$, $\hat{G}$) is defined as follows

$$M_j(\hat{F},\hat{G}) = \frac{\sum_{i=1}^{m}\left[\left|\bar{\mu}_{F(e_j)}(h_i) - \bar{\nu}_{F(e_j)}(h_i)\right| \wedge \left|\bar{\mu}_{G(e_j)}(h_i) - \bar{\nu}_{G(e_j)}(h_i)\right|\right]}{\sum_{i=1}^{m}\left[\left|\bar{\mu}_{F(e_j)}(h_i) - \bar{\nu}_{F(e_j)}(h_i)\right| \vee \left|\bar{\mu}_{G(e_j)}(h_i) - \bar{\nu}_{G(e_j)}(h_i)\right|\right]} \tag{3.1}$$

Where j = 1,2,3,…,n and

$$\bar{\mu}_{F(e_j)}(h_i) = \frac{1}{2}\left(\inf \mu_{F(e_j)}(h_i) + \sup \mu_{F(e_j)}(h_i)\right)$$

$$\bar{\nu}_{F(e_j)}(h_i) = \frac{1}{2}\left(\inf \nu_{F(e_j)}(h_i) + \sup \nu_{F(e_j)}(h_i)\right)$$

Then the similarity measure between the IVIFS-sets $\hat{F}$ and $\hat{G}$ is defined as,

$$M(\hat{F},\hat{G}) = \frac{\sum_{j=1}^{n} M_j(\hat{F},\hat{G})}{n} \quad (3.2)$$

Theorem 3.2 If $M(\hat{F},\hat{G})$ be the similarity measure between two IVIFS-sets (F,E) and (G,E) then

(i) $M(\hat{F},\hat{G}) = M(\hat{G},\hat{F})$

(ii) $0 \le M(\hat{F},\hat{G}) \le 1$

(iii) $M(\hat{F},\hat{G}) = 1$ if and only if (F,E) = (G,E).

Proof: Obvious from the definition 3.1 .

Example 3.3 Let U={h_1,h_2,h_3,h_4,h_5}be the universal set and A={e_1,e_2} be the set of parameters. Let (F,A) and (G,A) be two interval-valued intuitionistic fuzzy soft sets over U such that (F,A) is given by

	e_1	e_2
h_1	[0.1,0.7] , [0.1,0.3]	[0.3,0.4] , [0.2,0.5]
h_2	[0.3,0.5] , [0.3,0.4]	[0.2,0.3] , [0.4,0.5]
h_3	[0.2,0.6] , [0.1,0.3]	[0.1,0.3] , [0.2,0.3]
h_4	[0.3,0.4] , [0.1,0.5]	[0.1,0.6] , [0.2,0.3]
h_5	[0.1,0.3] , [0.2,0.3]	[0.4,0.5] , [0.1,0.4]

Table 1: Tabular presentation of (F,A).

(G,A) is given by

	e_1	e_2
h_1	[0.2,0.6] , [0.3,0.4]	[0.2,0.4] , [0.3,0.5]
h_2	[0.1,0.2] , [0.4,0.6]	[0.4,0.5] , [0.1,0.3]
h_3	[0.4,0.6] , [0.2,0.3]	[0.2,0.5] , [0.1,0.3]
h_4	[0.1,0.3] , [0.3,0.6]	[0.3,0.4] , [0.2,0.5]
h_5	[0.3,0.5] , [0.2,0.4]	[0.4,0.5] , [0.3,0.4]

Table 2: Tabular presentation of (G,A)

Now by equation **(3.1)** we have, $M_1(\hat{F},\hat{G}) = 0.35$, $M_2(\hat{F},\hat{G}) = 0.44$.

Therefore by equation **(3.2)** the similarity measure between the IVIFS-sets $\hat{F}$ and $\hat{G}$ is given by M($\hat{F}$, $\hat{G}$) = 0.39.

Example 3.4 Let U={h_1,h_2,h_3,h_4}be the universal set and E={e_1,e_2,e_3} be the set of parameters. Let (F_1,E) and (G_1,E) be two interval-valued intuitionistic fuzzy soft sets over U such that (F_1,E) is given by

	e_1	e_2	e_3
h_1	[0.1,0.4] , [0.5,0.6]	[0.6,0.7] , [0.1,0.2]	[0.1,0.3] , [0.5,0.7]
h_2	[0.2,0.5] , [0.4,0.5]	[0.3,0.4] , [0.5,0.6]	[0.6,0.7] , [0.2,0.3]
h_3	[0.6,0.8] , [0.0,0.1]	[0.7,0.8] , [0.1,0.2]	[0.2,0.3] , [0.4,0.6]
h_4	[0.5,0.6] , [0.1,0.2]	[0.4,0.5] , [0.2,0.3]	[0.1,0.6] , [0.3,0.4]

Table 3: Tabular presentation of (F_1,E).

(G_1,E) is given by

	e_1	e_2	e_3
h_1	[0.1,0.3] , [0.5,0.6]	[0.5,0.7] , [0.1,0.2]	[0.2,0.3] , [0.6,0.7]
h_2	[0.3,0.4] , [0.4,0.5]	[0.3,0.5] , [0.2,0.4]	[0.5,0.6] , [0.3,0.4]
h_3	[0.7,0.8] , [0.1,0.2]	[0.7,0.8] , [0.0,0.1]	[0.2,0.4] , [0.5,0.6]
h_4	[0.3,0.4] , [0.1,0.2]	[0.3,0.5] , [0.3,0.4]	[0.2,0.5] , [0.3,0.5]

Table 4: Tabular presentation of (G_1,E)

Now by equation **(3.1)** we have $M_1(\hat{F}_1,\hat{G}_1)=0.80, M_2(\hat{F}_1,\hat{G}_1)=0.75$ *and* $M_3(\hat{F}_1,\hat{G}_1)=0.77.$

Therefore by equation **(3.2)** the similarity measure between the IVIFS Sets $\hat{F}_1$ *and* $\hat{G}_1$ is given by $M(\hat{F}_1,\hat{G}_1)=0.77.$

Definition 3.5 Let (F,A) and (G,B) be two IVIFS-sets over U where A=B. Then (F,A) and (G,B) are said to be α-similar, denoted by $(F,A)\overset{\alpha}{\cong}(G,B)$ if and only if $M(\hat{F},\hat{G})>\alpha$ for $\alpha \in (0,1)$. We call the two IVFIS-sets significantly similar if $M(\hat{F},\hat{G})>\frac{1}{2}$.

Example 3.6 In example 3.3 , $M(\hat{F},\hat{G})=0.39<\frac{1}{2}$ and in example 3.4 , $M(\hat{F}_1,\hat{G}_1)=0.77>\frac{1}{2}$. Therefore the IVIFS-sets (F,E) and (G,E) are not significantly similar but (F_1,E) and (G_1,E) are significantly similar.

4 Application in a medical diagnosis problem

In this section we developed an algorithm based on similarity measure of two IVIFS sets to estimate the possibility that an ill person having certain symptoms is suffering from cancer. For this we first construct a model IVIFS set for illness and another IVIFS set for ill person . Then we find the similarity measure of these two IVIFS sets. We also assume that if the similarity measure between these two IVIFS sets is greater than or equal to 0.6 (which can be fixed with the help of medical expert) then we conclude that the person is possibly suffering from cancer.

The algorithm of this method is as follows:
Step 1: construct an model IVIFS set (F, A) over the universe U for illness.
Step 2: construct IVIFS set (G,B) over the universe U for the ill person.
Step 3: calculate similarity measure *M*(F,G)between (F, A) and (G,B)
Step 4: If M(F,G)≥ 0.6 then the person is possibly suffering from cancer and if M(F,G) < 0.6 then the person is not possibly suffering from cancer.

Example 4.1 Let U be the universal set , which contains only two elements x_1 (cancer) and x_2 (not cancer) i.e. U={x_1,x_2}. Here the set of parameters E is a set of certain visible symptoms. Let E = {e_1,e_2,e_3,e_4,e_5,e_6,e_7}, where e_1 = unexpected weight loss, e_2 = bone pain, e_3 = fatigue , e_4 = skin changes , e_5 = persistent fever e_6 = repeated infection and e_7 = lump any where on the body for no reason.

Step 1: Construct a IVIFS set (F,A) over U for cancer as given below, which can be prepared with the help of an expert (a doctor or a medical person).

(F,A)	e_1	e_2	e_3	e_4	e_5	e_6	e_7
x_1	[0.3,0.4],[0.4,0.5]	[0.4,0.5],[0.2,0.3]	[0.6,0.7],[0.1,0.2]	[0.6,0.7],[0.2,0.3]	[0.3,0.4],[0.4,0.5]	[0.4,0.5],[0.2,0.3]	[0.5,0.6],[0.1,0.2]
x_2	[0.2,0.3],[0.6,0.7]	[0.1,0.2],[0.6,0.7]	[0.2,0.3],[0.6,0.7]	[0.0,0.2],[0.4,0.6]	[0.0,0.1],[0.7,0.8]	[0.2,0.3],[0.6,0.7]	[0.0,0.2],[0.6,0.7]

Table 5: Tabular presentation of (F,A)

Step 2: Construct a IVIFS set (G,B) over U based on data of a ill person as given below

(G,B)	e_1	e_2	e_3	e_4	e_5	e_6	e_7
x_1	[0.3,0.45],[0.35,0.5]	[0.3,0.5],[0.25,0.35]	[0.55,0.7],[0.1,0.2]	[0.5,0.7],[0.1,0.3]	[0.25,0.4],[0.5,0.6]	[0.3,0.55],[0.25,0.4]	[0.45,0.6],[0.1,0.25]
x_2	[0.2,0.3],[0.65,0.7]	[0.05,0.2],[0.6,0.75]	[0.25,0.35],[0.6,0.65]	[0.05,0.25],[0.4,0.6]	[0.05,0.15],[0.65,0.8]	[0.2,0.35],[0.55,0.65]	[0.0,0.15],[0.6,0.75]

Table 6: Tabular presentation of (G,B) where A = B = E.

Step 3 calculate similarity measure between (F,A) and (G,B) : By equation (3.1) we have $M_1(F,G) = 0.86, M_2(F,G) = 0.61, M_3(F,G) = 0.83, M_4(F,G) = 0.94, M_5(F,G) = 0.78, M_6(F,G) = 0.71$ and $M_7(F,G) = 0.90$. Therefore by equation **(3.2)** the similarity measure between the IVIFS sets (F,A) and (G,B) is given by $M(F,G) = 0.80$.

Step 4 Here M(F,G) = 0.80 > 0.6 therefore we conclude that the person is possibly suffering from cancer.

Example 4.2 Now we consider example 4.1 with a different ill person.

Step 1: Construct a IVIFSS (F,A) over U for cancer as in example 5, which can be prepared with the help of a medical person.

Step 2: Construct a IVIFSS (H,C) over U based on data of a ill person as given below

(H,C)	e_1	e_2	e_3	e_4
x_1	[0.05,0.45],[0.7,0.8]	[0.5,0.6],[0.0,0.1]	[0.4,0.5],[0.3,0.4]	[0.1,0.2],[0.65,0.75]
x_2	[0.6,0.7],[0.1,0.2]	[0.2,0.3],[0.3,0.4]	[0.25,0.35],[0.6,0.65]	[0.3,0.45],[0.2,0.25]

e_5	e_6	e_7
[0.5,0.7],[0.2,0.3]	[0.1,0.2],[0.6,0.7]	[0.3,0.4],[0.5,0.6]

[0.3,0.4],[0.15,0.2]	[0.7,0.8],[0.1,0.2]	[0.4,0.5],[0.35,0.4]

Table 7: Tabular presentation of (H,C) where A = C = E.

Step 3 calculate similarity measure between (F,A) and (H,C) : By equation (3.1) we have $M_1(F,H) = 0.42, M_2(F,H) = 0.43, M_3(F,H) = 0.47, M_4(F,H) = 0.58, M_5(F,H) = 0.26, M_6(F,H) = 0.54$ and $M_7(F,H) = 0.29.$ Therefore by equation **(3.2)** the similarity measure between the IVIFS sets (F,A) and (H,C) is given by $M(F,H) = 0.43$.

Step 4 Here M(F,H) = 0.43 < 0.6 therefore we conclude that the person is not possibly suffering from cancer.

5. Conclusion

In this paper we have defined similarity measure between two IVIFS-sets based on set theoretic approach. Then we construct a decision making method based on similarity measures. Finally we give an example to show the possibilities of applications of similarity measure between two IVIFS sets in a medical diagnosis problem. Thus we can use the method to solve the problem that contain uncertainty such as problem in social, economic system, pattern recognition, medical diagnosis, game theory coding theory and so on.

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