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A Solution Proposal to Indefinite Quadratic Interval Transportation Problem

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Abstract: The data of real world applications generally cannot be expressed strictly. An efficient way of handling this situation is expressing the data as intervals. Thus, this paper focus on the Indefinite Quadratic Interval Transportation Problem (IQITP) in which all the parameters i.e. cost and risk coefficients of the objective function, supply and demand quantities are expressed as intervals. A Taylor series approach is presented for the solution of IQITP by means of the expression of intervals with its left and right limits. Also a numerical example is executed to illustrate the procedure.

Keywords: Quadratic Interval Transportation Problem, Interval coefficients, Taylor Series.

1. Introduction

Transportation Problem (TP) has wide practical applications in logistic systems, manpower planning, personnel allocation, inventory control, production planning, etc. and aims to find the best way to fulfill the demand of n demand points using the capacities of m supply points. The parameters of the transportation problem are unit costs (or profits), supply and demand quantities. The unit cost is the coefficient of the objective function and it could represent transportation cost, average delivery time of the commodities, number of goods transported unfulfilled demand, product deterioration, preference coefficient, and many others. The linear functions are the most useful and widely used in operational research. Also quadratic functions and quadratic problems are the least difficult ones to handle out of all nonlinear programming problems. A fair number of functional relationships occurring in the real world are truly quadratic. For example kinetic energy carried by a rocket or an atomic particle is proportional to the square of its velocity, in statistics, the variance of a given sample of observations is a quadratic function of the values that constitute the sample. So there are countless other non-linear relationships occurring in nature, capable of being approximated by quadratic functions.

Indefinite quadratic programming problems and Interval Transportation Problem have been extensively studied for several decades. A bibliography of Quadratic programming problems can be found in [11]. Using fuzzy triangular technique, [1] proposed a fuzzy method to solve interval transportation problems. Interval Fractional Transportation Problem (IFTP) in which all the parameters i.e. cost and preference coefficients of the objective function, supply and demand quantities are expressed as intervals. A Taylor series approach is presented for IFTP by means of the expression of intervals with its left and right limits in [2]. Sivri et al. [3] proposed a Taylor series based method to IFTP whose objective function coefficients are assumed as intervals. Also in [4], a new approach is proposed by the variable transformation for a linear fractional programming problem with interval coefficients in the objective function. In [5], a fuzzy multi objective linear fractional programming problem is reduced to a single objective problem using the Taylor series and an approximate solution is obtained. In [6,9,10,12], the authors studied fixed charge indefinite quadratic transportation problems and fixed charge bi-criterion quadratic transportation problems. Guzel and Sivri [8] concerned with the multi objective version of transportation problem, and proposed a solution procedure based on Taylor series expansion. In [13], using fuzzy technique, a new method is proposed for interval transportation problems by considering the right bound and midpoint of interval. Also in [14], fuzzy and interval programming technique is presented to deal with inexact coefficient in multi objective programming problem.

This paper dealt with the IQITP in which all the parameters are expressed as intervals. Expressing the parameters as interval makes Decision Maker (DM) more comfortable and this enables to consider tolerances for the model parameters in a more natural and direct way. Therefore, IQITP seems to be more realistic and reliable according to crisp values. In this paper, we present an iterative procedure based on the Taylor series expansion. Firstly, a feasible initial point is determined within the Northwest Corner method by means of expressing all the interval parameters as left and right limits. Then the objective function is linearized by using first order Taylor series expansion about the feasible initial point. Thus IQITP is transformed a traditional linear programming problem. And then an iterative procedure is presented in such a way that the optimal solution of lastly constructed linear programming problem is selected as the point where about the objective will be expanded to its first order Taylor series in the next iteration step. The stopping criterion of the proposed procedure is obtaining the same point for the last two iteration steps. A numerical example supports the proposed procedure.

2. Indefinite Quadratic Interval Transportation Problem

The mathematical formulation of IQITP can be stated as follows:

$$\min Z(\mathbf{x}) = \left(\sum_{i=1}^{m} \sum_{j=1}^{n} \left[c_{ij}^L, c_{ij}^R \right] x_{ij} \right) \left(\sum_{i=1}^{m} \sum_{j=1}^{n} \left[d_{ij}^L, d_{ij}^R \right] x_{ij} \right) \qquad \text{(P1)}$$

s.t.

$$\sum_{j=1}^{n} x_{ij} = \left[a_i^L, a_i^R \right] \quad i = 1, 2, \ldots, m,$$

$$\sum_{i=1}^{m} x_{ij} = \left[b_j^L, b_j^R \right] \quad j = 1, 2, \ldots, n,$$

$$x_{ij} \geq 0 \quad i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n.$$

x_{ij} is decision variable which refers to product quantity that transported from supply point *i* to demand point *j*. The closed interval $\left[c_{ij}^L, c_{ij}^R \right]$ denotes that the unit transportation cost from *i* th supply point to *j* th demand point lies between c_{ij}^L and c_{ij}^R. The closed interval $\left[d_{ij}^L, d_{ij}^R \right]$ denotes that the depreciation (risk) by transport from *i* th supply point to *j* th demand point lies between d_{ij}^L and d_{ij}^R. The closed interval $\left[a_i^L, a_i^R \right]$ represent that *i* th supply quantity lies between a_i^L and a_i^R. Similarly, the closed interval $\left[b_j^L, b_j^R \right]$ represent that *j* th demand quantity lies between b_j^L and b_j^R

In the above problem the cost of transporting one unit from *i* th origin to *j* th destination is $\sum_{i=1}^{m} \sum_{j=1}^{n} \left[c_{ij}^L, c_{ij}^R \right] x_{ij}$, but while transporting goods can get damaged so the total cost of damaged good is $\sum_{i=1}^{m} \sum_{j=1}^{n} \left[d_{ij}^L, d_{ij}^R \right] x_{ij}$. Our aim is to minimize the two cost simultaneously: therefore we consider the product of two cost.

We note that the objective function being the product of two affine function is a quasi concave function will have its optimal solution at an extreme point.

Correspondingly to the literature, the model in this paper has the following assumptions:

- $\sum_{i=1}^{m} a_i^L = \sum_{j=1}^{n} b_j^L$ and $\sum_{i=1}^{m} a_i^R = \sum_{j=1}^{n} b_j^R$ (Balance condition)
- The parameters $a_i^L, b_j^L, c_{ij}^L, d_{ij}^L, a_i^R, b_j^R, c_{ij}^R, d_{ij}^R$ are all nonnegative.

3. A Taylor Series Approach for IQITP

To apply the Taylor series approach, we need to specify an initial single point from the feasible region of (P1). First interval supply-demand quantities and interval quadratic objective function coefficients are converted into deterministic ones by means of the combination of their's left and right limit in the following way:

$$\left[a_i^L, a_i^R \right] \Rightarrow \bar{a}_i = \delta_i\, a_i^R + (1-\delta_i)\, a_i^L = a_i^L + \left(a_i^R - a_i^L \right) \delta_i \quad (i = 1, 2, \ldots, m). \qquad (1)$$

$$\left[b_j^L, b_j^R \right] \Rightarrow \bar{b}_j = \mu_j\, b_j^R + (1-\mu_j)\, b_j^L = b_j^L + \left(b_j^R - b_j^L \right) \mu_j \quad (j = 1, 2, \ldots, n). \qquad (2)$$

$$\left[c_{ij}^L, c_{ij}^R \right] \Rightarrow \bar{c}_{ij} = \theta_{ij}\, c_{ij}^R + (1-\theta_{ij})\, c_{ij}^L = c_{ij}^L + \left(c_{ij}^R - c_{ij}^L \right) \theta_{ij} \quad (i = 1, 2, \ldots, m)(j = 1, 2, \ldots, n) \quad (3)$$

$$\left[d_{ij}^L, d_{ij}^R \right] \Rightarrow \bar{d}_{ij} = \lambda_{ij}\, d_{ij}^R + (1-\lambda_{ij})\, d_{ij}^L = d_{ij}^L + \left(d_{ij}^R - d_{ij}^L \right) \lambda_{ij} \quad (i = 1, 2, \ldots, m)(j = 1, 2, \ldots, n) \qquad (4)$$

where $\delta_i, \mu_j, \theta_{ij}, \lambda_{ij} \in [0,1]$. With these equivalent expression of the interval parameters, (P1) is converted to the following IQITP:

$$\begin{aligned} \min Z &= Z_1(x).Z_2(x) \\ &= \left(\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^L + \theta_{ij}(c_{ij}^R - c_{ij}^L) x_{ij} \right) \left(\sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij}^L + \lambda_{ij}(d_{ij}^R - d_{ij}^L) x_{ij} \right) \qquad \text{(P2)} \\ &= \left(\sum_{i=1}^{m} \sum_{j=1}^{n} \bar{c}_{ij} x_{ij} \right) \left(\sum_{i=1}^{m} \sum_{j=1}^{n} \bar{d}_{ij} x_{ij} \right) \end{aligned}$$

s.t. $\sum_{j=1}^{n} x_{ij} = a_i^L + \left(a_i^R - a_i^L \right) \delta_i \quad i = 1, 2, \ldots, m,$

$$\sum_{i=1}^{m} x_{ij} = b_j^L + \left(b_j^R - b_j^L\right)\mu_j \qquad j = 1,2,\ldots,n,$$

$$x_{ij} \geq 0\,,\ \delta_i, \mu_j, \theta_{ij}, \lambda_{ij} \in [0,1]. \quad i = 1,2,\ldots,m, \quad j = 1,2,\ldots,n.$$

The main purpose here is to specify an initial feasible point, not an optimal one. Thus, the value of the combination parameters $\delta_i, \mu_j, \theta_{ij}$ and λ_{ij} ($i = 1,2,\ldots,m,\quad j = 1,2,\ldots,n$) can be chosen arbitrarily from the interval $[0,1]$. After substituting these arbitrary values in (P2), a traditional TP is obtained and then an initial basic feasible solution can be determined by Northwest Corner Method which ignores the objective function coefficients and compute a basic feasible solution of TP, where the basic variables are selected from the North – West corner (i.e. top left corner). Let denote the initial feasible solution as $\mathbf{X}^{(0)} = \left(\mathbf{x}^{(0)}, \boldsymbol{\theta}^{(0)}, \boldsymbol{\lambda}^{(0)}, \boldsymbol{\delta}^{(0)}, \boldsymbol{\mu}^{(0)}\right)$.

Using the first order Taylor series at the feasible point $\mathbf{X}^{(0)}$, the objective function of (P2) can be constructed approximately as follows:

$$Z \approx Z\left(\mathbf{X}^{(0)}\right) + \sum_{i=1}^{m}\sum_{j=1}^{n} \left.\frac{\partial Z}{\partial x_{ij}}\right|_{\mathbf{X}^{(0)}} \left(x_{ij} - x_{ij}^{(0)}\right) + \sum_{i=1}^{m}\sum_{j=1}^{n} \left.\frac{\partial Z}{\partial \theta_{ij}}\right|_{\mathbf{X}^{(0)}} \left(\theta_{ij} - \theta_{ij}^{(0)}\right) + \sum_{i=1}^{m}\sum_{j=1}^{n} \left.\frac{\partial Z}{\partial \lambda_{ij}}\right|_{\mathbf{X}^{(0)}} \left(\lambda_{ij} - \lambda_{ij}^{(0)}\right)$$

Hence the terms $Z\left(\mathbf{X}^{(0)}\right)$, $\sum_{i=1}^{m}\sum_{j=1}^{n} \left.\frac{\partial Z}{\partial x_{ij}}\right|_{X^{(0)}} \left(x_{ij}^{(0)}\right)$, $\sum_{i=1}^{m}\sum_{j=1}^{n} \left.\frac{\partial Z}{\partial \theta_{ij}}\right|_{X^{(0)}} \left(\theta_{ij}^{(0)}\right)$ and $\sum_{i=1}^{m}\sum_{j=1}^{n} \left.\frac{\partial Z}{\partial \lambda_{ij}}\right|_{X^{(0)}} \left(\lambda_{ij}^{(0)}\right)$ are constant value, all of these do not change the direction of minimization and can be eliminated. The first partial derivatives with respect to the variables x_{ij}, θ_{ij}, λ_{ij} in the Taylor series expansion are:

$$\frac{\partial Z}{\partial x_{ij}} = \frac{\partial Z_1}{\partial x_{ij}} Z_2 + \frac{\partial Z_2}{\partial x_{ij}} Z_1 = \bar{c}_{ij} Z_2 + \bar{d}_{ij} Z_1\,,$$

$$\frac{\partial Z}{\partial \theta_{ij}} = \frac{\partial Z_1}{\partial \theta_{ij}} Z_2 + \frac{\partial Z_2}{\partial \theta_{ij}} Z_1 = \left(c_{ij}^R - c_{ij}^L\right) x_{ij} Z_2\,,$$

$$\frac{\partial Z}{\partial \lambda_{ij}} = \frac{\partial Z_1}{\partial \lambda_{ij}} Z_2 + \frac{\partial Z_2}{\partial \lambda_{ij}} Z_1 = \left(d_{ij}^R - d_{ij}^L\right) x_{ij}\ Z_1\,.$$

Thus, an equivalent form of (P2) can be constructed as follows:

$$\min \bar{Z} \approx \sum_{i=1}^{m}\sum_{j=1}^{n} \left(\bar{c}_{ij} Z_2 + \bar{d}_{ij} Z_1\right)\Big|_{\mathbf{X}^{(0)}} x_{ij} + \sum_{i=1}^{m}\sum_{j=1}^{n} \left(\left(c_{ij}^R - c_{ij}^L\right) x_{ij} Z_2\right)\Big|_{\mathbf{X}^{(0)}} \theta_{ij} + \sum_{i=1}^{m}\sum_{j=1}^{n} \left(\left(d_{ij}^R - d_{ij}^L\right) x_{ij}\ Z_1\right)\Big|_{\mathbf{X}^{(0)}} \lambda_{ij} \qquad \left(\mathrm{P3}\big|_{\mathbf{X}^{(0)}}\right)$$

$$\text{s.t.} \qquad \sum_{j=1}^{n} x_{ij} = a_i^L + \left(a_i^R - a_i^L\right)\delta_i \quad i = 1,2,\ldots,m,$$

$$\sum_{i=1}^{m} x_{ij} = b_j^L + \left(b_j^R - b_j^L\right)\mu_j \qquad j = 1,2,\ldots,n,$$

$$x_{ij} \geq 0\ ,\ \delta_i, \mu_j, \theta_{ij}, \lambda_{ij} \in [0,1],\ \ i = 1,2,\ldots,m, \quad j = 1,2,\ldots,n,$$

$$\sum_{i=1}^{m} a_i \geq \sum_{j=1}^{n} b_j\ .$$

We note here that since the objective function does not depend on the variables δ_i and μ_j, The partial derivatives with respect to these variables are equal to zero and so it is not necessary to add these to the objective function.

The last constraint of equation $\left(\mathrm{P3}\big|_{\mathbf{X}^{(0)}}\right)$ guarantees that total demand is certainly met. Thus, IQITP is converted to a linear programming problem $\left(\mathrm{P3}\big|_{\mathbf{X}^{(0)}}\right)$ which can be easily solve with any computer packages. Let denote the optimal solution of equation $\left(\mathrm{P3}\big|_{\mathbf{X}^{(0)}}\right)$ as $\mathbf{X}^{(1)}$. If the objective function of (P2) is expanded to its first Taylor polynomial at the new point $\mathbf{X}^{(1)}$, the problem $\left(\mathrm{P3}\big|_{\mathbf{X}^{(1)}}\right)$ can be constructed, similarly. Let denote the optimal solution of $\left(\mathrm{P3}\big|_{\mathbf{X}^{(1)}}\right)$ by $\mathbf{X}^{(2)}$. The objective value at the point $\mathbf{X}^{(2)}$ is better than the value at $\mathbf{X}^{(1)}$. Hence the last obtained point is a closer extreme point to the optimal solution of (P2), this procedure can be continued until the last point is repeated. So the optimal solution of (P2) is obtained by repeating the given procedure.

The Taylor series approach can be summarized with the following algorithm:

Step 0: (Initialization) After constructing (P2), obtain a feasible initial point $\mathbf{X}^{(0)}$ with the Northwest Corner Method for any value of $\delta_i, \mu_j \in [0,1]$, $i = 1,2,\ldots,m,\quad j = 1,2,\ldots,n$. Set $k = 0$.

Step 1: (Generating a new point) With the aim of linearizing the indefinite quadratic objective, use first order Taylor Series Expansion about the point $\mathbf{X}^{(k)}$, build and solve the corresponding $\left(\text{P3}\big|_{\mathbf{X}^{(k)}}\right)$ and obtained its optimal solution set $\mathbf{X}^{(k+1)}$.

Step 2: (Stopping criterion) If $\mathbf{X}^{(k)} = \mathbf{X}^{(k+1)}$ then stop. Otherwise, set $k = k+1$ and go to Step 1.

4. A numerical example

Let us consider the following interval objective functions:

$$Z_1(\mathbf{x}) = [1,2]x_{11} + [2,4]x_{12} + [1,3]x_{13} + [3,5]x_{14} + [0,2]x_{21} + [2,5]x_{22} + [1,4]x_{23} + [3,4]x_{24} + [0,3]x_{31} + [1,2]x_{32} + [3,5]x_{33} + [2,4]x_{34}$$

$$Z_2(\mathbf{x}) = [2,3]x_{11} + [1,2]x_{12} + [3,5]x_{13} + [1,3]x_{14} + [0,4]x_{21} + [1,5]x_{22} + [2,3]x_{23} + [3,5]x_{24} + [0,5]x_{31} + [0,1]x_{32} + [1,3]x_{33} + [2,4]x_{34}$$

$$\sum_{j=1}^{4} x_{1j} = [18,24],\quad \sum_{j=1}^{4} x_{2j} = [10,17],\quad \sum_{j=1}^{4} x_{3j} = [20,26],$$

$$\sum_{i=1}^{3} x_{i1} = [10,19],\quad \sum_{i=1}^{3} x_{i2} = [7,12],\quad \sum_{i=1}^{3} x_{i3} = [16,20],\quad \sum_{i=1}^{3} x_{i4} = [15,19]$$

$$x_{ij} \ge 0 \quad i = 1,2,3 \quad j = 1,2,3,4.$$

After expressing all interval parameters in the form of (1)-(4), corresponding problem (P2) is constructed as follows:

$$Z_1(x) = (1+\theta_{11})x_{11} + (2+2\theta_{12})x_{12} + (1+2\theta_{13})x_{13} + (3+2\theta_{14})x_{14} + (2\theta_{21})x_{21} + (2+3\theta_{22})x_{22} + (1+3\theta_{23})x_{23} + (3+\theta_{24})x_{24} + (3\theta_{31})x_{31} + (1+\theta_{32})x_{32} + (3+2\theta_{33})x_{33} + (2+2\theta_{34})x_{34}$$

$$Z_2(x) = (2+\lambda_{11})x_{11} + (1+\lambda_{12})x_{12} + (3+2\lambda_{13})x_{13} + (1+2\lambda_{14})x_{14} + (4\lambda_{21})x_{21} + (1+4\lambda_{22})x_{22} + (2+\lambda_{23})x_{23} + (3+2\lambda_{24})x_{24} + (5\lambda_{31})x_{31} + (\lambda_{32})x_{32} + (1+2\lambda_{33})x_{33} + (2+2\lambda_{34})x_{34}$$

$$\min Z(\mathbf{x}) = \begin{pmatrix} (1+\theta_{11})x_{11} + (2+2\theta_{12})x_{12} + (1+2\theta_{13})x_{13} + (3+2\theta_{14})x_{14} \\ +(2\theta_{21})x_{21} + (2+3\theta_{22})x_{22} + (1+3\theta_{23})x_{23} + (3+\theta_{24})x_{24} \\ +(3\theta_{31})x_{31} + (1+\theta_{32})x_{32} + (3+2\theta_{33})x_{33} + (2+2\theta_{34})x_{34} \end{pmatrix} \cdot \begin{pmatrix} (2+\lambda_{11})x_{11} + (1+\lambda_{12})x_{12} + (3+2\lambda_{13})x_{13} + (1+2\lambda_{14})x_{14} \\ +(4\lambda_{21})x_{21} + (1+4\lambda_{22})x_{22} + (2+\lambda_{23})x_{23} + (3+2\lambda_{24})x_{24} \\ +(5\lambda_{31})x_{31} + (\lambda_{32})x_{32} + (1+2\lambda_{33})x_{33} + (2+2\lambda_{34})x_{34} \end{pmatrix} \text{ s.t.}$$

$$\sum_{j=1}^{4} x_{1j} = 18 + 6\delta_1,\quad \sum_{j=1}^{4} x_{2j} = 10 + 7\delta_2,\quad \sum_{j=1}^{4} x_{3j} = 20 + 6\delta_3$$

$$\sum_{i=1}^{3} x_{i1} = 10 + 9\mu_1,\quad \sum_{i=1}^{3} x_{i2} = 7 + 5\mu_2,\quad \sum_{i=1}^{3} x_{i3} = 16 + 4\mu_3,\quad \sum_{i=1}^{3} x_{i4} = 15 + 4\mu_4$$

$$x_{ij} \ge 0,\ \delta_i, \mu_j, \theta_{ij}, \lambda_{ij} \in [0,1].\quad i = 1,2,\quad j = 1,2,3$$

Assuming the arbitrary value of δ_i, μ_j as one for $\forall i, \forall j$, the following initial feasible solution set $\mathbf{X}^{(0)}$ is determined by Northwest Corner Method:

$$\mathbf{x}^{(0)} = \begin{bmatrix} 10 & 7 & 1 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 5 & 15 \end{bmatrix}, \boldsymbol{\theta}^{(0)} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \boldsymbol{\lambda}^{(0)} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \boldsymbol{\delta}^{(0)} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}, \boldsymbol{\mu}^{(0)} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}.$$

For the point $\mathbf{X}^{(0)}$, the values of objective is calculated as $Z_1\big|_{\mathbf{X}^{(0)}} = 80$, $Z_2\big|_{\mathbf{X}^{(0)}} = 85$ and so $Z\big|_{\mathbf{X}^{(0)}} = \left(Z_1\big|_{\mathbf{X}^{(0)}}\right)\left(Z_2\big|_{\mathbf{X}^{(0)}}\right) = 6800$. The corresponding problem $\left(\text{P3}\big|_{\mathbf{X}^{(0)}}\right)$ can be written as follows:

$$\min \bar{Z} \approx \left(\left(1+\theta_{11}\right) Z_2 + \left(2+\lambda_{11}\right) Z_1\right)\Big|_{\mathbf{X}^{(0)}} x_{11} + \left(\left(2+2\theta_{12}\right) Z_2 + \left(1+\lambda_{12}\right) Z_1\right)\Big|_{\mathbf{X}^{(0)}} x_{12}$$
$$+\left(\left(1+2\theta_{13}\right) Z_2 + \left(3+2\lambda_{13}\right) Z_1\right)\Big|_{\mathbf{X}^{(0)}} x_{13} + \left(\left(3+2\theta_{14}\right) Z_2 + \left(1+2\lambda_{14}\right) Z_1\right)\Big|_{\mathbf{X}^{(0)}} x_{14}$$
$$+\left(\left(2\theta_{21}\right) Z_2 + \left(4\lambda_{21}\right) Z_1\right)\Big|_{\mathbf{X}^{(0)}} x_{21} + \left(\left(2+3\theta_{22}\right) Z_2 + \left(1+4\lambda_{22}\right) Z_1\right)\Big|_{\mathbf{X}^{(0)}} x_{22}$$
$$+\left(\left(1+3\theta_{23}\right) Z_2 + \left(2+\lambda_{23}\right) Z_1\right)\Big|_{\mathbf{X}^{(0)}} x_{23} + \left(\left(3+\theta_{24}\right) Z_2 + \left(3+2\lambda_{24}\right) Z_1\right)\Big|_{\mathbf{X}^{(0)}} x_{24}$$
$$+\left(\left(3\theta_{31}\right) Z_2 + \left(5\lambda_{31}\right) Z_1\right)\Big|_{\mathbf{X}^{(0)}} x_{31} + \left(\left(1+\theta_{32}\right) Z_2 + \left(\lambda_{32}\right) Z_1\right)\Big|_{\mathbf{X}^{(0)}} x_{32} \qquad \text{s.t.}$$
$$+\left(\left(3+2\theta_{33}\right) Z_2 + \left(1+2\lambda_{33}\right) Z_1\right)\Big|_{\mathbf{X}^{(0)}} x_{33} + \left(\left(2+2\theta_{34}\right) Z_2 + \left(2+2\lambda_{34}\right) Z_1\right)\Big|_{\mathbf{X}^{(0)}} x_{34}$$
$$+\left(x_{11} Z_2\right)\Big|_{\mathbf{X}^{(0)}} \theta_{11} + \left(2x_{12} Z_2\right)\Big|_{\mathbf{X}^{(0)}} \theta_{12} + \left(2x_{13} Z_2\right)\Big|_{\mathbf{X}^{(0)}} \theta_{13} + \left(3x_{23} Z_2\right)\Big|_{\mathbf{X}^{(0)}} \theta_{23}$$
$$+\left(2x_{33} Z_2\right)\Big|_{\mathbf{X}^{(0)}} \theta_{33} + \left(2x_{34} Z_2\right)\Big|_{\mathbf{X}^{(0)}} \theta_{34} + \left(x_{11} Z_1\right)\Big|_{\mathbf{X}^{(0)}} \lambda_{11} + \left(x_{12} Z_1\right)\Big|_{\mathbf{X}^{(0)}} \lambda_{12}$$
$$+\left(2x_{13} Z_1\right)\Big|_{\mathbf{X}^{(0)}} \lambda_{31} + \left(x_{23} Z_1\right)\Big|_{\mathbf{X}^{(0)}} \lambda_{23} + \left(2x_{33} Z_1\right)\Big|_{\mathbf{X}^{(0)}} \lambda_{33} + \left(2x_{34} Z_1\right)\Big|_{\mathbf{X}^{(0)}} \lambda_{34}$$

$$\sum_{j=1}^{4} x_{1j} = 18+6\delta_1 \ , \quad \sum_{j=1}^{4} x_{2j} = 10+7\delta_2 \ , \quad \sum_{j=1}^{4} x_{3j} = 20+6\delta_3$$

$$\sum_{i=1}^{3} x_{i1} = 10+9\mu_1 \ , \quad \sum_{i=1}^{3} x_{i2} = 7+5\mu_2 \ , \quad \sum_{i=1}^{3} x_{i3} = 16+4\mu_3 \ , \quad \sum_{i=1}^{3} x_{i4} = 15+4\mu_4$$

$$x_{ij} \geq 0 \ , \ \delta_i, \mu_j, \theta_{ij}, \lambda_{ij} \in [0,1] . \quad i=1,2, \quad j=1,2,3 ,$$

$$\left(18+6\delta_1\right)+\left(10+7\delta_2\right)+\left(20+6\delta_3\right) \geq \left(10+9\mu_1\right)+\left(7+5\mu_2\right)+\left(16+4\mu_3\right)+\left(15+4\mu_3\right).$$

The optimal solution set $\mathbf{X}^{(1)}$ of the problem $\left(\text{P3}\big|_{\mathbf{X}^{(0)}}\right)$ is:

$$\mathbf{x}^{(1)} = \begin{bmatrix} 0 & 0 & 3 & 15 \\ 0 & 0 & 13 & 10 \\ 13 & 7 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{\theta}^{(1)} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{\lambda}^{(1)} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{\delta}^{(1)} = \begin{bmatrix} 0 & 0.4286 & 0 \end{bmatrix},$$

$$\boldsymbol{\mu}^{(1)} = \begin{bmatrix} 0.3333 & 0 & 0 & 0 \end{bmatrix}.$$

For the point $\mathbf{X}^{(1)}$, the values of objective is calculated as $Z_1\big|_{\mathbf{X}^{(1)}} = 68$, $Z_2\big|_{\mathbf{X}^{(1)}} = 50$ and so $Z\big|_{\mathbf{X}^{(1)}} = \left(Z_1\big|_{\mathbf{X}^{(1)}}\right)\left(Z_2\big|_{\mathbf{X}^{(1)}}\right) = 3400$. The next optimal solution set $\mathbf{X}^{(2)}$ is the same with the previous solution set $\mathbf{X}^{(1)}$, i.e . therefore $\mathbf{X}^{(2)} = \mathbf{X}^{(1)}$. Thus algorithm ends.The last solution implies following interval values for the two objective functions:

$$Z_1\big|_{\mathbf{X}^{(2)}} = [68, 80]$$
$$Z_2\big|_{\mathbf{X}^{(2)}} = [50, 85].$$

5. Conclusion

In this paper, we deal with IQITP whose objective coefficients and supply-demand quantities are given as intervals. In real life applications, this version of indefinite quadratic transportation problem is more realistic and reliable according to crisp ones. For the proposed solution procedure, all the interval parameters are handled by means of combination of left and right limits. And after determining a feasible initial point with the Northwest Corner method, then the indefinite quadratic objective is linearized by using first order Taylor series expansion about the feasible initial point. Thus IQITP is transformed a traditional linear programming problem. And then an iterative procedure is executed in such a way that the optimal solution of lastly constructed linear programming problem is selected as the point where about the nonlinear objective will be expanded to its first order Taylor series in the next iteration step. The stopping criterion of the proposed procedure is obtaining the same point for the last two iteration step. Finally, a numerical example is provided to illustrate the proposed procedure.

6. References

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