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Research Article

Randomized Decomposition Methods in Multi-objective Evolutionary Algorithm based on Decomposition for Many-objective Optimization Problems

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Abstract

As the number of objectives are increased in the optimization problem, the objective space is increased therefore it is not possible to use conventional methods to get answers for these problems. Therefore, some methods are proposed to solve this problem. As one of the solutions is called the decomposition. In decomposition the objectives are applied to the scalarization functions, and many sub-problems are obtained. Based on their neighborhood, the best members in the current generation of the evolutionary algorithm will be survived to the next generation. The algorithm which uses that idea is called Multi-objective Evolutionary Algorithm based on Decomposition (MOEA/D). Different typed of decomposition methods can be used with the MOEA/D algorithm. However, each of them has their own weaknesses or advantages. Therefore, to reduce the disadvantage of the decomposition methods, a hybrid approach is proposed in this research such that instead of a single decomposition method, two methods will be use randomly. The performance of the proposed hybrid method will be demonstrated on seven benchmark problems by using two metrics.

Keywords: multi-objective optimization, MOEA/D, many-objective optimization, evolutionary algorithms.

Çok Amaçlı Optimizasyon Problemleri için Ayrıştırmaya Dayalı Çok Amaçlı Evrimsel Algoritmada Rastgele Ayrıştırma Yöntemleri

Öz

Optimizasyon probleminde amaç sayısı arttıkça amaç uzayı da büyümektedir, bu nedenle bu problemlere cevap almak için geleneksel yöntemleri kullanmak mümkün değildir. Bu nedenle, bu sorunu çözmek için bazı yöntemler önerilmektedir. Çözümlerden birine ayrıştırma denir. Ayrıştırmada hedefler skalarizasyon fonksiyonlarına uygulanır ve birçok alt problem elde edilir. Komşuluklarına bağlı olarak, evrimsel algoritmanın mevcut neslindeki en iyi üyeler, bir sonraki nesile aktarılacaktır. Bu fikri kullanan algoritmaya Ayrıştırmaya Dayalı Çok Amaçlı Evrimsel Algoritma (MOEA/D) denir. MOEA/D algoritması ile farklı türde ayrıştırma yöntemleri kullanılabilir. Bununla birlikte, her birinin kendi zayıflıkları veya avantajları vardır. Bu nedenle, ayrıştırma yöntemlerinin dezavantajını azaltmak için bu araştırmada tek bir ayrıştırma yöntemi yerine rastgele iki yöntemin kullanılacağı hibrit bir yaklaşım önerilmiştir. Önerilen hibrit yöntemin performansı, iki metrik kullanılarak yedi test problemi üzerinde gösterilecektir.

Anahtar Kelimeler: çok amaçlı optimizasyon, MOEA/D, çok amaçlı optimizasyon, evrimsel algoritmalar.

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Table 1. Benchmark Problems [5]

Id.	Mathematical Expression
DTLZ1	$f_1 = \frac{1}{2}x_1x_2\ldots x_{M-1}(1+g(x_M))\ldots(1-x_{M-1})(1+g(x_M))\ldots f_M = \frac{1}{2}(1-x_1)(1+g(x_M))$ $g(x_M) = 100\left[\lvert x_M\rvert + \sum_{i=1}^{M}\left(\left(x_i-\frac{1}{2}\right)^2 + cos\left(20\pi\left(x_i-\frac{1}{2}\right)\right)\right)\right]$
DTLZ2	$f_1 = (1+g(x_M))cos\left(x_1\frac{\pi}{2}\right)\ldots cos\left(x_{M-2}\frac{\pi}{2}\right)cos\left(x_{M-1}\frac{\pi}{2}\right)\ldots cos\left(x_{M-2}\frac{\pi}{2}\right)sin\left(x_{M-1}\frac{\pi}{2}\right)\ldots$ $f_M = (1+g(x_M))sin\left(x_1\frac{\pi}{2}\right)\, g(x_M) = \sum_{i=1}^{M}\left(\left(x_i-\frac{1}{2}\right)^2\right)$
DTLZ3	$f_1 = (1+g(x_M))cos\left(x_1\frac{\pi}{2}\right)\ldots cos\left(x_{M-2}\frac{\pi}{2}\right)cos\left(x_{M-1}\frac{\pi}{2}\right)\ldots cos\left(x_{M-2}\frac{\pi}{2}\right)sin\left(x_{M-1}\frac{\pi}{2}\right)\ldots$ $f_M = (1+g(x_M))sin\left(x_1\frac{\pi}{2}\right)\, g(x_M) = 100\left[\lvert x_M\rvert + \sum_{i=1}^{M}\left(\left(x_i-\frac{1}{2}\right)^2 + cos\left(20\pi\left(x_i-\frac{1}{2}\right)\right)\right)\right]$
DTLZ4	$f_1 = (1+g(x_M))cos\left(x_1^{100}\frac{\pi}{2}\right)\ldots cos\left(x^{100}{}_{M-2}\frac{\pi}{2}\right)cos\left(x^{100}{}_{M-1}\frac{\pi}{2}\right)$ $\ldots f_M = (1+g(x_M))sin\left(x^{100}{}_1\frac{\pi}{2}\right),$ $g(x_M) = \sum_{i=1}^{M}\left(\left(x_i-\frac{1}{2}\right)^2\right)$
DTLZ5	$f_1 = (1+g(x_M))cos\left(\theta_1\frac{\pi}{2}\right)\ldots cos\left(\theta_{M-2}\frac{\pi}{2}\right)cos\left(\theta_{M-1}\frac{\pi}{2}\right)\ldots cos\left(\theta_{M-2}\frac{\pi}{2}\right)sin\left(\theta_{M-1}\frac{\pi}{2}\right)\ldots$ $f_M = (1+g(x_M))sin\left(\theta_1\frac{\pi}{2}\right)\, \theta_i = \frac{\pi}{4(1+g(x_M))}(1+2g(x_M)x_i), g(x_M) = \sum_{i=1}^{M}\left(\left(x_i-\frac{1}{2}\right)^2\right)$
DTLZ6	$f_1 = (1+g(x_M))cos\left(\theta_1\frac{\pi}{2}\right)\ldots cos\left(\theta_{M-2}\frac{\pi}{2}\right)cos\left(\theta_{M-1}\frac{\pi}{2}\right)$ $\ldots f_M = (1+g(x_M))sin\left(\theta_1\frac{\pi}{2}\right)$ $\theta_i = \frac{\pi}{4(1+g(x_M))}(1+2g(x_M)x_i), g(x_M) = \sum_{i=1}^{M}\left(x_i^{0.1}\right)$
DTLZ7	$f_1 = x_1, f_2 = x_2\ldots f_M = (1+g(x_M))h g(x_M) = 1 + \frac{9}{\lvert x_M\rvert}\sum x_i,$ $h = M - \sum_{i=1}^{M-1}\left(\frac{f_i}{1+g}(1+sin(3\pi f_i))\right)$

1. Introduction

The many-objective optimization problems generally referred as problems with number of objectives more than three. As the number of objectives are increased, more intelligence optimization algorithms are needed to solve these problems since relatively large computational power is needed by using the conventional and classical optimization methods. As a possible solution to problem decomposition (or scalarization) is used with evolutionary algorithm. The algorithm is called Multi-objective Evolutionary Algorithm based on Decomposition (MOEA/D) [1]. The performance of the MOEA/D is greatly depended on chosen decomposition method [2]. At the same paper [2] weighted sum method and Tchebycheff methods are changed through the implementation of the algorithm. In [3], a grid-based approach for multi usage of the decomposition method is proposed such that for each weight a different (one of two decomposition methods) decomposition method is used. Recently in [4], a combination of decomposition methods with a variable is proposed and showed the difference of the proposed method. In this research two decomposition/scalarization methods Penalty-based Boundary Intersection Scalarization (PBI) and Tchebycheff methods are preferred to be used as a hybrid decomposition method. One of these two methods are evaluated randomly. A uniform random number generator is preferred to generate a random number and based on this number one of the scalarization method evaluates. To present the performance difference of the proposed method seven benchmark problems with different objective numbers 10, 15, 20 and 25 objectives with five independent runs. Two properties of the solutions are compared which are accuracy and distribution of the solution by using two metrics which are IGD and Spread metrics. This research is organized as four sections beginning with the introduction. In the second section MOEA/D Algorithm, decomposition methods and benchmark problems are explained and then implementation results are reported at the next section and finally the conclusion of this study is presented.

2. Algorithms

In this section, optimization algorithm with decomposition methods -scalarization methods- are proposed. In addition to that metrics which are used to compare algorithms are proposed. Finally, the benchmark problems and their mathematical formulations are defined as a part of implementations. he definition of the multi-objective optimization algorithm is given as;

$$\min \quad F(x) = (f_1(x) \ldots f_M(x)) \qquad (1)$$
$$subject\ to\ x \in \Omega$$

Table 2. IGD Solutions

	Prob	M	D	PBI	0.4	0.5	0.6	0.7	Tchebycheff
M=10	DTLZ1	10	14	1.4198e-1 (6.16e-3) +	1.8162e-1 (2.98e-3) -	1.8023e-1 (2.82e-3) -	1.7782e-1 (3.49e-3) =	1.7192e-1 (3.44e-3) +	1.7881e-1 (6.60e-3)
	DTLZ2	10	19	5.0005e-1 (1.15e-3) +	1.0574e+0 (7.52e-2) -	1.0584e+0 (6.74e-2) -	1.0504e+0 (7.09e-2) -	1.0471e+0 (5.52e-2) -	7.2701e-1 (1.56e-2)
	DTLZ3	10	19	8.5641e-1 (3.36e-1) =	1.2044e+0 (6.71e-2) -	1.1904e+0 (7.57e-2) -	1.1998e+0 (5.20e-2) -	1.1600e+0 (8.84e-2) -	7.4049e-1 (3.59e-2)
	DTLZ4	10	19	6.7237e-1 (7.47e-2) +	1.1193e+0 (3.55e-2) -	1.0883e+0 (5.58e-2) -	1.0305e+0 (5.00e-2) -	9.5809e-1 (5.96e-2) -	8.3684e-1 (4.54e-2)
	DTLZ5	10	19	7.3730e-2 (9.76e-6) +	3.1647e-1 (5.39e-2) -	3.3521e-1 (7.46e-2) -	2.7439e-1 (6.06e-2) -	2.6142e-1 (5.76e-2) -	9.2888e-2 (9.95e-3)
	DTLZ6	10	19	7.4600e-2 (1.40e-3) -	4.2423e-1 (1.03e-1) -	4.4320e-1 (1.18e-1) -	3.7221e-1 (1.22e-1) -	3.6877e-1 (1.11e-1) -	6.9955e-2 (7.92e-3)
	DTLZ7	10	29	1.9257e+0 (2.54e-1) +	3.8710e+0 (1.23e+0) -	3.7117e+0 (1.40e+0) -	3.4964e+0 (1.04e+0) -	3.2663e+0 (1.33e+0) =	2.6649e+0 (8.01e-1)
M=15	DTLZ1	15	19	1.5916e-1 (7.96e-3) +	1.8371e-1 (2.29e-3) +	1.8285e-1 (3.06e-3) +	1.7958e-1 (2.05e-3) +	1.7398e-1 (2.87e-3) +	2.2589e-1 (9.65e-3)
	DTLZ2	15	24	6.2138e-1 (8.01e-4) +	1.1673e+0 (2.28e-2) -	1.2076e+0 (7.21e-2) -	1.1753e+0 (4.04e-2) -	1.1764e+0 (1.22e-2) -	8.9365e-1 (5.76e-2)
	DTLZ3	15	24	1.1143e+0 (2.75e-1) =	1.2668e+0 (1.07e-2) -	1.2656e+0 (2.82e-2) -	1.2615e+0 (2.64e-2) -	1.2224e+0 (6.54e-2) -	9.3163e-1 (9.09e-2)
	DTLZ4	15	24	7.1722e-1 (4.48e-2) +	1.2132e+0 (1.05e-2) -	1.1784e+0 (4.66e-2) -	1.1232e+0 (3.20e-2) -	1.0489e+0 (3.72e-2) -	9.6914e-1 (2.93e-2)
	DTLZ5	15	24	9.5739e-2 (1.02e-5) =	2.9748e-1 (3.78e-2) -	2.9899e-1 (9.06e-2) -	2.9522e-1 (4.75e-2) -	2.9920e-1 (1.13e-1) -	9.6429e-2 (8.81e-3)
	DTLZ6	15	24	9.5732e-2 (5.63e-6) =	3.9230e-1 (5.06e-2) -	3.2760e-1 (9.24e-2) -	3.3345e-1 (1.33e-1) -	3.1902e-1 (6.69e-2) -	8.1031e-2 (1.65e-2)
	DTLZ7	15	34	2.7766e+0 (3.23e-1) +	3.4469e+0 (5.27e-1) =	4.1344e+0 (1.67e+0) =	3.5433e+0 (9.32e-1) =	4.4657e+0 (1.71e+0) =	4.4884e+0 (7.43e-1)
M=20	DTLZ1	20	24	1.9488e-1 (9.11e-4) +	2.4609e-1 (1.20e-3) +	2.4565e-1 (2.43e-3) +	2.4312e-1 (2.23e-3) +	2.4188e-1 (1.76e-3) +	2.6417e-1 (8.59e-3)
	DTLZ2	20	29	7.5987e-1 (4.64e-4) +	1.2787e+0 (7.36e-3) -	1.2740e+0 (3.33e-2) -	1.2499e+0 (5.62e-2) -	1.1891e+0 (3.29e-2) -	1.1115e+0 (3.20e-2)
	DTLZ3	20	29	8.7618e-1 (2.54e-1) =	1.3282e+0 (3.89e-3) -	1.3274e+0 (4.75e-3) -	1.3196e+0 (8.42e-3) -	1.2992e+0 (9.41e-3) -	1.1001e+0 (3.16e-2)
	DTLZ4	20	29	8.8050e-1 (2.81e-2) +	1.3357e+0 (3.37e-5) -	1.2757e+0 (1.37e-2) -	1.1911e+0 (4.31e-2) =	1.1557e+0 (2.74e-2) =	1.1157e+0 (8.29e-3)
	DTLZ5	20	29	2.3879e-1 (6.63e-6) -	5.8749e-1 (1.18e-1) -	5.8803e-1 (9.65e-2) -	4.9027e-1 (1.79e-1) -	6.0614e-1 (7.82e-2) -	7.5367e-2 (4.33e-3)
	DTLZ6	20	29	2.3879e-1 (1.22e-6) -	6.4695e-1 (9.64e-2) -	6.1753e-1 (8.96e-2) -	6.0166e-1 (9.71e-2) -	5.8823e-1 (5.25e-2) -	7.2982e-2 (3.46e-3)
	DTLZ7	20	39	3.9573e+0 (1.06e+0) +	4.9145e+0 (7.96e-1) +	4.3803e+0 (1.45e+0) +	6.4104e+0 (3.21e+0) =	4.3851e+0 (8.15e-1) +	7.4403e+0 (2.10e+0)
M=25	DTLZ1	25	29	1.7300e-1 (4.86e-4) +	2.0981e-1 (2.00e-3) +	2.0939e-1 (1.61e-3) +	2.0842e-1 (2.42e-3) +	2.0909e-1 (1.72e-3) +	2.4524e-1 (2.72e-3)
	DTLZ2	25	34	7.7169e-1 (9.83e-4) +	1.2875e+0 (7.22e-3) -	1.3027e+0 (1.91e-2) -	1.2260e+0 (4.10e-2) -	1.2089e+0 (3.58e-2) -	1.1175e+0 (2.92e-2)
	DTLZ3	25	34	1.1058e+0 (3.10e-1) =	1.3350e+0 (5.15e-3) -	1.3184e+0 (1.37e-2) -	1.3300e+0 (9.55e-3) -	1.2791e+0 (3.99e-2) -	1.1375e+0 (1.96e-2)
	DTLZ4	25	34	9.0023e-1 (5.96e-2) +	1.3395e+0 (7.17e-7) -	1.2875e+0 (8.19e-3) -	1.2194e+0 (2.20e-2) -	1.1533e+0 (5.45e-2) =	1.1268e+0 (1.97e-2)
	DTLZ5	25	34	2.6392e-1 (1.03e-6) -	6.3760e-1 (1.51e-2) -	6.1557e-1 (1.49e-1) -	6.4484e-1 (1.94e-2) -	5.9947e-1 (3.50e-2) -	7.3612e-2 (4.57e-4)
	DTLZ6	25	34	2.6392e-1 (2.05e-7) -	6.7650e-1 (3.14e-2) -	7.1245e-1 (1.82e-2) -	6.3223e-1 (7.09e-2) -	6.0803e-1 (6.59e-2) -	7.4097e-2 (5.05e-3)
	DTLZ7	25	44	4.2149e+0 (2.82e-1) +	4.6013e+0 (6.34e-1) +	5.8644e+0 (1.70e+0) =	5.3035e+0 (1.09e+0) =	7.3928e+0 (2.51e+0) =	8.8337e+0 (2.60e+0)

where Ω is the decision space and F:Ω→RM is the real valued objective space [6, 7], where F is the objective function vector of real valued f. The best possible solution on the objective space is called Pareto Front (PF) [8].

2.1. Multi-objective Evolutionary Algorithm based on Decomposition (MOEA/D)

Multi-objective Evolutionary Algorithm Based on Decomposition (MOEA/D) is proposed by Zhang and Li in 2007 [1]. MOEAD is an evolutionary algorithm so that three operators which are Crossover, Mutation and Selection operators are also evaluated inside MOEA/D algorithm. For crossover operator, the weights are defined. These weights are reference points on the objective space and neighbourhood matrix is defined for the neighbourhood of each reference points. The parents from neighbourhood are selected and offspring is generated by using SBX method. After the offspring are generated mutation operator is applied and Polynomial Mutation is preferred in MOEA/D algorithm. Therefore, two populations are generated and the best members are survived to the next generator. For this purpose, the decomposition (scalarization) methods are applied to the members of these populations.

Table 3. Spread Solutions

	Prob	M	D	PBI	0.4	0.5	0.6	0.7	Tchebycheff
M=10	DTLZ1	10	14	1.3120e-1 (3.71e-2) +	1.0283e+0 (7.64e-3) +	1.0320e+0 (6.52e-3) +	1.0371e+0 (8.31e-3) =	1.0492e+0 (1.43e-2) =	1.0482e+0 (7.78e-2)
	DTLZ2	10	19	3.9726e-1 (6.68e-3) +	1.0217e+0 (1.95e-2) +	1.0243e+0 (2.25e-2) +	1.0198e+0 (1.13e-2) +	1.0190e+0 (1.07e-2) +	1.1519e+0 (7.57e-2)
	DTLZ3	10	19	7.7323e-1 (3.15e-1) +	1.0054e+0 (8.79e-3) +	1.0072e+0 (7.71e-3) +	1.0060e+0 (9.62e-3) +	1.0093e+0 (1.22e-2) +	1.1892e+0 (3.84e-2)
	DTLZ4	10	19	1.1835e+0 (2.12e-1) =	1.0404e+0 (3.49e-2) +	1.0398e+0 (4.08e-2) +	1.0595e+0 (5.05e-2) +	1.1048e+0 (5.65e-2) +	1.2027e+0 (5.84e-2)
	DTLZ5	10	19	1.8535e+0 (2.11e-3) -	1.0201e+0 (3.01e-2) +	1.0181e+0 (3.33e-2) +	1.0240e+0 (3.35e-2) +	1.0166e+0 (1.62e-2) +	1.8300e+0 (3.94e-2)
	DTLZ6	10	19	1.8810e+0 (5.27e-2) -	1.1112e+0 (1.93e-1) +	1.0240e+0 (6.16e-2) +	1.0287e+0 (4.59e-2) +	1.0172e+0 (3.06e-2) +	1.8257e+0 (6.55e-2)
	DTLZ7	10	29	1.0436e+0 (1.41e-2) -	1.0070e+0 (4.63e-3) -	1.0075e+0 (3.89e-3) -	1.0101e+0 (3.30e-3) -	1.0145e+0 (9.88e-3) -	9.4575e-1 (2.72e-2)
M=15	DTLZ1	15	19	1.8181e-1 (2.50e-2) +	1.0170e+0 (2.11e-3) +	1.0232e+0 (8.86e-3) +	1.0333e+0 (8.47e-3) +	1.0659e+0 (1.53e-2) +	1.1501e+0 (4.03e-2)
	DTLZ2	15	24	3.2317e-1 (9.63e-3) +	1.0205e+0 (1.78e-2) +	1.0035e+0 (6.50e-3) +	1.0148e+0 (1.55e-2) +	1.0041e+0 (1.44e-3) +	1.1562e+0 (8.89e-2)
	DTLZ3	15	24	9.1135e-1 (3.25e-1) +	1.0064e+0 (5.86e-3) +	1.0027e+0 (2.12e-3) +	1.0051e+0 (4.21e-3) +	1.0029e+0 (2.87e-3) +	1.1780e+0 (3.46e-2)
	DTLZ4	15	24	1.1647e+0 (2.10e-1) =	1.0513e+0 (1.80e-2) +	1.0180e+0 (2.05e-2) +	1.0376e+0 (2.87e-2) +	1.0738e+0 (4.41e-2) +	1.2151e+0 (3.84e-2)
	DTLZ5	15	24	1.9686e+0 (3.11e-5) =	1.0257e+0 (4.28e-2) +	1.0373e+0 (4.67e-2) +	1.0154e+0 (1.18e-2) +	1.0325e+0 (4.53e-2) +	1.9434e+0 (2.43e-2)
	DTLZ6	15	24	1.9685e+0 (1.42e-4) -	1.0893e+0 (1.80e-1) +	1.0249e+0 (3.26e-2) +	1.0295e+0 (5.16e-2) +	1.0053e+0 (5.08e-3) +	1.9088e+0 (1.58e-2)
	DTLZ7	15	34	1.0114e+0 (3.84e-3) -	1.0038e+0 (2.53e-3) -	1.0045e+0 (1.23e-3) -	1.0076e+0 (2.17e-3) -	1.0080e+0 (1.09e-3) -	9.5624e-1 (3.23e-3)
M=20	DTLZ1	20	24	1.1929e+0 (3.67e-2) -	1.0111e+0 (3.92e-3) =	1.0122e+0 (1.62e-3) =	1.0143e+0 (5.71e-3) =	1.0207e+0 (5.07e-3) =	1.0144e+0 (7.97e-3)
	DTLZ2	20	29	5.1445e-1 (9.79e-2) +	1.0084e+0 (5.07e-3) =	1.0062e+0 (4.28e-3) =	1.0099e+0 (1.03e-2) =	1.0101e+0 (5.94e-3) =	1.0306e+0 (2.85e-2)
	DTLZ3	20	29	7.7904e-1 (1.28e-1) +	1.0012e+0 (7.90e-4) +	1.0018e+0 (1.65e-3) +	1.0023e+0 (3.87e-3) +	1.4296e+0 (9.53e-1) =	1.0339e+0 (2.98e-2)
	DTLZ4	20	29	1.0746e+0 (2.88e-2) =	1.0000e+0 (2.71e-5) +	1.0161e+0 (2.37e-2) =	1.0304e+0 (4.97e-2) =	1.0421e+0 (3.70e-2) =	1.0329e+0 (2.38e-2)
	DTLZ5	20	29	3.5000e+0 (2.94e-5) -	1.0175e+0 (3.43e-2) +	1.0015e+0 (8.28e-4) +	1.0019e+0 (1.80e-3) +	1.0042e+0 (4.29e-3) +	2.8981e+0 (1.61e-4)
	DTLZ6	20	29	3.4999e+0 (4.51e-5) -	1.1564e+0 (3.47e-1) +	1.0774e+0 (1.57e-1) +	1.0149e+0 (3.14e-2) +	1.0013e+0 (5.98e-4) +	2.8980e+0 (1.79e-4)
	DTLZ7	20	39	1.0302e+0 (1.43e-2) -	1.0016e+0 (5.25e-4) =	1.0025e+0 (9.37e-4) -	1.0028e+0 (7.52e-4) -	1.0029e+0 (7.12e-4) -	1.0008e+0 (1.42e-3)
M=25	DTLZ1	25	29	1.1866e+0 (1.72e-2) -	1.0082e+0 (4.13e-3) =	1.0115e+0 (1.62e-3) =	1.0146e+0 (4.62e-3) =	1.0177e+0 (4.69e-3) -	1.0095e+0 (5.00e-3)
	DTLZ2	25	34	8.5247e-1 (4.25e-2) +	1.0058e+0 (1.85e-3) +	1.0045e+0 (2.45e-3) +	1.0096e+0 (6.96e-3) =	1.0117e+0 (3.01e-3) =	1.0240e+0 (1.02e-2)
	DTLZ3	25	34	1.0452e+0 (6.40e-2) =	1.0005e+0 (5.29e-4) +	1.0022e+0 (2.35e-3) +	1.0007e+0 (9.80e-4) +	1.0039e+0 (4.98e-3) +	1.0152e+0 (3.66e-3)
	DTLZ4	25	34	1.0680e+0 (1.81e-2) -	1.0000e+0 (4.26e-7) +	1.0138e+0 (1.34e-2) =	1.0335e+0 (2.24e-2) =	1.0537e+0 (3.59e-2) =	1.0277e+0 (1.99e-2)
	DTLZ5	25	34	3.6800e+0 (4.88e-5) -	1.0011e+0 (9.11e-4) +	1.0020e+0 (4.10e-3) +	1.0006e+0 (4.27e-4) +	1.0027e+0 (1.95e-3) +	3.0292e+0 (1.81e-4)
	DTLZ6	25	34	3.6795e+0 (5.29e-4) -	1.0006e+0 (8.05e-4) +	1.0039e+0 (8.42e-3) +	1.0033e+0 (4.03e-3) +	1.0038e+0 (3.48e-3) +	3.0341e+0 (1.03e-2)
	DTLZ7	25	44	1.0087e+0 (1.00e-3) -	1.0013e+0 (6.74e-4) -	1.0013e+0 (4.42e-4) -	1.0012e+0 (6.06e-4) -	1.0017e+0 (7.80e-4) -	1.0001e+0 (4.90e-4)

Based on scalarization values the smallest values are survived and based on survived members the neighbourhood matrix is updated. This process is repeated until the termination conditions are satisfied.

2.2. Decomposition Methods

Decomposition (or scalarization) is a function that converts multi-objective optimization to single objective optimization with a defined weight (w). The sum of these weights will be equal to one. However, if it is applied not only a single set of weights but also applied to many sets, matrix, a set of solution candidate with respect to the weights will be obtained. This is used as a selection operator in MOEA/D.

In this research two different decomposition methods will be used. The first one is named as Penalty-based Boundary Intersection Scalarization (PBI). In Eq. 2 the mathematical expression for the PBI is presented.

$$g(x) = d_1 + \theta d_2 \quad (2)$$

$$d_1 = \frac{\|w(z-F(x))\|}{\|w\|} \text{ and } d_2 = \|F(x) - (z - wd_1)\|$$

As the second decomposition method in this study Tchebycheff method is selected. Unlike previous studies in literature weighted sum and Tchebycheff is evaluated in a different aspect. However, in literature it is clearly demonstrated that weighted sum methods give the worst result on MOEA/D algorithm. However, in this study two best decomposition methods will be used and compared with each other. In Eq 3 the mathematical description of the Tchebycheff method presented.

$$g(x) = max(w_i|f_i(x) - z_i|) \quad (3)$$

2.3. Metrics

To compare the performance of the algorithms some functions are needed to evaluate the solutions on the objective space. Two important parameters are observed for comparison. These are accuracy and distribution of the solutions on objective space. For the accuracy, inverted generalized distance (IGD) metric is proposed and mathematical description of this metric is given as [9]

$$f_{IGD} = \frac{\sum ds(a,P)}{|P|} \quad (4)$$

The IGD metric is based on computing the average distance between obtained solution candidates and the Pareto Front where

$$ds(a,P) = \sqrt{\sum (a_i - p_i)^2}$$

The second metric is related to the distribution of the solution on the objective space. It is important because each point in objective space corresponds to the solution candidate. The spread metric is defined in Eq. 5 [10]. The metric is based on calculation of the normalized squared sum of the distance between maximum and minimum difference between produced solutions and PF.

$$f_{Spread} = \sqrt{\frac{1}{M}\sum\left(\frac{\max(a,PF)-\min(a,PF)}{PF_{max}-PF_{min}}\right)^2} \quad (5)$$

2.4. Benchmark Problems

Table 1 shows the benchmark problems used in this research which is proposed by Deb et. Al in [5]. These benchmark problems may have many numbers of objectives (M) and in this research 10, 15, 20 and 25 objectives are considered. There are seven benchmark problems are selected for this study. For DTLZ1 the dimension of the decision space is M+4, DTLZ2-DTLZ6 M+9 and for DTLZ7 the dimension is equal to M+19.

3. Implementation

In this research six different setups are implemented on seven benchmark problems (DTLZ1-7). The implementations are repeated 15 independent run and statistics are recorded as mean and standard deviation of the results of the metrics IGD and Spread metric. The population size of each implementation is equal to (M*10) and maximum number of function evaluations is equals to (M*104). The MOEA/D implementations are evaluated as 1) Just PBI, 2) Random number is smaller 0.4 than apply PBI else Tchebycheff, 3) same with smaller 0.5, 4) same with smaller 0.6, 5) same with smaller 0.7, and 6) Just Tchebycheff. Table 2 and 3 shows the performance of these algorithm with respect to the IGD and Spread metrics. From Table 2, the accuracy of the solutions is evaluated and PBI gives almost all cases and all objective dimensions. However, the case “0.7” gives the similar performance with the PBI. On contrary, in Table 3, it is not possible to mention the distribution of the solution for only one method. However, from the statistical results (Wilcoxon rank test) the case “0.7” gives the average results and statistically almost same performance with other cases in general.

4. Conclusion

In this research to improve the distribution property of the PBI decomposition method a hybrid decomposition method with PBI and Tchebycheff are used randomly. The results showed that PBI gives almost best results for the accuracy. However, PBI could not produce a well distributed solution, in this case Tchebycheff helps the PBI to distribute better solution just used approximately -randomly- %30 of the total implementation with respect to the given results.

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