



Solving Many-objective Constraint Real-World Optimization Problems with Multi-objective Optimization Algorithms

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Abstract

In real world engineering optimization problems many constraints must be considered due to the imperfect conditions of the systems or process. Therefore, constraint handling methods are integrated into optimization algorithms. However, since the constraints are succeeded by the algorithm, their value is not considered or watched. Alternatively, it is possible to convert constraints to objectives and these many-objective constraint real-world optimization problems are changed to many-objective optimization problems. In this research for this purpose five real world engineering design problems are converted into many-objective optimization problem which are Gear Train Design, Pressure Vessel Design, Two Bar Truss Design, Disc Brake Design and Vibrating Platform Design problems. The problems are solved by using multi-objective optimization algorithms (NSGA-II, MOEA/D, MOEA/D-DE, MPSO/D and MOPSO) and their performance is compared by using the hypervolume metric.

Keywords: Gear Train, Pressure Vessel, Two Bar Truss, Disc Brake, Vibrating Platform, multi-objective optimization, many-objective optimization, NSGA-II, MOEA/D, MOEA/D-DE, MPSO/D, MOPSO

Çok Amaçlı Kısıtlama Gerçek Dünya Optimizasyon Problemlerini Çok Amaçlı Optimizasyon Algoritmaları ile Çözme

Öz

Gerçek dünya mühendislik optimizasyon problemlerinde, sistemlerin veya süreçlerin kusurlu koşulları nedeniyle birçok kısıtlamanın dikkate alınması gerekir. Bu nedenle, kısıtlama işleme yöntemleri optimizasyon algoritmalarına entegre edilmiştir. Ancak kısıtlamalar algoritma tarafından başarılı olduğu için değerleri dikkate alınmaz veya izlenmez. Alternatif olarak, kısıtlamaları hedeflere dönüştürmek mümkündür ve bu çok amaçlı kısıtlamalı gerçek dünya optimizasyon problemleri, çok amaçlı optimizasyon problemlerine dönüştürülür. Bu amaçla bu araştırmada, beş gerçek dünya mühendislik tasarım problemi, Dişli Tren Tasarımı, Basınçlı Kap Tasarımı, İki Çubuk Kafes Tasarımı, Disk Fren Tasarımı ve Titreşimli Platform Tasarımı problemleri olan çok amaçlı optimizasyon problemine dönüştürülmüştür. Problemler çok amaçlı optimizasyon algoritmaları (NSGA-II, MOEA/D, MOEA/D-DE, MPSO/D ve MOPSO) kullanılarak çözülmüş ve hiperhacim metriği kullanılarak performansları karşılaştırılmıştır.

Anahtar Kelimeler: Dişli Treni, Basınçlı Kap, İki Çubuk Kafes, Disk Fren, Titreşimli Platform, çok amaçlı optimizasyon, çok amaçlı optimizasyon, NSGA-II, MOEA/D, MOEA/D-DE, MPSO/D, MOPSO.

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1. Introduction

The aim of the real-life engineering design problems is to determine some physical properties of the corresponding system to demonstrate a desired performance. A possible set of problems which are also considered in this research are Pressure Vessel Design [1], Vibrating Platform Design [2], Two Bar Truss Design [3], Disc Brake Design [4] and Gear Train Design [5] problems. The objectives of these problems are generally be the performance specifications and the decision variables are generally be the design properties. Therefore, it is possible to define these problems are multi-objective optimization problem. The definition of the multi-objective optimization problem is given as

$$\begin{aligned} \min F(x) &= (f_1(x) \dots f_M(x)) & (1) \\ \text{subject to } x &\in \Omega \\ g(x) &\leq 0 \\ h(x) &= 0 \end{aligned}$$

where $g(x)$ and $h(x)$ are constraints with the decision vector $x \in \Omega$ is the decision space and $F: \Omega \rightarrow \mathbb{R}^M$ is the real valued objective space [7,8], where F is the objective function vector of real valued f . Constraint handling methods are defined for multi-objective optimization algorithm such that admissible solutions may higher ranked [9], or crossover the admissible parents are two of some constraint handling techniques [10]. Alternatively, it is possible to convert constraints into objective functions and solved as a many-objective optimization problem. Therefore, in this research five different design problems are selected as the real-world constraint optimization engineering problems. These problems are converted into many objective optimization problems and solved by using nondominated Sorted Genetic Algorithm 2 (NSGA-II), Multi-objective Evolutionary Algorithm-based Decomposition (MOEA/D), Differential Evolution based MOEA/D (MOEA/D-DE), Decomposition-based multi-objective particle swarm optimization (MPSO/D) and multi-objective particle swarm optimization (MOPSO) and compared with each other.

This paper is organized as following, section two gives the mathematical description of the design problems and their properties, the following section gives the properties of the optimization algorithms. Next, implementation and implementation results are given and compared and finally the conclusion of the study is presented.

2. Engineering Problems

In this research five design problems are considered as the real-world engineering problems and converted these constrained real world bi-objective optimization problems into many objective optimization problems. By this way it will be certain that all the constraints are succeeded by the optimization algorithm also it is possible to compare possible solutions with respect to their remaining objective values. For this purpose, five problems Gear Train Design, Pressure Vessel Design, Two Bar Truss Design, Disc Brake Design and Vibrating Platform Design problems are selected and labeled as Design 1-5 for comparing at the implementation section.

2.1. Gear Train Design (Design 1) [5]

The (compound) gear train is a set of gears which are connected with each other and rotated at the same speed. The gear train design is based on given in [1] such that the expected gear

ratio is designed so that it will be as close as possible to 1/6.931. The constrained in this problem is the number of teeth should be between 12 and 60. For this design problem the following objective functions are defined.

$$\begin{aligned} f_1 &= \left| 6.931 - \frac{x_3 x_4}{x_1 x_2} \right| & (2) \\ f_2 &= \max(x_1, x_2, x_3, x_4) \\ f_3 &= \left| 1 - \frac{x_3 x_4}{6.931 x_1 x_2} \right| - 0.5 \end{aligned}$$

where all decision variables are inside [12,60].

2.2. Pressure Vessel Design (Design 2) [1]

Pressure vessels are storage devices to hold high pressured liquid or gas. The design of the vessel includes material, forming and welding which are needed to be minimized. The following objective functions are given for multi-objective optimization problem.

$$\begin{aligned} f_1 &= 0.11x_2x_3^2 + 0.04x_1x_3x_4 & (3) \\ &\quad + 0.012x_1^2x_4 + 0.075x_1^2x_3 \\ f_2 &= -\pi x_3^2x_4 - \frac{4}{3}\pi x_3^3 \\ f_3 &= 0.00954x_3 - 0.0625x_2 \\ f_4 &= 0.0193x_3 - 0.0625x_1 \end{aligned}$$

where the boundaries of the x_1 and x_2 in [1,99] and x_3 and x_4 in [10,200].

2.3. Two Bar Truss Design (Design 3) [3]

Two bar truss system is an experimental setup for holding a bar system which is connected to the floor. The design specifications as objective functions are given below;

$$\begin{aligned} f_1 &= x_1\sqrt{16 + x_3^2} + x_2\sqrt{1 + x_3^2} & (4) \\ f_2 &= \frac{20\sqrt{16 + x_3^2}}{x_1x_3} \\ f_3 &= x_1\sqrt{16 + x_3^2} + x_2\sqrt{1 + x_3^2} - 0.1 \\ f_4 &= \frac{20\sqrt{16 + x_3^2}}{x_1x_3} - 10^5 \\ f_5 &= \frac{80\sqrt{1 + x_3^2}}{x_2x_3} - 10^5 \end{aligned}$$

where the boundaries for x_1 and x_2 is in (0,100] and x_3 is defined in [1,3].

2.4. Disc Brake Design (Design 4) [4]

The braking is an important property for the security of vehicles and its design is another engineering problem. The important criteria are the mass and stopping time. Therefore, the inner and outer radius of the discs should be designed by using the engaging force and the number of friction surfaces. The objective functions are given as;

$$\begin{aligned} f_1 &= 4.9 * 10^{-5}(x_2^2 - x_1^2)(x_4 - 1) & (5) \\ f_2 &= 9.82 * 10^6 \left(\frac{x_2^2 - x_1^2}{x_3x_4(x_2^3 - x_1^3)} \right) \\ f_3 &= 20 - x_2 + x_1 \\ f_4 &= \frac{x_3}{\pi(x_2^2 - x_1^2)} - 0.4 \end{aligned}$$

Table 1. Hypervolume Metric Results for Real-Work Design Problems

Problem	M	D	NSGAI	MOEAD	MOEAD-DE	MPSOD	MOPSO
Design1	3	4	1.5593e+2 (2.87e-2) =	1.5580e+2 (1.30e-1) =	1.5647e+2 (2.10e-1) +	1.5288e+2 (2.29e+0) =	1.5470e+2 (1.16e+0)
Design2	4	4	2.6364e+9 (2.98e-3) +	2.6364e+9 (4.07e-1) +	2.6364e+9 (1.86e-3) +	2.6364e+9 (2.78e+1) +	2.6364e+9 (1.44e+3)
Design3	5	3	1.7182e+5 (1.06e-3) =	1.4691e+5 (1.24e+3) -	1.7182e+5 (2.04e-3) -	1.7181e+5 (8.84e-1) -	1.7182e+5 (1.56e-3)
Design4	6	4	1.5452e+1 (2.08e-3) =	1.5307e+1 (4.89e-2) =	1.5297e+1 (8.46e-2) =	1.5112e+1 (5.29e-2) +	1.5183e+1 (1.39e-1)
Design5	7	5	5.7789e+2 (3.01e-2) =	5.7805e+2 (8.85e-3) =	5.7782e+2 (1.72e-1) =	5.7799e+2 (5.00e-2) =	5.7791e+2 (2.19e-1)
+/-/=			1/0/4	1/1/3	2/1/2	2/1/2	

$$f_5 = \frac{2.22 * 10^{-3} x_3 (x_2^3 - x_1^3)}{(x_2^2 - x_1^2)^2} - 1$$

$$f_6 = 900 - 0.026 \frac{x_3 x_4 (x_2^3 - x_1^3)}{x_2^2 - x_1^2}$$

where the decision space dimension is four and their boundaries are defined as [55,80], [75,110], [1000,3000], [11,20] respectively.

2.5. Vibrating Platform Design (Design 5) [2][6]

Pinned-pinned sandwich beam design with vibrating motor is the vibrating platform design [6]. A vibratory disturbance is imparted from the motor onto the beam. The aim of this system is to minimize the vibration on the beam due to the motor disturbance. The objectives are defined as;

$$f_1 = -\frac{\pi}{2x_1} \sqrt{\frac{(0.16x_2^2 + 7(x_3^3 - x_2^3) + 20(x_4^3 - x_3^3))}{(30x_2 + 831(x_3 - x_2) + 2334(x_4 - x_3))}} \quad (6)$$

$$f_2 = 2x_5 x_1 (500x_2 + 1500(x_3 - x_2) + 800(x_4 - x_3))$$

$$f_3 = (30x_2 + 831(x_3 - x_2) + 2334(x_4 - x_3))x_1 - 2800$$

$$f_4 = x_2 - x_3$$

$$f_5 = x_3 - x_2 - 0.15$$

$$f_6 = x_3 - x_4$$

$$f_7 = x_4 - x_3 - 0.01$$

where the boundaries of the decision variables are [3,6], [0.05,0.5], [0.2,0.5], [0.2,0.6], [0.35,0.5] respectively.

3. Algorithms

In this research five different multi-objective optimization algorithms are used to solve the design problems. These algorithms are; nondominated Sorted Genetic Algorithm 2 (NSGA-II) [11], Multi-objective Evolutionary Algorithm-based Decomposition (MOEA/D) [12], Differential Evolution based MOEA/D (MOEA/D-DE) [13], Decomposition-based multi-objective particle swarm optimization (MPSO/D) [14] and multi-objective particle swarm optimization (MOPSO) [15].

NSGA-II is based on sorting the solution on the objective space with respect to their dominance. Like other evolutionary algorithms it has crossover mutation and selection operator. At the section phase nondominated solutions are sorting with respect to their form as a front and best solutions are survived to the next generation with the aid of other operators like crowding distance. In the literature the performance of this algorithm is reported as acceptable for especially objective with up to three. However, as

the number of objectives are increased in number the sorting algorithm needs more time and resources.

MOEA/D prefers different method to select the best members on the population called decomposition. Decomposition is a scalarization method that converts multi-objective optimization problem into single objective optimization problem with the aid of a set of reference points. In this algorithm offspring are generated from the neighbours of the current members and the best members are selected from a set of weights and corresponding objective values. In MOEA/D-DE algorithm the crossover and mutation operator are inherited from Differential Evolution algorithm, therefore it is claimed that the algorithm exhibits better performance since DE is presents better performance when compared to GA. Similarly, in MPSO/D algorithm the decomposition idea is joint with the Particle Swarm Optimization and the PSO algorithm is used to create new solution and best members are selected from decomposition idea. Finally, MOPSO is the multi-objective PSO algorithm so that positions and velocities for each objective are calculated and compared with each other with dominance principle that the best members survive to the next generation.

4. Implementation and Results

In this research five design problems are solved with five multi-objective optimization algorithms. Each algorithm is repeated 15 times (independent run) and their performance's statistical results mean and standard deviation is recorded into Table 1. The number of objectives for each design are 3,4,5,6 and 7 for Design 1-5 respectively and number of decision variable are 4, 4, 3 ,4 and 5 respectively. Therefore, it is possible to comment that the easiest problem is Design 1 and hardest is the Design 5 problem. To compare the performance Hypervolume is selected as the performance metric for each problem and algorithms. From Table 1, MOPSO gives the worst performance among all the algorithms, NSGAI, MOEAD and MPSOD algorithm presents only best performance for just one designs. However, among all these algorithms MOEA-DE gives the best results for three design problems. When considered the signed rank test, for design 1 only MOEA-DE gives the best performance and others presents almost same performance. For design 3, NSGA-II and MOPSO gives almost same performance and for Design 4 just MSPOD gives the best result and others presents the similar performance. Finally for the last design it is not possible to mention about the better performance since each algorithm gives almost the same performance. Also the description of the solution on the objective space is given in Fig. 1-5 for Design1-5 respectively.

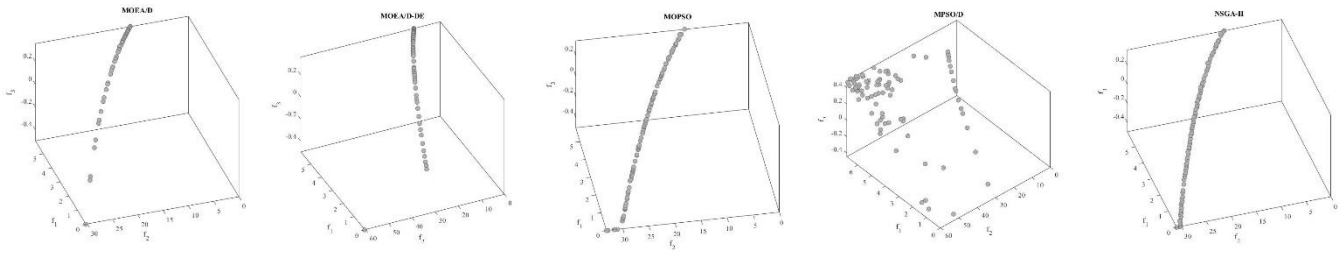


Fig. 1 Representation of the solutions on objective space for Design 1

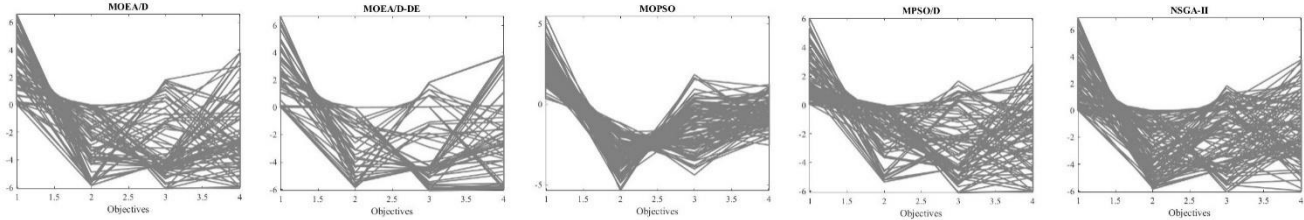


Fig. 2 Representation of the solutions on objective space for Design 2

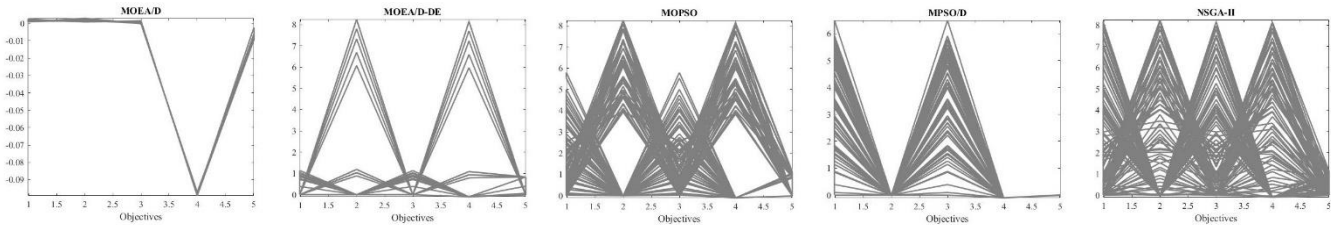


Fig.3. Representation of the solutions on objective space for Design 3

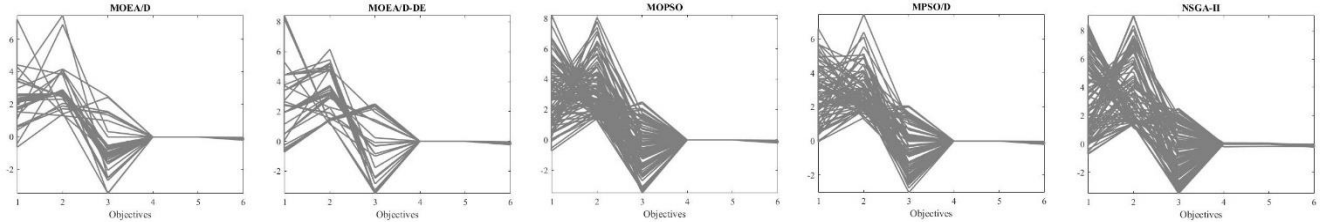


Fig. 4 Representation of the solutions on objective space for Design 4

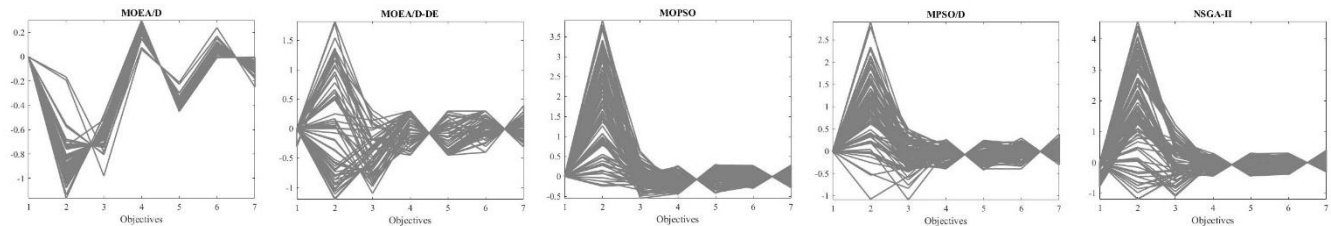


Fig. 5 Representation of the solutions on objective space for Design 5

5. Conclusion

In this study the constrained real-life engineering design problems are converted into many-objective optimization problems and they solved by using the multi-objective optimization algorithms. From the results it is not possible to mention about only one algorithm gives the best performance however in overall the MOEA/D-DE gives the best among them. Also, even if the number of objectives and decision space dimension is differing, the real-life design problems remain the hards problem when compared with the benchmark problems.

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