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A novel circular intuitionistic fuzzy AHP&VIKOR methodology: An application to a multi-expert supplier evaluation problem

Yeni bir dairesel sezgisel bulanık AHP&VIKOR metodolojisi: Çok uzmanlı tedarikçi değerlendirme problemine uygulama

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Abstract

VIKOR method being one of the frequently used Multi-Criteria Decision Making (MCDM) methods, is based on the distances of alternatives to positive and negative ideal solutions, and presents compromising solutions. AHP is another MCDM method dividing the big problem into small and manageable problems through pairwise comparisons of criteria and alternatives. In these methods, linguistic assessments are generally preferred since exact numerical assignments of criteria values are really difficult and experts can not reflect the thoughts in their minds with crisp numbers. The fuzzy set theory captures the vagueness and impreciseness in these linguistic assessments successfully thorough fuzzy numbers. Circular intuitionistic fuzzy sets (C-IFS) are the latest extension of ordinary fuzzy sets, which was introduced by Atanassov [1]. C-IFS help experts to define membership (belongingness) and non-membership (unbelongingness) degrees by incorporating the uncertainty of these degrees. In this paper, an integrated C-IF AHP & C-IF VIKOR methodology is developed and applied to a multi-expert supplier evaluation problem. The results obtained from the proposed methodology are compared with other methods, and a sensitivity analysis is performed as well.

Keywords: Fuzzy AHP, Fuzzy VIKOR, Circular intuitionistic fuzzy sets, MCDM, Negative and positive ideal solutions, Compromise solution.

Öz

Sık kullanılan Çok Ölçütlü Karar Verme (ÇÖKV) yöntemlerinden biri olan VIKOR yöntemi, alternatiflerin pozitif ve negatif ideal çözümlere olan uzaklıklarını temel alır ve uzlaşmacı çözümler sunar. AHP, ölçütlerin ve alternatiflerin ikili olarak karşılaştırılması yoluyla büyük bir problemi küçük ve yönetilebilir problemlere bölen bir başka ÇÖKV yöntemidir. Bu yöntemlerde, ölçüt değerlerinin kesin sayısal atamalarının gerçekten zor olması ve uzmanların düşüncelerini net rakamlarla yansıtamamaları gibi nedenlerle genellikle dilsel değerlendirmeler tercih edilmektedir. Bulanık küme teorisi, bu dilbilimsel değerlendirmelerdeki belirsizlik ve kesin olmama durumlarını bulanık sayıları kullanarak başarıyla ele alır. Dairesel sezgisel bulanık kümeler (D-SBK), Atanassov [1] tarafından tanıtılan sıradan bulanık kümelerin en son uzantısıdır. D-SBK, üyelik (aidiyet) ve üye olmama (aidiyetsizlik) derecelerindeki belirsizlikleri de göz önüne alarak uzmanların bu dereceleri tanımlamalarına yardımcı olur. Bu çalışmada, bütünleşik D-SB AHP ve D-SB VIKOR metodolojisi geliştirilmiş ve çok uzmanlı bir tedarikçi değerlendirme problemine uygulanmıştır. Önerilen metodolojiden elde edilen sonuçlar, diğer yöntemlerle karşılaştırılmakta ve duyarlılık analizi de yapılmaktadır.

Anahtar kelimeler: Bulanık AHP, Bulanık VIKOR, Dairesel sezgisel bulanık kümeler, ÇÖKV, Negatif ve pozitif ideal çözümler, Uzlaşık çözüm.

1 Introduction

When conflicting and incommensurable criteria exist in a decision making problem, MCDM methods help experts to evaluate a number of finite alternatives, and to reach the optimal solution. MCDM research area is categorized into two sub-research areas: Multiple Attribute Decision Making (MADM) and Multiple Objective Decision Making. When a discrete number of alternatives exists rather than a continuous number, MADM methods are used. Since 1980s, MADM methods have been frequently studied and employed for the solution of various problems.

Among classical MADM methods, two of the most used methods are Analytic Hierarchy Process (AHP) and VIekriterijumsko KOmpromisno Rangiranje (VIKOR). VIKOR method is a compromise solution technique based on optimization [2], and aims at providing a maximum level of group utility & a minimum level of individual regret.

As stated by the researchers, human judgments and preferences cannot be accurately expressed by crisp numbers. To deal with uncertainties and vaguness inherent in human judgments and incomplete information, the fuzzy set theory was proposed in 1965 [3]. Classical MCDM methods have been fuzzified by using the new types of fuzzy sets e.g. multi-sets [4], intuitionistic fuzzy sets (IFS) [5], picture fuzzy sets (PFS) [6], and spherical fuzzy (SF) sets [7]. AHP is among the most common and preferred MCDM methods [8]. Throughout the years, many researchers have modified the classical AHP method employing the mentioned extensions such as hesitant fuzzy (HF) AHP, SF AHP, and neutrosophic AHP. Likewise, VIKOR method is one of these MCDM methods modified by these extensions such as intuitionistic HF VIKOR [9], HF VIKOR [10], neutrosophic VIKOR, picture fuzzy VIKOR, and SF VIKOR methods.

C-IFS have recently introduced as an extension of IFS by [1]. A C-IFS is defined by all possible values of membership and non-membership degrees with a radius r. The originality of this

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study is the introduction of the Circular Intuitionistic Fuzzy AHP & VIKOR methodology.

The remaining of the paper is structured as follows: Section 2 gives an up-to-date literature review on fuzzy AHP and VIKOR methods. Section 3 includes the preliminaries of intuitionistic and circular intuitionistic fuzzy sets and also presents IF AHP and IF VIKOR methods. Section 4 includes the proposed C-IF AHP&VIKOR methodology. Section 5 gives the implementation of the proposed methodology together with analyses of sensitivity and comparison. Section 6 states managerial implications while Section 7 concludes the paper and presents suggestions for further studies.

2 Literature review

In this section, an extensive literature review on fuzzy versions of AHP and VIKOR methods based on a variety of fuzzy set extensions are presented. AHP is a systematic and structured approach used as a weighted factor-scoring model. Considering its simplicity, easiness to apply and interpret the solutions, the method has applied to various decision-making processes. Besides, decision makers prefer to use linguistic terms rather than exact numerical values. Fuzzy logic provides a mathematical tool used to represent reality better compared to the binary (crisp) sets [11].

Among several type-1 (ordinary) fuzzy AHP approaches, van Laarhoven and Pedrycz [12] extended Saaty's crisp AHP method by utilizing fuzzy priority theory considering triangular fuzzy numbers. To incorporate exact ratios for the alternatives, Buckley [13] proposed fuzzy AHP method for computing criteria weights and alternative scores, and proposed geometric mean method by means of trapezoidal fuzzy numbers. Since it was introduced in 1985, many of the researchers have applied the method considering its advantages such as simplicity and easiness to apply, and ability to provide efficient solutions. Chang [14] also developed a fuzzy AHP method known as extent analysis used to derive the fuzzy synthetic values obtained from pairwise comparisons. However, Chang's fuzzy AHP received a lot of criticism because it often produces the value of zero for criteria weights.

Some of the recent studies using fuzzy AHP method modified by fuzzy set extensions are presented as follows: Kahraman et al. [15] developed a new fuzzy AHP approach using type-2 fuzzy (T2F) sets and introduced two ranking methods called DTriT and DTraT for interval-valued (IV) T2F numbers. Ayodele et al. [16] implemented GIS based fuzzy AHP method with IVT2F numbers for wind farm site selection problem in Nigeria. Öztayşi et al. [17] proposed fuzzy AHP using HF sets and implemented it to a supplier evaluation problem. Senvar [18] used HF AHP method for evaluating performances of service departments. Sadiq and Tesfamariam [19] proposed a six level hierarchical decision making model for evaluating drilling fluids and brought solutions using IF AHP method. Xu and Liao [20] extended AHP employing IF sets, and implemented it for a global supplier evaluation problem. Rouyendegh [21] also implemented AHP method and integrated it with IF TOPSIS. Bolturk and Kahraman [22] applied an IV neutrosophic AHP method with cosine similarity index. Yazdani et al. [23] proposed IV neutrosophic framework for sustainable supplier selection problem by considering subjective judgments and uncertainty. Shete et al. [24] evaluated innovation of sustainable supply chain employing Pythagorean (PyF) AHP by taking into account social, environmental and economic aspects. Ayyildiz and Taskin Gumus [25] used PyF AHP method for prioritizing risk assessment methodologies for hazardous material transportation problem. The authors employed modified Delphi method to consolidate experts' judgments on factors. Kutlu Gündoğdu and Kahraman [26] developed a SF AHP method for industrial robot evaluation problem. Dogan [27] handled a technology selection problem for process mining operation by means of SF AHP. Otay and Kahraman [28] integrated one of the other extensions entitled as Z-fuzzy numbers, into fuzzy AHP method for solar energy PV plant selection problem. Shishavan et al. [29] extended traditional fuzzy AHP method by means of q-Rung Orthopair Fuzzy Sets (q-ROFS) for logistics center location problem. Kutlu Gündoğdu et al. [30] proposed a PF AHP method for evaluating public transport service quality. The authors integrated the multi expert PF method with linear assignment model.

On the other hand, VIKOR method aims to provide compromise solutions to multicriteria decision making problems evaluating conflicting as well as noncommensurable criteria ([31],[32]). Compromise solution incorporates with minimum regret and maximum utility. VIKOR method was also modified using the fuzzy sets to obtain fuzzy compromise solutions [33].

As new fuzzy sets extensions have been introduced, the academicians have modified VIKOR method based on these extensions in different MCDM problems. Below, some recent studies conducting fuzzy VIKOR with a various fuzzy set extensions are presented. Ghorabaee et al. [34] extended fuzzy VIKOR using IVT2F sets for multi-expert multi-criteria project evaluation and selection problem. Wang [35] developed a novel T2F VIKOR method with IVT2 trapezoidal fuzzy numbers and developed a signed area function and a new ranking method. Liao and Xu [36] introduced HF VIKOR method considering hesitant preference information, and used it for assessing service quality of domestic airlines. Dong et al. [37] used linguistic HF VIKOR for an intelligent transportation decision problem. Devi [38] utilized IF VIKOR to determine the best industrial robot for material handling processes. Chatterjee et al. [39] applied extended IF VIKOR method to evaluate strategic decisions on information systems. Hu et al. [40] developed an interval neutrosophic VIKOR to deal with doctor evaluation and selection problem on mobile healthcare. Abdel-Basset et al. [41] preferred to integrate neutrosophic sets into fuzzy VIKOR method to analyze e-government websites. Chen [42] proposed PyF VIKOR based on Minkowski distance for internet stock and R&D project selection problems. In the proposed model, the authors also considered several remoteness indices. Rani et al. [43] suggested adapting entropy and divergence into PyF VIKOR. As an application, the authors concentrated on renewable energy technology evaluation problem in India. Kutlu Gündoğdu et al. [44] introduced a SF VIKOR method and applied it to a waste disposal site evaluation problem. Akram et al. [45] introduced complex SF VIKOR method and developed a numerous weighted arithmetic and geometric aggregation operators. Krishankumar et al. [46] employed fuzzy VIKOR method using IV q-ROFS relying on evidence-based Bayes approximation for green supplier selection. Cheng et al. [47] developed fuzzy VIKOR with q-ROFS for risk assessment and management problem, and proposed q-ROF weighted averaging operator as well. Wang et al. [48] introduced PF VIKOR for construction project risk evaluation problem. Yu [49] also employed multi-expert PF normalized projection based VIKOR to a case study on software projects evaluation.

3 Preliminaries

3.1 Intuitionistic fuzzy sets (IFSs)

IFSs are described by both membership ($\mu_{\tilde{A}}(x)$) and non-membership ($\vartheta_{\tilde{A}}(x)$) values for any x in X where sum of membership and non-membership values is equal to or less than "1" ([5],[50],[51]).

Below, some basic definitions for IFS are presented:

Definition 1. An IFS $\tilde{A}$ in X ($X \neq \emptyset$) is an object described in Eq. (1).

$$\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), \vartheta_{\tilde{A}}(x)\rangle; x \in X\} \quad (1)$$

Where $\mu_{\tilde{A}}(x)$ and $\vartheta_{\tilde{A}}(x): X \to [0,1]$, and $0 \leq \mu_{\tilde{A}}(x) + \vartheta_{\tilde{A}}(x) \leq 1$, for every $x \in X$.

Definition 2. An intuitionistic fuzzy number (IFN) $\tilde{A}$ is described as follows [51]:

An IF subset of the real line

Normal, i.e., there is any $x_0 \in \mathbb{R}$ such that

$$\mu_{\tilde{A}}(x_0) = 1 \; (\vartheta_{\tilde{A}}(x_0) = 0) \quad (2)$$

A convex set for $\mu_{\tilde{A}}(x)$

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)) \quad (3)$$

$$\forall x_1, x_2 \in \mathbb{R}, \lambda \in [0,1]$$

A concave set for $\vartheta_{\tilde{A}}(x)$

$$\vartheta_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \leq max(\vartheta_{\tilde{A}}(x_1), \vartheta_{\tilde{A}}(x_2)) \quad (4)$$

$$\forall x_1, x_2 \in \mathbb{R}, \lambda \in [0,1]$$

Definition 3. The α-cut of an IFS of $\tilde{A}$ is stated in Eq.(5).

$$\tilde{A}_{\propto} = \{x \in X \,|\, \mu_{\tilde{A}}(x) \geq \propto, \vartheta_{\tilde{A}}(x) \leq 1 - \propto\} \quad (5)$$

Definition 4. Let $\tilde{A} = \{(x, \mu_{\tilde{A}}(x), \vartheta_{\tilde{A}}(x) \,|\, x \in X)\}$ and $\tilde{B} = \{(x, \mu_{\tilde{B}}(x), \vartheta_{\tilde{B}}(x) \,|\, x \in X)\}$ be two IFNs. Some arithmetic operations are given below [52]:

Addition:

$$\tilde{A} \oplus \tilde{B} = \left\{\begin{pmatrix} x, \mu_{\tilde{A}}(x) + \mu_{\tilde{B}}(x) - \mu_{\tilde{A}}(x).\mu_{\tilde{B}}(x), \\ \vartheta_{\tilde{A}}(x).\vartheta_{\tilde{B}}(x) \,|\, x \in X \end{pmatrix}\right\} \quad (6)$$

Multiplication:

$$\tilde{A} \otimes \tilde{B} = \left\{\begin{pmatrix} x, \mu_{\tilde{A}}(x).\mu_{\tilde{B}}(x), \\ \vartheta_{\tilde{A}}(x) + \vartheta_{\tilde{B}}(x) - \vartheta_{\tilde{A}}(x).\vartheta_{\tilde{B}}(x) \,|\, x \in X \end{pmatrix}\right\} \quad (7)$$

Subtraction:

$$\tilde{A} \ominus \tilde{B} = \left\{\left(x, \frac{\mu_{\tilde{A}}(x) - \mu_{\tilde{B}}(x)}{1 - \mu_{\tilde{B}}(x)}, \frac{\vartheta_{\tilde{A}}(x)}{\vartheta_{\tilde{B}}(x)}\right) \,|\, x \in X\right\} \quad (8)$$

satisfying the following conditions:

$$\tilde{A} \geq \tilde{B}, \mu_B(x) \neq 1, \vartheta_{\tilde{B}}(x) \neq 0, and$$
$$\mu_{\tilde{A}}(x).\vartheta_{\tilde{B}}(x) - \mu_{\tilde{B}}(x).\vartheta_{\tilde{A}}(x) \leq \vartheta_{\tilde{B}}(x) - \vartheta_{\tilde{A}}(x)$$

Division:

$$\tilde{A} \oslash \tilde{B} = \left\{\left(x, \frac{\mu_{\tilde{A}}(x)}{\mu_{\tilde{B}}(x)}, \frac{\vartheta_{\tilde{A}}(x) - \vartheta_{\tilde{B}}(x)}{1 - \vartheta_{\tilde{B}}(x)}\right) \,|\, x \in X\right\} \quad (9)$$

satisfying the following conditions:

$$\tilde{A} \leq \tilde{B}, \mu_{\tilde{B}}(x) \neq 0, \vartheta_{\tilde{B}}(x) \neq 1, and$$
$$\mu_{\tilde{A}}(x).\vartheta_{\tilde{B}}(x) - \mu_{\tilde{B}}(x).\vartheta_{\tilde{A}}(x) \geq \mu_{\tilde{A}}(x) - \mu_{\tilde{B}}(x)$$

Multiplication by a scaler:

$$\lambda.\tilde{A} = \left\{\left(x, 1 - (1 - \mu_{\tilde{A}}(x))^{\lambda}, (\vartheta_{\tilde{A}}(x))^{\lambda}\right) \,|\, x \epsilon X\right\} \quad (10)$$

Power operation:

$$\tilde{A}^{\lambda} = \left\{\left(x, (\mu_{\tilde{A}}(x))^{\lambda}, 1 - (1 - \vartheta_{\tilde{A}}(x))^{\lambda}\right) \,|\, x \epsilon X\right\} \quad (11)$$

3.2 Circular intuitionistic fuzzy sets (C-IFSs)

A C-IFS $\tilde{C}$ described by a circle indicating vagueness and impreciseness in membership ($\mu_{\tilde{C}}(x)$) and non-membership ($\vartheta_{\tilde{C}}(x)$) degrees, is represented in Eq.(12) [1]:

Definition 5: A C-IFS $\tilde{C}_r$ in E is an object having the form for a fixed universe E:

$$\tilde{C}_r = \{\langle x, \mu_{\tilde{C}}(x), \vartheta_{\tilde{C}}(x);\, r\rangle \,|\, x \in E\}$$

where $0 \leq \mu_{\tilde{C}}(x) + \vartheta_{\tilde{C}}(x) \leq 1$, (12)

$r \in [0,1]$, $\mu_{\tilde{C}}: E \to [0,1]$ and $\vartheta_{\tilde{C}}: E \to [0,1]$.

In Eq. (12), "r" defines a radius of the circle around each element x, $x \in E$ to the set $C \subseteq E$.

The degree of indeterminacy can be obtained as in Eq. (13):

$$\pi_{\tilde{C}}(x) = 1 - \mu_{\tilde{C}}(x) - \vartheta_{\tilde{C}}(x) \quad (13)$$

Definition 6: Let $\tilde{\alpha}_i = (\mu_{\tilde{\alpha}_i}, \vartheta_{\tilde{\alpha}_i})$ $(i = 1,2,\dots,n)$ be a set of IF pairs. Then, intuitionistic fuzzy pairs are aggregated using Intuitionistic Fuzzy Weighted Geometric (IFWG) operator as seen in Eq.(14), and the values of $\mu_{agg} = \prod_{j=1}^{m} \mu_{\tilde{\alpha}_j}{}^{w_j}$ and $\vartheta_{agg} = \prod_{j=1}^{m} \vartheta_{\tilde{\alpha}_j}^{w_j}$ for the aggregated fuzzy numbers are computed. Euclidean distances between judgments of each expert and the aggregated intuitionistic fuzzy sets are obtained by means of Eq. (15). The maximum of these distances gives the value of the radius for each criterion.

$$IFWG(\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_n) = \left(\prod_{j=1}^{m} \mu_{\tilde{\alpha}_j}{}^{w_j}, \prod_{j=1}^{m} \vartheta_{\tilde{\alpha}_j}^{w_j}\right) \quad (14)$$

where $w = (w_1, \dots, w_n)^T$ is the weight vector of $\tilde{\alpha}_i$ $(i = 1,2,\dots,n)$ with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$ [53].

$$r_i = \max_{1 \leq j \leq k_i} \sqrt{\left(\prod_{j=1}^{m} \mu_{\tilde{\alpha}_j}{}^{w_j} - \mu_{\tilde{\alpha}_i}\right)^2 + \left(\prod_{j=1}^{m} \vartheta_{\tilde{\alpha}_j}^{w_j} - \vartheta_{\tilde{\alpha}_i}\right)^2} \quad (15)$$

where k_i denotes decision makers.

Basic geometric interpretations of several forms of circles in C-IFSs are illustrated in Figure 1 ([1],[54]).

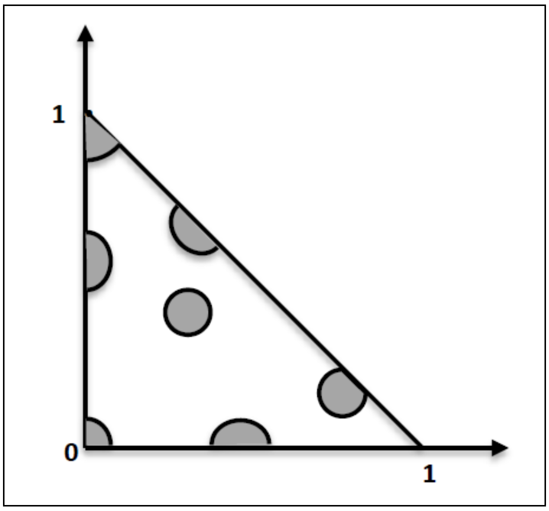

Figure 1. Geometrical representation of several C-IFS.

Definition 7: Let $\tilde{Q}_a = \langle \mu_{\tilde{Q}_a}(x), \vartheta_{\tilde{Q}_a}(x); r_a \rangle$ and $\tilde{Q}_b = \langle \mu_{Q_b}(x), \vartheta_{Q_b}(x); r_b \rangle$ be two circular intuitionistic fuzzy numbers (C-IFNs). For these C-IFNs, some of the arithmetic operations including union, intersection, addition and multiplication operations are presented in Eqs. (16)-(25), ([1], [54]):

Intersection:

$$\tilde{Q}_a \cap_{min} \tilde{Q}_b = \left\{ \begin{matrix} x, min\left(\mu_{\tilde{Q}_a}(x), \mu_{\tilde{Q}_b}(x)\right), \\ \langle\, max\left(\vartheta_{\tilde{Q}_a}(x), \vartheta_{\tilde{Q}_b}(x)\right); \rangle \mid x \epsilon X \\ min(r_a, r_b) \end{matrix} \right\} \quad (16)$$

$$\tilde{Q}_a \cap_{max} \tilde{Q}_b = \left\{ \begin{matrix} x, min\left(\mu_{\tilde{Q}_a}(x), \mu_{\tilde{Q}_b}(x)\right), \\ \langle\, max\left(\vartheta_{\tilde{Q}_a}(x), \vartheta_{\tilde{Q}_b}(x)\right); \rangle \mid x \epsilon X \\ max(r_a, r_b) \end{matrix} \right\} \quad (17)$$

Union:

$$\tilde{Q}_a \cup_{min} \tilde{Q}_b = \left\{ \begin{matrix} x, max\left(\mu_{\tilde{Q}_a}(x), \mu_{\tilde{Q}_b}(x)\right), \\ \langle\, min\left(\vartheta_{\tilde{Q}_a}(x), \vartheta_{\tilde{Q}_b}(x)\right); \rangle \mid x \epsilon X \\ min(r_a, r_b) \end{matrix} \right\} \quad (18)$$

$$\tilde{Q}_a \cup_{max} \tilde{Q}_b = \left\{ \begin{matrix} x, max\left(\mu_{\tilde{Q}_a}(x), \mu_{\tilde{Q}_b}(x)\right), \\ \langle\, min\left(\vartheta_{\tilde{Q}_a}(x), \vartheta_{\tilde{Q}_b}(x)\right); \rangle \mid x \epsilon X \\ max(r_a, r_b) \end{matrix} \right\} \quad (19)$$

Addition:

$$\tilde{Q}_a \oplus_{min} \tilde{Q}_b = \left\{ \begin{matrix} x, \mu_{\tilde{Q}_a}(x) + \mu_{\tilde{Q}_b}(x) - \mu_{\tilde{Q}_a}(x) * \mu_{\tilde{Q}_b}(x), \\ \langle\, \vartheta_{\tilde{Q}_a}(x) * \vartheta_{\tilde{Q}_b}(x); \rangle \mid x \epsilon X \\ min(r_a, r_b) \end{matrix} \right\} \quad (20)$$

$$\tilde{Q}_a \oplus_{max} \tilde{Q}_b = \left\{ \begin{matrix} x, \mu_{\tilde{Q}_a}(x) + \mu_{\tilde{Q}_b}(x) - \mu_{\tilde{Q}_a}(x) * \mu_{\tilde{Q}_b}(x), \\ \langle\, \vartheta_{\tilde{Q}_a}(x) * \vartheta_{\tilde{Q}_b}(x); \rangle \mid x \epsilon X \\ max(r_a, r_b) \end{matrix} \right\} \quad (21)$$

Multiplication:

$$\tilde{X}_a \otimes_{min} \tilde{X}_b = \left\{ \begin{matrix} x, \mu_{\tilde{Q}_a}(x) * \mu_{\tilde{Q}_b}(x), \\ \langle \vartheta_{\tilde{Q}_a}(x) + \vartheta_{\tilde{Q}_b}(x) - \vartheta_{\tilde{Q}_a}(x) * \vartheta_{\tilde{Q}_b}(x); \rangle \mid x \epsilon X \\ min(r_1, r_2) \end{matrix} \right\} \quad (22)$$

$$\tilde{X}_a \otimes_{max} \tilde{X}_b = \left\{ \begin{matrix} x, \mu_{\tilde{Q}_a}(x) * \mu_{\tilde{Q}_b}(x), \\ \langle \vartheta_{\tilde{Q}_a}(x) + \vartheta_{\tilde{Q}_b}(x) - \vartheta_{\tilde{Q}_a}(x) * \vartheta_{\tilde{Q}_b}(x); \rangle \mid x \epsilon X \\ max(r_1, r_2) \end{matrix} \right\} \quad (23)$$

Multiplication by a scaler:

$$\lambda . \tilde{Q}_a = \left\{ \langle x, 1 - (1 - \mu_{\tilde{Q}_a}(x))^{\lambda}, \left(\vartheta_{\tilde{Q}_a}(x)\right)^{\lambda}; r_a \rangle \mid x \epsilon X \right\} \quad (24)$$

Power operation:

$$\tilde{Q}_a^{\lambda} = \left\{ \langle x, \left(\mu_{\tilde{Q}_a}(x)\right)^{\lambda}, 1 - (1 - \vartheta_{\tilde{Q}_a}(x))^{\lambda}; r_a \rangle \mid x \epsilon X \right\} \quad (25)$$

3.3 Intuitionistic fuzzy AHP

In this sub-section, a fuzzy AHP method based on single-valued IFSs is presented [55]. As similar to other extensions of fuzzy AHP, initially IF pairwise comparison matrices ($\tilde{X}^k$) of criteria as in Eq. (26) are obtained; and then, IF pairwise comparison matrices of alternatives regarding to criteria are collected from l decision makers.

$$\tilde{X}^k = \begin{array}{c} \\ C_1 \\ C_2 \\ \vdots \\ C_i \\ \vdots \\ C_n \end{array} \begin{array}{c} \begin{matrix} C_1 & C_2 & \cdots & C_j & \cdots & C_n \end{matrix} \\ \begin{bmatrix} 1 & \tilde{x}_{12}^k & \cdots & \tilde{x}_{1j}^k & \cdots & \tilde{x}_{1n}^k \\ \tilde{x}_{21}^k & 1 & \cdots & \tilde{x}_{2j}^k & \cdots & \tilde{x}_{2n}^k \\ \vdots & \vdots & \ddots & \cdots & \ddots & \vdots \\ \tilde{x}_{i1}^k & \tilde{x}_{i2}^k & \cdots & \tilde{x}_{ij}^k & \cdots & \tilde{x}_{jn}^k \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ \tilde{x}_{n1}^k & \tilde{x}_{n2}^k & \cdots & \tilde{x}_{nj}^k & \cdots & 1 \end{bmatrix} \end{array} \quad (26)$$

Once the consistencies of the pairwise comparison matrices are checked, the aggregated pairwise comparison matrix ($\tilde{X}_{agg}$) is obtained using Intuitionistic Fuzzy Weighted Averaging (IFWA) operator ([56]) given in the following eqution.

$$\begin{aligned} \tilde{X}_{agg} &= IFWA_{\lambda}(\tilde{X}^1, \tilde{X}^2, \dots, \tilde{X}^l) \\ \tilde{X}_{agg} &= \lambda_1 \tilde{X}^1 \oplus \lambda_2 \tilde{X}^2 \oplus \dots \oplus \lambda_k \tilde{X}^k \oplus \dots \oplus \lambda_l \tilde{X}^l \end{aligned} \quad (27)$$

where the weight of k[th] decision maker is pointed out by λ_k .

Finally, the entropy weights of criteria ($\bar{\bar{w}}_i$) are calculated using Eqs.(28)-(29) ([55],[57]).

$$\bar{\bar{w}}_i = -\frac{1}{n\, ln2}[\mu_i \ln \mu_i + \vartheta_i \ln \vartheta_i - (1 - \pi_i)\ln(1 - \pi_i) - \pi_i ln2] \quad (28)$$

$$w_i = \frac{1 - \bar{\bar{w}}_i}{n - \sum_{j=1}^{n} \bar{\bar{w}}_i} \quad (29)$$

3.4 Intuitionistic fuzzy VIKOR

An MCDM model evaluates a finite set of alternatives A_i (i=1,2,..,m) based on a criteria set C_j (j=1,2,..,n). Assuming that a decision maker DM_k (k=1,2,...,l) has weights of λ_k where $\sum \lambda_k = 1$. Let $\tilde{w}_j^k = (\mu_j^k, \vartheta_j^k)$ be the weight of criterion C_j for the kth decision maker. Using IFWA operator as in Eq.(30), the aggregated IF weights of criteria are computed and normalized.

$$IFWA_w(\tilde{w}_j^1,\dots,\tilde{w}_j^l) = \left(1-\prod_{k=1}^{l}\left(1-\mu_j^k\right)^{\lambda_k}, \prod_{k=1}^{l}\left(\vartheta_j^k\right)^{\lambda_k}\right) \quad (30)$$

where $\tilde{w}_j = (\mu_j, \vartheta_j)$ (j=1,2,…,n).

An IF decision matrix is obtained based on each DM's judgments ($\tilde{x}_{ij}^k = (\mu_{ij}^k, \vartheta_{ij}^k)$). Then, IFWA operator (Eq. (31)) is used to aggregate the decision matrices including IF ratings of alternatives regarding to criteria.

$$IFWA(\tilde{x}_{ij}^1, \tilde{x}_{ij}^2, \dots, \tilde{x}_{ij}^l) = \left(1-\prod_{k=1}^{l}\left(1-\mu_{ij}^k\right)^{\lambda_k}, \prod_{k=1}^{l}\left(\vartheta_{ij}^k\right)^{\lambda_k}\right) \quad (31)$$

Afterwards, the IF best ($\tilde{x}_j^*$) and IF worst ($\tilde{x}_j^-$) values are obtained using Eqs. (32)-(33).

$$PIS = \begin{cases} \tilde{x}_j^* = \max_i \tilde{x}_{ij} & for\ benefit\ criteria \\ \tilde{x}_j^* = \min_i \tilde{x}_{ij} & for\ cost\ criteria \end{cases} \quad (32)$$

j=1,2,..,n

$$NIS = \begin{cases} \tilde{x}_j^- = \min_i \tilde{x}_{ij} & for\ benefit\ criteria \\ \tilde{x}_j^- = \max_i \tilde{x}_{ij} & for\ cost\ criteria \end{cases} \quad (33)$$

j=1,2,..,n

An intuitionistic fuzzy maximum level of group utility ($\tilde{S}_i$) and minimum individual level of regret of the opponent ($\tilde{R}_i$) are calculated employing Eqs.(34)-(36).

$$\tilde{S}_i = \sum \tilde{w}_i \frac{D(\tilde{x}_j^*, \tilde{x}_{ij})}{D(\tilde{x}_j^*, \tilde{x}_j^-)}, \tilde{R}_i = \max_j \left(\tilde{w}_i \frac{D(\tilde{x}_j^*, \tilde{x}_{ij})}{D(\tilde{x}_j^*, \tilde{x}_j^-)}\right) \text{ i=1,2,…,m} \quad (34)$$

$$D(\tilde{x}_j^*, \tilde{x}_{ij}) = \sqrt{\frac{1}{2}\left(\left(\mu_j^* - \mu_{ij}\right)^2 + (\vartheta_j^* - \vartheta_{ij})^2 + (\pi_j^* - \pi_{ij})^2\right)} \quad (35)$$

$$D(\tilde{x}_j^*, \tilde{x}_j^-) = \sqrt{\frac{1}{2}\left((\mu_j^* - \mu_j^-)^2 + (\vartheta_j^* - \vartheta_j^-)^2 + (\pi_j^* - \pi_j^-)^2\right)} \quad (36)$$

Then, $\tilde{Q}_i$ index which is a function of group utility and at the same time individual regret, is computed using Eq.(37). In the equation, v is the weight for the maximum level of group utility.

$$\tilde{Q}_i = v\frac{(\tilde{S}_i - \tilde{S}^*)}{(\tilde{S}^- - \tilde{S}^*)} + (1-v)\frac{(\tilde{R}_i - \tilde{R}^*)}{(\tilde{R}^- - \tilde{R}^*)} \quad \text{i=1,2,…,m} \quad (37)$$

where $\tilde{S}^* = \min_i \tilde{S}_i, \tilde{S}^- = \max_i \tilde{S}_i, \tilde{R}^* = \min_i \tilde{R}_i$, and $\tilde{R}^- = \max_i \tilde{R}_i$.

Based on the values of $\tilde{S}_i$, $\tilde{R}_i$ and $\tilde{Q}_i$, the alternatives are sorted in increasing order. The smallest values of the defuzzified $\tilde{S}_i$, $\tilde{R}_i$ and $\tilde{Q}_i$ indicate the best alternative.

In VIKOR method, a compromise solution satisfying the following conditions is obtained [58].

C1 Acceptable Advantage: $Q(A'') - Q(A') \geq (1/(m-1))$ where A″ is the second ranked alternative with the threshold value of $(1/(m-1))$; A′ is the first ranked alternative, and m is the number of alternatives.

C2 Acceptable Stability: The alternative A' has to be the best alternative with respect to the values of S or/and R.

If one of these conditions is not met, then a set of compromise solutions is proposed, consisting of

- A' and A'' if only condition C2 is not fulfilled, or
- A', A'',…, $A^{(M)}$ if condition C1 is not met; $A^{(M)}$ is defined by Q($A^{(M)}$)-Q(A')< $(1/(m-1))$ for maximum M.

4 Proposed circular ıntuitionistic fuzzy AHP-VIKOR methodology

The proposed C-IF AHP & C-IF VIKOR methodology is presented below and illustrated in Figure 2.

Step 1. Describe multi-criteria fuzzy decision making problem by clarifying a finite set of criteria (C_j, j = 1,2,..,n), sub-criteria and alternatives ($A_i, i = 1, 2, \dots, m$).

Phase 1: Prioritization of criteria (C-IF AHP)

Step 2. Collect pairwise comparison matrices of criteria.

In these matrices, the experts are demanded to fill out the pairwise comparisons employing linguistic terms with IF numbers as listed in Table 1. Herein, exactly equal is represented by the IF number (0.50, 0.50). In the proposed methodology, decision makers are also allowed to assign intermediate values if there is hesitation in between consecutive linguistic terms such as Low (L) and Medium Low (ML).

Table 1. Linguistic scale with IF numbers.

Linguistic Terms	(μ,v)	SI
Absolutely Low (AL)	(0.05, 0.85)	0.11
Very Low (VL)	(0.15, 0.75)	0.14
Low (L)	(0.25, 0.65)	0.20
Medium Low (ML)	(0.35, 0.55)	0.33
Almost Equal (AE)	(0.45, 0.45)	1.20
Medium High (MH)	(0.55, 0.35)	3.0
High (H)	(0.65, 0.25)	5.0
Very High (VH)	(0.75, 0.15)	7.0
Absolutely High (AH)	(0.85, 0.05)	9.0

Step 3. Perform consistency analysis using Saaty's approach. To convert the IF values (Table 1) into their equivalent crisp values which are right after called as Score Indices (SI), Eq. (38) is employed. The calculated SI values of the linguistic terms are presented in Table 1.

$$SI = \begin{cases} 1 + 10\left|\left(\mu(x) * \vartheta(x) - \mu(x) * \vartheta(x) * \pi(x)\right)\right| \\ \text{for AE, MH, H, VH, and AH,} \\ \dfrac{1}{\left(1 + 10\left|\left(\mu(x) * \vartheta(x) - \mu(x) * \vartheta(x) * \pi(x)\right)\right|\right)} \\ \text{for ML, L, VL and AL} \end{cases} \quad (38)$$

Step 4. Aggregate the evaluations of the experts in pairwise comparison matrices with regard to the weights of the experts as in Eq. (14), and compute radiuses "r" of criteria through taking the maximum value of the Euclidean distances from each expert's evaluation to the aggregated value of criteria as in Eq. (15).

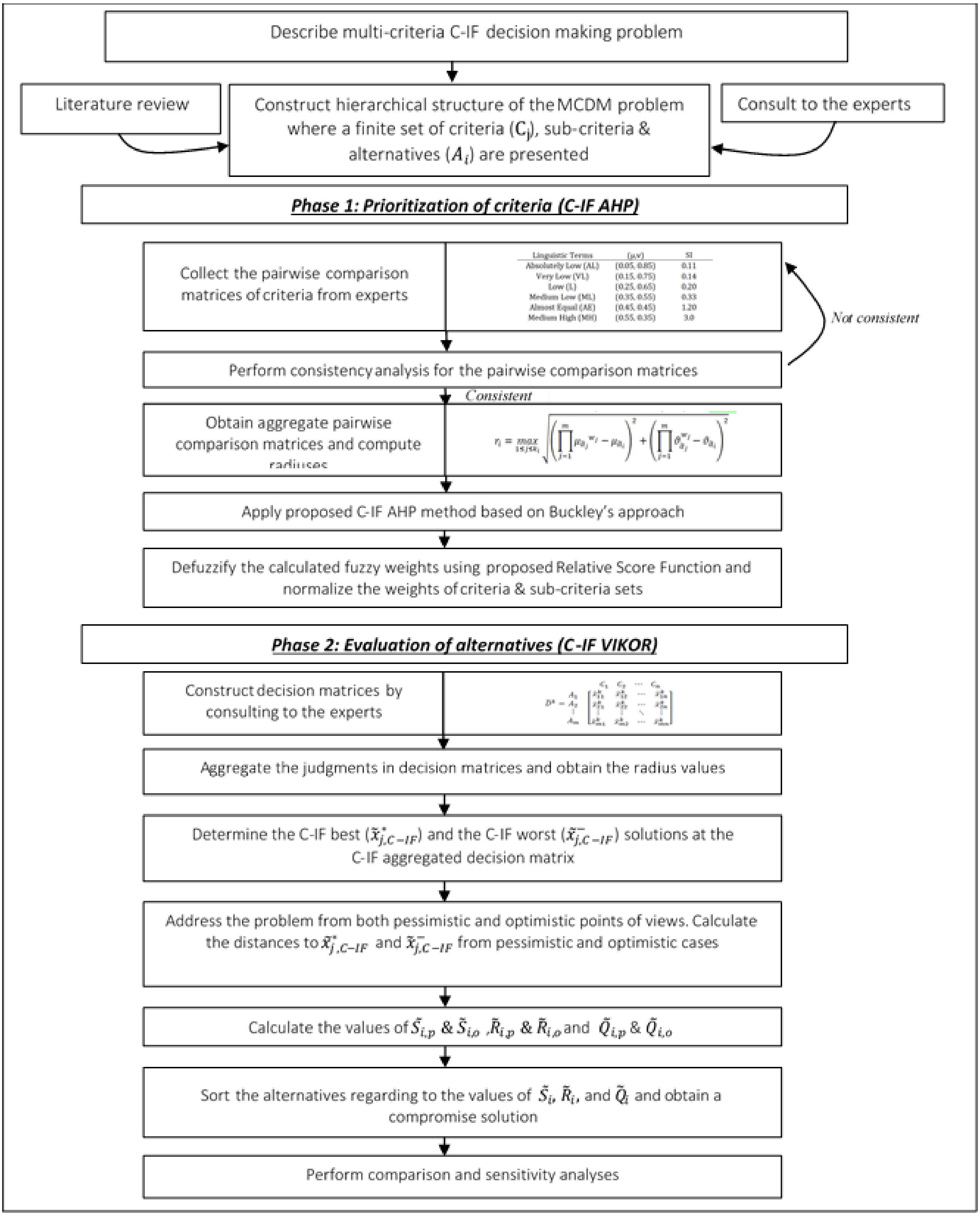

Figure 2. Proposed integrated fuzzy methodology.

Step 5. Compute the geometric mean of judgments employing multiplication of n IF judgments and power operation given in Eqs. (23)-(25), respectively.

Step 6. Defuzzify the calculated fuzzy weights of criteria using Relative Score Function (RSF) based on vector normalization as seen in Eq.(39) [54]. In the equation, τ is defined as a small number such as 0.01.

$$RSF_j = \frac{(1-v_j)(1+\mu_j)+\mu_j}{3} \times \left(\frac{\frac{1}{r_j}}{\sqrt{\frac{1}{r_1^2}+\frac{1}{r_2^2}+\frac{1}{r_3^2}}} \right)^{\tau} \tag{39}$$

Step 7. Normalize the defuzzified weights of criteria by dividing each value to the sum of the weights.

Similar procedure is implemented to calculate the weights of sub-criteria.

Phase 2: Evaluation of alternatives (C-IF VIKOR)

Step 8. Construct decision matrices after meetings with several experts by using the linguistic terms presented in Table 1.

$$\tilde{D}^k = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_n \end{array} \\ \begin{bmatrix} \tilde{x}_{11}^k & \tilde{x}_{12}^k & \cdots & \tilde{x}_{1n}^k \\ \tilde{x}_{21}^k & \tilde{x}_{22}^k & \cdots & \tilde{x}_{2n}^k \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{x}_{m1}^k & \tilde{x}_{m2}^k & \cdots & \tilde{x}_{mn}^k \end{bmatrix} \end{array} \tag{40}$$

$i = 1,2,\dots,m;\ \mathrm{j} = 1,2,..,\mathrm{n},;\ \mathrm{k} = 1,2,..\mathrm{l}$

Step 9. Similar to Step 4, aggregate the judgments in decision matrices using Eq.(14) and obtain the radius values using Eq.(15).

Step 10. Determine the C-IF best ($\tilde{x}_{j,C-IF}^{*}$) and the C-IF worst ($\tilde{x}_{j,C-IF}^{-}$) solutions at the C-IF aggregated decision matrix in Step 9. RSF function given by Eq.(39) is used as a guide to compare the C-IF values.

$$\tilde{x}_{j,C-IF}^{*} = Max_i \tilde{x}_{ij} \ , \tilde{x}_{j,C-IF}^{-} = Min_i \tilde{x}_{ij} \quad \mathrm{j} = 1,2,..,\mathrm{n} \tag{41}$$

Step 11. Address the problem from both pessimistic and optimistic points of views. Distances to $\tilde{x}_{j,C-IF}^{*}$ and $\tilde{x}_{j,C-IF}^{-}$ are derived using Eqs. (42)-(43) for pessimistic case and Eqs. (44)-(45) for optimistic case. Thereafter, to simplify the notation we use $\tilde{x}_j^*$ and $\tilde{x}_j^-$ in spite of $\tilde{x}_{j,C-IF}^{*}$ and $\tilde{x}_{j,C-IF}^{-}$, respectively. In the following equations, p and o indicate pessimistic and optimistic cases, respectively.

$$D_p(\tilde{x}_{ij}, \tilde{x}_j^*) = \sqrt{\frac{1}{2}\left(\begin{array}{c} \left((\mu_{\tilde{x}_{ij}} - r_{\tilde{x}_{ij}}) - (\mu_{\tilde{x}_j^*} + r_{\tilde{x}_j^*}) \right)^2 \\ + \left((\vartheta_{\tilde{x}_{ij}} + r_{\tilde{x}_{ij}}) - (\vartheta_{\tilde{x}_j^*} - r_{\tilde{x}_j^*}) \right)^2 + \left(\pi_{\tilde{x}_{ij}} - \pi_{\tilde{x}_j^*} \right)^2 \end{array} \right)} \tag{42}$$

$$D_p(\tilde{x}_j^-, \tilde{x}_j^*) = \sqrt{\frac{1}{2}\left(\begin{array}{c} \left((\mu_{\tilde{x}_j^-} - r_{\tilde{x}_j^-}) - (\mu_{\tilde{x}_j^*} + r_{\tilde{x}_j^*}) \right)^2 \\ + \left((\vartheta_{\tilde{x}_j^-} + r_{\tilde{x}_j^-}) - (\vartheta_{\tilde{x}_j^*} - r_{\tilde{x}_j^*}) \right)^2 + \left(\pi_{\tilde{x}_j^-} - \pi_{\tilde{x}_j^*} \right)^2 \end{array} \right)} \tag{43}$$

$$D_o(\tilde{x}_{ij}, \tilde{x}_j^*) = \sqrt{\frac{1}{2}\left(\begin{array}{c} \left((\mu_{\tilde{x}_{ij}} + r_{\tilde{x}_{ij}}) - (\mu_{\tilde{x}_j^*} - r_{\tilde{x}_j^*}) \right)^2 \\ + \left((\vartheta_{\tilde{x}_{ij}} - r_{\tilde{x}_{ij}}) - (\vartheta_{\tilde{x}_j^*} + r_{\tilde{x}_j^*}) \right)^2 + \left(\pi_{\tilde{x}_{ij}} - \pi_{\tilde{x}_j^*} \right)^2 \end{array} \right)} \tag{44}$$

$$D_o(\tilde{x}_j^-, \tilde{x}_j^*) = \sqrt{\frac{1}{2}\left(\begin{array}{c} \left((\mu_{\tilde{x}_j^-} + r_{\tilde{x}_j^-}) - (\mu_{\tilde{x}_j^*} - r_{\tilde{x}_j^*}) \right)^2 \\ + \left((\vartheta_{\tilde{x}_j^-} - r_{\tilde{x}_j^-}) - (\vartheta_{\tilde{x}_j^*} + r_{\tilde{x}_j^*}) \right)^2 + \left(\pi_{\tilde{x}_j^-} - \pi_{\tilde{x}_j^*} \right)^2 \end{array} \right)} \tag{45}$$

Step 12. Calculate the values of $\tilde{S}_{i,p}$ & $\tilde{S}_{i,o}$ and $\tilde{R}_{i,p}$ & $\tilde{R}_{i,o}$ with respect to pessimistic and optimistic view points (Eqs. (46)-(47)). Then, Eq.(48) is implemented to obtain $\tilde{Q}_{i,p}$ & $\tilde{Q}_{i,o}$ values.

$$\tilde{S}_{i,p} = \sum \tilde{w}_i \frac{D_p(\tilde{x}_{ij}, \tilde{x}_j^*)}{D_p(\tilde{x}_j^-, \tilde{x}_j^*)} \ , \ \tilde{S}_{i,o} = \sum \tilde{w}_i \frac{D_o(\tilde{x}_{ij}, \tilde{x}_j^*)}{D_o(\tilde{x}_j^-, \tilde{x}_j^*)} \tag{46}$$

$$\tilde{R}_{i,p} = max_j \left(\tilde{w}_i \frac{D_p(\tilde{x}_{ij}, \tilde{x}_j^*)}{D_p(\tilde{x}_j^-, \tilde{x}_j^*)} \right), \tilde{R}_{i,o} = max_j \left(\tilde{w}_i \frac{D_o(\tilde{x}_{ij}, \tilde{x}_j^*)}{D_o(\tilde{x}_j^-, \tilde{x}_j^*)} \right) \tag{47}$$

$$\tilde{Q}_{i,p} = v \frac{(\tilde{S}_{i,p} - \tilde{S}_p^*)}{(\tilde{S}_p^- - \tilde{S}_p^*)} + (1-v) \frac{(\tilde{R}_{i,p} - \tilde{R}_p^*)}{(\tilde{R}_p^- - \tilde{R}_p^*)} \ ,$$

$$\tilde{Q}_{i,o} = v \frac{(\tilde{S}_{i,o} - \tilde{S}_o^*)}{(\tilde{S}_o^- - \tilde{S}_o^*)} + (1-v) \frac{(\tilde{R}_{i,o} - \tilde{R}_o^*)}{(\tilde{R}_o^- - \tilde{R}_o^*)} \tag{48}$$

where $\tilde{S}_p^* = \min_i \tilde{S}_{i,p}, \tilde{S}_p^- = \max_i \tilde{S}_{i,p}$, $\tilde{R}_p^* = \min_i \tilde{R}_{i,p}$, $\tilde{R}_p^- = \max_i \tilde{R}_{i,p}$,$\tilde{S}_o^* = \min_i \tilde{S}_{i,o}$, $\tilde{S}_o^- = \max_i \tilde{S}_{i,o}$,$\tilde{R}_o^* = \min_i \tilde{R}_{i,o}$, $\tilde{R}_o^- = \max_i \tilde{R}_{i,o}$.

Step 13. Sort the alternatives in ascending order based on the values of $\tilde{S}_i$, $\tilde{R}_i$, and $\tilde{Q}_i$ by means of Eq.(49).

$$\tilde{S}_i = \tilde{S}_{i,p}/(\tilde{S}_{i,o} + \tilde{S}_{i,p}) \ , \tilde{R}_i = \tilde{R}_{i,p}/(\tilde{R}_{i,o} + \tilde{R}_{i,p}) \ ,$$
$$\tilde{Q}_i = \tilde{Q}_{i,p} /(\tilde{Q}_{i,o} + \tilde{Q}_{i,p}) \tag{49}$$

Step 14: Obtain a compromise solution as explained at the end of Section 3.4.

5 Implementation

5.1 Definition of the problem

The proposed integrated multi-criteria C-IF group decision making methodology is implemented to solve a multi-expert supplier evaluation and selection problem of an engineering company. In the study, among a range of supplied components only one of them is considered. Initially, once the alternative suppliers are listed, the primary evaluations are done based on company's environmental concerns. The supplier/s failing to meet environmental concerns with respect to pollution control system and ISO standards, are discarded from the analysis. Then, the remaining options (herein referred to Supplier 1, Supplier 2 and Supplier 3) are evaluated based on three main criteria which are "Cost", "Service", and "Technology & Quality", and nine sub-criteria such as price, flexibility and technological capability. The hierarchical structure is designed with respect to an extensive review of literature and the notes taken during the meetings with the decision makers in the company, as displayed in Figure 3.

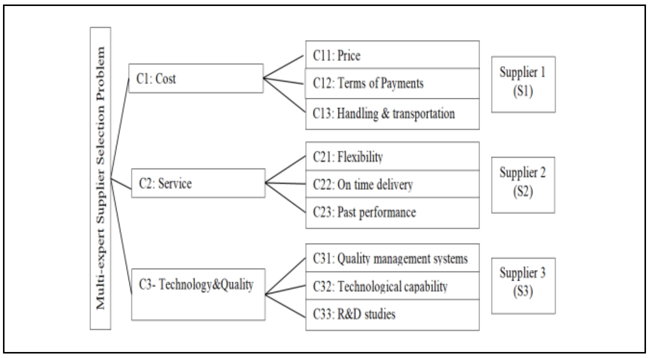

Figure 3. Hierarchical structure of the supplier selection problem.

5.2 Solutions of the integrated C-IF approach

In this sub-section, the criteria and sub-criteria are prioritized utilizing C-IF AHP. The calculated weights are integrated into C-IF VIKOR to evaluate the alternatives. In the proposed methodology, firstly the pairwise comparison matrices are collected from the three experts working in the procurement department of the company. The experts are asked to fill out the matrices using linguistic terms with their corresponding IF numbers in Table 1. Table 2 lists the pairwise comparison judgments of criteria, collected from the experts.

These judgments are aggregated employing Eq. (14) as presented in fuzzy aggregated pairwise comparison matrix in Table 3. In the calculations, the weights of the experts are attained as 0.50, 0.30 and 0.20 considering their expertise and know-how in the field. The radius values in the table are computed utilizing the Euclidean distance based formula given in Eq. (15).

Afterwards, geometric means of the judgments are calculated through Eqs. (23) and (25). The C-IF weights are found as follows:

$\widetilde{w}_{C1} = (0.578, 0.358; 0.112), \widetilde{w}_{C2} = (0.329, 0.585; 0.158)$ and $\widetilde{w}_{C3} = (0.450, 0.476; 0.158)$. These fuzzy weights are defuzzified for the value of τ set to 0.01, as shown below.

$$RSF_{C1} = \frac{(1-0.358)(1+0.578)+0.578}{3} \times \left(\frac{\frac{1}{0.112}}{\sqrt{\frac{1}{0.112^2} + \frac{1}{0.158^2} + \frac{1}{0.158^2}}} \right)^{\tau} = 0.528$$

$$RSF_{C2} = 0.292 \text{ and } RSF_{C3} = 0.400$$

Then, the defuzzified values are normalized by dividing each weight to the sum of the defuzzified weights. The defuzzified & normalized weights of the criteria (C1, C2 and C3) are obtained as 0.433, 0.239 and 0.328, respectively. By following the similar procedure, the pairwise comparison judgments of experts on sub-criteria are collected as given in Table 4.

Table 2. Pairwise comparison matrices of criteria.

Criteria	C1			C2			C3		
	DM1	DM2	DM3	DM1	DM2	DM3	DM1	DM2	DM3
C1	E	E	E	H	VH	H	MH	MH	H
C2	1/H	1/VH	1/H	E	E	E	ML	L	AE
C3	1/MH	1/MH	1/H	1/ML	1/L	1/AE	E	E	E

Table 3. Aggregated C-IF pairwise comparison matrix.

Criteria	C1	C2	C3
C1	(0.500,0.500;0)	(0.679,0.214;0.096)	(0.569, 0.327;0.112)
C2	(0.214,0.679;0.096)	(0.500,0.500;0)	(0.333,0.556;0.158)
C3	(0.327,0.569; 0.112)	(0.556,0.333; 0.158)	(0.500,0.500;0)

Table 4. Pairwise comparison matrices of sub-criteria.

Sub-criteria	C11			C12			C13		
	DM1	DM2	DM3	DM1	DM2	DM3	DM1	DM2	DM3
C11	E	E	E	H	VH	H	MH	H	AE
C12	1/H	1/VH	1/H	E	E	E	ML	ML	L
C13	1/MH	1/H	1/AE	1/ML	1/ML	1/L	E	E	E
Sub-criteria	C21			C22			C23		
	DM1	DM2	DM3	DM1	DM2	DM3	DM1	DM2	DM3
C21	E	E	E	ML	L	E	MH	MH	MH
C22	1/ML	1/L	1/E	E	E	E	H	VH	MH
C23	1/MH	1/MH	1/MH	1/H	1/VH	1/MH	E	E	E
Sub-criteria	C31			C32			C33		
	DM1	DM2	DM3	DM1	DM2	DM3	DM1	DM2	DM3
C31	E	E	E	VL	L	L	ML	ML	ML
C32	1/VL	1/L	1/L	E	E	E	MH	MH	MH
C33	1/ML	1/ML	1/ML	1/MH	1/MH	1/MH	E	E	E

After aggregation and geometric mean operations, the C-IF weights of sub-criteria are obtained as in Table 5. The decision matrices collected from three experts are presented in Table 6. When the proposed procedure is followed, the aggregated decision matrix with C-IF numbers is obtained as in Table 7. From Table 7, the C-IF best $(\tilde{x}^*_{j,C-IF})$ and C-IF worst $(\tilde{x}^-_{j,C-IF})$ solutions are determined by using RSF formulation given in Eq. (39). The results are displayed in Table 8.

As the following step, from Tables 7 and 8, distances to $\tilde{x}^*_j$ and $\tilde{x}^-_j$ are derived using Eqs. (42)-(43) and Eqs. (44)-(45) for pessimistic and optimistic cases, respectively. Using the distances in Table 9, we calculate the values of $\tilde{S}_{i,p}$, $\tilde{S}_{i,o}$, $\tilde{R}_{i,p}$ & $\tilde{R}_{i,o}$, $\tilde{Q}_{i,p}$ and $\tilde{Q}_{i,o}$ employing Eqs. (46-48). The fuzzy values of these parameters and their corresponding defuzzified values are displayed in Table 10.

Table 5. C-IF weights of sub-criteria and their defuzzified & normalized values.

Sub-criteria	C-IF weights	Defuzzified and normalized weights
C11	(0.573, 0.360; 0.158)	0.429
C12	(0.327, 0.589; 0.112)	0.238
C13	(0.456, 0.469; 0.158)	0.333
C21	(0.454, 0.480; 0.174)	0.328
C22	(0.571, 0.366; 0.174)	0.424
C23	(0.342, 0.574; 0.161)	0.248
C31	(0.324, 0.592; 0.074)	0.207
C32	(0.577, 0.360; 0.074)	0.380
C33	(0.458, 0.473; 0.000)	0.413

Table 6. Decision matrices.

DM1	C11	C12	C13	C21	C22	C23	C31	C32	C33
S1	ML	MH	ML	ML	MH	H	MH	H	MH
S2	MH	H	MH	VH	VH	VH	MH	MH	MH
S3	H	VH	H	MH	H	AH	VH	MH	H
DM2	C11	C12	C13	C21	C22	C23	C31	C32	C33
S1	L	ML	L	MH	H	MH	H	H	H
S2	VH	H	MH	H	H	H	H	H	MH
S3	MH	H	H	H	MH	VH	VH	MH	MH
DM3	C11	C12	C13	C21	C22	C23	C31	C32	C33
S1	MH	ML	MH	ML	MH	H	MH	ML	ML
S2	H	H	H	H	VH	AH	VH	H	H
S3	H	MH	MH	VH	H	H	H	H	MH

Table 7. Aggregated C-IF decision matrix.

Alternatives	C11	C12	C13
S1	(0.346,0.528;0.271)	(0.439,0.439;0.142)	(0.346,0.528;0.271)
S2	(0.624,0.254;0.163)	(0.65,0.25;0)	(0.569,0.327;0.112)
S3	(0.618,0.277;0.1)	(0.675,0.207;0.19)	(0.629,0.267;0.114)
Alternatives	C21	C22	C23
S1	(0.401,0.48;0.198)	(0.578,0.316;0.098)	(0.618,0.277;0.1)
S2	(0.698,0.194;0.074)	(0.718,0.175;0.102)	(0.737,0.14;0.145)
S3	(0.615,0.267;0.178)	(0.618,0.277;0.1)	(0.776,0.096;0.199)
Alternatives	C31	C32	C33
S1	(0.578,0.316;0.098)	(0.574,0.293;0.341)	(0.528,0.346;0.271)
S2	(0.615,0.267;0.178)	(0.598,0.296;0.072)	(0.569,0.327;0.112)
S3	(0.729,0.166;0.115)	(0.569,0.327;0.112)	(0.598,0.296;0.072)

Table 8. C-IF best and C-IF worst solutions.

$\tilde{x}^-_j$ / $\tilde{x}^*_j$	C11	C12	C13
$\tilde{x}^-_j$	(0.346,0.528;0.271)	(0.439,0.439;0.142)	(0.346,0.528;0.271)
$\tilde{x}^*_j$	(0.624,0.254;0.163)	(0.675,0.207;0.19)	(0.629,0.267;0.114)
$\tilde{x}^-_j$ / $\tilde{x}^*_j$	C21	C22	C23
$\tilde{x}^-_j$	(0.401,0.48;0.198)	(0.578,0.316;0.098)	(0.618,0.277;0.1)
$\tilde{x}^*_j$	(0.698,0.194;0.074)	(0.618,0.277;0.1)	(0.776,0.096;0.199)
$\tilde{x}^-_j$ / $\tilde{x}^*_j$	C31	C32	C33
$\tilde{x}^-_j$	(0.578,0.316;0.098)	(0.569,0.327;0.112)	(0.528,0.346;0.271)
$\tilde{x}^*_j$	(0.729,0.166;0.115)	(0.598,0.296;0.072)	(0.598,0.296;0.072)

Table 9. Distances based on optimistic and pessimistic cases.

Alternatives	$D_p(\tilde{X}_{ij},\tilde{X}_j^*)$			Alternatives	$D_o(\tilde{X}_{ij},\tilde{X}_j^*)$		
	C11	C12	C13		C11	C12	C13
S1	0.710	0.566	0.657	S1	0.158	0.098	0.115
S2	0.326	0.225	0.286	S2	0.326	0.157	0.166
S3	0.278	0.380	0.228	S3	0.249	0.380	0.228
Alternatives	C21	C22	C23	Alternatives	C21	C22	C23
S1	0.564	0.238	0.469	S1	0.022	0.158	0.132
S2	0.148	0.101	0.386	S2	0.148	0.303	0.302
S3	0.331	0.200	0.398	S3	0.175	0.200	0.398
Alternatives	C31	C32	C33	Alternatives	C31	C32	C33
S1	0.363	0.425	0.403	S1	0.062	0.404	0.283
S2	0.401	0.145	0.215	S2	0.187	0.145	0.154
S3	0.230	0.215	0.145	S3	0.230	0.154	0.145

Table 10. Results of C-IF VIKOR.

Alternatives	$\tilde{S}_{i,p}$	$S_{i,p}$	$\tilde{R}_{i,p}$	$R_{i,p}$	$Q_{i,p}$	Rank
S1	(0.908, 0.035; 0.174)	0.912	(0.448, 0.446; 0.158)	0.414	1.00	3
S2	(0.677, 0.200; 0.174)	0.669	(0.183, 0.759; 0.158)	0.155	0.00	1
S3	(0.716,0.166; 0.174)	0.712	(0.259, 0.665; 0.158)	0.226	0.23	2
Alternatives	$\tilde{S}_{i,o}$	$S_{i,o}$	$\tilde{R}_{i,o}$	$R_{i,o}$	$Q_{i,o}$	Rank
S1	(0.924, 0.027; 0.174)	0.927	(0.545, 0.343;0.158)	0.518	0.00	1
S2	(0.986, 0.002; 0.174)	0.984	(0.666, 0.193;0.174)	0.666	0.81	2
S3	(0.993, 0.001; 0.174)	0.989	(0.725, 0.144;0.174)	0.730	1.00	3
Alternatives	S_i	Rank	R_i	Rank	Q_i	Rank
S1	0.496	3	0.445	3	1.000	3
S2	0.405	1	0.189	1	0.000	1
S3	0.419	2	0.236	2	0.184	2

Table 10 demonstrates that with respect to the $Q_{i,p}$ and $Q_{i,o}$ values the best alternatives are S2 and S1 based on pessimistic and optimistic cases, respectively. When the alternatives are sorted based on the combined defuzzified values of S_i , R_i, and Q_i using Eq. (49), alternative S2 is found as the best alternative. However, when the conditions for compromise solutions are checked with respect to Q_i values given in Table 10, it is obtained that alternatives S2 and S3 are the compromise solutions.

5.3 Comparison & Sensitivity analyses

In this section, first comparison analysis is conducted, and then sensitivity analysis is performed. The results are displayed in the following tables, and discussions on the results are also presented.

5.3.1 Comparison analysis

The proposed C-IF AHP & C-IF VIKOR methodology is compared with crisp AHP-VIKOR and ordinary fuzzy AHP-VIKOR methodology based on triangular fuzzy numbers.

By using crisp SI values given in Table 1, we apply the classical AHP and VIKOR to prioritize the alternative suppliers. The results illustrate that the best alternative is found as "S2" and is sequentially followed by S3 and S1.

The proposed integrated C-IF methodology is also compared with ordinary fuzzy AHP-VIKOR methodology. In fuzzy AHP part, the triangular fuzzy numbers presented in Table 11 are used. The results show that the rankings of the alternatives are found same in both of the methodologies. However, this does not necessarily mean that this is valid for any case.

The results in Table 12 indicate that the proposed C-IF AHP & C-IF VIKOR methodology is consistent with the results of the other methodologies. This also shows the validity and applicability of the developed C-IF methodology.

Table 11. Triangular fuzzy numbers (TFNs) used in ordinary fuzzy AHP.

Linguistic Terms	Triangular Fuzzy Number (l,m,u)
Equal (E)	(1,1,1)
Slightly High (SH)	(1,1,3)
Medium High (MH)	(1,3,5)
High (H)	(3,5,7)
Very High (VH)	(5,7,9)
Absolutely High (AH)	(7,9,9)
Reciprocals are taken as (1/u, 1/m, 1/l)	

Table 12. Results of comparison analysis.

Crisp AHP-VIKOR methodology				
Alternatives	Si	Ri	Qi	Rank
S1	1.00	0.420	1.00	3
S2	0.047	0.035	0.00	1
S3	0.152	0.051	0.076	2
Ordinary fuzzy AHP-VIKOR methodology				
Alternatives	Si	Ri	Qi	Rank
S1	1.00	0.398	1.00	3
S2	0.05	0.036	0.00	1
S3	0.155	0.049	0.073	2

5.3.2 Sensitivity analysis

The proposed C-IF model is run for 90 (9×10) times by changing the weights of each sub-criterion, ranging from 0.1 to

1.0. The sensitivity analysis is based on the principle that once the weight of a sub-criterion is assigned, the remaining weights are equally distributed among the other sub-criteria.

The findings illustrate that when the weight of C11 (Price) is equal to or greater than 0.40, the ranking of optimistic case has changed from S1-S2-S3 to S1-S3-S2 while the ranking of pessimistic case remains same. The same process has been applied to the other sub-criteria. For C12 (Terms of Payments), C21 (Flexibility) and C23 (Past performance), the ranking of alternatives in pessimistic cases becomes S2-S3-S1 for the weights above 0.10, while it is S3-S2-S1 for the weight of 0.10. Nothing has changed for optimistic cases. The findings indicate that different weights of C13 (Handling & transportation) have not affected the results for both of the cases. In terms of C22 (On time delivery), the weights equal to or greater than 0.20, the rank of the alternatives changed from S3-S2-S1 to S2-S3-S1 in pessimistic cases whereas the weights equal to or greater than 0.30, the rank has differed from S1-S2-S3 to S1-S3-S2 in optimistic cases.

It is determined that the decisions are the most sensitive to the changes in the weights of the sub-criteria of C3 (Technology&Quality). Table 13 illustrates the ranking results of the alternatives for both of the cases depending on different weights of C31 (Quality management systems), C32 (Technological capability), and C33 (R&D studies). It is worth to mention that specifically for C33, the alternatives may have all the rankings from 1 to 3 in the optimistic cases. The proposed integrated C-IF model is also run 10 more times for different values of "v" between 0.1 and 1.0. The rankings remain same for all the cases, which are S2-S3-S1 and S1-S2-S3 for pessimistic and optimistic cases, respectively.

The proposed C-IF model allows decision makers to find solutions based on both optimistic and pessimistic points of view whereas crisp AHP-VIKOR and ordinary fuzzy AHP-VIKOR methods do not provide that opportunity. As seen from the results, optimistic and pessimistic decision makers prefer S1 and S2, respectively, while the combined solution is dominated by pessimistic view; so that S2 is selected as the best alternative. In addition to these, it is observed that the distinctions between the alternatives are more obvious in the proposed C-IF model.

6 Managerial implications

The supplier selection decision is one of the strategic and complex problems that companies face as it affects their long-term performance and efficiencies. For this reason, the uncertainty factor inherent in supplier selection problems is a factor that should be carefully handled. The supplier selection decision-making process, which is generally based on intuitive and subjective evaluations, is transformed into an objective structure with the proposed C-IF multi-criteria decision-making model. Thus, managers are provided with a mathematical model that they can use in such decision-making processes including vagueness and impreciseness.

Multi-criteria decision-making models under uncertainty, are important tools that enable decision makers to cope with intangible and tangible criteria simultaneously. For this reason, they are often used in real-life problems involving uncertain evaluations. Quantification of intangible criteria is generally a difficult step in the operational decision-making processes for managers. Quantifying and incorporating the vague judgments represented by linguistic expressions, also becomes an important problem for managers. For instance, in the supplier selection problem, when evaluating alternative suppliers in terms of "Technological Capability" sub-criterion, decision makers may prefer linguistic expressions such as "Medium Low", "Absolutely Low", or "Very High" instead of using exact numerical values. This requires the decision-making model to capture the ambiguity in these linguistic expressions.

The proposed integrated decision-making model can capture the uncertainty in linguistic evaluations as well as the hesitancy of decision makers. With C-IF numbers, our model can take into account the truthiness and falsity of the evaluations in the decision matrix according to the criteria set, as well as the deviations that may occur in these judgments. Through a systematic perspective, managers are provided all possible outcomes in the considered MCDM problem before the final decision is made. The C-IF proposed integrated fuzzy model can be utilized within a decision support system by managers to obtain more reliable solutions.

7 Conclusion & Future remarks

C-IF sets have been recently introduced as an extension of IF sets to handle the uncertainty by using a radius around the membership and non-membership degrees. The proposed C-IF AHP & C-IF VIKOR methodology has successfully captured the uncertainty in linguistic assessments. The proposed RSF function has enabled decision makers to defuzzify the C-IF judgments and has given their relative assessments. In addition to that, the study contributes to the literature by introducing compromise solutions using C-IF sets based on pessimistic and optimistic points of views. Herein, the radius values play a key role on calculating distances from assessments to positive and negative ideal solutions.

Table 13. Sensitivity results of the sub-criteria of C31, C32, and C33.

The weights of C31	Pessimistic			Optimistic			The weights of C32	Pessimistic			Optimistic			The weights of C33	Pessimistic			Optimistic		
	S1	S2	S3	S1	S2	S3		S1	S2	S3	S1	S2	S3		S1	S2	S3	S1	S2	S3
0.1	3	1	2	1	2	3	0.1	3	2	1	1	2	3	0.1	3	2	1	1	2	3
0.2	3	2	1	1	2	3	0.2	3	1	2	1	2	3	0.2	3	2	1	1	2	3
0.3	3	2	1	1	2	3	0.3	3	1	2	2	1	3	0.3	3	2	1	1	2	3
0.4	3	2	1	1	2	3	0.4	3	1	2	2	1	3	0.4	3	2	1	1	2	3
0.5	3	2	1	1	2	3	0.5	3	1	2	3	1	2	0.5	3	2	1	2	1	3
0.6	3	2	1	1	2	3	0.6	3	1	2	3	1	2	0.6	3	2	1	2	1	3
0.7	3	2	1	1	2	3	0.7	3	1	2	3	1	2	0.7	3	2	1	2	1	2
0.8	2	3	1	1	2	3	0.8	3	1	2	3	1	2	0.8	3	2	1	3	1	2
0.9	2	3	1	1	2	3	0.9	3	1	2	3	1	2	0.9	3	2	1	3	2	1
1	2	3	1	1	2	3	1	3	1	2	3	1	2	1	3	2	1	3	2	1

The considered supplier selection problem has been solved by the integrated C-IF model. The findings have been compared with crisp AHP-VIKOR and fuzzy AHP-VIKOR with triangular fuzzy sets. In the study, sensitivity analysis is also performed for both optimistic and pessimistic cases. The comparison and sensitivity analyses show that the proposed method provides reliable and robust solutions. The results also demonstrate that different rankings may be obtained from optimistic/pessimistic cases. Besides, the proposed method explicitly displays differences between the alternatives when compared with the other methods.

The proposed integrated C-IF model can be employed in various emerging application areas such as digital transformation problems, augmented reality, intelligent computing systems, and IoT applications.

For future studies, we suggest extending the same methodology by using the other fuzzy set extensions such as fermatean fuzzy sets or PyF sets considering radius values together with membership and non-membership degrees. Further studies can employ IV or triangular C-IF numbers and compare the findings. Also, instead of C-IF AHP & C-IF VIKOR methodology, future studies can develop C-IF AHP & C-IF ELECTRE or C-IF AHP & C-IF TOPSIS methodologies, and their solutions can be compared with this study. In addition to that, future studies can also consider applying other decision making methods such as a utility range-based interactive method with multiple experts ([59]). It is also suggested integrating the proposed C-IF MCDM model into a mathematical model with multiple objectives ([60]).

Finally, some other techniques such as fuzzy best and worst method (BWM) [61] by considering the limitation on the number of criteria with respect to measuring consistencies of pairwise comparison matrices in fuzzy AHP, and fuzzy ANP [62] when there is interaction among the set of criteria, can be used and integrated with multi-expert C-IF VIKOR method.

8 Author contribution statements

In the scope of this study, Cengiz KAHRAMAN contributed to the ideation, and design of the proposed integrated C-IF MCDM methodology. İrem OTAY contributed to the literature review, design of the proposed approach and revision of the article. The application of the proposed integrated C-IF model is realized and results are discussed together. Also, sensitivity and comparison analyses are performed together.

9 Ethics committee approval and conflict of interest statement

For this paper, it is not necessary to get permission from the ethics committee. The authors also state that there is no conflicting interest between authors or with any institution/s.

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