Minimization of Labor Costs in Textile Manufacturing with Dynamic Programming

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Abstract

In the increasingly competitive world today, it has become an inevitable necessity to use labor-intensive businesses by keeping their employees at a minimum in order to maintain their profitability and production and ensure their continuity. Thus, the aim is to ensure that the production process is of the desired time and quality while optimizing the labor costs, which constitute a large part of the total costs. In recent years, dynamic programming has been widely used in problems such as optimization of labor costs, shift planning, inventory control, and transportation problems. In this study, after giving general information about the minimization of labor costs and Dynamic programming, some Dynamic Programming problems were introduced and the optimization of the labor costs of the contract sewing workshop operating in Istanbul was made using Dynamic Programming and the results were suggested to the business.

Keywords: Dynamic programing, Labor cost minimization, Labor force planning, Textile factory, Optimization

Dinamik Programlama ile Tekstil İşçilik Maliyetlerinin Minimizasyonu

Öz

Günümüz artan rekabet ortamında emek yoğun işletmelerin karlıklarını ve üretimlerini sürdürürek devamlılıklarını sağlamak için çalışanların minimum düzeyde tutarak etkin bir şekilde kullanılması kaçınılmaz bir zorunluluk haline gelmiştir. Böylece verilen sipariş istenen zamanda ve kalitede üretildiğinden aynı zamanda maliyetlerinin büyük kısmını oluşturan işgücü maliyetleri optimize edilecektir. Son yıllarda, işgücü maliyetlerinin optimizasyonu, varlık planlaması, envanter kontrolü, nakliye problemleri gibi sorunlarda yaygın olarak dinamik programlama kullanılmaktadır. Bu çalışmada, işgücü maliyetlerinin minimizasyonu ve Dinamik programlama hakkında genel bilgi verildikten sonra bazı Dinamik Programlama problemleri tartışılacak İstanbul’da faaliyet gösteren fason dikim atölyesinin işgücü maliyetlerinin optimizasyonu Dinamik Programlama kullanılarak yapılmış ve elde edilen sonuçlar işletmeye önerilmiştir

Anahtar Kelimeler: Dinamik programlama, İşgücü maliyetlerinin optimizasyonu, İşgücü planlama, Tekstil işletmesi, Optimizasyon
1. Introduction

In order for businesses to operate successfully in today’s competitive environment, the production of goods and services must take place without interruption, and the costs of this production process must be minimized. Today, the production factor inputs are grouped under four main sections. These are:

- Natural resources
- Capital
- Labor
- Entrepreneur

This study focuses on the labor factor, which is extensively used in ready-made clothing manufacturers, which are also known as contract manufacturing textile workshops. Contract manufacturing workshops are required to respond to the daily, weekly, or monthly orders from clothing wholesalers and companies in a timely manner, ensuring a high quality of workmanship. Disruptions during this process result not only in a decline in future orders, but also in payment deductions for each missing or defective product. Thus, the profit of such production workshops is determined not only by the number of goods they produce, but also by the quality and completeness of their output. Therefore, the labor factor which ensures the careful and waste-free production of goods with the technological devices utilized in such enterprises is considered to be the largest cost input. The correct calculation of these labor costs will both minimize the most important cost and maximize the profits of the subcontracting workshops with the effect of the labor force, which is the most important factor in increasing the profits. In other words, for the optimization of production, determining the correct size of workforce and optimal work plans are crucial for such businesses. Therefore, planning of labor costs using dynamic programming method has gained great popularity in recent years. For example, despite the possibility of formulating production and workforce planning as a linear program, (Aardal and Ari, 1987) increased the efficiency of this program to solve problems that are more complex due to their large scale by using a decomposition approach. (Okakun, 1998) studied the internal and external factors that affect Human Resources Planning applications in Turkey. (Patr, 2010) investigated the shortest path with Dynamic programming in the distribution of a pharmaceutical warehouse. (Nirmala & Jeeva, 2010) developed a mathematical model with the objective of minimizing the manpower system cost during the recruitment and promotion period which are determined by the changes that take place in the system. Developed a mathematical model with the objective of minimizing the manpower system cost during the recruitment and promotion period which are determined by the changes that take place in the system. (Küçükışile and Güngör, 2009) investigated the number of workers needed in supermarkets for each hourly period, for each day of the week. Using dynamic programming, (Yücel and Ulutaş, 2010) studied the optimization of labor costs in construction industry. (Shi and Landa-Silva, 2016) investigated the problem in nurse scheduling using approximate dynamic programming, which a problem addressed in workforce scheduling. For the assignment and scheduling of caregivers, (Restrepo et al, 2019) developed a two-stage scholastic model which focuses on the distribution of caregivers in certain geographical areas in the first stage, and the temporary reallocation of caregivers to nearby districts and contacting caregivers to work on their days off in the second stage. (Alam and Habib, 2021) developed a framework which optimizes the effective use of the available modes of transport including public transport and school buses with the knapsack problem approach.

The remainder of this study is organized as follows: In Chapter 2, the minimization of labor costs is briefly described. In Chapter 3, dynamic programming is explained in detail. In Chapter 4, the practical problem of the study is defined and solved. In the final Chapter, solutions and suggestions for the problem are given.

2. Minimization of Labor Costs

By focusing on the minimization of the labor factor in production, businesses aim to determine the necessary planning that is needed to produce a certain amount of goods or services using the least amount of labor with the utilization of advanced technological systems. Thus, by reducing the labor costs, which is the most important input in the production of goods and services, businesses can significantly increase their profits. The studies on the planning of labor costs, two main factors were determined, namely workforce planning and optimal shift planning. To define planning, it is the adjustment of personnel, equipment, finances to determine the direction a business will take in the future. The most important step of planning is workforce planning in resources, human resources and personnel planning which is also known as employee planning. Personnel planning refers to the training of the necessary number of staff members to give them the required skills and abilities at a level that is in line with the future goals of the enterprises, or the employment of individuals with these skills where they are needed. Shift planning, on the other hand, is one of the factors that has a direct impact on operational efficiency and labor costs for businesses that operate or provide services 24/7 and need production and service planning in accordance with the number and the quality of their workforce. As shift based operational systems lead to extra costs due to labor laws on weekly and monthly limitations to work periods, overtime work principles etc., incorrect planning will increase the costs of businesses. For this reason, optimal shift planning is vital for businesses operating in shifts.

3. Dynamic Programming

Dynamic programming is a mathematical programming (planning) method that has gained importance since 1920 and reached its peak in the 1940s when it was utilized by the American mathematician Richard Ernes Bellman to solve the problems in successive decision-making processes. Richard Ernest Bellman revised his approach in 1953 to solve optimization problems (Bellmann, 1957). The problems that can be solved by using dynamic programming in businesses can be listed as follows (Tütek and Gümüşoğlu, 2000):

- Determining the variables in setting the commission rules,
- Planning of production in changing demand conditions and determination of labor requirements,
- Determining the level of spare parts to avoid malfunction costs,
- Determination of resource distribution in new production areas,
3.1. The Basic Concept of Dynamic Programming

Dynamic programming is an approach where optimal solution of a problem is determined after the decision variable has been identified and divided into independent subproblems as in all dynamic programming problems. The terminology of this approach includes the concepts of stage, state, policy decision, optimal policy, and recursive relationship. To summarize these concepts briefly:

- Stage: Depending on the process of the problem, the breaking down of the problem into smaller sub-problems is defined as the steps or the stages. These steps are defined in different ways according to the problem. For instance, if the problem is about a long-term system, the steps are time frames, or if it is a spatial problem, the steps are the places involved.
- State: As the decision made at each step leads to a different situation in the next, the value assigned to variables differs at every step. The concept of “state” changes according to each problem. For example, for an inventory problem it refers to the stock level.
- Policy decision: Choosing the most optimal decision at each step is called policy decision. This decision is used in the making of the decision to be taken in the next step.
- Optimal policy: This concept refers to the policy set for the totality of the problem, and it is the sequence of decisions made at every step. In other words, it is the sequence of optimal decisions.
- Recursive relationship: The iteration function that takes the solution to the most optimal decision by iterating at each stage.

The formula for discretely timed simple dynamic system to understand the structure of dynamic programming is as follows (Bertsekas, 1995):

\[ x_{k+1} = f_k(x_k, u_k, w_k), k = 0, 1, 2, ... N - 1 \]  

where,
- \( k \), is the discrete time index,
- \( x_k \), is the prior information relevant to the state variable and future optimization,
- \( u_k \), is the control or decision variable to be chosen at \( k \)
- \( w_k \), is a random parameter (it is also referred to as distortion or noise depending on the context),
- \( N \), is the number of times the iteration or stages are executed,
- \( f_k \), is a function that defines the system and specifically the mechanism by which the state is updated.

The cost of the problem or system is displayed with \( g_k(x_k, u_k, w_k) \), and it is the cost that occurs in the stage \( k \). Here, due to the presence of \( w_k \) and as cost is usually a random variable, it cannot be optimized in a significant way. Therefore, the optimization of the expected cost of the system can be formulized as:

\[ E \left( g_N(x_N) + \sum_{k=1}^{N-1} g_k(x_k, u_k, w_k) \right) \]  

where, \( g_N(x_N) \) is a cost that occurs at the end of the process. To visualize this formulation (Bertsekas, 1995).

3.2. Advantages and Disadvantages of Dynamic Programming

There are some advantages to solving problems using dynamic programming. These are:
- It is a programming system that can be applied to deterministic and stochastic processes.
- It offers solutions to recurring problems.
- It has a flexible structure that can be adjusted specifically for each problem.
- It provides the opportunity to solve complex problems by separating them into interrelated sub-problems. This advantage of dynamic programming provides the opportunity to solve both complex and large problems involving sequential decisions more easily.
- It is an optimization approach that can be applied to mathematical programming as well as other programming problems.
- Dynamic programming problems involving the solution of coefficient values can allow for the application of sensitivity analysis.
- Problems such as chemical processes can only be solved by means of dynamic programming (Halaç, 1995)

Although few in number, there are also some disadvantages to using dynamic programming. Briefly, these disadvantages are:
- Since dynamic programming does not have a specific algorithm, dynamic programming packages which differ...
for each problem can be insufficient. Therefore, it is necessary to work with an expert both in the formulation and the coding process of the problem.

- Since dynamic programming is programmed individually for each problem, it involves more complex concepts than mathematical programs.
- The excess of sub-problems and variables in dynamic programming causes dimensionality problems.

### 3.3. Dynamic Programming Solution with Iteration Method

In dynamic programming problems, the problem is decomposed on the basis of the given constraints. The decomposed problem is repeated for each sub-problem, and the optimal value obtained continues as the input of the next step and iterated. Thus, the optimal solution is reached with the iteration process that continues until the last step.

Solution methods of dynamic programming by iteration can be done in two ways, namely:

- Backward iteration method
- Forward iteration method

Although the backward iteration method is used more extensively in the literature due to its advantages, forward iteration method also gives the same results. In backward iteration method, the solution to the problem which is divided into \( n \) number of sub-problems is reached by moving back from stage \( n \) to stage 1; whereas with forward iteration method, optimal solution is reached from stage 1 to \( n \).

The dynamic programming problems which can be solved through the iteration method and are used extensively, are problems such as knapsack, stock control, equipment renewal, investment, and determination of the number of employees. The most popular of these problems, which are the knapsack and stock control problems, will be briefly introduced and their dynamic programming algorithm structures will be given.

**The knapsack problem:** Refers to the loading of products into sacks, warehouses, or delivery trucks in the most optimal way by maximizing the number or value of products that can be fit in such spaces with limited storage volume. This type of problem is named and modeled differently according to the desired constraints. These models include the fractional knapsack problem, \((0/1)\) (binary) knapsack problem, and unbounded knapsack problem where any number of product types can be selected. The simple knapsack problem is formulated as:

\[
g(w) = \max \left \{ b_j + g(w - w_j) \right \} \tag{3.3}
\]

where, \( b_j \) refers to the benefit, \( w_j \) to the weight of the product, and \( g \) refers to the maximum benefit that can be attained by adding 1 kilogram to the knapsack. The calculation is repeated until the \( g(w) \) capacity of the bag is full in the recurrence equation given in Equation (3.3).

**Stock (inventory) problem:** Inventory is the number of products an entrepreneur or manufacturer must keep in stock in order to meet the possible demands in the future. In other words, this problem is aimed to ensure the minimization of cost by keeping in balance the supply and demand of the products an entrepreneur or manufacturer has. The expected cost in such problems is (Bertsekas, 1995):

\[
E \left \{ R(x_N) \sum_{k=0}^{N-1} c u_k + r(x_k + u_k - w_k) \right \} \tag{3.4}
\]

where, \( x_k \) shows the stock level, \( u_k \) the number of orders, \( w_k \) the amount of demand, \( c \) coefficient the cost per product ordered, and \( R \) shows the terminal cost. In addition, customer holding costs arise in case of depletion of stocks.

### 4. Identification of the problem and suggestions for solution

The aim here is to optimize the cost of labor, which is the biggest cost item for the newly established contract sewing workshop operating in the sewing of sports jackets in Istanbul. In order to achieve that, the number of workers and the labor cost which is calculated by taking the cost of service, meals, and other expenses into account, are aimed to be minimized in the first 6 months of the establishment. The optimization of workforce number is achieved through the utilization of dynamic programming method.

Figure 4.1 demonstrates simple structure of deterministic Dynamic programming schema.

![Fig. 4.1. Simple structure of deterministic Dynamic programming](image)

Each stage in the figure is shown as an independent optimization model. Here \( n \) is the stage, \( s_n \) is the current state value and \( x_n \) is the decision variable.

### 4.1. Identifying the Problem

In workforce planning, the number of recruitments and dismissals are to be balanced. The dynamic programming function for the optimization of the workforce throughout the trial period is as follows,

\[
f_i(x_{i-1}) = \min_{x_i} \left \{ c_1(x_i - b_i) + c_2 + c_3(x_i - x_{i-1}) + f_{i+1}(x_i) \right \} \tag{4.1}
\]

\[
f_{i+1}(x_n) = 0 \tag{4.2}
\]

The calculation in the Eqs. (4.1) and (4.2) start from stage \( n \), or the step \( x_n = b_n \) and the solution is reached by regression. Where,

- \( n \), is the number of stages
- \( b_i \), is the minimum number of workers at month \( i \),

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4.2. The Solution of the Problem

As the problem involves a 6-month trial period, the solution is obtained by regressing back from the 6th month (step or stage) with the backwards iteration method. The solution will be presented through tables to ensure visibility and comprehensibility. The solution will commence from the last month which is January.

Table 4.4 demonstrates the solution of November, which is the fourth stage.

Table 4.3 demonstrate the solution of December, which is the fifth stage.

Table 4.5. Third stage (month) solution

<table>
<thead>
<tr>
<th>f(x_2) = \left{ \begin{array}{l} c_1(x_2 - b_2) \ + c_2 + c_3(x_3 - x_2) + f_3(x_3) \end{array} \right}</th>
<th>Optimal Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>x_2</td>
<td>x_3 = 55, 56, 57; b_3 = 55</td>
</tr>
<tr>
<td>60</td>
<td>250(55-55)+0+0(55-60)+2550</td>
</tr>
<tr>
<td>56</td>
<td>250(56-55)+0+0(56-60)+1750</td>
</tr>
<tr>
<td>57</td>
<td>250(57-55)+0+0(57-60)+250</td>
</tr>
</tbody>
</table>

In the table, value x_2 is the number of workers in October, x_3 is the number of workers in the 3rd month, f(x_2) value shows the cost in stage three, and x_3^* is the optimal number of workers. The optimal number of workers for the third month was found to be 57.

In the table, x_3 value is the number of workers in November, x_5 is the number of workers in the 5th month, f(x_4) value shows the cost in stage five, and x_5^* is the optimal number of workers. As there were no recruitments in the transition from the fourth to the fifth month, c_2 and c_3 values were 0 in the calculation for stage five. The optimal number of workers for the fifth month was found to be 51.

Table 4.2. Sixth stage (month) solution

<table>
<thead>
<tr>
<th>f(x_5) = \left{ \begin{array}{l} c_1(x_5 - b_5) \ + c_2 + c_3(x_5 - x_4) + f_5(x_4) \end{array} \right}</th>
<th>Optimal Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>x_5</td>
<td>x_4 = 50, 51; b_5 = 50</td>
</tr>
<tr>
<td>57</td>
<td>250(50-50)+0+0(50-57)+1500</td>
</tr>
</tbody>
</table>

In the table, value x_4 is the number of workers in November, x_5 is the number of workers in the 6th month, f(x_4) value shows the cost in stage four, and x_5^* is the optimal number of workers. The optimal number of workers for the sixth month was found to be 51.

Table 4.1. Number of minimum employees per month

<table>
<thead>
<tr>
<th>Stages</th>
<th>Months</th>
<th>Employees per Month</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>August</td>
<td>57</td>
</tr>
<tr>
<td>2</td>
<td>September</td>
<td>60</td>
</tr>
<tr>
<td>3</td>
<td>October</td>
<td>55</td>
</tr>
<tr>
<td>4</td>
<td>November</td>
<td>57</td>
</tr>
<tr>
<td>5</td>
<td>December</td>
<td>50</td>
</tr>
<tr>
<td>6</td>
<td>January</td>
<td>51</td>
</tr>
</tbody>
</table>
Table 4.6 demonstrates the solution of September, which is the second stage.

**Table 4.6. Second stage (month) solution**

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_2(x_1) = \left{ \frac{c_1(x_2 - b_2) + c_2}{+c_s(x_2 - x_1) + f_2(x_2)} \right}$</td>
<td><strong>Optimal Solution</strong></td>
</tr>
<tr>
<td>$x_1$</td>
<td>$x_2 = 60; b_2 = 60$</td>
<td>$f_2(x_1) \quad x_2$</td>
</tr>
<tr>
<td>57</td>
<td>250(60-60)+700+800(60-57)+750</td>
<td>3850  60</td>
</tr>
<tr>
<td>58</td>
<td>250(60-60)+700+800(60-58)+750</td>
<td>3050</td>
</tr>
<tr>
<td>59</td>
<td>250(60-60)+700+800(60-59)+750</td>
<td>2250</td>
</tr>
<tr>
<td>60</td>
<td>250(60-60)+0+800(60-60)+750</td>
<td>750</td>
</tr>
</tbody>
</table>

In the table, $x_1$ value is the number of workers in August, $x_2$ is the number of workers in the 2nd month, $f_2(x_1)$ value shows the cost in stage two, and $x_1^*$ is the optimal number of workers. The optimal number of workers for the second stage was found to be 60.

Table 4.7 demonstrates the solution August, which is the first stage.

**Table 4.7. First stage (month) solution**

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f_1(x_0) = \left{ \frac{c_1(x_1 - b_1) + c_2}{+c_s(x_1 - x_0) + f_2(x_1)} \right}$</td>
<td><strong>Optimal Solution</strong></td>
</tr>
<tr>
<td>$x_0$</td>
<td>$x_1 = 57,58,59,60; b_1 = 57$</td>
<td>$f_1(x_0) \quad x_1^*$</td>
</tr>
<tr>
<td>57</td>
<td>250(57-57)+700+800(57-0)+3850</td>
<td>50150  57</td>
</tr>
<tr>
<td></td>
<td>250(58-57)+700+800(58-0)+3050</td>
<td>50400</td>
</tr>
<tr>
<td></td>
<td>250(59-57)+700+800(59-0)+2250</td>
<td>50650</td>
</tr>
<tr>
<td></td>
<td>250(60-57)+700+800(60-0)+750</td>
<td>50200</td>
</tr>
</tbody>
</table>

In the table, $x_0$ value is the initial number of workers, $x_1$ is the number of workers in the 1st month, $f_1(x_0)$ value shows the cost at the final stage, and $x_1^*$ is the number of workers initially hired. As a result of the minimization process performed in the last stage the optimal number of workers was found to be 57.

### 5. Conclusion and Suggestions

As a result of this study which aimed to minimize the labor costs of a contract sewing workshop, the number of workers to be hired for this establishment and their costs were summarized in Table 5.1. In addition, the monthly employees and optimal monthly employees are visualized graphically in Figure 5.1. According to these results, the workshop started with 57 employees in August. In September, 4 more workers were recruited to the workshop, and the number of workers was increased to 60 by hiring 3 more workers. At the beginning of October, it was determined with dynamic programming that the number of workers were too high, and with the dismissal of 3 employees the number was reduced to 57. As the number of workers, which was reduced to 57, was found to be optimal, there were no changes to the workforce. As it was understood with the dynamic programming that the number of workers working in the workshop exceeded the optimal amount in December, 6 workers were dismissed, and the number of workers was reduced to 51. In the following month, the number of employees in the workshop was maintained. Table 5.1 and Figure 5.1 demonstrate the optimal results and decisions of this problem.

**Table 5.1. Optimal decision table**

<table>
<thead>
<tr>
<th>Months</th>
<th>Monthly of Employees</th>
<th>Optimal of Employees</th>
<th>Decision</th>
</tr>
</thead>
<tbody>
<tr>
<td>August</td>
<td>57</td>
<td>57</td>
<td>Hire 57 employees</td>
</tr>
<tr>
<td>September</td>
<td>60</td>
<td>60</td>
<td>Hire 3 more employees</td>
</tr>
<tr>
<td>October</td>
<td>55</td>
<td>57</td>
<td>Dismiss 3 employees</td>
</tr>
<tr>
<td>November</td>
<td>57</td>
<td>57</td>
<td>No change</td>
</tr>
<tr>
<td>December</td>
<td>50</td>
<td>51</td>
<td>Dismiss 6 employees</td>
</tr>
<tr>
<td>January</td>
<td>51</td>
<td>51</td>
<td>No change</td>
</tr>
</tbody>
</table>

When analyzed in terms of cost, Table 5.1 shows that if 55 workers were employed for the month of October, an extra cost of 2300€ would arise. However, although there were 2 more employees in October and a total of 57, the cost was only 750€. Likewise, had the company employed 50 workers in December, the cost would be 1250€ more. The rest of the optimal decision table shows that if there were 55 workers employed in October and 50 in December, the company would have incurred the cost of 2 workers for October and 1 worker for December.

As a result, in this study, Dynamic Programming problems were introduced and applied for the optimization of the labor costs of the workers working in the contract sewing workshop. Thus, the labor costs of the contract sewing workshop were optimized, and the enterprise was not exposed to both high labor costs and labor production disruption. In addition, it can be suggested that this Dynamic Programming study for reducing labor costs can be applied considering that high rates of labor costs will be saved if it is applied in enterprises with a large number of employees.
References


