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Research Article

A computer program for linear analysis of two-dimensional semi-rigid frames

Senem YILMAZ ÇETİN [1]*, Halil GÖRGÜN [2], Derman KAYA [3]

1 Dicle University ,Department of Civil Engineering, senyilmaz@dicle.edu.tr, Orcid No: 0000-0002-7282-6413

2 Dicle University ,Department of Civil Engineering, hgorgun@dicle.edu.tr, Orcid No: 0000-0002-4357-0009

3 Dicle University ,Department of Civil Engineering,17806503@org.dicle.edu.tr , Orcid No: 0000-0002-5779-736X

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* Corresponding author

ABSTRACT

It is usual to assume that a displacement caused at any point in a structure is linearly dependent on the magnitude of the loads applied. This paper focuses on the linear analysis of 2-D frames with flexural connected beam-column members considering shear displacements. A computer program was written in MATLAB for this purpose. To achieve the above purpose, first, the element stiffness matrix with linear flexural springs at its ends has been obtained by using relevant differential equations, considering shear deformations. In the analysis of the stiffness methods, it has been observed that the loading vector can be obtained by means of the loads applied between the joint points. It is found that the presents of an axial load in a member affect the values of the fixed-end forces, and these are the subject of another paper. For linear cases, the semi-rigid end forces have been obtained for a uniformly distributed load, an unsymmetrical point load, a linearly distributed load, an unsymmetrical trapezoidal distributed load, and an unsymmetrical triangular distributed load. To prove the validity of the computer program, some problems in the literature have been solved differently. There was a good agreement between the relevant results.

Introduction

Both clockwise and anti-clockwise bending moments *M* affects the connections in precast concrete frames as shown in Figure 1 that induces relative rotations *Φ*, between the beam end and the face of the adjacent column. In this type of semi-rigid connection, rotational stiffness and finite strength are substituted for rigid connections to perform frame analysis. Strength, stiffness, and ductility (deformation capacities) are important properties of connections. The idealized behaviour shown in Figure 2 may be described by moment-rotation M- *Φ* curve. In Figure 2, the rotational stiffness of the joint *J*, is shown by the gradient of the *M- Φ* curve. Expressing stiffness K_s as a non-dimensional term, where

$$K_s = \frac{J}{4\, E_c I/l} \qquad (1)$$

is the ratio of the stiffness of the connection to the flexural stiffness of the beam that it is connected, while E_c is the modulus of elasticity of concrete, *l* is the effective span of the beam, and *I* is the second moment of inertia of the beam.

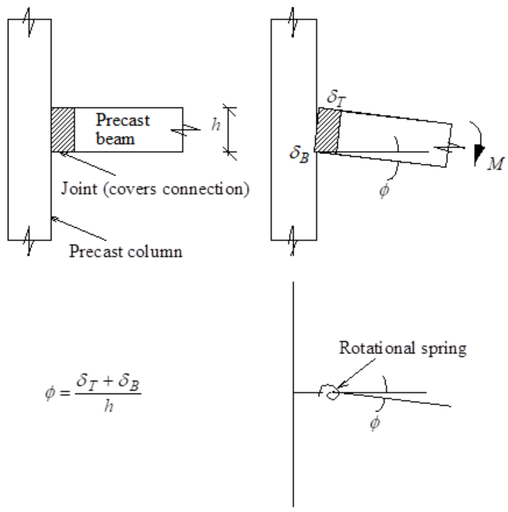

Figure 1: Simplified definition of joint rotation [1]

Generally, only the traditional identification and analysis methods for pin and rigid connections are used in the construction of steel and precast structures. However the actual behaviour of these types of structures is not that simple. The actual behaviour of most column-to-beam connections is known to be non-linear. At braced frames, the attitude of a single span beam can be used in observing the effect of semi-rigid connections on beam model.

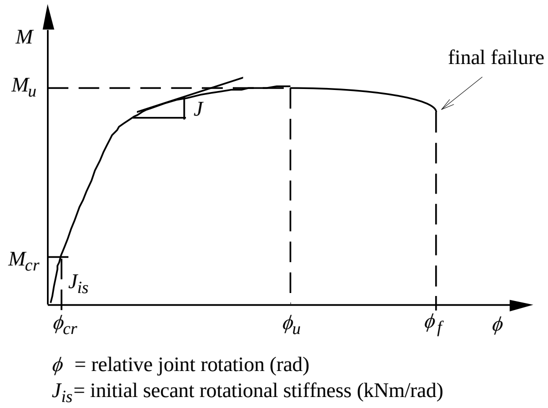

Figure 2: Relationship between Moment-rotation [1]

A simply supported beam subjected to a uniformly distributed load is seen in Figure 3(a). As can be seen from the figure, the maximum bending moment occurs in the mid-span of the beam. Now the simple supports are replaced by fixed ones in Figure 3(b). In this case, the maximum elastic moment occurs at the supports.

A beam that has semi-rigid end connections is seen in Figure 3(c). The maximum elastic bending moment occurs at the supports or at mid-span depending on the flexural stiffness of the connection, however, it always permits a reduction at support. The optimum connections are that going to be the connections those let adequate end rotation to balance the end and mid-span moments ($\frac{ql^2}{16}$). The semi-rigid connection concept is related to such cases as seen in Figure 3(c). The practical assumption about the connections of the above-mentioned structures would be the reflection of non-linear rotational springs at the ends of beams while presenting actual pinned and fully rigid connections by appropriate spring constants. As is well known, iterative solutions with non-linear springs will be required. In this study, nonlinear springs were used in the linear analysis of the planar frame. The analytical results obtained with the formulas were translated and analyzed in the MATLAB programming language.

In the rest of the analytical study, the stiffness matrix coefficients of a straight constant prismatic member of the plane frame have been obtained. A computer program was prepared in MATLAB language for numerical handling of plane frames with semi-rigid connections.

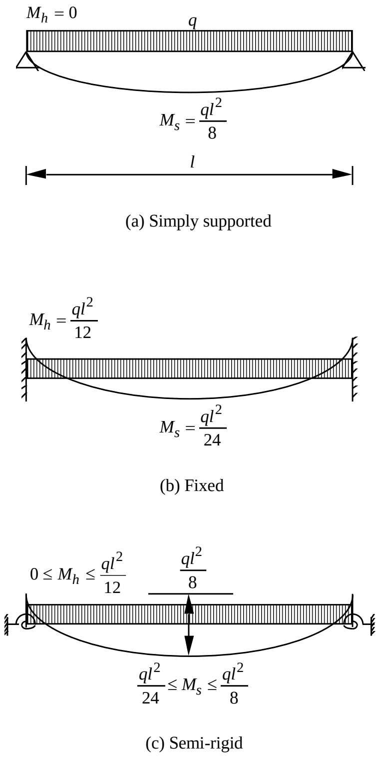

Figure 3. Beam with various end conditions [1]

Previous Studies

Monforton and Wu [2] performed a linear analysis of flexibly connected frames to obtain the stiffness matrix from the relationship between forces and displacements. Livesley [3] examined the element with rotating springs at the ends using the stiffness matrix. Romstad and Subramanian [4] performed the frame analysis for pin, rigid, and semi-rigid (flexible) connections. In their work, they presented moment-relative rotation graphs of flexibly simple frames. Ackroyd and Gerstle [5] investigated a portal frame under vertical loads. It has been observed that the critical load value increases proportionally with the spring constant. Steelmack et al. [6] made some experiments to observe its performability to steel frames by making use of the literature results. When they compared the experimental results with the other results, they observed a satisfactory match. Yu and Shanmugam [7] used a two-story one-bay frame to examine the stability of flexible coupled frames in which they consider the effect of flexure on axial stiffness. They reported that the discrepancy between experimental and analytical results would not exceed 19 percent. Cunningham [8] made some experiments to examine the moment-relative rotation relationships for different connection types between steel elements. Azizinamini and Radzirninski [9] investigated beam-column connections in semi-rigid steel frames. They observed cyclic and static behaviour in semi-rigid steel frames. Aksoğan and Akkaya [10] investigated the linear analysis of planar frames consisting of flexible connected elements with rotating springs at the ends using differential equations with the computer program REDUCE. Using differential equations, they found the stiffness matrix for a single bar with rotating springs at its ends. For various loading types with the help of stiffness matrix; fixed end forces for uniformly distributed load, linearly distributed load, concentrated load, unsymmetrical triangular distributed load, and symmetrical trapezoidal distributed load were found. Aksoğan and Görgün [11] worked on the nonlinear analysis of semi-rigid coupled frames. They obtained the fixed end forces for various intermediate loads and prepared a computer program on this subject. Aksoğan et al. [12] studied the stability analysis of planar frames consisting of rotational springs with rigid regions at their ends. In this study, the element stiffness matrix based on the element elasticity modulus, a moment of inertia, length, and axial force is given and a computer program for both subjects is prepared. Erdem [13] studied the analysis of frames consisting of elements connected by nonlinear rotational springs with rigid regions at their ends and prepared a computer program. On the other hand, Aksoğan et al. [14] have prepared a computer program for frames with rigid ends, considering nonlinear analysis and nonlinear semi-rigid connections. Ochoa [15] studied the stability and second-order analysis of elastically supported semi-rigid connected planar frames by taking shear force into account. Domenico et al. [16] have proposed a purely probabilistic approach to describe the structural response of beams and frames characterized by indefinite semi-rigid connections and noted that resorting to deterministic approaches can lead to misleading design implications. Artar and Daloğlu [17] obtained a more economical design by using the genetic algorithm method for different structures. Ihaddoudène et al. [18] proposed a mechanical model considering elastic buckling load in plane steel frames and stated that elastic buckling load acts strongly in semi-rigid structures. Ghassemieh et al. [19] included P-Delta effects and material and geometric nonlinearity in all models to investigate the effect of flexibility of extended end plate connections in steel moment frames. And they revealed that there are big differences in the behavior of fully rigid modeled structures in terms of natural periods, strength, and maximum inter-story drift. Du et al. [20] developed a modified low-speed fatigue model for precast concrete frames with semi-rigid connections.

3. Material and Method

In this article, when the material is questioned, the study only includes analytical work and computer programming. This study includes two parts. The first part is composed of the analytical study that operates the matrix method for analysis, usually used in structural analysis. Here, the stiffness matrix of the structure is found. The additives of varied types of loads to the loading vector are obtained. Additionally, the unknown displacements were clarified by the formulation of the equilibrium equations. The second part of the study was composed of preparing a relevant computer program written in MATLAB.

4. Findings, Results, and Discussions

4.1 Analysis

In the analytical study, the matrix analysis for the formulation of the relations between the nodal displacements and the applied loads which cause the nodal displacements is made clear. For this goal, the sign convention for different quantities that belong to the ends of a member should first be specified. The specified sign convention shown in Figure 4, where the six arrows show the positive senses of all quantities at the ends of a member were used in this study. Besides the sign convention, some of the other symbols used in this analysis are also shown in Figure 4.

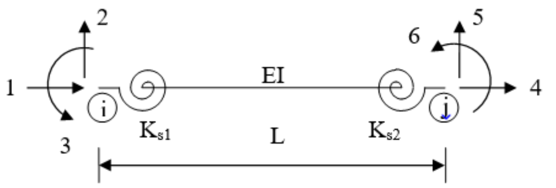

Figure 4. Notation and sign convention.

The semi-rigid end forces *p* of a straight member, having a constant cross-section, of a plane frame (see Figure 4) in terms of the member end displacements *d* and fixed end forces *f*, because of loads between the ends of the member, is given by the well-known formula,

$$p = Kd + f \qquad (2)$$

where *K* is the member stiffness matrix whereas *p*, *d* and *f* are vectors of member end forces, member end displacements, and member fixed end forces, respectively. *x* shows the distance from the left end of the member while *y* shows the downward displacements when the others are zero and solving the differential equations,

$$\frac{d^4y}{dx^4} = 0 \qquad (3)$$

for every case. The formula given below is taken into account while solving Equation 3.

$$y = Ax^3 + Bx^2 + Cx + D \qquad (4)$$

while solving the *y* value, modified boundary conditions are taken into account, excluding the spring constants.

With the above-stated procedure for a plane frame, member stiffness coefficients are obtained based on its submatrices K_{11}, K_{12}, and K_{22} in

$$K = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \qquad (5)$$

and defining

$H_1 = (\beta_1 + \beta_2 + 1)$,

$H_2 = (2\beta_2 + 1)$,

$H_3 = 3(\beta_2 + \beta_3) + 1$,

$H_4 = (2\beta_1 + 1)$,

$H_5 = (1 - 6\beta_3)$,

$H_6 = 3(\beta_1 + \beta_3) + 1$ and

$H = 4(3\beta_1\beta_2 + \beta_1 + \beta_2) + 12(\beta_1 + \beta_2 + 1)\beta_3 + 1$

Because of its symmetry the sub-matrices of the member of length *L*, shear modulus *G*, cross-sectional area *A* and uniform flexural rigidity *EI*, stiffness matrix are given as follow:

$$K_{11} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EIH_1}{L^3H} & \frac{6EIH_2}{L^2H} \\ 0 & \frac{6EIH_2}{L^2H} & \frac{4EIH_3}{LH} \end{bmatrix} \qquad (6)$$

$$K_{12} = \begin{bmatrix} -\frac{EA}{L} & 0 & 0 \\ 0 & -\frac{12EIH_1}{L^3H} & \frac{6EIH_4}{L^2H} \\ 0 & -\frac{6EIH_2}{L^2H} & \frac{2EIH_5}{LH} \end{bmatrix} \qquad (7)$$

$$K_{22} = \begin{bmatrix} \frac{EA}{L} & 0 & 0 \\ 0 & \frac{12EIH_1}{L^3H} & -\frac{6EIH_4}{L^2H} \\ 0 & -\frac{6EIH_4}{L^2H} & \frac{4EIH_6}{LH} \end{bmatrix} \qquad (8)$$

where

K_{s1}: Moment value at left support to rotate the sprint one radian

K_{s2}: Moment value at right support to rotate the spring one radian

k_t = kGA

k= A cross-section constant for elements

$$\beta_1 = 1/(4K_{s1}),\ \beta_2 = 1/(4K_{s2}),\ \beta_3 = EI/(L^2 k_t)$$

As the stiffness matrix mentioned above is symmetrical, K_{21} is the transpose state of K_{12}. In the next step, fixed end forces are determined for different loading conditions on the elements. In Figure 5, semi-rigid end forces are found by applying Equation 2 for a uniformly loaded straight member. Beside the boundary conditions at the ends of the member the compatibility conditions at the loaded point are also used to tackle the problem for semi-rigid end forces. The outcome results will be given later in the paper. Applying Equation (2) the semi-rigid end forces for various types of loads on a member can be determined. Semi-rigid end quantities alone have been given in the following, with respect to Figures. 5-9.

Uniformly Distributed Load

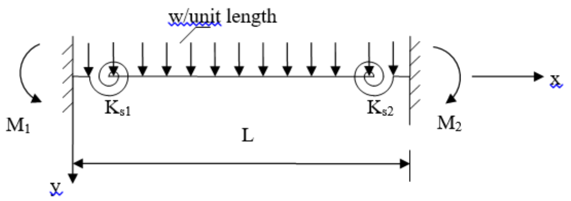

Figure 5. A beam with semi-rigid end connections subjected to uniformly distributed load.

The elastic bending moments occur at the supports:

$$M_1 = \frac{wL^2}{12}\frac{[(6\beta_2+1)+12\beta_3]}{4(3\beta_1\beta_2+\beta_1+\beta_2)+12(\beta_1+\beta_2+1)\beta_3+1} \quad (9)$$

$$M_2 = \frac{wL^2}{12}\frac{[(6\beta_1+1)+12\beta_3]}{4(3\beta_1\beta_2+\beta_1+\beta_2)+12(\beta_1+\beta_2+1)\beta_3+1} \quad (10)$$

Unsymmetrical point load

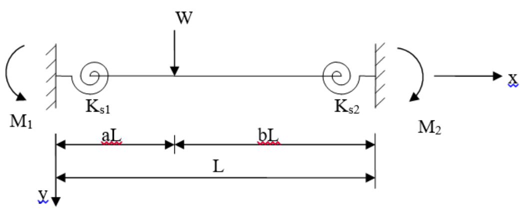

Figure 6. An unsymmetrical point load exposed to a beam with semi-rigid end connections

$$M_1 = WLa\frac{[2\beta_2(a^2-3a+2)+a^2-2a+1]}{4(3\beta_1\beta_2+\beta_1+\beta_2)+1} \quad (11)$$

$$M_2 = WLb\frac{[2\beta_1(b^2-3b+2)+b^2-2b+1]}{4(3\beta_1\beta_2+\beta_1+\beta_2)+1} \quad (12)$$

Linearly Distributed Load

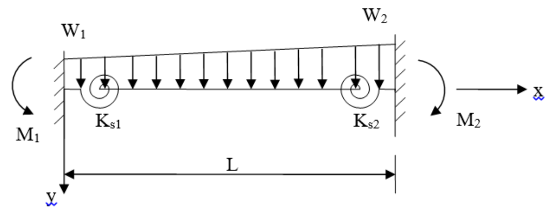

Figure 7. A linearly distributed load exposed to a beam with semi-rigid end connections

$$M_1 = \frac{L^2}{180}\frac{[6(8W_1+7W_2)\beta_2+3(3W_1+2W_2)+90(W_1+W_2)\beta_3]}{4(3\beta_1\beta_2+\beta_1+\beta_2)+12(\beta_1+\beta_2+1)\beta_3+1} \quad (13)$$

$$M_2 = \frac{L^2}{180}\frac{[6(8W_2+7W_1)\beta_1+3(3W_2+2W_1)+90(W_1+W_2)\beta_3]}{4(3\beta_1\beta_2+\beta_1+\beta_2)+12(\beta_1+\beta_2+1)\beta_3+1} \quad (14)$$

Unsymmetrical Trapezoidal Load

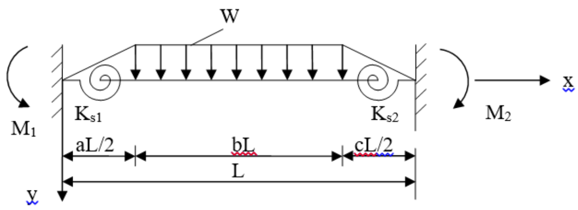

Figure 8. An unsymmetrical trapezoidal load exposed to a beam with semi-rigid end connections

$$M_1 = \frac{WL^2}{96}\frac{[a^3-4a^2+8]}{(2\beta_1+1)} \quad (15)$$

$$M_1 = \frac{WL^2}{96}\frac{[c^3-4c^2+8]}{(2\beta_1+1)} \quad (16)$$

Unsymmetrical Triangular Load

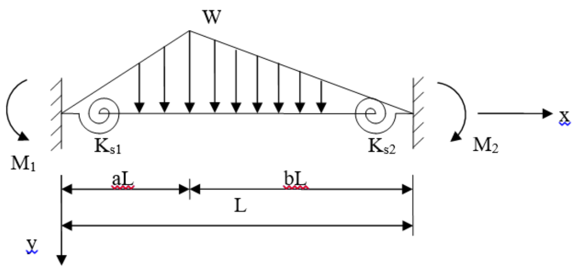

Figure 9. An unsymmetrical triangular load exposed to a beam with semi-rigid end connections

$$M_1 = \frac{WL^2}{60}\frac{[[6a^2(a-4)+16(a+1)]\beta_2+3(a^3+a+1)-7a^2+30(ab+1)\beta_3]}{4(3\beta_1\beta_2+\beta_1+\beta_2)+12(\beta_1+\beta_2+1)\beta_3+1} \quad (17)$$

$$M_2 = \frac{WL^2}{60}\frac{[[6b^2(b-4)+16(b+1)]\beta_2+3(b^3+b+1)-7b^2+30(ab+1)\beta_3]}{4(3\beta_1\beta_2+\beta_1+\beta_2)+12(\beta_1+\beta_2+1)\beta_3+1} \quad (18)$$

4.2 Programming

After the analytical explanations for the problem type shown in Figure 10 a computer program in MATLAB programming language was prepared to solve the problems.

The computer program is basically a supplication of stiffness matrix method to plane frames. The prime difference is the existence of the lengthless flexural springs take place at the ends of beams and the calculation of the relevant stiffness and the semi-rigid end forces correspondingly.

4.3. Discussion and Results

A nonrealistic loading is chosen involving all possible types of loadings, as an example. All necessary information of the structure are given in Figure 10. The joints and members are shown in Figure 11. In the example problem, the standard I section steel frame is used in all beams and columns.

The element data information used in the example is taken from studies in the literature. The cross-sectional area used in all elements is 0.48 ft^2 and the moment of inertia is 0.00722 m^4. In addition, the spring constants (K_{s1} and K_{s2}) in all elements are given as 0.5 for external connections and 0.6 for internal connections for typical beams.

The results of semi-rigid end quantities for the beam and columns are shown in Table 1. The rest of results as joint displacements and mid-span sagging moments could not be given because of restricted page numbers.

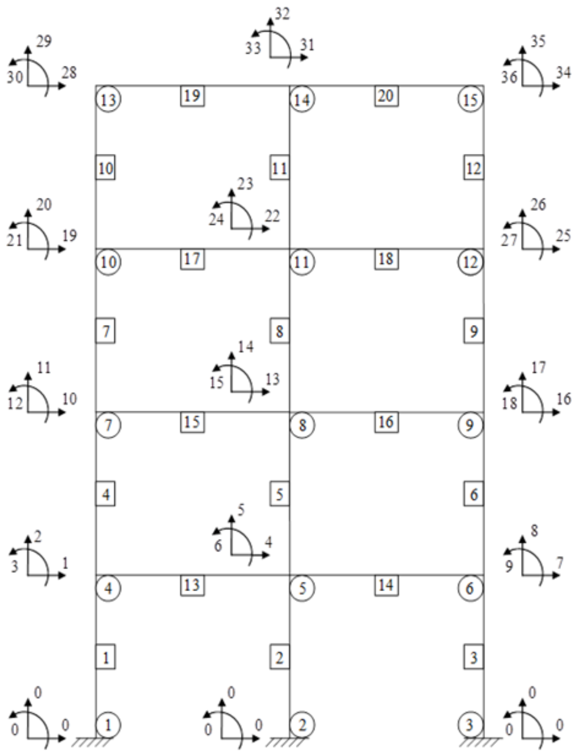

Figure 11. Coding of joints and numbering of members

Table 1. Member end forces

No	M1 (kNm)	M2 (kNm)	T1 (kN)	T2 (kN)	N2 (kN)
1	-15.70	108.40	26.49	-26.49	-158.75
2	16.21	119.28	38.71	-38.71	-355.57
3	6.06	115.76	34.81	-34.81	-260.68
4	15.59	38.54	15.46	-15.46	-113.78
5	61.60	79.50	40.32	-40.32	-297.44
6	52.87	66.90	34.22	-34.22	-183.79
7	21.19	-0.03	6.07	-6.07	-71.45
8	70.56	49.14	34.20	-34.20	-178.87
9	62.54	41.50	29.73	-29.73	-104.68
10	8.64	-20.72	-3.45	3.45	-16.73
11	58.61	26.04	24.18	-24.18	-55.29
12	50.26	17.17	19.27	-19.27	-27.98
13	-22.84	-67.32	44.98	45.03	1.02
14	-28.39	-72.96	13.11	76.89	-0.58
15	-15.55	-90.49	42.33	77.67	-10.61
16	-20.26	-94.37	40.90	79.11	-4.50
17	-0.58	-76.10	54.72	80.28	-20.48
18	-20.50	-79.71	43.30	76.70	-10.46
19	-8.64	-40.98	16.73	33.27	-43.45
20	-17.63	-50.26	22.02	27.98	-19.27

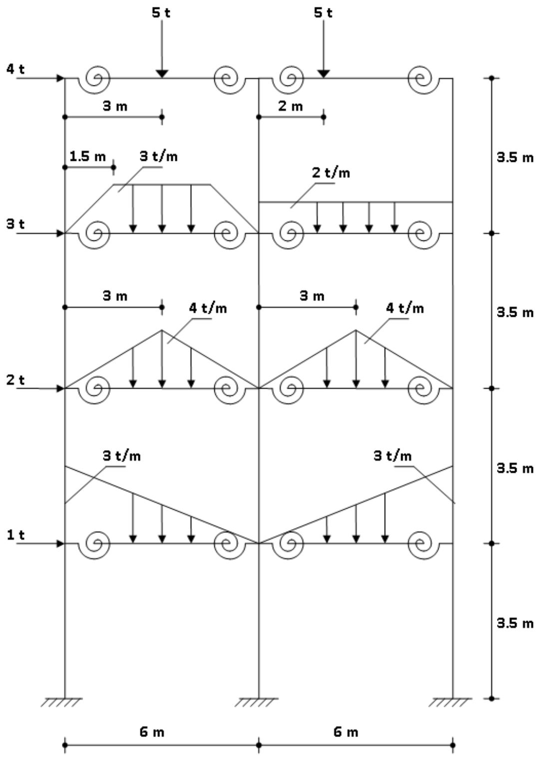

Figure 10. An example of geometry and loading

In the figures given below, the effect of the spring constants on the displacements in Figure 12, the variation of the bending moments at the supports with the spring constants K_s in the Figure 13, and the effect of the spring constants K_s on the height of the structure are shown in Figure 14, respectively. Table 2 shows the effect of spring constants on displacements.

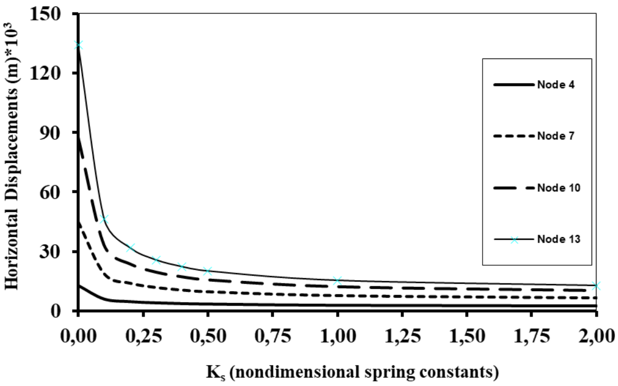

Figure 12. In the example problem, the spring constants (K_s) and displacement relation at each floor level

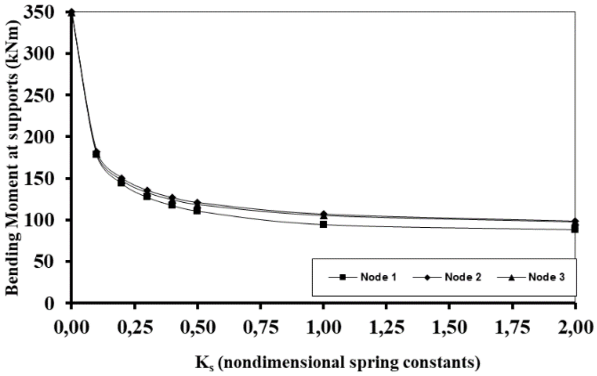

Figure 13. Variation of bending moments at the supports with spring constants (K_s) in the example problem

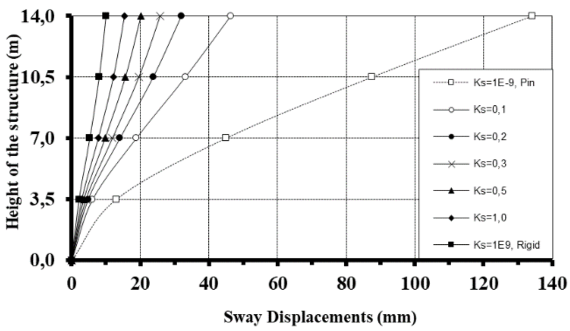

Figure 14. In the example problem, the spring constants (K_s) and height of the structure

Table 2. Variations in the horizontal displacements due to spring constants K_s at floor levels

Height of the structure (m)	Horizontal displacements (mx10^3)								
	K_s								
	Pinned	Semi-Rigid							Rigid
	1E-9	0,1	0,2	0,3	0,4	0,5	1,0	2,0	1E9
14,00	134,220	46,358	31,885	25,780	22,380	20,202	15,462	12,854	10,038
10,50	87,527	33,134	23,694	19,565	17,204	15,663	12,214	10,245	8,048
7,00	44,958	18,840	14,043	11,865	10,587	9,738	7,784	6,630	5,300
3,00	12,992	6,100	4,771	4,148	3,775	3,524	2,931	2,570	2,143

Conclusion

In this study by taking into account the linear analysis of plane frames made of flexibly connected straight prismatic element, the efficacy of shear deformations is handled. Then for the numerical calculations a computer programme is prepared. Various kinds of span loadings were taken into account. When the literature was reviewed, no study related to aperture loading was found. The results were checked and analyzed according to the loading conditions given in the article. In the analyzed example, it was found that as the spring constants of the flexible connections get smaller, the displacements increase and at the same time the critical extreme values of the bending moments increase.

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