Bitlis Eren Üniversitesi Fen Bilimleri Dergisi
BİTLİS EREN UNIVERSITY JOURNAL OF SCIENCE
ISSN: 2147-3129/e-ISSN: 2147-3188
VOLUME: 11 NO: 2 PAGE: 678-693 YEAR: 2022
DOI: 10.17798/bitlisfen.1088988

An Integrated Fuzzy Approach Based Failure Mode and Effects Analysis for a Risk Assessment

Burak EFE[1*]

[1]*Necmettin Erbakan University, Department of Industrial Engineering, Konya, Türkiye*
(ORCID: 0000-0001-7092-3001)

Keywords: Risk evaluation, mathematical model, FMEA, fuzzy TOPSIS, fuzzy extension of AHP

Abstract

This paper provides to cope with the limitations of traditional FMEA by using an integrated fuzzy multi-criteria decision making method, which considers fuzzy extension of AHP (Analytic Hierarchy Process) and fuzzy TOPSIS (Technique for Order Preference by Similarity to Ideal Solution), and a linear programming. The proposed method is shown for an application to failure mode and effects analysis (FMEA) based risk assessment of a construction firm. Firstly, fuzzy extension of AHP approach is utilized to define the weights of criteria in risk evaluation. Secondly, fuzzy TOPSIS approach is used to determine the most important failure mode in the construction firm. This work handles a sensitivity analysis and a comparison with the other methods. FMEA related papers in the literature presents only ranking of failure modes by using various methods. This study aims to handle the limited resources such as budget and time in a linear programming to establish a suitable occupational health and safety policy.

1. Introduction

Failure Mode and Effects Analysis (FMEA) is a systematic quality improvement technique to prevent any possible malfunctions that may occur in the system, design, process or services in advance [1]. The technique focuses on improving safety and increasing customer satisfaction, while eliminating defects and preventing potential errors [2]. Any undesirable situation regarding the process, such as the structural disorder of the process, the irregularity in its functioning, the irregularity in its implementation, and the output not meeting the expectations, are considered as "errors". These may be previously known events, or they may be events that have never been encountered but are likely to happen. Failure mode is a short and general statement that summarizes the physical conditions in which the failure occurs with correct adjectives. The probability of failure is a frequency of how often any error can occur. The effect of the failure is the result of a failure that will occur in a system, design, process or service. It is necessary to consider this effect not only as the consequences of the failure in the system, but also in terms of its effect on other systems and components [1]. The process to be improved with the FMEA technique is examined in detail by the FMEA team. By identifying and prioritizing improvement opportunities, it is determined where to start the work. Then, it is questioned what kind of problems may occur in the process. If there are issues that need to be taken into account, such as customer expectations, it is examined whether they can be met or not [2]. Risk priority number (RPN) is a numerical value calculated by multiplying the the probability of failure (O), severity of failure (S) and non-detection of failure (D). RPN is calculated as O*x*S*x*D. FMEA technique uses RPN value as a practical tool in order to rank the failure modes in terms of their risk [3].

The interpretation of the RPN value is made on the basis of the definition of these multipliers and the scales used. The increase in the probability of

---

[*] Corresponding author: *burakefe0642@gmail.com*

Received: 16.03.2022, Accepted: 13.06.2022

failure or the severity of the effect and the difficulty of determination generally mean an increased risk. If the relevant scales are prepared accordingly, a high RPN value will indicate high risk, and a low RPN will indicate low risk [1]. Table 1 presents some of FMEA based on multi-criteria decision making (MCDM) papers. The proposed method has some contributions as follows:

1. This paper ensures to reduce the loss of information by using fuzzy number instead of crisp number.
2. Each decision maker can have different judgments about a selection process due to their experience and knowledge. In group decision making environment, each decision maker presents their judgments to achieve a group judgment in evaluation process. The weights of the decision makers, which depend on characteristics of the decision makers, are important to achieve a group judgment. Inaccurate weights of the decision makers generate the mistaken group judgment. This situation generates the mistaken decision and inherently loss of cost and waste of time. This paper presents an attribute based aggregation technique (ABAT) proposed by Olcer and Odabasi [17] to cope with this limitation.
3. This paper presents 45 different situations with regard to an occupational health and safety policy. This study uses a linear programming due to limited budget, time properties. The construction firm could select the most appropriate situation according to its conditions.

Table 1. Papers about FMEA

Methods	Authors
Fuzzy evidential reasoning and belief rule-based approach	Liu et al. [4]
DEMATEL and TODIM	Ulu and Şahin [5]
Fuzzy inference system	Jee et al. [6]
Fuzzy PROMETHEE	Efe et al. [7]
Fuzzy and grey theories	Zhou and Thai [8]
Hesitant 2-tuple linguistic term sets and an extended QUALIFLEX approach	Liu et al. [9]
Z numbers based AHP, entropy and VIKOR methods	Mohsen and Fereshteh [10]
Intuitionistic fuzzy AHP - VIKOR methods	Efe et al. [3]
Fuzzy best-worst, relative entropy, VIKOR	Tian et al. [11]
MULTIMOORA (Multi-objective Optimization By Ratio Analysis) and AHP	Fattahi and Khalilzadeh [12]
Quality function deployment and intuitionistic fuzzy VIKOR	Efe [13]
Fuzzy ANP	Yazdani et al. [14]
Intuitionistic fuzzy best-worst method	Yazdi et al. [15]
Regret theory and PROMETHEE under linguistic neutrosophic context	Zhu et al. [16]
Double upper approximated rough number, FUCOM and TOPSIS	Dhalmahapatra et al. [28]

This paper aims to provide an integrated multi-criteria decision making approach to define the most important failure mode for a construction firm. Priority values of criteria, which are O, S and D, have been defined by utilizing fuzzy extension of AHP approach. The rankings of failure modes in the construction firm are defined by using fuzzy TOPSIS method based on an ABAT. The results of the proposed method are compared with results of the different methods, which are FAHP-fuzzy VIKOR and FAHP-fuzzy GRA. A sensitivity analysis can be realized under different β coefficients. A linear programming is suggested to form an occupational health and safety policy. This mathematical model is solved in GAMS software program.

2. The Proposed approach

The suggested integrated multi-criteria decision making approach is utilized to rank the failure modes in a risk assessment. Firstly, the priorities of criteria in risk evaluation will be defined by fuzzy extension of AHP. Decision makers present the pair wise comparison matrixes to acquire the priorities of O, S and D criteria so that they use the linguistic statements in Table 2.

Table 2. Linguistic terms for O, S and D

Terms	Fuzzy numbers
Absolutely strong (AS)	(7/2,4,9/2)
Very strong (VS)	(5/2,3,7/2)
Few strong (FS)	(3/2,2,5/2)
Poor (P)	(2/3,1,3/2)
Equal (E)	(1,1,1)

An ABAT is utilized to degrade to a group judgment the judgments of three decision makers. The ranking of the failure modes in the construction firm are determined by using fuzzy TOPSIS approach. Decision makers present their judgments for the values of failure modes based on criteria by using linguistic statements in Table 3.

Table 3. Linguistic terms for failure modes

Terms	Fuzzy numbers
Absolute Poor (AP)	(0,0.1,0.2)
Very Poor (VP)	(0.1,0.2,0.3)
Poor (P)	(0.1,0.3,0.5)
Fairly Poor (FP)	(0.4,0.45,0.5)
Medium (M)	(0.3,0.5,0.7)
Fairly Good (FG)	(0.5,0.55,0.6)
Good (G)	(0.5,0.7,0.9)
Very Good (VG)	(0.8,0.9,1)
Absolute Good (AG)	(0.9,1,1)

2.1. Fuzzy extension of AHP

AHP, which simultaneously considers qualitative and quantitative data, was developed by Saaty [18]. Fuzzy extension of AHP approach was developed by Chang [19]. Chang integrated fuzzy logic with Saaty's AHP. The weights of O, S and D criteria in risk assessment are calculated by utilizing fuzzy extension of AHP systematically in an uncertain environment. Fuzzy extension of AHP approach is defined in Eqs. (1)-(9) [19, 20]:

When $m_1^- \geq m_2^-, m_1 \geq m_2, m_1^+ \geq m_2^+$

The degree of the possibility is defined as one [21]:

$$V(M_1 \geq M_2) = 1 \tag{1}$$

The ordinate of the highest intersection point is determined as follows [19, 21]:

$$\begin{aligned} &V(M_2 \geq M_1) = hgt(M_1 \cap M_2) \\ &= \mu(d) = \frac{m_1^- - m_2^+}{(m_2 - m_2^+) - (m_1 - m_1^-)} \end{aligned} \tag{2}$$

The value of the fuzzy synthetic extent can be determined as follows [19, 21]:

$$F_i = \sum_{j=1}^{m} M_{gi}^j \otimes \left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^j \right]^{-1}, \ i = 1,2,\ldots,n \tag{3}$$

$$\sum_{j=1}^{m} M_{gi}^j = \left(\sum_{j=1}^{m} m_{ij}^-, \sum_{j=1}^{m} m_{ij}, \sum_{j=1}^{m} m_{ij}^+\right), j = 1,2,\ldots,m \tag{4}$$

$$\begin{aligned} &\left[\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gi}^j \right]^{-1} \\ &= \left(\frac{1}{\sum_{i=1}^{n} \sum_{j=1}^{m} M_{ij}^+}, \frac{1}{\sum_{i=1}^{n} \sum_{j=1}^{m} M_{ij}}, \frac{1}{\sum_{i=1}^{n} \sum_{j=1}^{m} M_{ij}^-} \right) \end{aligned} \tag{5}$$

$$\begin{aligned} &V(F \geq F_1, F_2, \ldots, F_k) \\ &= \min V(F \geq F_i), i = 1,2,\ldots,k \end{aligned} \tag{6}$$

$$\begin{aligned} &d(F_i) = \min V(F_i \geq F_k) = W_i^{'} \\ &k = 1,2,\ldots,n \ and \ k \neq i \end{aligned} \tag{7}$$

The weights of criteria are as follows after above procedure:

$$W^{'} = (W_1^{'}, W_2^{'}, \ldots, W_n^{'})^T \tag{8}$$

$$W = (W_1, W_2, \ldots, W_n)^T \tag{9}$$

The consistency ratio is calculated for the pair-wise comparison matrix, which shows relationship between O, S and D criteria. Fuzzy numbers of the pair-wise comparison matrix must be defuzzied into crisp numbers to calculate the consistency ratio. Crisp number is obtained from fuzzy number $\tilde{X} = (l, m, u)$ by using Eq.(10) [22]:

$$P(\tilde{X}) = \frac{1}{6}(l + 4 \times m + u) \tag{10}$$

The relative importance is calculated by using Eq.(11).

$$Aw = \lambda_{\max} w \tag{11}$$

The consistency index (*CI*) is defined as indicated in Eq. (12) [23]:

$$CI = (\lambda_{\max} - n) / (n - 1) \tag{12}$$

The consistency ratio (*CR*) considers the consistency of the assessments. It must be smaller than 0.1 and is calculated by using Eq.(13). The assessment, which CR is bigger than 0.1, must be revised to correct it. (Wang and Yang, 2007).

$$CR = CI / RI \tag{13}$$

Random consistency index (*RI*) can be acquired from Table 4.

Table 4. Random consistency index.

N	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49

The additive weighted aggregation (AWA) operator is presented in Eq.(14) [24]:

$$g_i = \lambda_k \times g_{ik} \tag{14}$$

2.2. Attribute based aggregation technique

Chen [25] suggested an aggregation approach with fuzzy logic for homo/heterogeneous group of experts. Homogeneous group of experts means that the importance degree of each expert is equal. Heterogeneous group of experts means that the importance degree of each expert is not equal. This phase aims to acquire a group judgment by combining the judgments of homo/heterogeneous group f experts. Let be the relative importance of each expert $E_k(k=1,2,...,M)$ we_k, where $we_k \in [0,1]$ and $\sum_{k=1}^{M} we_k = 1.$

The aggregation method for homo/heterogeneous groups of experts is introduced below [17, 20]:

Step 1: Determine the degree of similarity of E_u expert's opinions to E_v expert's opinions as in Eq.(15) . Let U=(u_1, u_2, u_3) and V=(v_1, v_2, v_3) be two standardised triangular fuzzy numbers where $0 \le u_1 \le u_2 \le u_3 \le 1$ and $0 \le v_1 \le v_2 \le v_3 \le 1$

$$S(U,V) = 1 - \frac{|u_1 - v_1| + |u_2 - v_2| + |u_3 - v_3|}{3} \tag{15}$$

where $S(U,V) \in [0,1]$.

Step 2: Define the agreement matrix (AM).

$$AM = \begin{bmatrix} 1 & S_{12} & \cdots & S_{1v} & \cdots & S_{1M} \\ \cdots & \cdots & & \cdots & & \cdots \\ S_{u1} & S_{u2} & \cdots & S_{uv} & \cdots & S_{uM} \\ \cdots & \cdots & & \cdots & & \cdots \\ S_{M1} & S_{M2} & \cdots & S_{Mv} & \cdots & 1 \end{bmatrix} \tag{16}$$

where $S_{uv} = S(R_u, R_v)$, if $u \neq v$ and $S_{uv} = 1$, if *u*=*v*.

Step 3: Determine the average degree of similarity *AA(E_u)*.

$$AA(E_u) = \frac{1}{M-1} \sum_{v=1, v\neq u}^{M} S(R_u, R_v) \tag{17}$$

Step 4: Determine the relative importance of agreement RA (E_u).

$$RA(E_u) = \frac{AA(E_u)}{\sum_{u=1}^{M} AA(E_u)} \tag{18}$$

Step 5: Determine the consensus degree coefficient CC(E_u).

$$CC(E_u) = \beta we_u + (1-\beta) RA(E_u) \tag{19}$$

where $\beta(0 \le \beta \le 1)$. When $\beta = 0,$ a homogeneous group of experts problem is considered.

Step 6: Aggregate the fuzzy opinions.

$$R_{AG} = CC(E_1) \otimes R_1 \oplus CC(E_2) \otimes R_2 \oplus \\ \ldots \oplus CC(E_M) \otimes R_M \tag{20}$$

2.3. Fuzzy TOPSIS

Hwang and Yoon suggested TOPSIS approach, which ranks alternatives. TOPSIS method aims to find solution [26]. Fuzzy TOPSIS method integrates fuzzy logic to classical TOPSIS method thus it ensures to ease a decision making process in ambiguous environment. The stages of fuzzy TOPSIS method are presented as follows [20, 27]:

Step 1: Defining the fuzzy decision matrix $\tilde{A}$: The decision maker defines $\tilde{A}_{ij}$ matrix as a beginning matrix and this matrix is shown in Eq.(21):

$$\tilde{A}_{ij} = \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\ \tilde{a}_{21} & \tilde{a}_{22} & \cdots & \tilde{a}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{a}_{m1} & \tilde{a}_{m2} & \cdots & \tilde{a}_{mn} \end{bmatrix} \tag{21}$$

Step 2: Determining the normalized fuzzy decision matrix $(\tilde{R})$: Eq.(22) is used to normalize the beginning matrix.

$$\tilde{r}_{ij} = \frac{\tilde{a}_{ij}}{\sqrt{\sum_{k=1}^{m} a_{kj}^2}} \tag{22}$$

Eq.(23) is utilized to acquire $\tilde{R}_{ij}$ matrix [20, 27]:

$$\tilde{R}_{ij} = \begin{bmatrix} \tilde{r}_{11} & \tilde{r}_{12} & \cdots & \tilde{r}_{1n} \\ \tilde{r}_{21} & \tilde{r}_{22} & \cdots & \tilde{r}_{2n} \\ \vdots & \ddots & \cdots & \vdots \\ \tilde{r}_{m1} & \tilde{r}_{m2} & \cdots & \tilde{r}_{mn} \end{bmatrix} \qquad (23)$$

where

$$\tilde{r}_{ij} = \left(\frac{a_{ij}}{c_j^+}, \frac{b_{ij}}{c_j^+}, \frac{c_{ij}}{c_j^+}\right) and\, c_j^+ = \max_i c_{ij} \text{ (Benefit criteria)}$$

$$\tilde{r}_{ij} = \left(\frac{a_j^-}{c_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{a_{ij}}\right) and\, a_j^- = \min_i a_{ij} \text{ (Cost criteria)}$$

Step 3: Defining the weighted normalized fuzzy decision matrix $(\tilde{v})$: Eqs.(24)-(25) is employed to calculate $\tilde{v}_{ij}$ matrix which shows the multiplication of $\tilde{r}_{ij}$ matrix and the weights of assessment criteria (w_i) [27]:

$$\sum_{i=1}^{n} w_i = 1 \qquad (24)$$

$$\tilde{v}_{ij} = \begin{bmatrix} w_1\tilde{r}_{11} & w_2\tilde{r}_{12} & \cdots & w_n\tilde{r}_{1n} \\ w_1\tilde{r}_{21} & w_2\tilde{r}_{22} & \cdots & w_n\tilde{r}_{2n} \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ w_1\tilde{r}_{m1} & w_2\tilde{r}_{m2} & \cdots & w_n\tilde{r}_{mn} \end{bmatrix} \qquad (25)$$

$$i = 1,2,\ldots,m; j = 1,2,\ldots,n \; where\; \tilde{v}_{ij} = \tilde{r}_{ij}(.)w_j$$

$\tilde{v}_{ij}$ is expressed by $\left(\tilde{a}_{ijk}, \tilde{b}_{ijk}, \tilde{c}_{ijk}\right)$.

Step 4: Defining the fuzzy ideal solution (FPIS) and fuzzy negative ideal solution (FNIS): Eqs.(26)-(27) are utilized to compute the FPIS and FNIS of the alternatives :

$$A^+ = \tilde{v}_1^+, \tilde{v}_2^+, \ldots, \tilde{v}_n^+ \quad where\, \tilde{v}_j^+ = 1,1,1 \qquad (26)$$

$$A^- = \tilde{v}_1^-, \tilde{v}_2^-, \ldots, \tilde{v}_n^- \quad where\, \tilde{v}_j^- = 0,0,0 \qquad (27)$$

Step 5: Defining the separation scales of each alternative: Eqs.(28)-(29) are employed to calculate the distance measure d_i^+, d_i^- from the FPIS and the FNIS for each alternative:

$$d_i^+ = \sum_{j=1}^{n} dv(\tilde{v}_{ij}, \tilde{v}_j^+), i = 1,2,\ldots,m \qquad (28)$$

$$d_i^- = \sum_{j=1}^{n} dv(\tilde{v}_{ij}, \tilde{v}_j^-), i = 1,2,\ldots,m \qquad (29)$$

If $\tilde{v}_{ij} = \tilde{a}_{ij}, \tilde{b}_{ij}, \tilde{c}_{ij}$ and $\tilde{v}_j^+ = 1,1,1$ and $\tilde{v}_j^- = 0,0,0$:

$$dv(\tilde{v}_{ij}, \tilde{v}_j^+) = \sqrt{\frac{1}{3}\left[(\tilde{a}_{ij} - 1)^2 + (\tilde{b}_{ij} - 1)^2 + (\tilde{c}_{ij} - 1)^2\right]},$$

$$dv(\tilde{v}_{ij}, \tilde{v}_j^-) = \sqrt{\frac{1}{3}\left[(\tilde{a}_{ij} - 0)^2 + (\tilde{b}_{ij} - 0)^2 + (\tilde{c}_{ij} - 0)^2\right]}.$$

Step 6: Defining the closeness coefficient (CC$_i$) of each alternative: Eq.(30) is used to calculate CC$_i$ for each alternative:

$$CC_i = \frac{d_i^-}{d_i^- + d_i^+} \qquad (30)$$

Step 7: Rank the alternatives: CC$_i$ values according to its increasing values are employed to rank the alternatives.

3. An application for risk evaluation

3.1. Implementation

After examining the process and identifying potential problems or areas for improvement, a risk assessment is made. Risk assessment is the scoring of failures in terms of probability of occurrence, severity of impact and determination. This scoring is done through ready-made scales or new scales to be prepared. There is no single and standardized scale to be used in FMEA applications, but low scores on the scales are attributed to low risk levels, and high scores are attributed to high risk levels. Existing information is used while scoring. If such a resource is not available, the scoring is done based on the experience and foresight of the team members. Another requirement at this stage is FMEA forms in which study-related records will be processed. These forms are also non-standard and FMEA teams can adapt these forms according to their own work. Risk assessment procedure covers determining of decision makers for risk assessment, defining failure modes, defining O, S and D criteria that presented in assessment stage, defining the weights of the O, S and D criteria and ranking orders of failure modes, solving a mathematical model phases. A real life application in a construction firm is presented to show the efficiency of the suggested method. Fig. 1 indicates the flowchart of the suggested method. Thirty-five failure modes in the construction firm are presented in Table 5. In order to conclude a comprehensive study such as FMEA in the most effective way, consultation with others and cooperation when necessary will be needed. It is extremely beneficial to conduct FMEA with teamwork, as it will be possible for everyone to benefit from the experience and knowledge of each other in teamwork. For this reason, after the critical problems to be worked on are determined, the first thing to do is to form this team. All team members should be selected from individuals who are knowledgeable about the process to be worked on and even about group behavior, are directly or indirectly related to the problem, and are willing to participate in the study. Team members should be people who are familiar with the process to be worked on, and all of them should have been given the necessary training for this job before starting to work. This paper collected the judgments of three experts. In this study, the suggested integrated fuzzy extension of AHP-fuzzy TOPSIS approach is utilized to order the failure modes in the construction firm. The priorities of criteria in risk assessment will be defined by fuzzy extension of AHP. Pair wise comparison matrixes of decision makers' judgments are considered to acquire priorities of criteria by utilizing the linguistic variables, which is indicated in Table 2.

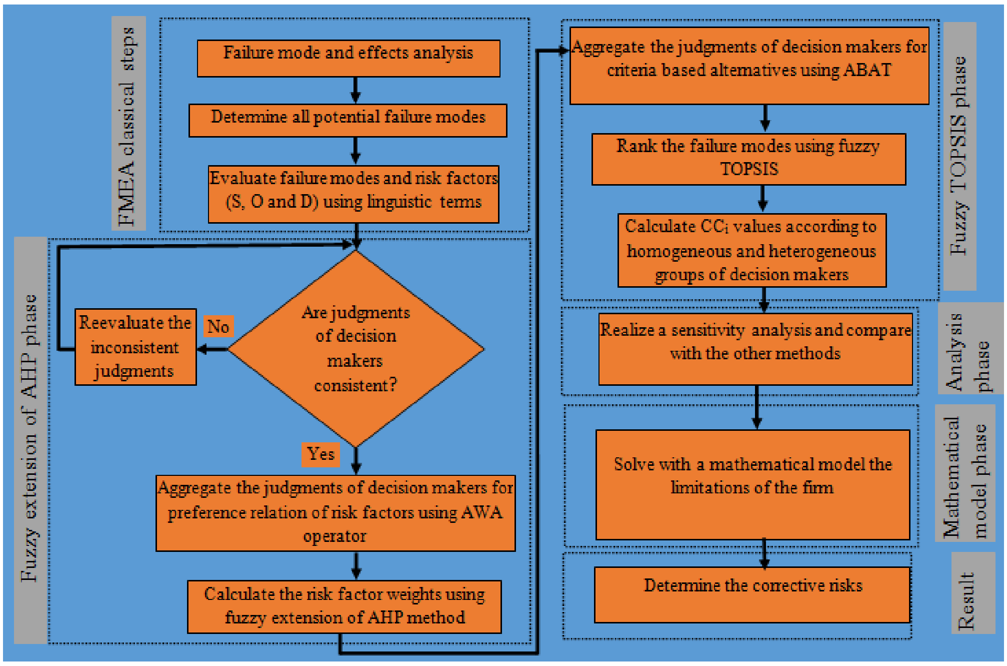

Figure 1. Flowchart of the suggested method.

Thirty-five failures based on O, S and D criteria are ranked. Fuzzy extension of AHP approach is used to define the weights of criteria and fuzzy TOPSIS approach is used to order criteria based failure modes. The three experts present the pair-wise comparison matrix of criteria as showing in Table 3.

Table 5. Failure modes in the construction firm

FM1	Working without prevention of staff in T shaft cavity
FM2	Non-running with water the pumice cutting machine during cutting
FM3	Injuring the foot of sharp materials in places
FM4	Working without prevention of staff building wall in square shaft cavity
FM5	Unsuitability of isolation and grounding of pumice grooving machine
FM6	Loss of balance
FM7	The broken stems of the mechanical hand tools
FM8	Open ends of electrical cables
FM9	Falling of the materials below when workers go from insecure places
FM10	Inappropriate utilization of pumice grooving machine
FM11	Hasty and careless working during building wall

FM12	Overthrowing the ladder in the edges of balcony and fronts of window
FM13	Holding the sharp edges of materials
FM14	Deformed railings located on the floors
FM15	Manual handling of heavy loads
FM16	Rubbish shot used to pour material and filth
FM17	Utilization of damaged and deformed cables
FM18	Utilization without protection the pumice grooving machine
FM19	Wrong utilization of pumice cutting machine
FM20	Noncovering the cavities in the ground during laying brick
FM21	Loading over material to the scaffolding
FM22	Absence of emergency stop button of the pumice cutting machine
FM23	Overthrowing the material from palet
FM24	Utilization of nonstandart scaffolding
FM25	Noncovering the cavities after building wall
FM26	Irritation of the skin
FM27	Utilization without protection the pumice cutting machine
FM28	Attempting to break the material with hands
FM29	Noncovering the shaft cavities in downstairs operations
FM30	Entering plaster to eye
FM31	Electrical leakage in the pumice cutting machine
FM32	Staggering to the material during manual handling the material
FM33	Working without prevention of staff building wall in elevator and shaft cavity
FM34	Absence of warning signboards of the pumice cutting machine
FM35	Availability of dust in the environment

The overall weights of O, S and D criteria are calculated in fuzzy extension of AHP stage by using AWA operator, which is presented in Eq.(14). Fuzzy extension of AHP is utilized to calculate the importance degrees of O, S, and D criteria by helping of Eqs. (1)-(9). The results of computation are indicated in Tables 6.-7. The weights of criteria is calculated detailly and indicated below. The weights of criteria can be defined according to decision maker E3 below:

Table 6. Pairwise comparison matrix of O, S and D criteria.

		O	S	D	CR
E1	O	(1.000,1.000,1.000)	(0.667,1.000,1.493)	(0.667,1.000,1.500)	
	S	(0.670,1.000,1.500)	(1.000,1.000,1.000)	(1.000,1.000,1.000)	
	D	(0.667,1.000,1.500)	(1.000,1.000,1.000)	(1.000,1.000,1.000)	0.032
		O	S	D	
E2	O	(1.000,1.000,1.000)	(1.500,2.000,2.500)	(0.670,1.000,1.500)	
	S	(0.400,0.500,0.667)	(1.000,1.000,1.000)	(0.670,1.000,1.500)	
	D	(0.667,1.000,1.493)	(0.667,1.000,1.493)	(1.000,1.000,1.000)	0.084
		O	S	D	
E3	O	(1.000,1.000,1.000)	(0.400,0.500,0.670)	(0.667,1.000,1.500)	
	S	(1.493,2.000,2.500)	(1.000,1.000,1.000)	(0.667,1.000,1.500)	
	D	(0.667,1.000,1.500)	(0.667,1.000,1.500)	(1.000,1.000,1.000)	0.084

Table 7. Total weight of criteria for each decision maker.

	O	S	D	Weights of experts
E1	0,333	0,333	0,333	0,35
E2	0,451	0,225	0,323	0,25
E3	0,226	0,450	0,324	0,40
Overall weight	0,320	0,353	0,327	

$$\sum_{i=1}^{n}\sum_{j=1}^{m} M_{gi}^{j} = \left(1,1,1\right) + (0.4, 0.5, 0.67) + \ldots + (1,1,1)$$

$$= (7.56, 9.50, 12.17)$$

$$\left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{gi}^{j}\right]^{-1} = \left(\frac{1}{12.17}, \frac{1}{9.50}, \frac{1}{7.56}\right) = \ 0.082, 0.105, 0.132$$

$$\sum_{j=1}^{m} M_{g1}^{j} = (1,1,1) + (0.4, 0.5, 0.67) + (0.67,1,1.5) = (2.07, 2.50, 3.17)$$

$$\sum_{j=1}^{m} M_{g2}^{j} = (3.16, 4.00, 5.00) \quad \sum_{j=1}^{m} M_{g3}^{j} = (2.33, 3.00, 4.00)$$

$$F_1 = \sum_{j=1}^{m} M_{g1}^{j} \otimes \left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{gi}^{j}\right]^{-1} = (2.07, 2.50, 3.17) \otimes \ 0.082, 0.105, 0.132$$

$$= (0.170, 0.263, 0.419)$$

$$F_2 = \sum_{j=1}^{m} M_{g2}^{j} \otimes \left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{gi}^{j}\right]^{-1} = (0.260, 0.421, 0.661)$$

$$F_3 = \sum_{j=1}^{m} M_{g3}^{j} \otimes \left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{gi}^{j}\right]^{-1} = (0.192, 0.316, 0.529)$$

$$V(F_1 \geq F_2) = 0.503, V(F_1 \geq F_3) = 0.812,$$
$$V(F_2 \geq F_1) = 1.000, V(F_2 \geq F_3) = 1.000,$$
$$V(F_3 \geq F_1) = 1.000,\ V(F_3 \geq F_2) = 0.719,$$

The weight vectors are determined as follows.

$$d(F_1) = Min V(F_1 \geq F_2, F_3) = Min(0.503, 0.812) = 0.503$$
$$d(F_2) = Min V(F_2 \geq F_1, F_3) = Min(1.000, 1.000) = 1.000$$

$$d(F_3) = MinV(F_3 \geq F_1, F_2) = Min(1.000, 0.719) = 0.719$$
$$W' = (d(F_1), d(F_2), d(F_3))^T = (0.503, 1.000, 0.719)^T$$
$$W = (0.226, 0.450, 0.324)$$

We could say that the weights of O, S, and D criteria according to expert 3 are 0.226, 0.450 and 0.324, respectively. The CRs is smaller than 0.1 for the pair-wise comparison matrix of O, S, and D criteria. This means that judgments of three decision makers are rather consistent and are suitable to use in the next phases. The CRs, which are calculated by helping of Eqs.(10)-(13), are presented in the last column of Table 6. The weights of decision makers, which are indicated in the last column of Table 7, can be rather important while aggregating the judgments of the decision makers.

The outcomes of fuzzy extension of AHP approach are presented in Table 7 so that these results are used in next phase of risk assessment. This paper used fuzzy TOPSIS based on an ABAT to rank the failure modes in the construction firm. The judgments of three decision makers are indicated as linguistic terms in Table 8 about criteria based failure modes. Eqs.(15)-(20) are utilized to obtain the aggregated fuzzy decision matrix of the criteria based failure modes and the results according to homogeneous and heterogeneous groups of decision makers are in Table 9 and Table 10, respectively. β that shows the moderator's dominance in the ABAT, is considered as 0.4 in fuzzy TOPSIS calculation. For example, aggregation calculations for occurrence criteria are indicated in Table 11 detailly. Degree of agreement (S), average degree of agreement (AA), relative degree of agreement (RA), consensus degree coefficient (CC) are introduced in Table 11.

Table 8. The criteria based failure modes with linguistic variables

	E1			E2			E3		
	O	O	O	O	S	D	O	S	D
FM1	M	G	FP	VG	FG	P	G	FP	AP
FM2	M	G	FP	FG	M	FP	M	FP	FG
FM3	M	FP	G	FP	FP	M	VP	P	FG
FM4	M	AG	VP	FP	AG	AP	FP	VG	AP
FM5	G	FP	G	VG	FP	M	M	P	FG
FM6	M	G	M	G	FG	G	G	AG	FG
FM7	G	FP	G	VG	FP	VG	P	VP	AG
FM8	G	FP	G	G	FP	FG	G	VP	G
FM9	G	FP	G	VG	FP	FG	M	M	AG
FM10	M	FG	FG	VP	G	G	FP	M	VP
FM11	G	FP	G	FG	VP	M	FG	FP	FP
FM12	M	M	M	VG	G	G	VG	FP	FP
FM13	G	FP	G	P	FP	VG	M	AP	G
FM14	G	FP	G	VG	VP	FG	VG	FP	FP
FM15	G	FP	G	FP	VP	M	M	FP	FG
FM16	G	FP	G	VG	FP	VG	FG	M	AG
FM17	G	FP	G	VG	FP	G	FG	FP	VG
FM18	M	G	FG	M	FG	VG	VP	G	VP
FM19	M	G	FG	P	FG	FP	FP	G	M
FM20	M	G	M	FP	VG	G	G	FG	VP
FM21	M	G	FG	M	M	P	FP	AG	M
FM22	FG	G	M	P	FG	FP	P	G	FG
FM23	M	G	G	VG	VP	FP	FG	FP	AG
FM24	M	FP	FG	VP	G	P	M	FP	FG
FM25	FG	G	M	FP	M	M	M	M	P
FM26	M	FP	G	VP	P	G	FP	VP	FG
FM27	M	G	M	FG	FG	FP	P	FG	FG
FM28	M	VP	FG	FG	P	FP	FG	AP	FP
FM29	M	FG	M	VG	M	VG	G	FP	FG
FM30	FG	FG	M	FP	G	FG	FP	M	P
FM31	M	M	FP	VP	FG	VP	VP	P	FG
FM32	M	M	G	FG	M	FG	FP	M	G
FM33	FG	G	FP	FP	FG	P	P	AG	P
FM34	M	G	FP	M	M	G	M	M	FP
FM35	M	G	FP	VP	FG	VP	VP	VG	FG

Table 9. Aggregated fuzzy decision matrix with homogeneous groups of decision makers.

	O	S	D
FM1	(0.532,0.700,0.868)	(0.467,0.565,0.663)	(0.166,0.286,0.405)
FM2	(0.364,0.516,0.668)	(0.397,0.547,0.697)	(0.432,0.482,0.532)
FM3	(0.273,0.389,0.506)	(0.306,0.403,0.500)	(0.434,0.582,0.731)
FM4	(0.368,0.466,0.564)	(0.867,0.967,1.000)	(0.032,0.132,0.232)
FM5	(0.532,0.700,0.868)	(0.306,0.403,0.500)	(0.434,0.582,0.731)
FM6	(0.438,0.638,0.838)	(0.622,0.742,0.831)	(0.434,0.582,0.731)
FM7	(0.489,0.656,0.822)	(0.310,0.375,0.440)	(0.741,0.872,0.969)
FM8	(0.500,0.700,0.900)	(0.310,0.375,0.440)	(0.500,0.653,0.806)
FM9	(0.532,0.700,0.868)	(0.368,0.466,0.564)	(0.622,0.742,0.831)
FM10	(0.273,0.389,0.506)	(0.434,0.582,0.731)	(0.385,0.500,0.615)
FM11	(0.500,0.597,0.694)	(0.310,0.375,0.440)	(0.397,0.547,0.697)
FM12	(0.664,0.791,0.918)	(0.397,0.547,0.697)	(0.397,0.547,0.697)
FM13	(0.300,0.500,0.700)	(0.287,0.351,0.415)	(0.592,0.762,0.931)
FM14	(0.708,0.838,0.969)	(0.310,0.375,0.440)	(0.467,0.565,0.663)
FM15	(0.397,0.547,0.697)	(0.310,0.375,0.440)	(0.434,0.582,0.731)
FM16	(0.595,0.714,0.834)	(0.368,0.466,0.564)	(0.741,0.872,0.969)

FM17	(0.595,0.714,0.834)	(0.400,0.450,0.500)	(0.592,0.762,0.931)
FM18	(0.242,0.413,0.583)	(0.500,0.653,0.806)	(0.470,0.550,0.630)
FM19	(0.269,0.418,0.566)	(0.500,0.653,0.806)	(0.400,0.500,0.600)
FM20	(0.397,0.547,0.697)	(0.595,0.714,0.834)	(0.305,0.475,0.645)
FM21	(0.332,0.484,0.636)	(0.558,0.727,0.866)	(0.303,0.453,0.603)
FM22	(0.220,0.375,0.530)	(0.500,0.653,0.806)	(0.400,0.500,0.600)
FM23	(0.518,0.635,0.752)	(0.338,0.450,0.563)	(0.592,0.714,0.806)
FM24	(0.242,0.413,0.583)	(0.430,0.525,0.620)	(0.380,0.475,0.570)
FM25	(0.400,0.500,0.600)	(0.362,0.562,0.762)	(0.238,0.438,0.638)
FM26	(0.273,0.389,0.506)	(0.196,0.315,0.434)	(0.500,0.653,0.806)
FM27	(0.303,0.453,0.603)	(0.500,0.597,0.694)	(0.400,0.500,0.600)
FM28	(0.436,0.534,0.632)	(0.067,0.200,0.333)	(0.432,0.482,0.532)
FM29	(0.532,0.700,0.868)	(0.400,0.500,0.600)	(0.518,0.635,0.752)
FM30	(0.432,0.482,0.532)	(0.434,0.582,0.731)	(0.303,0.453,0.603)
FM31	(0.158,0.288,0.417)	(0.303,0.453,0.603)	(0.342,0.408,0.473)
FM32	(0.400,0.500,0.600)	(0.300,0.500,0.700)	(0.500,0.653,0.806)
FM33	(0.337,0.435,0.533)	(0.622,0.742,0.831)	(0.194,0.347,0.500)
FM34	(0.300,0.500,0.700)	(0.362,0.562,0.762)	(0.430,0.525,0.620)
FM35	(0.158,0.288,0.417)	(0.595,0.714,0.834)	(0.342,0.408,0.473)

Table 10. Aggregated fuzzy decision matrix with heterogeneous groups of decision makers.

	O	S	D
FM1	(0.521,0.692,0.863)	(0.464,0.564,0.664)	(0.166,0.281,0.395)
FM2	(0.358,0.515,0.671)	(0.402,0.548,0.694)	(0.435,0.485,0.535)
FM3	(0.262,0.381,0.499)	(0.295,0.398,0.500)	(0.440,0.585,0.731)
FM4	(0.367,0.467,0.566)	(0.864,0.964,1.000)	(0.033,0.133,0.233)
FM5	(0.517,0.688,0.859)	(0.295,0.398,0.500)	(0.440,0.585,0.731)
FM6	(0.435,0.635,0.835)	(0.637,0.758,0.845)	(0.432,0.577,0.723)
FM7	(0.459,0.629,0.799)	(0.298,0.365,0.432)	(0.738,0.871,0.967)
FM8	(0.500,0.700,0.900)	(0.298,0.365,0.432)	(0.500,0.657,0.813)
FM9	(0.517,0.688,0.859)	(0.365,0.468,0.570)	(0.637,0.758,0.845)
FM10	(0.280,0.396,0.511)	(0.428,0.576,0.725)	(0.367,0.479,0.591)
FM11	(0.500,0.599,0.699)	(0.316,0.380,0.444)	(0.402,0.548,0.694)
FM12	(0.648,0.779,0.909)	(0.394,0.540,0.686)	(0.394,0.540,0.686)
FM13	(0.308,0.508,0.708)	(0.268,0.335,0.401)	(0.585,0.757,0.928)
FM14	(0.703,0.835,0.968)	(0.316,0.380,0.444)	(0.464,0.564,0.664)
FM15	(0.396,0.551,0.706)	(0.316,0.380,0.444)	(0.440,0.585,0.731)
FM16	(0.587,0.704,0.822)	(0.365,0.468,0.570)	(0.738,0.871,0.967)
FM17	(0.587,0.704,0.822)	(0.400,0.450,0.500)	(0.603,0.769,0.934)
FM18	(0.233,0.400,0.566)	(0.500,0.657,0.813)	(0.448,0.529,0.610)
FM19	(0.277,0.423,0.568)	(0.500,0.657,0.813)	(0.398,0.502,0.606)
FM20	(0.400,0.555,0.710)	(0.587,0.704,0.822)	(0.291,0.457,0.623)
FM21	(0.335,0.482,0.630)	(0.579,0.744,0.876)	(0.310,0.459,0.608)
FM22	(0.228,0.380,0.532)	(0.500,0.657,0.813)	(0.402,0.503,0.604)
FM23	(0.513,0.629,0.745)	(0.347,0.460,0.574)	(0.609,0.732,0.819)
FM24	(0.245,0.418,0.590)	(0.428,0.520,0.612)	(0.388,0.480,0.572)
FM25	(0.398,0.502,0.606)	(0.365,0.565,0.765)	(0.231,0.431,0.631)
FM26	(0.280,0.396,0.511)	(0.200,0.314,0.428)	(0.500,0.648,0.795)
FM27	(0.290,0.445,0.600)	(0.500,0.599,0.699)	(0.402,0.503,0.604)
FM28	(0.434,0.533,0.633)	(0.064,0.194,0.324)	(0.433,0.483,0.533)
FM29	(0.521,0.692,0.863)	(0.404,0.499,0.594)	(0.513,0.629,0.745)
FM30	(0.433,0.483,0.533)	(0.428,0.576,0.725)	(0.290,0.445,0.600)
FM31	(0.163,0.295,0.426)	(0.290,0.445,0.600)	(0.351,0.416,0.480)
FM32	(0.396,0.497,0.598)	(0.300,0.500,0.700)	(0.500,0.657,0.813)
FM33	(0.328,0.431,0.534)	(0.637,0.758,0.845)	(0.199,0.349,0.500)
FM34	(0.300,0.500,0.700)	(0.365,0.565,0.765)	(0.428,0.520,0.612)
FM35	(0.163,0.295,0.426)	(0.605,0.725,0.846)	(0.351,0.416,0.480)

Eqs.(21)-(25) are utilized to transform to the weighted normalized fuzzy decision matrix the aggregated fuzzy decision matrix. In this paper occurrence and severity criteria are the cost criteria and detection criterion is a benefit criteria. FPIS A^+ and FNIS A^- are $\tilde{v}^+ = (1,1,1)$ and $\tilde{v}^- = (0,0,0)$ for this benefit criteria, respectively. Eqs. (26)-(29) are used to measure the distance of each failure mode from FNIS and FPIS concurrently. For example, the values of FPIS and FNIS d_1^+, d_1^- for failure mode 1 are computed as follows. Eq.(30) is utilized to calculate CC$_1$ as an example as follows. Similarly, calculations (d_i^+, d_i^-, CC_i) can be done for the other situations.

$$d_1^+ = \sqrt{\frac{1}{3}[(0-0.1723)^2 + (0-0.2288)^2 + (0-0.2854)^2]}$$
$$+\sqrt{\frac{1}{3}[(0-0.1638)^2 + (0-0.1990)^2 + (0-0.2342)^2]}$$
$$+\sqrt{\frac{1}{3}[(1-0.0560)^2 + (1-0.0948)^2 + (1-0.1335)^2]} = 1{,}3403$$

$$d_1^- = \sqrt{\frac{1}{3}[(1-0.1723)^2 + (1-0.2288)^2 + (1-0.2854)^2]}$$
$$+\sqrt{\frac{1}{3}[(1-0.1638)^2 + (1-0.1990)^2 + (1-0.2342)^2]}$$
$$+\sqrt{\frac{1}{3}[(0-0.0560)^2 + (0-0.0948)^2 + (0-0.1335)^2]} = 1{,}6740$$

$$CC_1 = \frac{d_1^-}{d_1^+ + d_1^-} = \frac{1{,}6740}{1{,}3403 + 1{,}6740} = 0{,}5553$$

We specified thirty-five failure modes to define the most important failure mode. Distance based separation scales values d_i^+, d_i^- and closeness coefficient (CC$_i$) of the thirty-five failure modes are showed at Table 12.

Table 11. Aggregation calculations for occurrence criterion

	FM1	FM2	... FM34	FM35
E1	(0.3,0.5,0.7)	(0.3,0.5,0.7)	... (0.3,0.5,0.7)	(0.3,0.5,0.7)
E2	(0.8,0.9,1.0)	(0.5,0.55,0.6)	... (0.3,0.5,0.7)	(0.1,0.2,0.3)
E3	(0.5,0.7,0.9)	(0.3,0.5,0.7)	... (0.3,0.5,0.7)	(0.1,0.2,0.3)
S				
S12	0.60	0.88	... 1.00	0.70
S13	0.80	1.00	... 1.00	0.70
S23	0.80	0.88	... 1.00	1.00
AA				
AA(E1)	0.70	0.94	... 1.00	0.70
AA(E2)	0.70	0.88	... 1.00	0.85
AA(E3)	0.80	0.94	... 1.00	0.85
RA				
RA(E1)	0.318	0.340	... 0.333	0.292
RA(E2)	0.318	0.319	... 0.333	0.354
RA(E3)	0.364	0.340	... 0.333	0.354
CC				
CC(E1)	0.331	0.344	... 0.340	0.315
CC(E2)	0.291	0.292	... 0.300	0.313
CC(E3)	0.378	0.364	... 0.360	0.373
R_{AG}^{HM}	(0.532,0.700,0.868)	(0.364,0.516,0.668)	... (0.300,0.500,0.700)	(0.158,0.288,0.417)
R_{AG}^{HT}	(0.521,0.692,0.863)	(0.358,0.515,0.671)	... (0.300,0.500,0.700)	(0.163,0.295,0.426)

Table 12. The separation scales and the closeness coefficient.

	Heterogeneous				Homogeneous			
	d_i^+	d_i^-	CC$_i$	Rank	d_i^+	d_i^-	CC$_i$	Rank
FM1	1,3403	1,6740	0,5553	2	1,3410	1,6733	0,5551	2
FM2	1,2094	1,8031	0,5985	13	1,2104	1,8021	0,5982	12

FM3	1,0765	1,9367	0,6427	32	1,0818	1,9310	0,6409	32
FM4	1,4454	1,5665	0,5201	1	1,4459	1,5658	0,5199	1
FM5	1,1788	1,8358	0,6090	20	1,1850	1,8292	0,6069	19
FM6	1,2879	1,7281	0,5730	4	1,2821	1,7342	0,5749	4
FM7	1,0539	1,9565	0,6499	33	1,0652	1,9447	0,6461	33
FM8	1,1473	1,8678	0,6195	24	1,1519	1,8628	0,6179	23
FM9	1,1481	1,8632	0,6187	23	1,1562	1,8547	0,6160	22
FM10	1,1811	1,8323	0,6081	19	1,1741	1,8393	0,6104	20
FM11	1,1512	1,8579	0,6174	22	1,1491	1,8602	0,6181	24
FM12	1,2735	1,7407	0,5775	6	1,2777	1,7369	0,5762	6
FM13	1,0419	1,9759	0,6548	34	1,0435	1,9740	0,6542	34
FM14	1,2237	1,7834	0,5931	9	1,2225	1,7844	0,5934	8
FM15	1,1256	1,8867	0,6263	28	1,1233	1,8889	0,6271	29
FM16	1,1133	1,8951	0,6299	30	1,1153	1,8929	0,6293	30
FM17	1,1362	1,8728	0,6224	25	1,1420	1,8673	0,6205	25
FM18	1,1972	1,8188	0,6030	18	1,1930	1,8228	0,6044	18
FM19	1,2121	1,8027	0,5979	12	1,2099	1,8048	0,5987	13
FM20	1,2860	1,7307	0,5737	5	1,2804	1,7362	0,5755	5
FM21	1,2726	1,7444	0,5782	7	1,2701	1,7477	0,5791	7
FM22	1,1988	1,8171	0,6025	16	1,1970	1,8190	0,6031	17
FM23	1,1332	1,8759	0,6234	26	1,1372	1,8721	0,6221	26
FM24	1,1693	1,8443	0,6120	21	1,1712	1,8426	0,6114	21
FM25	1,2320	1,7929	0,5927	8	1,2275	1,7971	0,5942	9
FM26	1,0324	1,9821	0,6575	35	1,0296	1,9857	0,6586	35
FM27	1,1969	1,8151	0,6026	17	1,1993	1,8120	0,6017	15
FM28	1,0933	1,9202	0,6372	31	1,0960	1,9176	0,6363	31
FM29	1,1997	1,8118	0,6016	15	1,2007	1,8110	0,6013	14
FM30	1,2190	1,7943	0,5955	11	1,2178	1,7949	0,5958	11
FM31	1,1266	1,8890	0,6264	29	1,1294	1,8856	0,6254	28
FM32	1,1315	1,8877	0,6252	27	1,1338	1,8851	0,6244	27
FM33	1,2933	1,7202	0,5708	3	1,2898	1,7238	0,5720	3
FM34	1,2062	1,8166	0,6010	14	1,2033	1,8196	0,6019	16
FM35	1,2218	1,7896	0,5943	10	1,2181	1,7932	0,5955	10

3.2. Comparison and discussion

The most important failure mode is defined the FM4 according to the heterogeneous results of fuzzy TOPSIS in Table 10. The risk assessment process is also presented with different approaches, which are fuzzy AHP- fuzzy VIKOR and fuzzy AHP- fuzzy GRA. The ranking orders results of thirty-five failure modes are acquired by utilizing these methods and the results are shown in Table 13. FM4 is selected the most important failure mode.

Table 13. Result comparison with other methods.

	FAHP-FVIKOR	FAHP-FGRA	The proposed approach	
			Heterogeneous	Homogeneous
FM1	3	2	2	2
FM2	22	10	13	12
FM3	33	29	32	32
FM4	1	1	1	1
FM5	11	16	20	19
FM6	5	3	4	4
FM7	29	35	33	33
FM8	12	22	24	23
FM9	15	23	23	22
FM10	26	20	19	20
FM11	25	18	22	24
FM12	4	5	6	6
FM13	34	33	34	34
FM14	2	11	9	8
FM15	30	24	28	29
FM16	17	30	30	30
FM17	13	27	25	25
FM18	19	17	18	18
FM19	14	14	12	13
FM20	8	4	5	5
FM21	7	6	7	7
FM22	18	19	16	17
FM23	24	26	26	26
FM24	27	21	21	21
FM25	16	8	8	9
FM26	35	34	35	35
FM27	21	15	17	15
FM28	31	32	31	31
FM29	9	12	15	14
FM30	20	9	11	11
FM31	28	31	29	28

FM32	32	25	27	27
FM33	6	7	3	3
FM34	23	13	14	16
FM35	10	28	10	10

A sensitivity analysis is implemented to define the effect of β coefficient on the CC$_i$ value, which defines the final ranking of the failure modes. β coefficient begins as 0.1 value and ends as 1 value with increasing 0.1 value so that β coefficient changes as in Fig. 2. It is obvious that distance measure based CC$_i$ is not sensitive for varying β coefficient according to Fig. 2. FM4 remains the most important failure mode in all computations so the firm must consider to eliminate FM4 from work environment firstly. FM26 remains the most unimportant failure mode in all computations so the firm must consider eliminating FM26 from work environment finally.

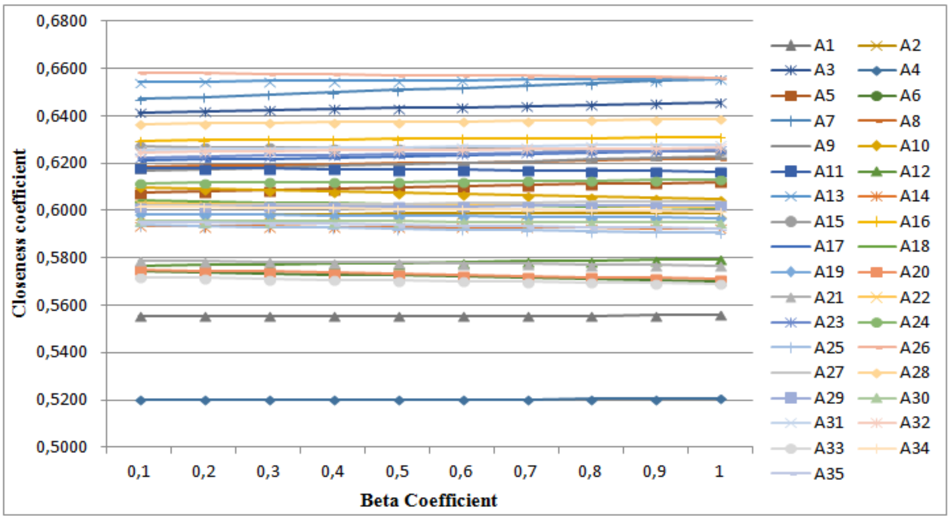

Figure 2. Sensitivity analysis due to exchanging of β value

3.3. Linear programming

The managers of the firm aim to handle limited resources such as budget and time. This study suggests a linear programming including these constraints. The firm planned to allocate 22500-27500 Turkish Liras (TL), 16-24 weeks, 11-15 correctable risks as constraints. We defined 3 different budget values, which are 22500, 25000 and 27500 TL. We defined 3 different times, which are 16, 20 and 24 weeks. We presented 11, 12, 13, 14, and 15 values for number of corrective risk. Cost and time data of failure modes in the construction firm are presented in Table 14.

Table 14. The additional data of the failure modes

FM	c$_i$	b$_i$	t$_i$	FM	c$_i$	b$_i$	t$_i$
FM1	750	1200	2	FM19	150	250	1
FM2	500	800	2	FM20	1000	1250	2
FM3	300	750	1	FM21	200	400	1
FM4	750	1200	2	FM22	250	500	1
FM5	1500	2500	3	FM23	200	400	1
FM6	1000	2000	1	FM24	1500	2500	3
FM7	150	250	1	FM25	1000	1250	2
FM8	150	250	1	FM26	150	350	1
FM9	750	1200	1	FM27	150	250	1
FM10	150	300	1	FM28	250	500	1
FM11	150	300	1	FM29	1000	1250	2
FM12	200	300	1	FM30	250	400	1
FM13	100	200	1	FM31	350	750	1
FM14	750	1000	2	FM32	200	300	1
FM15	500	750	2	FM33	750	1200	2
FM16	200	250	1	FM34	300	600	1
FM17	600	1000	1	FM35	1000	1500	3
FM18	150	250	1				

Notations

x_i : Binary variable, equal to 1 when failure mode i is corrective

c_i : Total cost after corrective action

b_i : Total cost without corrective action

t_i : Necessary time to correct failure mode i

q^* : CC value of failure mode, which has the highest CC value in Table 12

q_i : CC value of failure mode i in Table 12

$|q_i - q^*|$ defines the absolute value of difference between q_i and q^* values. Eq. (31) aim to maximize the impact values of corrective risks as possible.

$$\max \sum_{i=1}^{n} |q_i - q^*|.x_i \qquad (31)$$

st.

$$\sum_{i=1}^{n} c_i.x_i + b_i.(1 - x_i) \leq budget \qquad (32)$$

$$\sum_{i=1}^{n} t_i.x_i \leq time \qquad (33)$$

$$\sum_{i=1}^{n} x_i \leq number\ of\ correctable\ risk \qquad (34)$$

$$x_i = 0 - 1 \qquad (35)$$

Eq. (32) means the limited budget. Eq. (33) presents the limited time. Eq. (34) shows maximum number of correctable risk. Eq. (35) shows binary variable.

Table 15 presents the results of linear programming in GAMS software program. Though FM12, FM14, FM20, FM25, FM33 and FM35 are one of the first ten failures according to the result of the proposed method with heterogeneous groups of decision makers. FM4 is the most important failure mode for the results of FAHP-FTOPSIS method and mathematical model.

Table 15. The results of linear programming

Budget	Time	NOCR	CR
22500	16	11	1,4,5,6,9,17,21,24,31,34
22500	16	12	3,4,5,6,9,17,21,22,24,30,34
22500	16	13	3,4,5,6,9,17,21,22,24,30,34
22500	16	14	3,4,5,6,9,17,21,22,24,30,34
22500	16	15	3,4,5,6,9,17,21,22,24,30,34
22500	20	11	1,2,4,5,6,9,12,20,21,24,33
22500	20	12	1,4,5,6,9,12,20,21,24,30,33,34
22500	20	13	1,4,5,6,9,12,20,21,24,30,33,34
22500	20	14	1,4,5,6,9,12,17,19,20,21,30,31,33,34
22500	20	15	1,4,5,6,9,12,17,19,21,22,27,30,31,33,34
22500	24	11	1,4,5,6,12,14,20,21,24,33,35
22500	24	12	1,4,5,6,12,14,20,21,24,25,33,35
22500	24	13	1,2,4,5,6,12,14,20,21,24,25,30,33
22500	24	14	1,4,5,6,12,14,19,20,21,24,25,27,30,33
22500	24	15	1,2,4,5,6,9,12,14,19,20,21,25,30,33,34
25000	16	11	1,4,6,12,14,19,20,21,27,30,33
25000	16	12	1,4,6,12,19,20,21,22,27,30,33,34
25000	16	13	1,4,6,10,12,18,19,21,22,27,30,33,34
25000	16	14	1,4,6,10,12,18,19,21,22,27,30,33,34
25000	16	15	1,4,6,10,12,18,19,21,22,27,30,33,34
25000	20	11	1,4,6,12,14,20,21,25,30,33,35
25000	20	12	1,4,6,12,14,19,20,21,25,30,33,35
25000	20	13	1,2,4,6,12,14,19,20,21,25,27,30,33
25000	20	14	1,4,6,12,14,19,20,21,22,25,27,30,33,34
25000	20	15	1,4,6,10,12,14,18,19,20,21,22,27,30,33,34
25000	24	11	1,4,6,12,14,20,21,25,30,33,35
25000	24	12	1,2,4,6,12,14,20,21,25,30,33,35
25000	24	13	1,2,4,6,12,14,19,20,21,25,30,33,35
25000	24	14	1,2,4,6,12,14,19,20,21,25,29,30,33,35
25000	24	15	1,2,4,6,12,14,19,20,21,25,27,30,33,34,35
27500	16	11	1,4,6,12,14,19,20,21,27,30,33
27500	16	12	1,4,6,12,19,20,21,22,27,30,33,34
27500	16	13	1,4,6,10,12,18,19,21,22,27,30,33,34
27500	16	14	1,4,6,10,12,18,19,21,22,27,30,33,34
27500	16	15	1,4,6,10,12,18,19,21,22,27,30,33,34
27500	20	11	1,4,6,12,14,20,21,25,30,33,35
27500	20	12	1,4,6,12,14,19,20,21,25,30,33,35
27500	20	13	1,2,4,6,12,14,19,20,21,25,27,30,33
27500	20	14	1,4,6,12,14,19,20,21,22,25,27,30,33,34
27500	20	15	1,4,6,10,12,14,18,19,20,21,22,27,30,33,34

27500	24	11	1,4,6,12,14,20,21,25,30,33,35
27500	24	12	1,2,4,6,12,14,20,21,25,30,33,35
27500	24	13	1,2,4,6,12,14,19,20,21,25,30,33,35
27500	24	14	1,2,4,6,12,14,19,20,21,25,29,30,33,35
27500	24	15	1,2,4,6,12,14,19,20,21,25,27,30,33,34,35

*Corrective risks: CR, Number of corrective risk: NOCR

The managers should ensure a safe workplace for their employess. This study presents an occupational health and safety approach by using fuzzy logic, multi criteria decision making methods and linear programming. The firms can pay compensation in the result of occupational accident. Firms should make necessary precautions by performing a risk evaluation. This study handles fuzzy logic and multi criteria decision making methods for a risk evalution. This approach can be insufficient due to budget and time constraints of the firm. This paper examines a linear programming for these constraints.

4. Conclusion

Fuzzy extension of AHP has little calculation time and is very simpler than other fuzzy AHP procedures. The weights of O, S and D criteria are calculated by utilizing fuzzy extension of AHP. The failure modes are ranked in uncertain environment by using fuzzy TOPSIS. Classical TOPSIS method uses crisp number in evaluation procedures and this situation can generate information loss in ambiguous environment. Fuzzy TOPSIS method, which uses linguistic variables in uncertain environment, is proposed to overcome this drawback in this paper.
The weights obtained from fuzzy extension of AHP are employed in fuzzy TOPSIS computations and the thirty-five failure modes are ranked for defining the most important failure mode. The suggested model is implemented within a construction firm and shows that it can be efficiently utilized in risk evaluation problem. This paper presented a linear programming due to some limitations of the firm. The most important failure mode is defined the FM4 according to the homogeneous and heterogeneous results of fuzzy TOPSIS. The proposed approach is compared with different approaches. FM4 is selected the most important failure mode according to the results of the handled methods.

Several decision makers participate to evaluate the problem in group decision making so that each decision maker can have the prejudice about the problem. This situation can cause inaccurate solutions and inherently damage to the firm. A group judgment is usually preferred to decrease the siding and to prevent the prejudice in group decision making. Each decision maker can have different judgments about a selection process due to their experience and knowledge. The weights of the decision makers, which depend on characteristics of the decision makers, are important to achieve a group judgment. Inaccurate weights of the decision makers generate the wrong group judgment and inherently damage to the firm. This study proposes an ABAT presented by Olcer and Odabasi [17] to cope with this drawback. The quality and efficiency of the proposed method is considered by helping of a sensitivity analysis and other comparision methods. The results of FMEA can’t meet the demands of the firms in long period. This paper proposed a mathematical model to overcome this limitation. This mathematical model is solved in GAMS software program. The presented method needs some experts about risk evaluation area. It takes time and is very difficult. The constraints of linear programming can be insufficient to handle a comprehensive analysis. This paper handles Type 1 fuzzy numbers, which present crisp membership degrees, to define the judgments of decision makers. Interval type-2 fuzzy numbers handles more ambiguity of the real life world. In future paper, interval type 2 fuzzy numbers based multi-criteria decision making methods can be considered for the risk assessment. Furthermore, additional constraints for linear programming can be handled in future papers.

Statement of Research and Publication Ethics

The study is complied with research and publication ethics

References

[1] J.B. Bowles and C.E. Peláez, “Fuzzy logic prioritization of failures in a system failure mode, effects and criticality analysis,” *Reliability Engineering & System Safety*, vol. 50, no. 2, pp. 203–213, 1995.

[2] N.R. Sankar and B. S. Prabhu, “Modified approach for prioritization of failures in a system failure mode and effects analysis,” *International Journal of Quality & Reliability Management*, vol. 18, no. 3, pp. 324–336, 2001.

[3] B. Efe, M. Kurt and Ö. F. Efe, “An Integrated Intuitionistic Fuzzy Set And Mathematical Programming Approach For An Occupational Health And Safety Policy,” *Gazi University Journal of Science*, vol. 30, no. 2, pp. 73-95, 2017.

[4] H. C. Liu, L. Liu and Q. L. Lin, “Fuzzy failure mode and effects analysis using fuzzy evidential reasoning and belief rule-based methodology,” *IEEE Transactions on Reliability*, vol. 62, no. 1, pp. 23-36, 2013.

[5] M. Ulu and H. Şahin, “An integrated approach for fire extinguishers selection with DEMATEL and TODIM methods,” *Business & Management Studies: An International Journal*, vol. 9, no. 4, pp. 1696-1707, 2021.

[6] T. L. Jee, K. M. Tay and C. P. Lim, “A New Two-Stage Fuzzy Inference System-Based Approach to Prioritize Failures in Failure Mode and Effect Analysis,” *IEEE Transactions on Reliability*, vol. 64, no. 3, pp. 869-877, 2015.

[7] B. Efe, M. A. Yerlikaya and Ö. F. Efe, “İş Güvenliğinde Bulanık Promethee Yöntemiyle Hata Türleri ve Etkilerinin Analizi: Bir İnşaat Firmasında Uygulama,” *Gümüşhane Üniversitesi Fen Bilimleri Enstitüsü Dergisi*, vol. 6, no. 2, pp. 126-137, 2016.

[8] Q. Zhou and V. V. Thai, “Fuzzy and grey theories in failure mode and effect analysis for tanker equipment failure prediction,” *Safety Science*, vol. 83, pp. 74-79, 2016.

[9] H. C. Liu, J. X. You, P. Li and Q. Su, “Failure mode and effect analysis under uncertainty: An integrated multiple criteria decision making approach,” *IEEE Transactions on Reliability*, vol. 65, no. 3, pp. 1380-1392, 2016.

[10] O. Mohsen and N. Fereshteh, “An extended VIKOR method based on entropy measure for the failure modes risk assessment – A case study of the geothermal power plant (GPP),” *Safety Science*, vol. 92, pp. 160-172, 2017.

[11] Z. P. Tian, J. Q. Wang and H. Y. Zhang, “An integrated approach for failure mode and effects analysis based on fuzzy best-worst, relative entropy, and VIKOR methods,” *App. S. Comp.* vol. 72, pp. 636–646, 2018.

[12] R. Fattahi and M. Khalilzadeh, “Risk evaluation using a novel hybrid method based on FMEA, extended MULTIMOORA, and AHP methods under fuzzy environment,” *Saf. Sci.* vol. 102, pp. 290–300, 2018.

[13] B. Efe, “Analysis of operational safety risks in shipbuilding using failure mode and effect analysis approach,” *Ocean Engineering*. vol. 187, 106214, 2019.

[14] M. Yazdani, M. R. Abdi, N. Kumar, M. Keshavarz-Ghorabaee and F. T. Chan, “Improved decision model for evaluating risks in construction projects,” *J. Cons. Eng. Man.* vol.145, no. 5, 04019024, 2019.

[15] M. Yazdi, A. Nedjati, E. Zarei and R. Abbassi, “A reliable risk analysis approach using an extension of best-worst method based on democratic-autocratic decision-making style,” *Journal of Cleaner Production*, vol. *256*, 120418, 2020.

[16] J. Zhu, B. Shuai, G. Li, K. S. Chin and R. Wang, “Failure mode and effect analysis using regret theory and PROMETHEE under linguistic neutrosophic context,” *Journal of Loss Prevention in the Process Industries*, 104048, 2020.

[17] A. I. Olcer and A. Y. Odabasi, “A new fuzzy multiple attributive group decision making methodology and its application to propulsion/manoeuvring system selection problem,” *European Journal of Operational Research*, vol. 166, pp. 93–114, 2005.

[18] T. L. Saaty, “The analytic hierarchy process,” New York: McGraw-Hill, 1980.

[19] D. Y. Chang, “Applications of the extent analysis method on fuzzy AHP,” *European Journal of Operational Research*, vol. 95, no. 3, pp. 649–655, 1996.

[20] B. Efe, “An integrated fuzzy multi criteria group decision making approach for ERP system selection,” *Applied Soft Computing*, vol. *38*, pp. 106-117, 2016.

[21] K. Shaw, R. Shankar, S. S. Yadav and L. S. Thakur, “Supplier selection using fuzzy AHP and fuzzy multi-objective linear programming for developing low carbon supply chain,” *Expert Systems with Applications*, vol. 39, pp. 8182-8192, 2012.

[22] Y. Deng and F. T. S. Chan, “A new fuzzy dempster MCDM method and its application in supplier selection,” *Expert Systems with Applications*, vol. 38, no. 8, pp. 9854–9861, 2011.

[23] J. J. Wang and D. L. Yang, “Using a hybrid multi-criteria decision aid method for information systems outsourcing,” *Computers & Operations Research*, vol. 34, no. 12, pp. 3691 – 3700, 2007.

[24] Z. Xu, “An automatic approach to reaching consensus in multiple attribute group decision making,” *Computers & Industrial Engineering*, vol. 56, pp. 1369–1374, 2009.

[25] S. M. Chen, “Aggregating fuzzy opinions in the group decision-making environment,” *Cybernetics and Systems*, vol. 29, pp. 363–376, 1998.

[26] C. L. Hwang and K. Yoon, “Multiple attributes decision making methods and applications,” Berlin: Springer, 1981.

[27] F. R. L. Junior and L. Osiro L. C. R. Carpinetti “A comparison between Fuzzy AHP and Fuzzy TOPSIS methods to supplier selection,” *Applied Soft Computing*, vol. 21, pp. 194–209, 2014.

[28] K. Dhalmahapatra, A. Garg, K. Singh, N. F. Xavier and J. Maiti, “An integrated RFUCOM–RTOPSIS approach for failure modes and effects analysis: A case of manufacturing industry,” *Reliability Engineering & System Safety*, 108333, 2022.