

OPTIMAL HEDGE RATIOS FOR TURKISH MORTALITY

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Abstract

The increase in life expectancy of individuals poses a risk for insurance companies. If people live longer than anticipated, insurance companies make losses on their annuity books. The risk that survivor rates might be higher than anticipated is called the longevity risk.

In this paper, a pension plan whose aim is to hedge its longevity risk with longevity hedging instrument such as vanilla swap has been considered. We find the optimal hedge ratio which is defined as the number of units held of the hedging instrument. The optimal hedge ratio is calculated under minimum variance hedging and exponential utility. For the hedge ratio we need the value of the swap. In order to price the swap, we modelled Turkish mortality by using the Lee-Carter model and the Cairns-Blake-Dowd model. We find optimal hedge ratios for female and male populations of Turkey for different mortality models and different risk criteria. The analysis showed that the hedge ratios do not change significantly for different mortality models. However, as we change the risk criteria we observe quite different optimal hedge ratios.

Keywords: Longevity risk, hedge ratios, vanilla survivor swap, Turkey life tables.

Jel Codes: G12, G22, G23

TÜRKİYE'DEKİ ÖLÜMLÜLÜK İİN OPTİMAL KORUNMA ORANLARI

Öz

Bireylerin beklenen yařam sürelerinin artması sigorta řirketleri için büyük bir risk oluřturmaktadır. Tahmin edilenden daha uzun süre yapılması gereken ödemeler sigorta řirketlerinin anüite hesaplarında kayıplara yol açmaktadır. Bireylerin beklenenden daha uzun süre yařamaları riski uzun ömürlülük riski olarak adlandırılmaktadır. Bu alıřmada, uzun ömürlülük riskine karşı vanilla yařam swapları ile korunma sađlamak isteyen bir emeklilik planı ele alınmıřtır. Uzun ömürlülük riskinden korunma sađlanması için

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elde bulundurulması gereken swap sözleşmesi miktarı minimum varyans korunma oranı ve üstel fayda varsayımı altında hesaplanmıştır. Korunma oranlarının hesaplanabilmesi için swapın değerinin hesaplanması gerekmektedir. Swapın değeri, Türkiye verisi kullanılarak Lee-Carter ve Cairns-Blake-Dowd ölümlülük modelleri ile hesaplanmıştır. Korunma oranları farklı ölümlülük modelleri ve farklı risk kriterleri dik-kate alınarak kadın ve erkek popülasyonlar için ayrı ayrı hesaplanmıştır. Elde edilen sonuçlar, korunma oranlarının farklı ölümlülük modelleri için önemli ölçüde değişmediği ancak farklı risk kriterleri için değiştiğini göstermektedir.

Anahtar Kelimeler: Uzun ömürlülük riski, korunma oranları, vanilla yaşam swapı, Türkiye hayat tabloları.

Jel Kodları: G12, G22, G23

Introduction

The longevity risk is the risk that individuals may live longer than anticipated. Pension providers commit periodical payments to policy holders from their retirement to death. The increase in life expectancy poses a risk for insurance companies. Payments that have to be paid over a longer period cause losses on pension providers' annuity book. As more pension plans close to future accrual plan, sponsors face to choice of selling pension liabilities or retaining them on their books and managing them. So that pension plans and insurers have begun to focus on the measurement and management of longevity risk.

The world's first publicly announced longevity swap took place in April 2007. It was between Swiss Re and Friends' Provident, a UK life assurer. It was a pure longevity risk transfer and was not tied to another financial instrument or transaction. Friends' Provident retains administration of policies. Swiss Re makes payments and assumes longevity risk in exchange for an undisclosed premium. However, this swap was not a capital market instrument. It was an insurance contract (Blake, Cairns and Dowd, 2008).

A buy-out market for pension liabilities and a longevity swap market to help sponsors hedge longevity risk as a part of an asset-liability strategy were developed in the UK. An insurance company, in exchange for a buy-out fee, takes over the plan liabilities and assets on the responsibility for making the pension payments until the last plan member dies with a buy-out transaction (Cairns et al., 2012:217-235).

In this paper we consider the longevity risk and hedge its risk using longevity swaps. We focus on the optimal hedge ratios for a longevity swap. An asset-liability strategy includes the use of longevity swaps. However, there are different types of longevity swaps and different levels of basis risk. We use vanilla survivor swap in this study. Optimal hedge ratios are calculated by using fixed and floating legs of the swap.

Coughlan et al. (2011) proposed a framework for (1) developing an informed understanding of the basis risk, (2) calibrating the hedging instrument and (3) evaluating the hedge ratios and hedging effectiveness. They used model-free bootstrapping approach to the evaluation of the hedge ratios and hedging effectiveness. Cairns et al. (2012) used Coughlan et al. (2011) framework for the calibrating the hedging instrument and evaluating the hedge effectiveness, however, different from this study, they used a model-based simulation approach instead of the model-free bootstrapping approach.

Cairns (2013: 621-648) focused on quantification of the optimal hedge ratios and hedge effectiveness. They investigated the robustness of these quantities relative to inclusion of recalibration risk, parameter uncertainty and Poisson risk.

In this paper we use the hedging objectives described in Cairns et al. (2013). We compare the hedge ratios for different mortality models and different hedging objectives by using Turkish mortality data. To the best of the authors' knowledge the hedge ratios for Turkish mortality calculated in this paper have not been published in the literature before. The paper is organized as follows. Section 1 introduces two stochastic mortality models. In section 2, vanilla survivor swaps are explained. In chapter 3, optimal hedge ratio is defined and minimum variance risk measure and exponential utility function are examined. Section 4 presents a case study for calculating optimal hedge ratio for Turkish mortality and Section 5 concludes.

I. Stochastic Mortality Models

Over the twentieth century there has been a great improvement in mortality. Therefore the rates of improvement in different years and at different ages become unpredictable. Since the early 1990s various stochastic models have been developed to analyze and forecast these mortality improvements.

In this paper, we use the Lee-Carter (LC) model and the Cairns-Blake-Dowd (CBD) model for forecasting survivor process.

I.1. The Lee-Carter Model

The Lee-Carter model is a simple bilinear model with the variables x (age) and t (time) for forecasting mortality rates and defined as:

$$\ln(m_{x,t}) = a_x + b_x k_t + e_{x,t} \quad (1)$$

where

$m_{x,t}$: Central death rate at age x and in year t ,

a_x : Average age-specific pattern of mortality,

b_x : Age-specific parameter,

k_t : A time-trend index of general mortality level,

$\varepsilon_{x,t}$: The residual term at x and time t , which has no long-term trend.

The time-varying index k_t captures the main time trend on the logarithmic scale in mortality rates at all ages. The age component b_x tells us which rates decline rapidly or which rates decline slowly in response to changes in k_t (Lee and Carter, 1992:659-675).

In order to obtain a unique solution for the system of equations of the model, a_x is set equal to the averages over time of the $\ln(m_{x,t})$, the square values of sum to unity, and k_t values sum to zero, these are of the forms:

$$a_x = \frac{1}{T} \sum_t \ln(m_{x,t}), \sum_x b_x^2 = 1, \sum_t k_t = 0 \quad (2)$$

Lee and Carter (1992:659-675) applied a two-stage estimation method. In the first stage, singular value decomposition (SVD) is applied to the matrix of $\{\log(m_{x,t}) - a_x\}$ to obtain estimates of b_x and k_t . In the second stage, the time series of k_t is re-estimated. Lee and Carter noticed that once k_t and b_x have been estimated, the observed total number of deaths $D_t \equiv \sum_x D_{x,t}$ is not guaranteed to be equal to fitted number of deaths. Therefore, they made a second stage estimation of k_t by finding a value that makes the observed number of deaths equal to the predicted number of deaths (Wang, 2007).

1.2. The Cairns-Blake-Dowd Model

Cairns et al. (2006:684-718) proposed a two-factor stochastic mortality model and show how the calibrated model can be used to price mortality-linked financial instruments. Cairns, Blake and Dowd (2006: 684-718) looked at the algorithm of the ratio of the mortality rate to the survival rate in their model. Different from the natural logarithm as in the LC and Renshaw and Haberman model, they used the logistic function. The model is designed for modelling mortality at higher ages and for modelling longevity risk in pensions and annuities.

The model can be described by a parameter related to the year of interest t , and the interaction between another year-related parameter and the deviation of the age of the individual x from the average age in the population.

$q(x, t)$ is the realized mortality rate in year t and the CBD model is defined as follows:

$$q(x, t) = \frac{\exp[A_1(t+1) + A_2(t+1)(x - \bar{x})]}{1 + \exp[A_1(t+1) + A_2(t+1)(x - \bar{x})]} \quad (3)$$

where

$A_1(t)$ and $A_2(t)$ are discrete-time stochastic processes and \bar{x} is the mean of the range of ages used in the calibration of the model. Assuming that $A(t) = (A_1(t), A_2(t))$ is a 2-dimensional random walk with drift under real world measure P

$$A(t) = A(t-1) + \lambda + CZ(t) \quad (4)$$

where

λ is a constant 2×1 vector of drift parameters, C is a constant 2×2 lower triangular Cholesky square matrix of the covariance matrix V (that is $V = CC^T$), and $Z(t)$ is an i.i.d. sequence of random variables with a 2-dimensional standard normal distribution under P .

In order to estimate the mean and the variance-covariance matrix in equation (3), the mortality rates for each t are transformed from $q(x, t)$ to

$$\log\left(\frac{q(x, t)}{p(x, t)}\right) = \kappa_1^t + \kappa_2^t(x - \bar{x}) \quad (5)$$

The parameters in equation (5) are estimated by using the least squares method (Cairns et al., 2006: 684-718).

2. Vanilla Survivor Swaps (VSS)

A survivor swap is an agreement to exchange one or more cash flows in the future based on the outcome of at least one (random) survivor or mortality index. Survivor swaps are similar to the reinsurance contracts. Both involve swaps of anticipated payments for actual payments and both could be used for similar purposes. However, there are differences between survivor swaps and insurance contracts. Particularly, survivor swaps are not insurance contracts in the legal sense of the term and therefore not affected by some of the distinctive legal features of insurance contracts. Instead, mortality swaps are subjected to the requirements of securities law. Similarly, an insurance contract requires the policyholder to have an insurable interest, but a mortality swap does not.

It is known from industry contracts that some insurance companies have already entered into survivor swaps on an over-the-counter (OTC) basis. The counterparties are usually life

companies and some investment banks are also interested in swap transactions. The attractions of these arrangements are the obvious ones of risk mitigation and capital release for those laying off longevity risk, and low-beta risk exposures for those taking it on (Blake, Cairns and Dowd, 2006:153-228).

We consider vanilla survivor swaps as the hedging instrument. A vanilla survivor swap (VSS) involves the exchange of a single preset payment for a single random mortality-dependent payment periodically until the swap matures in its period. As in Dowd et al. (2006:51-557), $H(t)$ be the preset payments which are equal to the product of the rate anticipated in the current mortality table for the reference population, and $S(t)$ is a random amount which connects to the survivor index at a future time t . We need to model the mortality first to price the VSS.

The values of fixed and floating payments of swap are calculated as follows;

$$a(T, x) = \exp(-Tr) \{E_Q[S(T, x)] | M_t\} \quad (6)$$

where M_t represents the filtration generated by the evolution of the force of mortality up to and including time t . It follows that, given M_t , $S(t, x)$ represents the probability that an individual aged x at time 0 survived to age $x+t$. $a(T, x)$ is the value of a level annuity of 1 per annum payable annually of an individual aged x at time 0 to swap maturity time T under Q risk neutral probability measure.

3. Optimal Hedge Ratio

Portfolio managers, individual investors and corporations use different hedging techniques to reduce their exposure to various risks. Hedging against investment risk means that using instruments in the market strategically to offset the risk of any adverse price movements. In other words, investors hedge one investment by making another (Smith et al., 1985:391-405).

An investor can hedge her whole portfolio or some portion of it in hedging. The hedge ratio is the size of the survivor swap contract relative to the cash transaction. In other words optimal hedge ratio shows the number of units held of the hedging instrument.

In this study, we calculate optimal hedge ratios for VSS transaction by using hedging objectives such as minimum variance hedging and utility-based hedging with exponential utility function.

3.1. Optimal Hedge Ratio under Minimum Variance Hedge

Suppose the hedger has a liability with value $H(t)$ and a hedging instrument has a value $S(t)$ at time t . Our hedging portfolio consists of the liability plus h units (the hedge ratio or number of units held of the hedging instrument) of $S(t)$ and its value at t is $P(h) = H + hS$. The hedger

wishes to minimise the variance of $P(h)$. By solving this, the optimal hedge ratio is found as follows (Cairns, 2013:621-648):

$$h = \frac{Cov(H, S)}{Var(S)} \tag{7}$$

3.2. Optimal Hedge Ratio under Exponential Utility Hedge

In this subsection, we assume that the hedger has an exponential utility function and wishes to maximize her expected utility. To do this, below assumptions must be made.

- It is one-period model.
- Initial wealth at time 0 is W_0 .
- The hedger has a liability to pay an amount $H(t)$ at time t .
- The hedger invests from time 0 to 1 in the hedging instrument, that is vanilla survivor swap and its value is $S(t)$ at time t .
- $(H(t), S(t))$ is assumed to be bivariate normal.

$$\begin{aligned} S(t) &= \mu_1 + \sigma_{11}Z_1 \\ H(t) &= \mu_2 + \sigma_{21}Z_1 + \sigma_{22}Z_2 \end{aligned}$$

Z_1 and Z_2 are independent standard normal random variables.

$$\sigma_{11}^2 = Var(S(t)), \sigma_{11}\sigma_{21} = Cov(S(t), H(t)) \text{ and } \sigma_{21}^2 + \sigma_{22}^2 = Var(H(t)).$$

The hedger has an exponential utility function:

$$u(w) = -\exp(-\gamma w) \tag{8}$$

where $\gamma > 0$.

It follows that

$$E[u(W(h))] = -\exp[-\gamma(w_0 e^r - \mu_2 - h(\mu_1 - S(t)e^r))] + \frac{1}{2}\gamma^2((h\sigma_{11} - \sigma_{21})^2 + \sigma_{22}^2) \tag{9}$$

The hedger wishes to maximize her expected utility and this is equivalent to minimizing

$$f(h) = h(S(t)e^r - \mu_1) + \frac{1}{2}\gamma(h\sigma_{11} - \sigma_{21})^2 \tag{10}$$

Then the optimal hedge ratio is,

$$h^* = \frac{\sigma_{21}}{\sigma_{11}} \left(1 - \frac{(S(t)e^r - \mu_1)}{\gamma\sigma_{11}\sigma_{21}} \right) \quad (11)$$

γ is the hedger's risk aversion parameter. As risk aversion increases, the $\frac{(S(t)e^r - \mu_1)}{\gamma\sigma_{11}\sigma_{21}}$ term becomes smaller. For extremely large values of risk aversion, it will approach to zero. In other words the higher γ is, the higher is the hedger's risk aversion and the hedge ratio will be closer to the minimum variance hedge ratio, $\frac{\sigma_{21}}{\sigma_{11}}$ (Cairns, 2013:621-648).

4. Case Study

We use Turkish Mortality Tables constructed in 2010 by using the data from 1938 to 1995 to calculate the hedge ratios.

Starting from age 2, five year age groups are arranged, yet to increase the reliability of the data of 80 year old and older is excluded.

We fitted the LC and the CBD models to the Turkish female and male data. We forecast future mortality rates for male and female population and value the survivor swap.

The floating payments of the swap $S(t)$ are calculated by using projected survivor rates which are estimated via the LC model and the CBD model.

The fixed payments of the swap $H(t)$ are calculated with the standard present value method based on "2010 Turkey Annuity Table". Interest rate is taken as 0.04. The swap is priced for ten years maturity as in Değirmenci (2014) and Değirmenci and Şahin (2015:89-112).

Estimated parameter values of the LC model for male and female populations are shown in Figure 1.

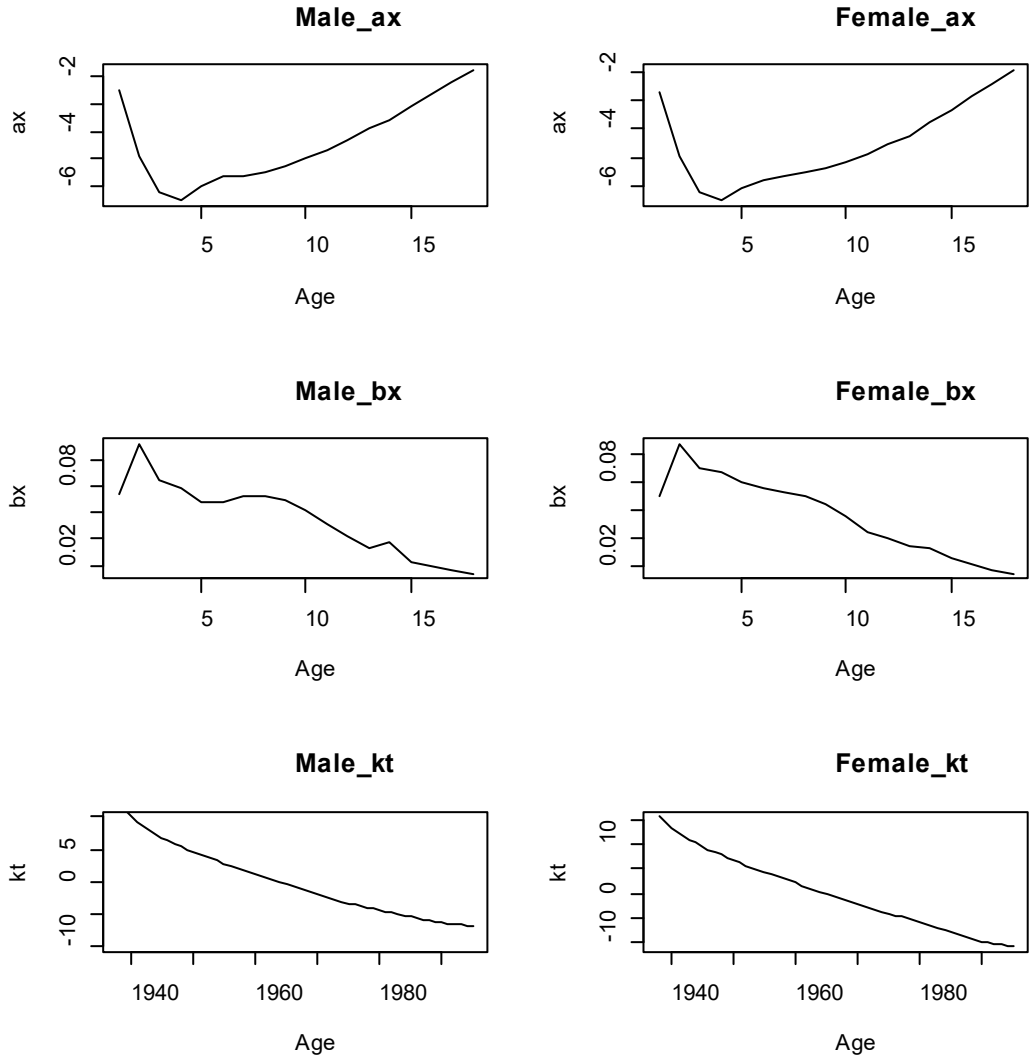


Figure 1: Estimated Parameter Values for the LC Model.

Parameter a_x represents the general age shape of mortality. Figure 1 shows that the mortality has an upward trend in general for both female and male populations, whereas the younger ages have lower mortality and the older ages have higher mortality. Parameter b_x reflects the tendency of mortality at age x to change as the general level of mortality index changes. The figure shows that b_x values are positive which indicates that the mortality is decreasing for all ages for female and male populations. b_x reaches the highest value for the young age groups and this implies that

the mortality is decreasing faster for this specific age group. The mortality index k_t shows the changes in mortality over time.

Projected k_t values that are obtained by using LC model from 1995 to 2020 are shown in Figure 2.

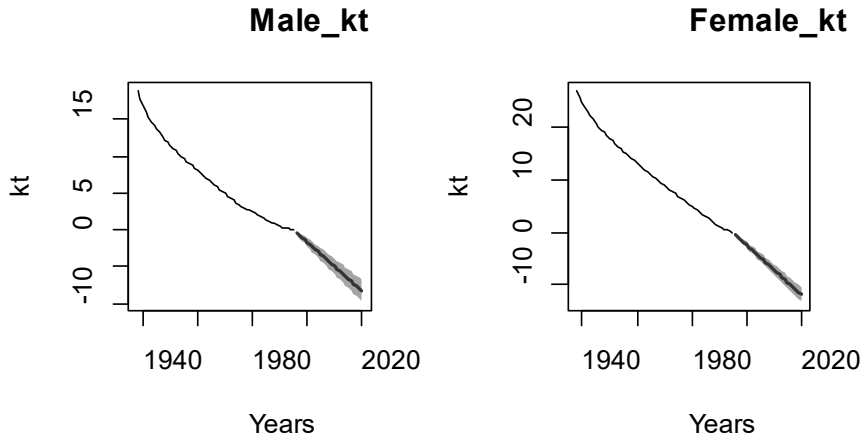


Figure 2: Projected k_t for LC Model.

According to Figure 2, the mortality is decreasing for both males and females over time.

Estimated values that are obtained by using CBD model from 1938 to 1995 are shown in Figure 3 and Figure 4. In the CBD model the first parameter affects mortality rate dynamics at all ages in the same way. The downward trend in $\kappa_t^{(1)}$ shows general improvements in mortality over time at all ages. A reduction in $\kappa_t^{(1)}$ represents an overall mortality improvement.

The second factor $\kappa_t^{(2)}$ shows the steepness of the logit-transformed mortality curve. The increasing trend in $\kappa_t^{(2)}$ reflects that the mortality at younger ages improves faster than older ages (Cairns et al., 2006).

Moreover, in any given year in the future, we expect that mortality rates for older cohorts are higher than those for younger cohorts. This criterion requires $\kappa_t^{(2)}$ remain positive.

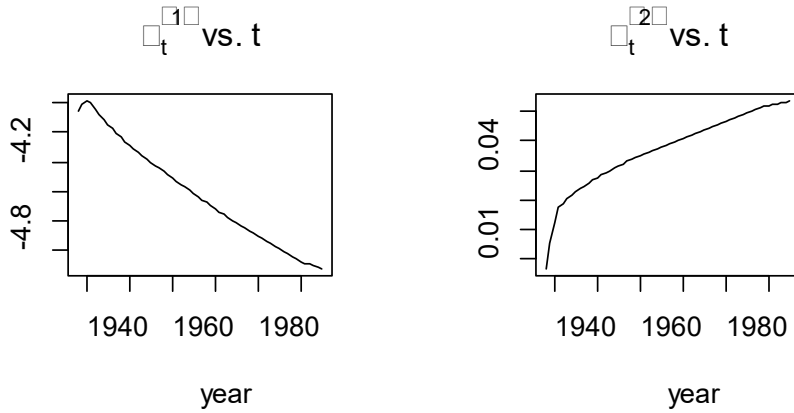


Figure 3: Estimated κ_t values for male population.

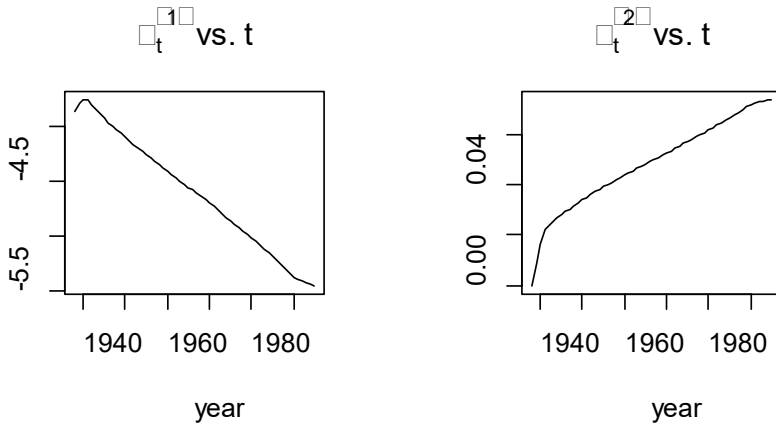


Figure 4: Estimated κ_t values for female population.

Projected κ_t values that are obtained by using CBD model from 1995 to 2010 are shown in Figure 5 and Figure 6. As expected, the projected $\kappa_t^{(1)}$ values are decreasing over time at all ages and the projected $\kappa_t^{(2)}$ values remain positive.

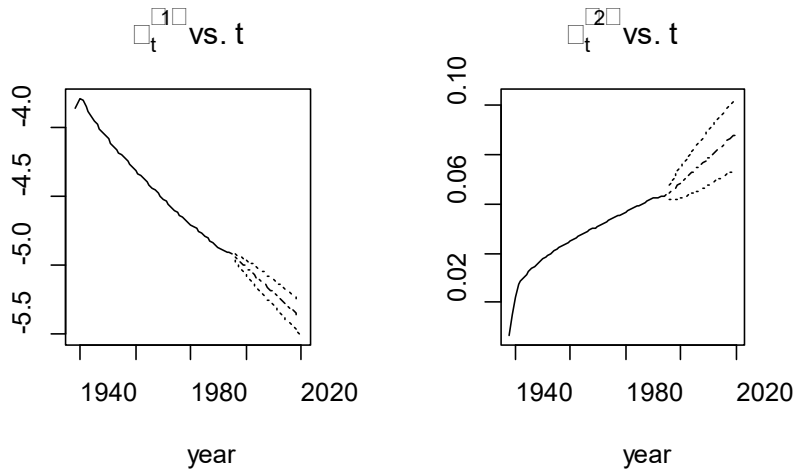


Figure 5: Projected K_t values for male population.

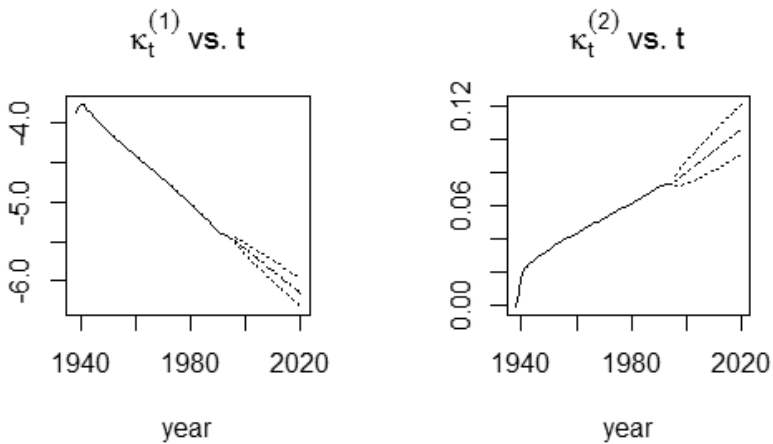


Figure 6: Projected K_t values for female population.

Projected mortality rates for LC and CBD Model with male and female populations are shown in Figure 7. According to the LC model the mortality rate is decreasing and the survivorship is increasing. However in the CBD model the mortality rate is increasing and the survivorship is decreasing for both male and female populations for an individual who aged 65. According to the

CBD model Turkish mortality is decreasing until age 55. In other words, the mortality rate at ages > 55 is rising over time than lowering. In addition, as in Cairns, Blake and Dowd (2006), the mortality curve in each calender year must increase at higher ages.

The LC model produces $m(x, t)$ whereas the CBD model produces $q(x, t)$. For a valid comparison we need to convert $q(x, t)$ to $m(x, t)$ by using the assumption $\mu(x, t) = m(x, t) = -\log(1 - q(x, t))$.

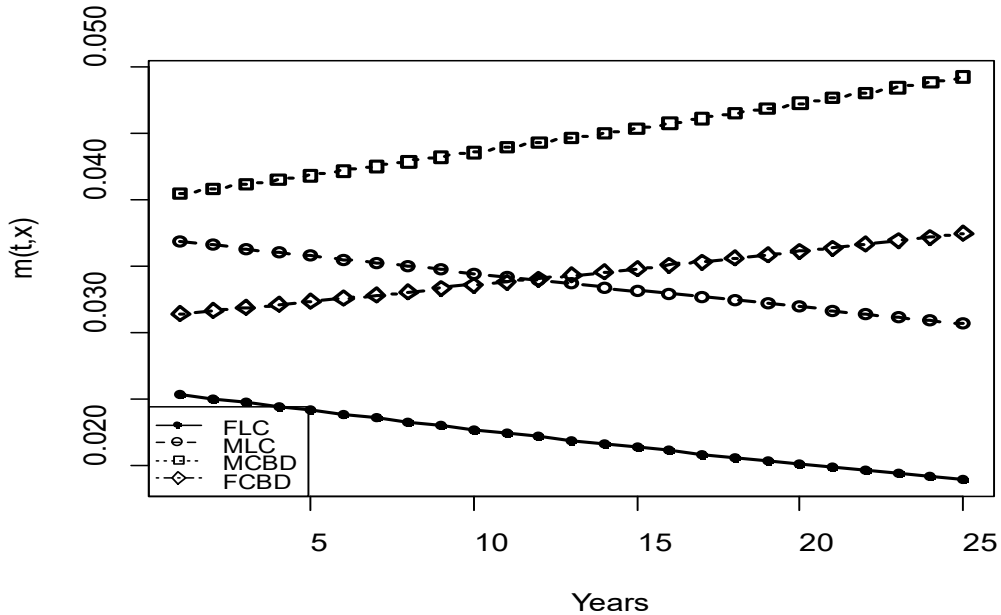


Figure 7: Projected Death Rates for LC and CBD Model with male and female population.

Table 1: Optimal Hedge Ratios

h^*	Minimum Variance Hedging	Utility Based Hedging					
		$\gamma=100$	$\gamma=500$	$\gamma=1000$	$\gamma=10000$	$\gamma=100000$	$\gamma=1000000$
Cairns-Blake-Dowd Model							
Female	1,2687	-6,8783	-0,3607	0,4540	1,1873	1,2606	1,2679
Male	1,5106	-6,6784	-0,1272	0,6917	1,4288	1,5024	1,5098
Lee-Carter Model							
Female	1,4131	-8,0182	-0,4732	0,4699	1,3188	1,4036	1,4121
Male	1,773	-8,2488	-0,2311	0,7711	1,6730	1,7632	1,7723

The optimal hedge ratios are given in Table 1. They are based on different risk measures, risk appetite of hedger and different mortality models. The optimal hedge ratios show that the number of units held of the swap contract. For instance, according to CBD model for female population we must hold 1, 2687 unit VSS for hedging our longevity risk. A negative optimal hedge ratio implies selling of VSS. As the risk aversion parameter increases, utility based hedge ratio approaches to the optimal hedge ratio which has the minimum variance. Optimal hedge ratios vary for the two models.

The source of the difference between the hedge ratios is the projected mortality rates obtained from different mortality models. For the LC model the mortality rates are decreasing over time, however according to the CBD model the mortality rates are decreasing until age 55, then increasing. Therefore, the longevity risk implied by the LC model is higher than the longevity risk implied by the CBD model. This means that, an investor whose mortality estimated by LC model must hold more unit VSS than an investor whose mortality modelled by the CBD model.

Another reason of the difference between the hedge ratios is the risk appetite of the investor. As the γ increases, the more units of VSS must be hold. The higher γ is, the higher is the hedger's risk aversion and the hedge ratio close to the variance hedge ratio.

Conclusions

Considering the improvement in mortality rates over time, the longevity risk has become an important risk for an insurance company or a pension plan.

In this paper, we obtain the optimal hedge ratio for a pension plan for its longevity risk. The optimal hedge ratios show the number of units held of the swap contract. The hedge ratios are calculated for minimum variance hedge and exponential utility function by using projected mortality rates. In order to obtain the projected rates, we use the LC and the CBD mortality models. The mortality rates which are obtained by using the LC model are decreasing over time, however according to the CBD model the mortality rates are decreasing until age 55, then increasing. Thus, an investor whose mortality has been estimated by the LC model must hold more unit VSS than an investor whose mortality has been modelled by the CBD model. In addition to that we examine the ratios for the risk appetite of the hedger for Turkish male and female populations. As the risk aversion parameter increases, utility based hedge ratio approaches to the optimal hedge ratio which has the minimum variance.

The analysis showed that the hedge ratios do not change significantly for different mortality models. However, as we change the risk criteria and risk aversion parameter we observe quite different optimal hedge ratios.

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