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Research Article

A Practical Approach to Implement Releases and Partial Fixities in Finite Elements Using Already Existing Stiffness Equations

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ABSTRACT

The usefulness of Finite Element (FE) models for many engineering purposes depends on the element's ability to support a variety of end-connection types including releases and partial-fixities. However, adding such features to a FE model would require additional theoretical effort in the element development process. Alternatively, zero-length external connector-elements can be used in the mesh structure, but this will complicate both mesh definition and assemblage operations. This study shows that the existing stiffness equations of any FE model with regular rigid connections can be effectively employed to automatically define both end-releases and end-partial-fixities by simply applying a basic matrix-equation modification process without the need for any additional theoretical development on the element itself. Our process can be summarized in three basic steps. Firstly, element equations are separated from the system equation by defining element's own degree-of-freedoms (DOFs). Secondly, elastic springs are introduced between the element and the system. Finally, the element is merged back into the system by eliminating its newly defined DOFs from the emerged equations. It has been verified by examples that, using these steps results in a new set of element equations with the desired end-releases/partial fixities and can be used in custom FE models.

Introduction

The Finite Element Method (FEM) is a very popular, even industry standard, method used to solve many different engineering problems. Within the scope of the method, it is possible to develop different element models for different problem types. However, there exist edge cases that require the fulfillment of special conditions that must be satisfied in the problem domain and/or boundaries. For example, it is nearly impossible to perform a realistic structural analysis with FEM without the use of special end connections, especially without releases and/or partial-fixities (R/PFs). This is mostly because the support conditions and element connections are far from ideal in most cases [1]. Bridges [2], multistorey buildings [3], truss and precast structures [4, 5], aircraft wings [6], wearable structures [7], furniture construction [8] are among many others that detailed analysis is carried out using R/PFs while also proving that these connections have a big impact on the overall structural behavior [9, 10]. In this context, R/PFs have also been an important element of various structural design [11, 12, 13, 14] and structural optimization [15, 16, 17, 18] studies.

The theory and applications of R/PFs in 1D elements are essentially well known. [19] is one of the earliest papers to incorporate R/PFs into the matrix stiffness method. [20] proposed the use of connection elements for R/PFs to establish connection effects. [21] used element's stiffness equations and [22] used energy approach to merge R/PFs into the target 1D element. Both provided solution only for rotational degree of freedoms DOFs. In [23], existing stiffness equations are used to introduce rotational R/PFs into a beam element. [24] briefly mentioned a simple formulation of 1D elements with all DOFs connected to the system via springs. More advanced applications, inter-element connections [25] and closed form solutions [26] are also available. A good overview of the literature on R/PFs is given in [27] under the nomenclature of semi-rigid connections.

Custom software developments for R/PFs applications are also implemented in [27, 28, 29]. In [30], a new software is presented for the analysis of body-in-white structures consisting of a custom super beam element to simulate the joint flexibility of the rods. In practice, most commercial software has built-in support for R/PFs in 1D elements. Even if the software does not support, it is always possible to perform the calculations by separating the nodes and adding spring-equivalent elements between them. Although relatively difficult to implement, such a method is used in this paper for verification purposes.

The reason why we revisit such a well-studied and old topic is that, we feel it is a shortcoming that R/PFs is only addressed as a 1D element specific issue in the literature. The main objective of this paper is to bring a different perspective to the practical application of R/PFs on custom FE models. In doing so, we include a more detailed inference of the method with a simple but generic formulation. In addition, we present numerical error control strategies and the usability of the method in elasticity elements, with theoretical explorations and sample analysis.

Material and methods

A general FE equation can be written in its well-known compact form as given in Equation (1)

$$[\mathrm{K}]\mathbf{U} = \mathbf{B} + \mathbf{Q} \tag{1}$$

where [K] represents the element stiffness matrix, $\mathbf{U}$ is the DOF vector which corresponds to the main unknowns of the problem (displacements of the system nodes in structural analysis) and $\mathbf{B}$ and $\mathbf{Q}$ are the body force vector and the boundary force vector respectively.

In order to separate the element from the rest of the system we define the independent equation of the element based on its own variables as given in Equation (2).

$$[\mathrm{K}]\widehat{\mathbf{U}} = \mathbf{B} + \widehat{\mathbf{Q}} \tag{2}$$

In the equation, $\widehat{\mathbf{U}}$ and $\widehat{\mathbf{Q}}$ are the element's newly defined DOF vector and boundary force vector respectively. Fig. 1 shows one of the nodes of the element, which is to be connected back to the system via elastic springs.

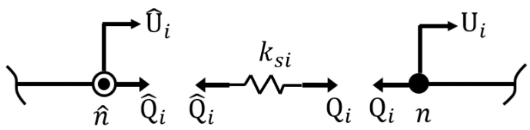

Figure 1. Disconnecting an element node from the system and inserting a spring between them ($\hat{n}$: Element node. n: System node).

Since the element is left with its own DOFs, $\widehat{\mathbf{Q}}$ remains the only way to interact with the element. Based on Fig. 1, one can re-integrate the element into the system with elastic springs considering the corresponding spring equilibrium and constitutive equations given in Equation (3)

$$\mathrm{Q}_i = \widehat{\mathrm{Q}}_i = k_{si}(\mathrm{U}_i - \widehat{\mathrm{U}}_i) \tag{3}$$

and re-writing the element equation in the following form as shown in Equation (4).

$$[\mathrm{K}]\widehat{\mathbf{U}} = \mathbf{B} + [\mathrm{k}_s](\mathbf{U} - \widehat{\mathbf{U}}) \tag{4}$$

Here, $[\mathrm{k}_s]$ is the diagonal matrix that includes the spring coefficients for the corresponding DOFs. As an example, Fig. 2 illustrates a frame element that is re-integrated to the system with different springs assigned for each DOF.

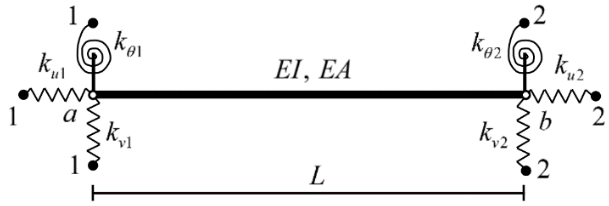

Figure 2. Beam element with end-springs [9].

In the figure, a and b represent the element's own independent nodes and the numbers 1 and 2 represent the corresponding system nodes (Note: System nodes with the same number are the same points, drawn as separate points for clarity of shape). Applying the Equation (4) on the frame element yields the following explicit form of the element equation, as given in Equation (5).

$$[K]\begin{Bmatrix} \hat{u}_a \\ \hat{v}_a \\ \hat{\theta}_a \\ \hat{u}_b \\ \hat{v}_b \\ \hat{\theta}_b \end{Bmatrix} = \mathbf{B} + \begin{Bmatrix} k_{u1}(u_1 - \hat{u}_a) \\ k_{v1}(v_1 - \hat{v}_a) \\ k_{\theta 1}(\theta_1 - \hat{\theta}_a) \\ k_{u2}(u_2 - \hat{u}_b) \\ k_{v2}(v_2 - \hat{v}_b) \\ k_{\theta 2}(\theta_2 - \hat{\theta}_b) \end{Bmatrix} \tag{5}$$

In order to retain the original form of the element equation (the form that contains only the system DOFs), we first solve $\widehat{\mathbf{U}}$ from the Equation (4) as

$$\widehat{\mathbf{U}} = [\mathrm{K}_s](\mathbf{B} + [\mathrm{k}_s]\mathbf{U}) \tag{6}$$

Here, $[K_s]$ takes the following form.

$$[\mathrm{K}_s] = ([\mathrm{K}] + [\mathrm{k}_s])^{-1} \tag{7}$$

Equation (7) can be transformed into an equivalent but more convenient expression, as given in Equation (8).

$$[\mathrm{K}][\mathrm{K}_s] + [\mathrm{k}_s][\mathrm{K}_s] = [\mathrm{I}] \tag{8}$$

Next, we consider the equilibrium of the spring as $\widehat{\mathbf{Q}} = \mathbf{Q}$ and substitute Equation (6) into Equation (2). Using Equation (8) for the final adjustments, the new stiffness equation given in Equation (9) is obtained for the element with embedded springs on its ends.

$$[\mathrm{K}][\mathrm{K}_s][\mathrm{k}_s]\mathbf{U} = [\mathrm{k}_s][\mathrm{K}_s]\mathbf{B} + \mathbf{Q} \tag{9}$$

Note that using Equation (9), end springs can be easily attached to any FE model, not just 1D elements. The remainder of the text deals with the details of the practical use of the equation.

Implementation details and error control

The simplest application of Equation (9) is the axial bar element where there exist only two DOFs. Such an element's well-known stiffness equation is as follows.

$$\begin{bmatrix} \frac{EA}{L} & -\frac{EA}{L} \\ -\frac{EA}{L} & \frac{EA}{L} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} B_1 \\ B_2 \end{Bmatrix} + \begin{Bmatrix} Q_1 \\ Q_2 \end{Bmatrix} \tag{10}$$

Elastic springs can be defined on both DOFs with the matrix given in Equation (11).

$$[\mathrm{k}_s] = \begin{bmatrix} k_{u1} & 0 \\ 0 & k_{u2} \end{bmatrix} \tag{11}$$

By applying Equation (9), the new element stiffness matrix can be obtained as

$$[\mathrm{K}][\mathrm{K}_s][\mathrm{k}_s] = \begin{bmatrix} \frac{\alpha EA}{\alpha L + \beta EA} & -\frac{\alpha EA}{\alpha L + \beta EA} \\ -\frac{\alpha EA}{\alpha L + \beta EA} & \frac{\alpha EA}{\alpha L + \beta EA} \end{bmatrix} \tag{12}$$

and the new body force vector as

$$[\mathrm{k}_s][\mathrm{K}_s]\mathbf{B} = \begin{Bmatrix} \frac{\alpha L B_1 + k_{u1} EA (B_1 + B_2)}{\alpha L + \beta EA} \\ \frac{\alpha L B_2 + k_{u2} EA (B_1 + B_2)}{\alpha L + \beta EA} \end{Bmatrix} \tag{13}$$

where;

$$\alpha = k_{u1} k_{u2}, \qquad \beta = k_{u1} + k_{u2} \tag{14}$$

It is easy to verify that the new element equation converges to its original rigid-connected form, by taking limits in the Equation (15).

$$\lim_{k_{u1}, k_{u2} \to \infty} ([\mathrm{K}_s][\mathrm{k}_s]) = [\mathrm{I}], \quad \lim_{k_{u1}, k_{u2} \to \infty} ([\mathrm{k}_s][\mathrm{K}_s]) = [\mathrm{I}] \tag{15}$$

For an exact stiffness equation, Equation (9) can be used directly only when the numerical values of all spring coefficients are known (including releases with 0 (zero) stiffness). If one or more connections are rigid, then the limit operation must be applied for the respective spring-coefficients. The result of such a limit for the case where the left side of the axial bar is rigid and the right side has an elastic connection is given below as an example by Equations (16) and (17).

$$[\mathrm{K}][\mathrm{K}_s][\mathrm{k}_s] = \begin{bmatrix} \frac{k_{u2} EA}{EA + k_{u2} L} & -\frac{k_{u2} EA}{EA + k_{u2} L} \\ -\frac{k_{u2} EA}{EA + k_{u2} L} & \frac{k_{u2} EA}{EA + k_{u2} L} \end{bmatrix} \tag{16}$$

$$[\mathrm{k}_s][\mathrm{K}_s]\mathbf{B} = \begin{Bmatrix} \frac{k_{u2} L B_1 + EA (B_1 + B_2)}{EA + k_{u2} L} \\ \frac{k_{u2} L B_2}{EA + k_{u2} L} \end{Bmatrix} \tag{17}$$

The disadvantage of solving the problem with the limit operation is evident in computer implementations, since one must consider all different sets of exact stiffness equations for all possible connection types. To overcome this difficulty and implement the problem in a more practical way, the spring coefficients at the rigid ends can be adjusted to large numerical values to obtain an approximate result of the limit operation, such that;

$$k_{u1}, k_{u2} \gg \gamma \frac{EA}{L}, \qquad \gamma = 10^4 \tag{18}$$

This approach is evident from the rearrangement of the first stiffness term of Equation (12) as follows.

$$\mathrm{K}_{11} = \frac{EA}{L + (\beta/\alpha) EA} \tag{19}$$

Equation (19) indicates that a good approximation of the fully-rigid connection case can be achieved by choosing the numerical values of the spring coefficients according to the inequality given in Equation (20).

$$L \gg \frac{\beta}{\alpha} EA \quad \rightarrow \quad \frac{\alpha}{\beta} = \frac{k_{u1} k_{u2}}{k_{u1} + k_{u2}} \gg \frac{EA}{L} \tag{20}$$

This inequality is automatically satisfied by setting the spring coefficients to large numerical values, as suggested in Equation (18). To verify this, one can substitute the suggested values into Equation (20) and find that $\gamma \gg 2$.

A similar approach can be used for the mixed connection cases. Such examples are investigated in the next section.

More implementation details

As a more comprehensive example for different connection types, we selected the well-known 2D frame element that is already depicted in Fig. 2 of the previous section. The short version of the stiffness matrix of that frame with rigid connections has the form given in Equation (21).

$$\begin{bmatrix} \frac{EA}{L} & 0 & 0 & . \\ 0 & \frac{12EI}{L^3} & \frac{6EI}{L^2} & . \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & . \\ . & . & . & . \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ \theta_1 \\ . \\ . \\ . \end{Bmatrix} = \begin{Bmatrix} q_x L/2 \\ q_y L/2 \\ \frac{q_y L^2}{12} \\ . \\ . \end{Bmatrix} + \mathbf{Q} \tag{21}$$

To illustrate the strength and usefulness of the proposed method, we define an elastic rotational spring with $k_{\theta 2}$ at the right end of the element. For this, it will be sufficient to apply the following limit operations given in Equation (22)

$$\lim_{k_{u1},k_{u2},k_{v1},k_{v2},k_{\theta 1}\to\infty}([\mathrm{K}][\mathrm{K}_s][\mathrm{k}_s]),\quad \lim_{k_{u1},k_{u2},k_{v1},k_{v2},k_{\theta 1}\to\infty}([\mathrm{k}_s][\mathrm{K}_s]\mathbf{B}) \tag{22}$$

which yields the stiffness matrix given in Equation (23)

$$\begin{bmatrix} \frac{EA}{L} & 0 & 0 & . \\ 0 & \frac{12EI(EI+k_{\theta 2}L)}{L^3(4EI+k_{\theta 2}L)} & \frac{6EI(2EI+k_{\theta 2}L)}{L^2(4EI+k_{\theta 2}L)} & . \\ 0 & \frac{6EI(2EI+k_{\theta 2}L)}{L^2(4EI+k_{\theta 2}L)} & \frac{4EI(3EI+k_{\theta 2}L)}{L(4EI+k_{\theta 2}L)} & . \\ . & . & . & . \end{bmatrix} \tag{23}$$

and the body force vector given in Equation (24).

$$[\mathrm{k}_s][\mathrm{K}_s]\mathbf{B} = \begin{Bmatrix} q_x L/2 \\ \frac{q_y L(5EI+k_{\theta 2}L)}{8EI+2k_{\theta 2}L} \\ \frac{q_y L^2(6EI+k_{\theta 2}L)}{12(4EI+k_{\theta 2}L)} \\ . \\ . \end{Bmatrix} \tag{24}$$

It is easy to see that, end releases can also be defined by setting $k_{\theta 2}$ = 0 in the matrices above, as shown in Equation (25).

$$\begin{bmatrix} \frac{EA}{L} & 0 & 0 & . \\ 0 & \frac{3EI}{L^3} & \frac{3EI}{L^2} & . \\ 0 & \frac{3EI}{L^2} & \frac{3EI}{L} & . \\ . & . & . & . \end{bmatrix}\mathbf{U} = \begin{Bmatrix} q_x L/2 \\ 5q_y L/8 \\ \frac{q_y L^2}{8} \\ . \\ . \end{Bmatrix} + \mathbf{Q} \tag{25}$$

As can be deduced from the given examples, the element equation of any end connection type is fairly easy to derive. However, as mentioned before, it would be more practical to carry out these calculations numerically in computer practice. The algorithm of such an application is depicted in Fig. 3.

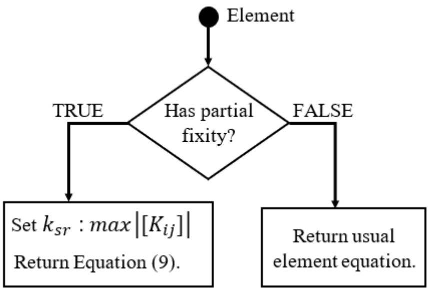

Figure 3. Numerical evaluation of a mixed type end-connection case.

Here, k_{sr} represents the selected numerical values for the rigid connections. The algorithm suggests a relatively conservative selection for the numerical values as they are set to the maximum absolute value extracted from the element stiffness matrix. We observed in the examples that selecting $k_{sri} \gg \gamma K_{ii}$ is more than enough for acceptable numerical results.

2D Frame example

As a more comprehensive example, we selected a 2D Frame structure with different types of partial-fixities/releases at different end locations as depicted in Fig 4.

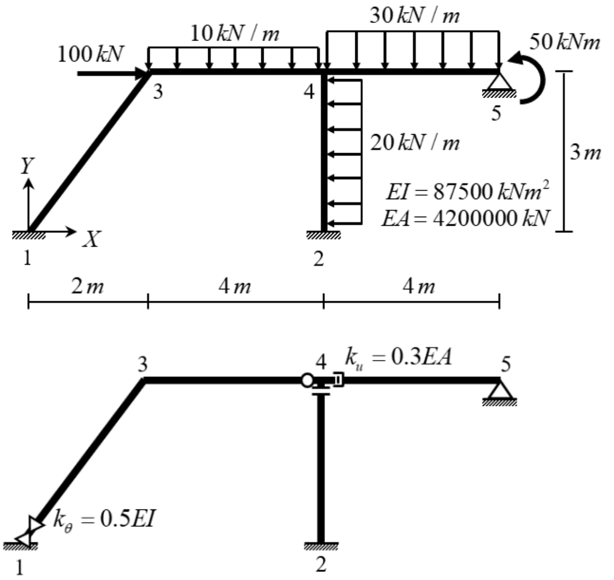

Figure 4. A 2D Frame structure with various types of external loadings, boundary conditions and end-springs and releases cases (Note: Top figure: loading conditions. Bottom figure: Releases and partial fixities).

According to the figure, two end-releases (moment and shear in elements 3-4 and 2-4 respectively) and two partial fixities (moment and axial fixities in elements 1-3 and 4-5) are placed in the structure. In the analysis, the numerical values of the rigid-end springs are chosen to be 10000 times the maximum values of the element stiffness coefficients. The results obtained by Equation (9) are compared with commercial software SAP2000 V18.2.0 [31] as shown in Table 1.

Table 1. Horizontal displacements and moment support reactions of the 2D Frame structure with releases and partial fixities.

Horizontal Disp.[mm]	Proposed Method	SAP2000 V18.2.0
Node #3	0.310**702**	0.310**670**
Node #4	0.2010**32**	0.2010**22**

Moment Reactions [kNm]	Proposed Method	SAP2000 V18.2.0
Node #1	2.016**5**	2.016**1**
Node #2	-43.448**2**	-43.448**0**

The results indicate that accuracy can be obtained at the level of the selected magnification factor.

Elasticity element example (plane-stress)

In order to demonstrate that the proposed method can be used for other element types, an example application will be given on the well-known plane stress element. The virtual work equation of the element is given in Equation (26).

$$\iint_A \delta\mathbf{u}^T.\boldsymbol{\sigma}\, hdA = \iint_A \delta\mathbf{u}^T.\mathbf{b}\, hdA + \oint\!\!\!\oint \delta\mathbf{u}^T.\mathbf{q}\, dL \quad (26)$$

The first two integrals form the stiffness matrix and the body force vector, respectively. h: is the thichness and **q** represents the distributed boundary force per unit length of the edges. The essential term, which is important here for the purposes of this study, is the boundary integral term (the last integral) which forms the boundary force vector **Q** of the FE model.

Fig. 5 depicts the discretization of an elastic edge spring based on the discretization of the uniformly distributed **q**.

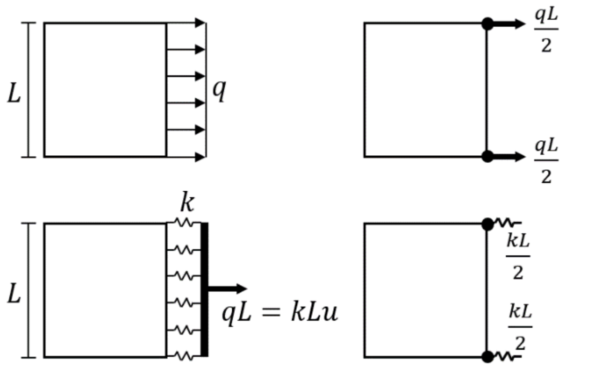

Figure 5. Discretization of a continuous edge spring in a plane element (Note: $u = 1$).

The figure suggests that, a continuous spring (spring-constant per unit length) can be discretized just like **q**.

The selected plane-stress problem is shown in Fig. 6. An irregular mesh is used to include the effects of all possible variables in the problem. Body forces are also included as self-weight. The elastic spring connection is inserted only in the horizontal direction. Vertical connection between the parts is rigid.

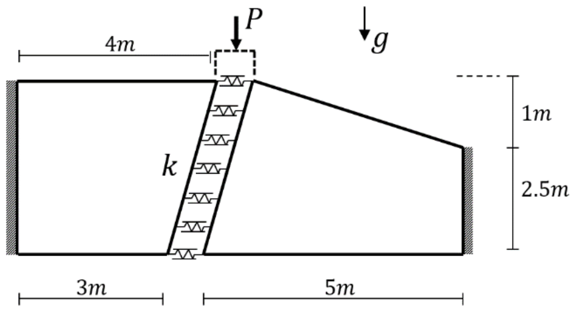

Figure 6. Two plane-stress elements connected to each other by horizontal elastic springs ($P = 1000\ kN$).

Spring coefficient, thickness, elasticity modulus, Poisson's ratio and unit weight are selected as; $k = 68680.282\ kN/m$, $h = 0.01m$, $E = 70GPa$, $v = 0.3$ and $\rho_w = 77\ kN/m^3$.

Based on the spring discretization mentioned, the classical FE model of the system can be set up as shown in Fig. 7.

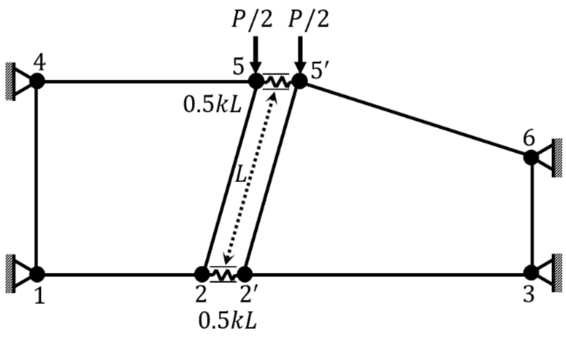

Figure 7. Classic FE model of the example ($kL = 250000\ kNm/m$).

In order to implement the classical method, node points that coincide with the elastic connection region must first be separated from each other. Then, springs or equivalent truss elements should be placed between these nodes. As these points are separated, they will move independently in the vertical direction. In order to prevent this, constraints must be defined to enforce the vertical displacement equivalence between these nodes as a final processing step.

However, instead of all these steps, the method described in this study provides a very practical solution to the problem, as depicted in Fig 8.

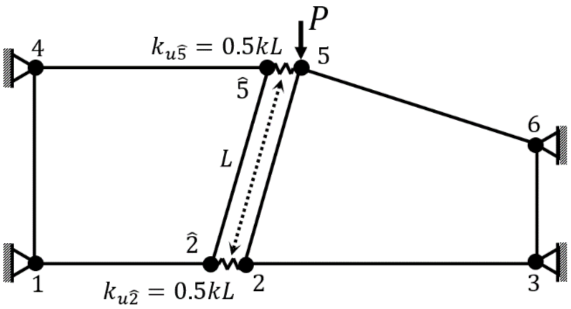

Figure 8. FE model of the proposed method ($kL = 250000\ kNm/m$).

Note that, in the proposed method the original system mesh is preserved as the springs are embedded directly in the element on the left (see the local nodes of the element $\hat{5}$ and $\hat{2}$). This embedding can be easily accomplished by defining the matrix $[k_s]$ containing the increased stiffnesses for rigid connections and the actual spring constants for elastic ones. Afterwards, it will be sufficient to perform the analysis using Equation (9) for the left element (see the diagram previously given in Fig. 3). The same procedure could be followed similarly for the element on the right, which would yield the same numerical results. It should also be noted that there is no need to define extra constraints for the vertical direction as the system DOFs are already preserved.

The analysis results of both methods are given in Table 2. In order to verify the results in SAP2000 [31], one should turn-off the so called "incompatible modes" of the plane element, which is active by default in the software.

Table 2. Analysis results for the elastic interconnected plane-stress problem (γ = 10000).

(SAP2000 results are obtained with the classical FE model and without incompatible modes).

Vert. Disp. [mm]	Proposed Method	SAP2000 V18.2.0
Node #2	-2.480**91**	-2.480**67**
Node #5	-3.315**69**	-3.315**42**

Support React. Horiz. [kN]	Proposed Method	SAP2000 V18.2.0
Node #1	450.**447**	550.**423**
Node #3	-374.5**17**	-374.5**47**
Node #4	-388.5**72**	-388.5**45**
Node #6	312.6**41**	312.6**69**

Support React. Vert. [kN]	Proposed Method	SAP2000 V18.2.0
Node #1	299.6**42**	299.6**53**
Node #3	414.**707**	414.**682**
Node #4	283.1**99**	283.1**58**
Node #6	22.**490**	22.**525**

One last example (plane-stress)

Fig. 9 shows a plane-stress beam connected at its midline by horizontal-springs (vertically rigid).

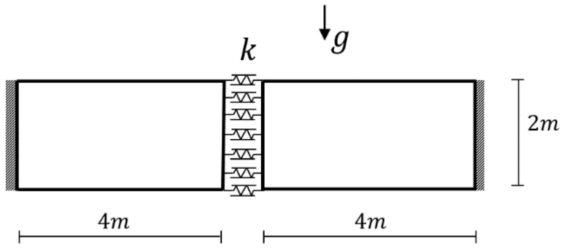

Figure 9. A 2D beam connected by a horizontal continuous elastic spring.

Spring coefficient, thickness, elasticity modulus, Poisson's ratio and unit weight are selected as; $k = 100000\ kN/m$, $h = 0.01m$, $E = 70GPa$, $v = 0.3$ and $\rho_w = 77\ kN/m^3$.

The beam is loaded only by its own weight. Fig. 10 shows the displacement and stress fields obtained in three different analyses for a selected 40x10 FE mesh.

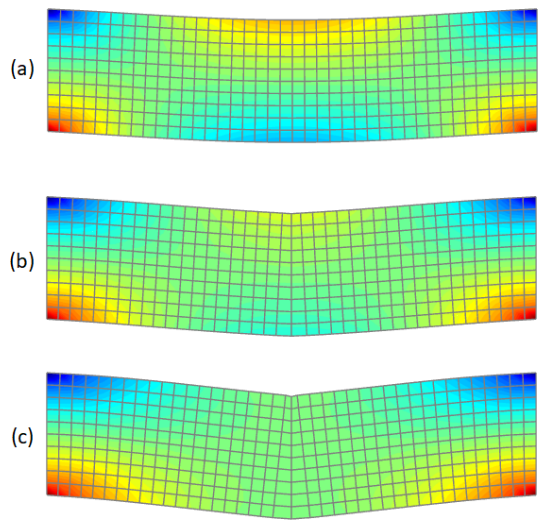

Figure 10. **Presented Method**: Displacement (x3000) and stress (σ_X) fields. a) Rigid connection ($k = \infty$). b) Horizontal partial fixity (k). c) Horizontal release ($k = 0$). (Note: Plots are created with the software presented in [32]).

The solutions are carried out using Equation (9) for the elements adjacent to the spring region. It is worth mentioning that in order to obtain the stress field, the DOFs in Equation (6) must be calculated first. The displacement and stress results for each case are given in Table 3.

Table 3. **Presented Method**: Analysis results for the beam with elastic springs in Fig. 9 (γ = 10000).

Analysis ID	Max. Vert. Disp. [mm]	Max. Horiz. Stress. [kPa]
(a) $k = \infty$	0.061065	1675.74
(b) k	0.0930**94**	2000.**02**
(c) $k = 0$	0.130**483**	2374.**87**

Table 4. **SAP2000**: Analysis results for the beam with elastic springs in Fig. 9.

Analysis ID	Max. Vert. Disp. [mm]	Max. Horiz. Stress. [kPa]
(a) $k = \infty$	0.061065	1675.74
(b) k	0.0930**75**	1999.**51**
(c) $k = 0$	0.130**394**	2374.**30**

Table 4. and Fig. 11 show the displacement and stress fields obtained in SAP2000 [31]. The analysis in SAP2000 is performed using spring-equivalent truss elements and vertical constraints at the elastic midline.

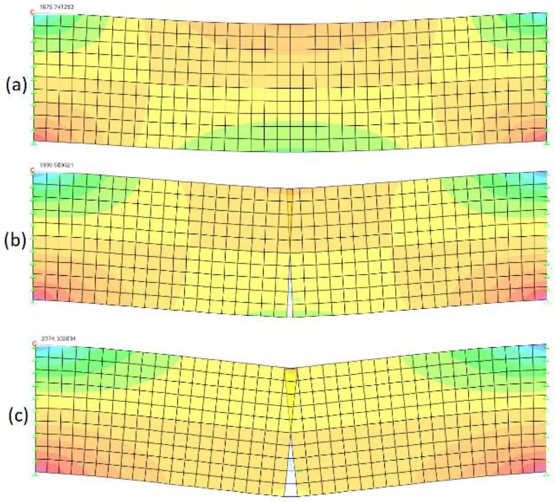

Figure 11. **SAP2000**: Displacement (x3000) and stress (σ_X) fields. a) Rigid connection ($k = \infty$). b) Horizontal partial fixity (k). c) Horizontal release ($k = 0$).

Concluding remarks

This paper presented a practical calculation method for equipping existing FE models with edge partial-fixities and releases. Demonstrated with examples that the application of the method consists only of simple matrix operations. The numerical results showed that the accuracy can be controlled with a single magnification factor. The method requires matrix inversion to generate the stiffness equations, even for a single spring case, so its implementation in commercial software may not be an optimal option in terms of speed, but still preferable. On the other hand, being able to instantly model existing elements to support different types of edge connections will be invaluable, especially for researchers who develop custom code in their work.

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