Sigma J Eng Nat Sci, Vol. 40, No. 1, pp. 188–195, March, 2022

Sigma Journal of Engineering and Natural Sciences
Web page info: https://sigma.yildiz.edu.tr
DOI: 10.14744/sigma.2022.00015

Research Article

A new method for conversion between pythagorean fuzzy sets and intuitionistic fuzzy sets

Gürkan IŞIK[1,*]

[1]*Department of Business Development, Valuable Touch Energy Services, Sakarya, Turkey*

ARTICLE INFO

Article history
Received: 03 May 2021
Accepted: 01 August 2021

Key words:
Fuzzy sets; Intuitionistic fuzzy sets; Pythagorean fuzzy sets; Fuzzy set conversion

ABSTRACT

Modeling with inconsistent fuzzy information is not possible for some problem types. For such cases, pythagorean fuzzy sets (PFSs) cannot be used in problem formulations and a conversion to another fuzzy set extension is needed. As a new conversion between PFSs and intuitionistic fuzzy sets (IFSs), the projective relation was proposed in the literature and its results were compared with the normalization that is the conversion method used by all. However, projective relation conversion is not valid. This conversion is based on the approach of subtraction of the part causing the inconsistency from the membership, non-membership and indeterminacy grades equally. This is not a proper approach because a negative grade is obtained when one of the membership and non-membership grades of PFS is smalller than the equally substracted part. In this study, the error in the proof of the projective relation has been discussed by presenting a counterexample. A new conversion namely "square-scaled normalization" (SSNORM) which converts PFSs to IFSs by rescaling the grades depending on the relative greatness of their squares has been offered and its score and accuracy functions have been formulated. SSNORM method has been examined on a numerical example from the manufacturing industry and the obtained results have been compared with the normalization. Although both methods obtained results close to each other, SSNORM yielded more cautious results. It reached a bigger score function value but a smaller accuracy function value compared to the normalization. SSNORM method can be preferrable alternative of the normalization if the approximation errors caused by the linear rescaling is high.

Cite this article as: Işık G. A new method for conversion between pythagorean fuzzysets and intuitionistic fuzzy sets. Sigma J Eng Nat Sci 2022;40(1): 188–195.

INTRODUCTION

The uncertainty is modeled with the term "membership function" and the sum of membership and non-membership grades of each set element is equal to 1 for standard fuzzy sets (FSs) [2]. However, it may not always be equal to 1. Intuitionistic fuzzy sets (IFSs) were offered by Atanassov [3] as a generalization of FSs. The sum of membership and

*Corresponding author.
*E-mail address: gurkan_isik@msn.com

This paper was recommended for publication in revised form by Regional Editor Ferkan Yilmaz

Published by Yıldız Technical University Press, İstanbul, Turkey
Copyright 2021, Yıldız Technical University. This is an open access article under the CC BY-NC license (http://creativecommons.org/licenses/by-nc/4.0/).

non-membership grades of the set elements is less than or equal to 1 for IFSs. The difference, which causes the inequality, is called the indeterminacy grade. Pythagorean fuzzy sets (PFSs) offered by Yager [4] [5] are a class of non-standard fuzzy sets (FSs) which are more general than intuitionistic fuzzy sets (IFSs) because of giving ability to model with inconsistent data. PFSs are characterized by the condition that the sum of squares of membership and non-membership grades is less than or equal to 1. Which means that the sum of membership and non-membership grades can exceed 1.

Working with inconsistent data may not be acceptable for some problem formulations such as acceptance sampling plans. In such scenarios, the only thing that can be done to use the available inconsistent data in formulations is to convert it to consistent data. There are several studies on the conversion methods between the FS extensions in the literature. The most common conversion approach to convert all non-standard fuzzy sets (FSs) into standard FSs is the normalization. For example, it is preferred for the conver-sion between Neutrosophic sets and IFSs by Smarandache [6]. Wang et al. [7] proposed a conversion between vague sets (It is proven that the vague set concept is identical with IFS concept [8].). Liu et al. [9] studied a conversion between soft sets and standart FSs, Deschrijver & Kerre [10] investigated the relationship and isomorphism between L-fuzzy sets, FSs, IFSs and interval valued ver-sions of them. Transformation between probabilistic and fuzzy information is also a widely researched in the literature [11] [12] [13] [14] [15]. There are also some studies about the conversion between PFSs and IFSs. Beliakov & James [16] discussed the mappings from IFSs to PFSs by analyzing multiple dilation spaces crossing at the point where the membership and non-membership grades are the same. Instead of presenting a clear conversion method, this analysis results were used for partial and total ordering of membership pairs belonging to different FS families in an aggregation process. Tao et al. [1] employed the normalization for the conversion between PFSs and IFSs and proposed an alternative conversion method named "projective relation", then used these two methods to derive the information measures of PFSs for multi criteria decision making (MCDM) problems.

According to the projective relation, the part of the total grades exceeding 1 is divided by 3 and subtracted from membership, non-membership and indeterminacy grades. It is obvious that one of the grades of the PFS is smalller than the equally substracted part, the method may yield a negative grade. For this reason, this method is not a valid conversion between PFSs and IFSs. This problem is caused by an error in the proof of the projective relation. In this study, this error has been presented with a counter example and disproof, then a new conversion method has been offered. The offered conversion method is a considerable alternative of the normalization because it reduces the approximation errors caused by the linear rescaling. An important advantage of this method over the mentioned studies is giving ability to continue modeling with precise fuzzy modeling and so having application area for the other type of problems apart from MCDM problems.

This paper is organized as follows. In Section 2, some notations and definitions have been presented. The error about the proof has been pointed out, a counterexample has been presented, and disproof for the theorem has been submitted in Section 3. A new conversion has been offered in Section 4, findings have been illustrated on a numerical example in Section 5, and the concluding remarks have been discussed in Section 6.

PRELIMINARIES

A variable can be both partially member and non-member at the same time in fuzzy set theory. The sum of the membership and non-membership grades of a set element is equal to 1 for standard FSs.

Definition 1: Let X be a given reference universe, $\tilde{A}$ be a standard FS on X, $\mu_{\tilde{A}}(x) \in [0,1]$ be the membership function of $x \in X$ into the set $\tilde{A}$, and $\vartheta_{\tilde{A}}(x)$ be the non-membership function which is complement of $\mu_{\tilde{A}}(x)$. $\tilde{A}$ is represented as $\tilde{A} = \{X, \mu_{\tilde{A}}(x) \mid X \in X\}$ satisfying Eq. (1) [2].

$$\mu_{\tilde{A}}(x) + \vartheta_{\tilde{A}}(x) = 1 \tag{1}$$

Standard FSs require overall information about the event. It is named as complete information case. However, it may not always be possible to decide the membership and non-membership grades of the set elements satisfy-ing Eq. (1). For example, some factors such as physi-cal obstacles or instructional inabilities can cause hesitation and it can hinder the complete information case. IFSs have been offered to model the uncertainties including such incomplete information cases.

Definition 2: Let X be a given reference universe. An IFS $\tilde{A}$ on X is represented as in Eq. (2) and indeterminacy grade of an element $x \in X$ ($\pi_{\tilde{A}}(x)$) is defined as in Eq (3)[3].

$$\begin{aligned} \tilde{A} = \{x, \mu_{\tilde{A}}(x), \vartheta_{\tilde{A}}(x) | x \in X\},\ \ \mu_{\tilde{A}}(x) + \vartheta_{\tilde{A}}(x) \le 1, \\ \mu_{\tilde{A}}(x) + \vartheta_{\tilde{A}}(x) + \pi_{\tilde{A}}(x) = 1 \end{aligned} \tag{2}$$

$$\Rightarrow \pi_{\tilde{A}}(x) = 1 - \left(\mu_{\tilde{A}}(x), \vartheta_{\tilde{A}}(x)\right) \tag{3}$$

In some cases, the sum of the membership, non-membership and indeterminacy grades can exceed 1. This circumstance is named as inconsistent information. PFS is a type of FS allows modeling with inconsistent data.

Definition 3: Let $\tilde{A}$ be a PFS, $r(x) \in [0,1]$ be the strength of commitment at x, $d(x) \in [0,1]$ be the direction of commitment, $A_Y(x)$ be the support for membership of x in $\tilde{A}$,

$A_N(x)$ be the support against membership of x in $\tilde{A}$ and, $\theta(x) \in [0, \pi/2]$ be a raidan angle. $A_Y(x)$ and $A_N(x)$ are defined as in Eq. (4) and a Pythagorean membership grade represented with a pair of values $r(x)$ and $d(x)$ for each $x \in X$ while $r(x)$ and $d(x)$ are associated with a pair of $A_Y(x)$ and $A_N(x)$ as in Eqs. (5) and (6) [4].

$$A_Y(x) = r(x) \times \cos(\theta(x)), \quad A_N(x) = r(x) \times \sin(\theta(x)) \tag{4}$$

$$r(x) = \sqrt{\left(A_Y^2(x) + A_N^2(x)\right)} \tag{5}$$

$$d(x) = \frac{\pi - 2\theta(x)}{\pi} \tag{6}$$

Definition 4: Let $\tilde{A}$ be a PFS, $r_{\tilde{A}}(x) \in [0,1]$ be the strength of commitment at x in $\tilde{A}$, $A_Y(x)$ be the support for membership of x in $\tilde{A}$, $A_N(x)$ be the support against membership of x in $\tilde{A}$ and, $\theta \in [0,\pi/2]$ be a raidan angle. PFS $\tilde{A}$ is defined as in Eq. (7) when it is written with the same terminology and letter symbols with IFSs and the indeterminacy grade of an element $x \in X$ ($\pi_{\tilde{A}}(x)$) is defined as in Eq. (8).

$$\tilde{A} = \left\{x, \mu_{\tilde{A}}(x) = A_Y(x), \vartheta_{\tilde{A}}(x) = A_N(x) | x \in X\right\}, \quad \mu_{\tilde{A}}^2(x) + \vartheta_{\tilde{A}}^2(x) \le 1, \quad \mu_{\tilde{A}}^2(x) + \vartheta_{\tilde{A}}^2(x) + \pi_{\tilde{A}}^2(x) = 1 \tag{7}$$

$$\pi_{\tilde{A}}(x) = \sqrt{1 - r_{\tilde{A}}^2(x)} = \sqrt{1 - \left(\mu_{\tilde{A}}^2(x) + \vartheta_{\tilde{A}}^2(x)\right)} \tag{8}$$

Definition 5: Let $\tilde{A}$ be a PFS. Denotation of $\tilde{A}$ as a pair of values such that $\alpha = \langle \mu_{\tilde{A}} \in [0,1], \vartheta_{\tilde{A}} \in [0,1] \rangle$, $\mu_{\tilde{A}}^2 + \vartheta_{\tilde{A}}^2 \le 1$ is called as Pythagorean fuzzy number (PFN) [5].

ERROR IN TAO ET AL'S PROOF OF THEOREM

Tao et al. [1] presented Theorem 1 (Theorem 3.1 in the original paper) to propose the "projective relation". Nevertheless, this method yields negative grades in some cases. This problem is caused by an error in the proof of Theorem 1.

Theorem 1: Eq. (9) provides a conversion from a Pythagorean fuzzy number (PFN) $\langle \mu_P, \vartheta_P, \pi_P \rangle$ to and intuitionistic fuzzy number (IFN) $\langle \mu_I^P, \vartheta_I^P, \pi_I^P \rangle$ [1].

$$\mu_I^P = \mu_P - \frac{\mu_P + \vartheta_P + \pi_P - 1}{3}, \quad \vartheta_I^P = \vartheta_P - \frac{\mu_P + \vartheta_P + \pi_P - 1}{3}, \quad \pi_I^P = \pi_P - \frac{\mu_P + \vartheta_P + \pi_P - 1}{3}, \tag{9}$$

Theorem 1 assumes that Eq. (10) is verified for all PFSs. However, it is not satisfied if $\vartheta_P + \pi_P - 1$ is greater than $2\mu_P$.

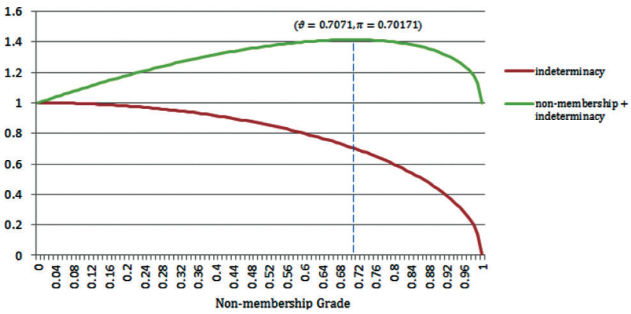

Figure 1. Non-membership and indeterminacy grades when membership is equal to 0.

$$\frac{2\mu_P - \vartheta_P - \pi_P + 1}{3} \ge \frac{2\mu_P - \vartheta_P{}^2 - \pi_P{}^2 + 1}{3} \tag{10}$$

Disproof of Theorem 1 has been presented below. Only the problematic part of the theorem has been considered to earn from the space. According to Tao et al [1], Eq. (10) is satif-sied for all PFNs $\tilde{P}$:$\langle \mu_P, \vartheta_P, \pi_P \rangle$.

Disproof: In order to find the lower limit of the statement shown in Eq. (9), membership grade should be minimized, and the sum of the non-membership and inde-terminacy grades should be maximized. Figure 1 shows the maximum value of the sum of the non-membership and indeterminacy grades is equal to 1.4142 when the membership grade is equal to 0.

$$\lim_{\substack{\mu_P \to 0 \\ (\vartheta_P + \pi_P) \to 2}} \left(\mu_P - \frac{\mu_P + \vartheta_P + \pi_P - 1}{3}\right) = \lim_{\substack{\mu_P \to 0 \\ (\vartheta_P + \pi_P) \to 2}} \left(\frac{2\mu_P - \vartheta_P - \pi_P + 1}{3}\right) = \frac{0 - 0.7071 - 0.7071 + 1}{3} = -0.138$$

The upper limit of the statement is found by maximizing the membership grade.

$$\lim_{\substack{\mu_P \to 1 \\ (\vartheta_P + \pi_P) \to 0}} \left(\mu_P - \frac{\mu_P + \vartheta_P + \pi_P - 1}{3}\right) = 1 - \frac{1 + 0 + 0 + 1}{3} = 1$$

Thus, we have:

$$-0.138 \le \mu_P - \frac{\mu_P + \vartheta_P + \pi_P - 1}{3} \le 1$$

As seen above, the statement shown in Eq. (9) can take negative values. This result shows, the Theorem 1 pre-sented by Tao et all. [1] is not valid. Example 1 supports this finding.

Example 1: Let $\tilde{P} = \langle \mu_P = 0.0866, \vartheta_P = 0.95, \pi_P = 0.3 \rangle$ be a PFN satisfying $0 \le \mu_P^2 + \vartheta_P^2 + \pi_P^2 \le 1$. The projection of $\tilde{P}$ is found as $\langle \mu_I^P = -0.0256, \vartheta_I^P = 0.8378, \pi_I^P = 0.1878 \rangle$ depending on the Eq. (9). Negative membership grade is unpermitted for IFNs, so the projection of $\tilde{P}$ is not an

IFS. In addition, it does not ensure Eq. (10) too as shown below:

$$\frac{2\mu_P - \vartheta_P - \pi_P + 1}{3} = -0.0256 \not\geq 0.06$$
$$= \frac{2\mu_P - \vartheta_P{}^2 + \pi_P{}^2 + 1}{3}$$

A NEW CONVERSIONS BETWEEN PYTHAGOREAN FUZZY SETS AND INTUITIONISTIC FUZZY SETS

Normalization is a popular approach to convert non-standard FSs into standard ones. For example, Smarandache [6] has offered to use the normalization to convert the Neutrosophic sets (NSs) into IFSs. Tao et al. [1] have suggested to use it for conversion between PFS and IFSs. The l ogic b ehind t he n ormalization i s s imple. It rescales the membership, non-membership, and indeterminacy grades with a ratio to satisfy the condition given in Eq. (2). Normalization is formulated as in Definition 6.

Definition 6: Let $\tilde{P} = \langle \mu_P, \vartheta_P, \pi_P \rangle$ be a PFN, $\tilde{A} = \langle \mu_I^P, \vartheta_I^P, \pi_I^P \rangle$ be an IFN. Normalization of $\tilde{P}$ defined by Eq. (11) provides a conversion from $\tilde{P}$ to $\tilde{A}$ [1].

$$\mu_I^P = \frac{\mu_P}{\mu_P + \vartheta_P + \pi_P},\quad \vartheta_I^P = \frac{\vartheta_P(x)}{\mu_P + \vartheta_P + \pi_P},\quad \pi_I^P = \frac{\pi_P(x)}{\mu_P + \vartheta_P + \pi_P} \qquad (11)$$

Normalization scale downs the grades by dividing them with the same ratio. For this reason, it protects the relative greatness of the grades between each other and always produces an IFS. While the main condition of PFSs shown in Eq. (7) is considering the squares of the grades, the normalization rescales the grades by using the sum of them. Accuracy of the normalization approach is negotiable because protecting the relative greatness of the grades. This approach may not always be acceptable for some situations related to PFSs. The definition space [which is limited by Eq. (7)] consumption increases exponentially while a grade is getting bigger. Based on this, rescaling can be done by considering the relative greatness of the squares of the grades as an alternative approach to the normalization. In this way, the rescaling procedure reduces large grades more than small grades. This approach is formulated as in Definition 7.

Definition 7: Let $\tilde{P} = \langle \mu_P, \vartheta_P, \pi_P \rangle$ be a PFN, $\tilde{A} = \langle \mu_I^P, \vartheta_I^P, \pi_I^P \rangle$ be an IFN. Square-scaled normalization of $\tilde{P}$ defined by Eq. (12) provides a conversion from $\tilde{P}$ to $\tilde{A}$.

$$\begin{aligned}
\mu_I^P &= \mu_P - (\mu_P + \vartheta_P + \pi_P - 1) \times \mu_P{}^2 \\
&= \mu_P\left(1 - \mu_P(\mu_P + \vartheta_P + \pi_P - 1)\right), \\
\vartheta_I^P &= \vartheta_P - (\mu_P + \vartheta_P + \pi_P - 1) \times \vartheta_P{}^2 \\
&= \vartheta_P\left(1 - \vartheta_P(\mu_P + \vartheta_P + \pi_P - 1)\right), \\
\pi_I^P &= \pi_P - (\mu_P + \vartheta_P + \pi_P - 1) \times \pi_P{}^2 \\
&= \pi_P\left(1 - \pi_P(\mu_P + \vartheta_P + \pi_P - 1)\right)
\end{aligned} \qquad (12)$$

Theorem 2: Eq. (12) produces an IFN for all PFNs.
Proof: $\mu_I^P + \vartheta_I^P + \pi_I^P = 1$ should be satisfied for all IFNs.

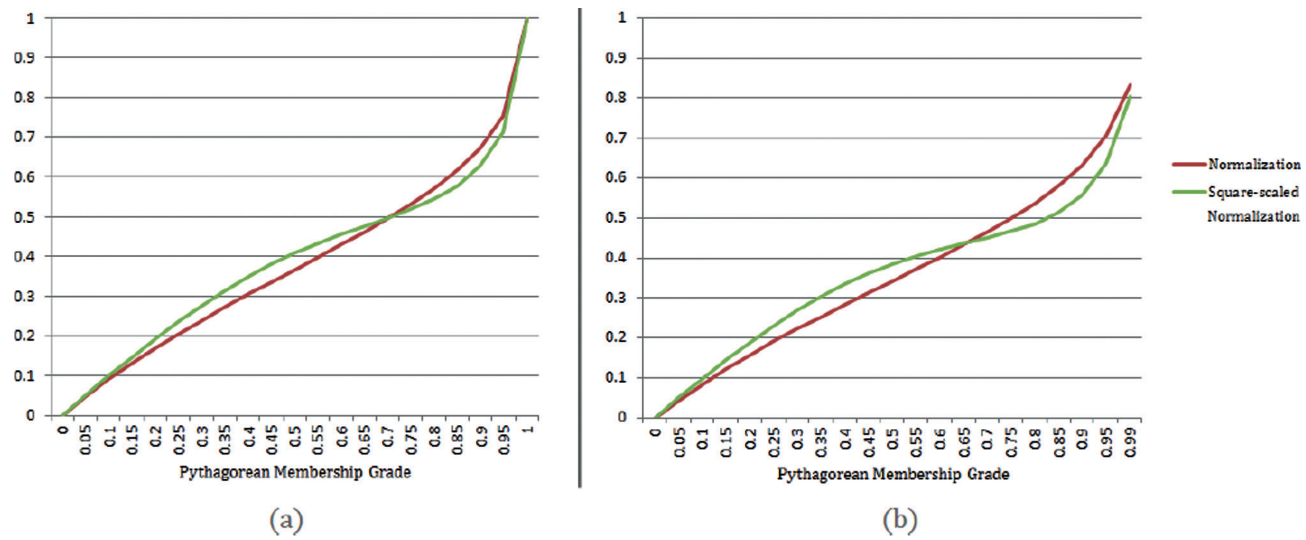

Figure 2. Membership grades for standard and square-scaled normalizations when (a) $\boldsymbol{\pi} = \mathbf{0}$ and (b) $\boldsymbol{\pi} = \mathbf{0.1}$.

$$\mu_I^P + \vartheta_I^P + \pi_I^P = \left(\mu_P - (\mu_P + \vartheta_P + \pi_P - 1) \times \mu_P{}^2\right) + \left(\vartheta_P - (\mu_P + \vartheta_P + \pi_P - 1) \times \vartheta_P{}^2\right) + \left(\pi_P - (\mu_P + \vartheta_P + \pi_P - 1) \times \pi_P{}^2\right) = (\mu_P + \vartheta_P + \pi_P) - (\mu_P + \vartheta_P + \pi_P - 1) \times (\mu_P{}^2 + \vartheta_P{}^2 + \pi_P{}^2) = (\mu_P + \vartheta_P + \pi_P) - (\mu_P + \vartheta_P + \pi_P) \times 1 + 1 \times 1 = 1$$

It must be ensured that $\mu_I^P, \vartheta_I^P, \pi_I^P \in [0,1]$ is satisfied. As we know that $\mu_P + \vartheta_P + \pi_P \leq 2$ is satisfied for all PFNs, the statement $\mu_I + \vartheta_I + \pi_I - 1$ is between 0 and 1. We also have $\mu_P + \vartheta_P + \pi_P \in [0,1]$ from Definition 5. Depending on these prerequisites, the following conditions are satisfied.

$$\mu_I^P = \mu_P - (\mu_P + \vartheta_P + \pi_P - 1) \times \mu_P{}^2 < \mu_P - \mu_P{}^2 > 0,$$
$$\vartheta_I^P = \vartheta_P - (\mu_P + \vartheta_P + \pi_P - 1) \times \vartheta_P{}^2 < \vartheta_P - \vartheta_P{}^2 > 0,$$
$$\pi_I^P = \pi_P - (\mu_P + \vartheta_P + \pi_P - 1) \times \pi_P{}^2 < \pi_P - \pi_P{}^2 > 0$$

The square-scaled normalization (SSNORM) produces IFSs close to the ones obtained by the normalization (NORM). Figure 2 shows the comparison between membership grades of the IFSs against the non-membership grade change procured by these two methods for the cases in which the indeterminacy grade is equal to 0 and 0.1.

As seen in Figure 2 the difference between NORM and SSNORM increases depending on the increase of the indeterminacy grade. The membership grade obtained by SSNORM is observed smaller than the one obtained by NORM for big PFN membership grades. SSNORM reduces the small grades less and big grades more. The results of the two conversion methods are same when the PFN membership and non-membership grades are equal.

Based on the IFSs, some metrics have been discussed in the literature. Score function, one of these metrics, is a measure of the degree of the suitability of the fuzzy information for decision maker's requirement. Score function is formulated as the difference between membership and non-membership grades [17]. Another metric is the accuracy function which measures the degree of certainty of the fuzzy information. Accuracy function is formulated as the sum of membership and non-membership degrees [18].

Definition 8: Let $\tilde{P} = \langle \mu_P, \vartheta_P, \pi_P \rangle$ be a PFN, $\tilde{A} = \langle \mu_I^P, \vartheta_I^P, \pi_I^P \rangle$ be an IFN obtained by the SSNORM conversion of $\tilde{P}$. The score function $S(\tilde{A})$ of $\tilde{A}$ is obtained as in Eq. (13).

$$S(\tilde{A}) = \mu_I^P - \vartheta_I^P = \mu_P\left(1 - \mu_P(\mu_P + \vartheta_P + \pi_P - 1)\right) - \vartheta_P\left(1 - \vartheta_P(\mu_P + \vartheta_P + \pi_P - 1)\right) \tag{13}$$

Definition 9: Let $\tilde{P} = \langle \mu_P, \vartheta_P, \pi_P \rangle$ be a PFN, and $\tilde{A} = \langle \mu_I^P, \vartheta_I^P, \pi_I^P \rangle$ be an IFN obtained by the SSNORM conversion of $\tilde{P}$. The accuracy function $H(\tilde{A})$ of $\tilde{A}$ is obtained as in Eq. (14).

$$H(\tilde{A}) = \mu_I^P - \vartheta_I^P = 1 - \pi_I^P = \mu_P\left(1 - \mu_P(\mu_P + \vartheta_P + \pi_P - 1)\right) - \vartheta_P\left(1 - \vartheta_P(\mu_P + \vartheta_P + \pi_P - 1)\right) = 1 - \pi_P\left(1 - \pi_P(\mu_P + \vartheta_P + \pi_P - 1)\right) \tag{14}$$

Example 2: For the PFN given in Example 1, NORM and SSNORM are found as follows:

$$\mu_{I\ NORM}^P = \frac{\mu_P}{\mu_P + \vartheta_P + \pi_P} = \frac{0.0866}{1.3366} = 0.0648$$
$$\vartheta_{I\ NORM}^P = \frac{\vartheta_P(x)}{\mu_P + \vartheta_P + \pi_P} = \frac{0.95}{1.3366} = 0.7108$$
$$\pi_{I\ NORM}^P = \frac{\pi_P(x)}{\mu_P + \vartheta_P + \pi_P} = \frac{0.3}{1.3366} = 0.2244$$
$$\mu_{I\ NORM}^P + \vartheta_{I\ NORM}^P + \pi_{I\ NORM}^P = 1$$
$$S_{NORM} = -0.646,\ H_{NORM} = 0.7756$$
$$\mu_{I\ SSNORM}^P = \mu_P - (\mu_P + \vartheta_P + \pi_P - 1) \times \mu_I^P = 0.0866 - 0.3366 \times 0.0075 = 0.0841$$
$$\vartheta_{I\ SSNORM}^P = \vartheta_P - (\mu_P + \vartheta_P + \pi_P - 1) \times \vartheta_I^P = 0.95 - 0.3366 \times 0.9025 = 0.6462$$
$$\pi_{I\ SSNORM}^P = \pi_P - (\mu_P + \vartheta_P + \pi_P - 1) \times \pi_I^P = 0.3 - 0.3366 \times 0.09 = 0.2697$$
$$\mu_{I\ SSNORM}^P + \vartheta_{I\ SSNORM}^P + \pi_{I\ SSNORM}^P = 1$$
$$S_{SSNORM} = -0.5621,\ H_{SSNORM} = 0.7303$$

The score and accuracy functions can be used for comparison and ordering of multiple IFNs in the context of MCDM problems. The comparison law offered by [19] is applicable for the IFNs produced by the SSNORM conversion too.

Definition 10: Let $\tilde{A}_1$, $\tilde{A}_2$ be two IFNs, $S(\tilde{A}_1)$, $S(\tilde{A}_2)$ be the score functions, and $H(\tilde{A}_1)$, $H(\tilde{A}_2)$ be the accuracy functions. The IFNs are ordered by the following comparison rules [19]:

- If $S(\tilde{A}_1) < S(\tilde{A}_2)$, then $\tilde{A}_1 < \tilde{A}_2$
- If $S(\tilde{A}_1) > S(\tilde{A}_2)$, then $\tilde{A}_1 > \tilde{A}_2$
- If $S(\tilde{A}_1) = S(\tilde{A}_2)$, then:
  - If $H(\tilde{A}_1) < H(\tilde{A}_2)$, then $\tilde{A}_1 = \tilde{A}_2$
  - If $H(\tilde{A}_1) > H(\tilde{A}_2)$, then $\tilde{A}_1 < \tilde{A}_2$
  - If $H(\tilde{A}_1) = H(\tilde{A}_2)$, then: $\tilde{A}_1 > \tilde{A}_2$

A NUMERICAL EXAMPLE FROM MANUFACTURING INDUSTRY

The proposed conversion method has been demonstrated on a numerical example from a pen manufacturer company in this section. The company produces pens; and the body of plastic pens are purchased from a supplier. Acceptance sampling procedure has been applied to the pen bodies. Pen bodies might have different types of defects having different significances. For example, while the screw area defects are considered totally defective, major color defects and minor form defects are considered acceptable. The defectiveness levels also vary inside the defect classes thus deciding the defectiveness class of the pen bodies may not be possible because of indetermination.

The parameters of the traditional ASPs are decided by using a single numerical acceptable quality level (AQL) [20]. For this reason, they do not give the ability to model this variability of item defectiveness. Multiple defect types and relevant AQL levels are defined, and multiple separate ASP procedure are conducted in practice as an alternative solution to overcome this limitation [21]. However, there is no need to organize multiple ASPs because the ASP formulation based on IFS proposed by Işık & Kaya [22] fits well with this scenario.

Definition 11: Let $\tilde{p}$ be the fuzzy defective item proportion and, $\tilde{q}$ be the fuzzy non-defective item proportion in a lot, n be the sample size, c be the maximum allowed defective item count, and τ be the maximum allowed indeterminate item count. An intuitionistic fuzzy ASP is a set of rules, in which the defectiveness of the items is an IFS such that $\tilde{A} = \{x, \mu_{\tilde{A}}(x) = \tilde{P}, \vartheta_{\tilde{A}}(x) = \tilde{q} \mid x \in X\}$ satisfying $\tilde{p} + \tilde{q} \leq 1$ with the hesitancy/non-determinacy degree $\tilde{\pi} = 1 - \tilde{p} - \tilde{q}$ [22].

Definition 12: Lot acceptance probability ($\tilde{P}$) and lot rejection probability ($\tilde{P}_r$) for intuitionistic fuzzy ASPs are calculated as shown in Eqs. (15) and (16) [22].

$$\tilde{P}_a = \sum_{d=0}^{c} \binom{n}{d} \otimes \tilde{p}^d \otimes \left[\sum_{i=0}^{\tau} \binom{n\Theta d}{i} \otimes \left(1\Theta \tilde{p} \Theta \tilde{q}\right)^i \otimes \tilde{q}^{(n\Theta i\Theta d)} \right] \quad (15)$$

$$\tilde{P}_r = \sum_{d=c\oplus 1}^{n} \binom{n}{d} \otimes \tilde{p}^d \otimes \left[\sum_{i=0}^{n\Theta d} \binom{n\Theta d}{i} \otimes \left(1\Theta \tilde{p} \Theta \tilde{q}\right)^i \otimes \tilde{q}^{(n\Theta i\Theta d)} \right] \quad (16)$$

The symbols $\oplus$, Θ and $\otimes$ have been used for addition, subtraction, and multiplication operations, respectively related to FSs.

Definition 13: Average outgoing quality $\left(\widetilde{AOQ}\right)$ and average total inspection $\left(\widetilde{ATI}\right)$ are calculated for the lots having N items as in Eqs. (17) and (18) [22].

$$\widetilde{AOQ} = \tilde{P}_a \otimes \tilde{P} \quad (17)$$

$$\widetilde{ATI}_{(0)} = n \oplus \left(1\Theta \tilde{P}_a\right) \otimes \left(N\Theta n\right) \quad (18)$$

In some cases that require working with multiple experts such as purchasing the pen bodies from an alternative supplier in case of urgency, the assessments for the item defectiveness may contain inconsistency. In such cases, the ASP formulation offered by Işık & Kaya [22] cannot be used and conversion to the IFSs is required.

Assume that the company wants to perform ASP having parameters $n = 50$, $c = 5$ and $\tau = 4$ for the lots having 500 items. The experts assessed the defective item proportion as 0.15, non-defective item proportion as 0.98 and indeterminate item proportion as 0.13. For this assessment, the main condition of the PFSs presented in Eq. (7) is satisfied. Membership, non-membership, and indeterminacy grades are found as in Table 1 for NORM and SSNORM.

As seen in Table 1, the defective item proportion have been obtained close to each other for two conversion meth-ods. However, the difference of defectiveness and non-defec-tiveness grades have been obtained bigger for NORM in comparison with SSNORM. The ASP results for the conver-sion methods have been presented in Table 2. SSNORM has

Table 1. Membership, non-membership and indeterminacy grades for the conversion methods

Conversion Method	Defectiveness	Non-defectiveness	Indeterminacy
Normalization	0.1190	0.7778	0.1032
Square-scaled Normalization	0.1441	0.7303	0.1256

Table 2. ASP results for the conversion methods

Conversion Method	Acceptance Probability	Rejection Probability	Average Outgoing Quality	Average Total Inspection
Normalization	0.1598	0.5561	0.2842	0.0190
Square-scaled Normalization	0.0453	0.7453	0.2093	0.0065

given more precautionary results for ASPs because a bigger defective item proportion has been reached by SSNORM than NORM. Acceptance probability has been obtained smaller by using SSNORM conversion. Depending on this, smaller AOQ and ATI values have been found.

CONCLUSION

PFSs are the generalized version of IFSs allow to model the uncertainty with inconsistent data. However, working with inconsistent data may not be applicable for some real-life problems such as ASPs. For these cases, it is required to convert the PFS statements into IFSs. The most popu-lar conversion method named normalization depends on scaling down the membership, non-membership, and indeterminacy grades by linear proportioning. Thence, it protects the relative greatness of the grades between each other. There is another conversion named "projective rela-tion" offered in the literature as an alternative of the nor-malization. However, this conversion method is not valid because it can cause negative membership grades. In this study, the error about the proof of the projective relation between PFNs and IFNs offered by Tao et al. [1] has been pointed out, a disproof and a counter example have been presented. As an alternative conversion, a modified ver-sion of the normalization named "square-scaled normaliza-tion" has been offered. A numerical ASP example has also been presented and the results for the suggested conversion method have been compared with the normalization. The proposed conversion method has provided more cautious results for ASPs.

As future directions, the suggested conversion method can be combined with the linguistic fuzzy approach and used in MCDM problems, fuzzy distance measures can be formulated for the IFSs produced by the suggested conver-sion, and new conversions can be examined and compared with the suggested one.

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports th e findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

REFERENCES

[1] Tao Z, Zhu J, Zhou L, Liu J, Chen H. Multi-attribute decision making with Pythagorean fuzzy sets via conversions to intuitionistic fuzzy sets and ORESTE method. J Control Decis 2020;8:1–22. [CrossRef]

[2] Zadeh LA. Fuzzy sets. Inform Control 1965;8:338–353. [CrossRef]

[3] Atanassov KT. Intuitionistic fuzzy sets: theory and applications. Berlin: Physica, Heidelberg; 1999. [CrossRef]

[4] Yager RR. Pythagorean fuzzy subsets. 2013 Joint IFSA World Congress and NAFIPS Annual Meeting (IFSA/NAFIPS), 2013. [CrossRef]

[5] Yager RR. Pythagorean membership grades in mul-ticriteria decision making. IEEE Trans Fuzzy Syst 2014;22:958–965. [CrossRef]

[6] Smarandache F. Introduction to neutrosophic statis-tics. Infinite Study; 2014.

[7] Wang XJ, Hao YW, Zhao RH. Method, model and application for the conversion from vague sets to fuzzy sets. 2009 International Conference on Artificial Intelligence and Computational Intelligence, Shanghai, China, 2009. [CrossRef]

[8] Bustince H, Burillo P. Vague sets are intuitionistic fuzzy sets. Fuzzy Sets Syst 1996;79:403–405. [CrossRef]

[9] Liu Z, Alcantud JCR, Qin K, Pei Z. The relationship between soft sets and fuzzy sets and its application. J Intell Fuzzy Syst 2019;36:3751–3764. [CrossRef]

[10] Deschrijver G, Kerre EE. On the relationship between some extensions of fuzzy set theory. Fuzzy Sets Syst 2003;133:227–235. [CrossRef]

[11] Dubois D, Foulloy L, Mauris G, Prade H. Probability-possibility transformations, triangular fuzzy sets, and probabilistic inequalities. Reliab Comput 2004;10;273–297. [CrossRef]

[12] Geer JF, Klir GJ. A mathematical analysis of infor-mation-preserving transformations between proba-bilistic and possibilistic formulations of uncertainty. Int J Gen Syst 1992;20:143–176. [CrossRef]

[13] Yamada K. Probability-possibility transforma-tion based on evidence theory. Proceedings Joint 9th IFSA World Congress and 20th NAFIPS International Conference, 2001.

[14] Pota M, Esposito M, Pietro GD. Transformation of probability distribution into fuzzy set interpretable with likelihood view. 11th International Conference on Hybrid Intelligent Systems (HIS), 2011. [CrossRef]

[15] Florea MC, Jousselme AL, Grenier D, Bossé É. Approximation techniques for the transforma-tion of fuzzy sets into random sets. Fuzzy Sets Syst 2008;159:270–288. [CrossRef]

[16] Beliakov G, James S. Averaging aggregation func-tions for preferences expressed as Pythagorean membership grades and fuzzy orthopairs. 2014

IEEE International Conference on Fuzzy Systems (FUZZ-IEEE), Beijing, China, 2014. [CrossRef]

[17] Chen SM, Tan JM. Handling multicriteria fuzzy decision-making problems based on vague set theory. Fuzzy Sets Syst 1994;67:163−172. [CrossRef]

[18] Hong DH, Choi CH. Multicriteria fuzzy decision-making problems based on vague set theory. Fuzzy Sets Syst 2000;114:103−113. [CrossRef]

[19] Xu Z. Intuitionistic fuzzy aggregation operators. IEEE Trans Fuzzy Syst 2007;15:1179−1187. [CrossRef]

[20] Montgomery DC. Introduction to statistical quality control. 6 ed. Hoboken, New Jersey: Wiley, 2009: 631−670.

[21] Anjoran R. QualityInspection.org. Available: https://qualityinspection.org/what-is-the-aql/. Accessed on Apr 1 2021.

[22] Işık G, Kaya İ. Design and analysis of acceptance sampling plans based on intuitionistic fuzzy linguistic terms," Iranian Journal of Fuzzy Systems, 2021;18:101−118.