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Applications Of Soft Mapping

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ÖZET

Bu çalışmada grup soft dönüşümlerin tanımını verdik ve onların bazı cebirsel özelliklerini ve cebirsel yapılar üzerinde uygulamalarını çaıştık. Ayrıca grup soft dönüşümler altında grupların ve soft grupların görüntülerini verdik.

Anahtar Kelimeler: Soft kümeler, Soft gruplar, grup soft dönüşümler

Soft dönüşüm uygulamaları

ABSTRACT

In this paper we give a denition of group soft mappings and study their some properties and applications on algebraic structures. We also introduce images of groups and soft groups under group soft mapping.

Key Words: Soft sets, soft groups, soft group mappings

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1. Introduction

Molodtsov [1] initiated the theory of soft sets a new mathematical toll for dealing with uncertainties because classical tools are not always successful. Soft set theory has a rich potential for applications in economics, engineering, environmental sciences and medical sciences. Recently, research on soft set theory has been progressing rapidly. Maji et al.[2] introduced several operators for soft set theory: equality of two soft sets, complement of a soft set, null soft set, absolute soft set and subset of soft set. They also dened soft binary operations such as AND, OR, and the operation of union and intersection. Babitha and Sunil [3] dened soft set relations and functions. Aktas and Cagman [4]compared soft sets to the related concepts of fuzzy sets and rough sets. They also introduced a basic version of soft group theory. Feng et al.[5] studied soft set combined with fuzzy sets and rough sets. Majumdar and Samanta [6] introduced the notion of soft mappings. They also dened images and inverse images of crisp sets and soft sets under soft mapping.

The rest of paper is organized as follows. We express brief historical of soft set theory in the rst section. In the second section we give the basic principles of soft sets and soft group. In the third section we introduce a denition of group soft mappings and study some of their properties. Images of groups and soft groups under group soft mappings are also studied.

2. Preliminaries

Let us rst recall some concepts related soft set and soft group.

Denition 2.1. [1] Let U be an initial universal set and let E be a set of parameters. Let $P(U)$ denote the power set of U and $A \subset E$. A pair (F,A) is called a soft set over U i F is a mapping given by $F: A \to P(U)$.

Denition 2.2. [2] A soft set $(F;A)$ over U is said to be a null soft set denoted by $\emptyset$, if $\forall e \in A,\ F(e) = \emptyset$.

Denition 2.3. [2] Let $(F;A)$ over U is said to be an absolute soft set denoted by A, if $\forall e \in A,\ F(e) = U$.

Denition 2.4. [2] Let (F,A) and (G,B) be two soft sets over U. Then (F,A) is called a soft subset of (G,B), denoted by $(F,A) \subset (G,B)$, if

i. $A \subset B$

ii. $\forall \propto \in A,\ F(\propto) \subset G(\propto)$.

Denition 2.5. [2] The intersection of two soft sets (F,A) and (G,B) over U is the soft set (H,C), where $C = A \cap B$ and $\forall \propto C,\ H(\propto) = F(\propto)$ or $G(\propto)$ (as both are same set). This is denoted by $(F,A) \bar{\cap} (G,B) = (H,C)$.

Denition 2.6. [2] The union of two soft sets (F,A) and (G,B) over U is the soft set (H,C), where $C = A \cup B$ and $\forall \propto \in C$,

$$H(\propto) = \{F(\propto)\,;\ if\ \propto \in A - B\}$$

Denition 2.7. [2] If (F,A) and (G,B) are two soft sets then $(F,A) AND (G,B)$ is denoted $(F,A) \wedge (G;B)$. $(F,A) \wedge (G;B)$ is defined as $(H, A \times B)$, where $H(\alpha,\beta) = F(\alpha) \cap G(\beta)$, $\forall (\alpha,\beta) \epsilon A \times B$.

Throught this paper, G is a group and A is any non-empty set.

Denition 2.8. [4] Let (F,A) be a soft set over G. Then (F,A) is said to be a soft group over G if and only if $F(x) < G$ for all $x \in A$.

Denition 2.9. Let (F,A) be a soft set over G. Then (F,A) is said to be a soft semigroup over G if and only if $F(x)$ is a semigroup for all $x \in A$.

Denition 2.10. [4] Let (F,A) and (H,K) be two soft groups over G. Then (H,K) is a soft subgroup of (F,A), written $(H,K) \precsim (F,A)$, if

i. $K \subset A$

ii. $H(x) < F(x)$ for all $x \in K$.

Theorem 2.11. [4] Let (F,A) and (H,A) be two soft group over G. Then their intersection $(F;A) \bar{\cap} (H;A)$ is a soft group over G.

Theorem 2.12. [4] Let (F,A) and (H,A) be two soft group over G. If $A \cap B = \emptyset$ then $(F,A) \cup (H,B)$ is a soft group over G.

Theorem 2.13. [4] Let (F,A) and (H,A) be two soft group over G. Then $(F,A) \wedge (H,B)$ is soft group over.

2. Group Soft Mapping

In this section we introduce a notion of group soft mapping and study its properties.

Definition 3.1. Let G, H be two groups and E be a parameter set. Then the mapping $F: E \to P(H^G)$ is called a group soft mapping from G to H under E, where H^G is the collection of all homomorphism from G to H.

Example 3.2. Let $E = \{e_1, e_2\}$ is a parameter set and $G = (R^2, +)$ and $H = (R, +)$ be two groups. Let $f_1,\ f_2,\ f_3 : G \to H$ homomorphism be defined as follows:
If $x = (x_1, x_2) \in R^2,\ f_1(x) = x_1 - x_2,\ f_2(x) = x_1 + x_2$ and $f_3(x) = 3x_2$. Let $F: E \to P(H^G)$ be defined as follows $F(e_1) = \{f_1\},\ F(e_2) = \{f_2, f_3\}$. Then F is a group soft mapping from G to H under E.

Proposition 3.3. A group soft mapping is a soft semigroup.

Proof For all elements of parameter set, product of two homomorphisms is a homomorphism owing to group theory. Therefore homomorphism set is closed. Homomorphism set also provide associativity property for all parameter set. Because of two properties, group soft mapping is a soft semigroup. In addition if the homomorphism in Example3.2 is injective and bijective that which isomorphism, group soft mapping is a soft group.

Definition 3.4. Let E, E_1, E_2 be a parameter set sets and G, H and K be groups.

1. $F: E \to P(G^G)$ is called a identity group soft mapping from G to G under E, if $F(x) = \{I_G\}$ for all $x \in E$. Where I_G is defined as group homomorphism from G to G.
2. For two soft mappings with $F: E_1 \to P(H^G)$ and $J: E_2 \to P(K^H)$ with $E_1 \cap E_2 \neq \emptyset$ their composition $J * F: E_1 \cap E_2 \to P(K^G)$ is defined by $(J * F)(e) = \{h = j^\circ f : A \to C \mid j(e)^\circ f(e) = h(e), j(e) \in J(e), f(e) \in F(e)\}$ for all $e \in E_1 \cap E_2$.

Example 3.5. Let $E_1 = \{e_1, e_2\} = E_2,\ G = (R^2, +),\ H = (R, +)$ and $K = (R^2, +)$. Let $f_1, f_2 : G \to H$ and $j_1,\ j_2 : H \to K$ be group homomorphism. Let $J: E_2 \to P(K^H)$ be a group soft mapping defined as $J(e_1) = \{j_1\},\ J(e_2) = \{j_2\}$. Let $F: E_1 \to P(H^G)$ be another group soft mapping defined as $F(e_1) = \{f_1,\ f_2\},\ F(e_2) = \{f_2\}$. Then the composition of F and J is possible and $H = J * F: E_1 \cap E_2 \to P(K^G)$ where $H(e_1) = \{j_1{}^\circ f_1,\ j_1{}^\circ f_2\},\ H(e_2) = \{j_2{}^\circ f_2\}$.

Proposition 3.6. Let for two soft mappings with $F: E_1 \to P(H^G)$ and $J: E_2 \to P(K^H)$ with $E_1 \cap E_2 \neq \emptyset$ their composition $J * F: E_1 \cap E_2 \to P(K^G)$.

1. Identity group soft mapping is a soft semigroup over G^G.

2. $J * F$ is also a group soft mapping.

Proof This is easily obtained from Definition3.4 and Definition3.1.

Definition 3.7. A group soft mapping $F : E \to P(H^G)$ is said to be weakly injective if $\forall e_1, e_2 \in E, e_1 \neq e_2$ ise $F(e_1) \neq F(e_2)$ and a group soft mapping F is said to be strongly injective if $\forall e_1, e_2 \in E,$ $e_1 \neq e_2$ ise $F(e_1) \cap F(e_2) = \emptyset$.

Proposition 3.8. If $F : E_1 \to P(H^G)$ and $J : E_2 \to P(K^H)$ are two weakly injective group soft mappings, $J * F$ is also weakly injective.

Proof This is easily obtained from Definition3.4 and Definition3.7.

Definition 3.9. [Image of a Group Under a Group Soft Mapping] Let $F : E \to P(H^G)$ be a group soft mapping and $T < G$. Then $F(T)$ is a mapping from E to $P(H)$ such that

$$F(T)(e) = \bigcap_{f_e \in F(e)} f_e(T) = \bigcap_{f_e \in F(e)} \{f_e(t) : t \in T\}$$

Theorem 3.10. Image of any group under a group soft mapping is a group.

Proof Let $F : E \to P(H^G)$ be a group soft mapping and $T < G$. From Definetion3.9 we write

$$F(T)(e) = \bigcap_{f_e \in F(e)} f_e(T) = \bigcap_{f_e \in F(e)} \{f_e(t) : t \in T\}$$

Since f_e is a group homomorphism from G to H, $f_e(T) < H$. The intersection of subgroups is a group. Hence we find that image of any group under a group soft mapping is a group.

Theorem 3.15. Let Let $F : E \to P(H^G)$ be a group soft mapping and $K_1, K_2 < G$

i.$K_1 < K_2 \Longrightarrow F(K_1) < F(K_2)$

ii. $F(K_1 \cap K_2) < F(K_1) \cap F(K_2)$.

Proof Let $F : E \to P(H^G)$ and for $e \in E,$ $F(e) = \{f_e : f_e : G \to H\}$ where f_e grup homomorphism,

i. $K_1 < K_2 \Longrightarrow f_e(K_1) < f_e(K_2)$, $\bigcap_{f_e \in F(e)} f_e(K_1) < \bigcap_{f_e \in F(e)} f_e(K_2) \Longrightarrow F(K_1)(e) < F(K_2)(e)$. Hence the theorem is proved

ii. $f_e(K_1 \cap K_2) < f_e(K_1) \cap f_e(K_2)$,

$$\bigcap_{f_e \in F(e)} f_e(K_1 \cap K_2) < \bigcap_{f_e \in F(e)} \left(f_e(K_1) \cap f_e(K_2)\right)$$

$$= \left(\bigcap_{f_e \in F(e)} f_e(K_1)\right) \cap \left(\bigcap_{f_e \in F(e)} f_e(K_2)\right)$$

$$= F(K_1) \cap F(K_2).$$

Hence $F(K_1 \cap K_2) < F(K_1) \cap F(K_2)$.

Definition 3.14. Let $F : E \to P(H^G)$ be a group soft mapping. Let $S = (K, E_1)$ be a soft group over G, where $E_1 \subset E$. Then the image of S under F, denoted by $F(S)$, is defined as follows: For $e \in E_1$,

$$F(S)(e) = \begin{cases} \bigcap_{f_e \in F(e)} \{f_e(t) : t \in K(e)\} & if\ K(e) \neq \emptyset \\ \emptyset & if\ K(e) = \emptyset \end{cases}$$

Theorem 3.18. Let $F : E \to P(H^G)$ be a group soft mapping and $S_1 = (K_1, E_1)$, $S_2 = (K_2, E_2)$ be a soft roup over G.

i. $S_1 \precsim S_2 \Longrightarrow F(S_1) \precsim F(S_2)$

ii. $F(S_1 \tilde{\cap} S_2) \precsim F(S_1) \tilde{\cap} F(S_2)$.

iii. $F(S_1 \wedge S_2) \precsim F(S_1) \wedge F(S_2)$

Proof i and ii are easily obtained from Definition2.10, Definition2.5 and Theorem2.11.

iii. For all $(x; y) \in E_1 \times E_2$,

$$F(S_1 \wedge S_2) = \bigcap_{f_e \in F(e)} (f_e((K_1, E_1) \cap (K_2, E_2)))$$

$$= \bigcap_{f_e \in F(e)} f_e\,(T, E_1 \times E_2)$$

$$= \bigcap_{f_e \in F(e)} f_e\,(K_1(x) \cap K_2(y))$$

$$\precsim \bigcap_{f_e \in F(e)} f_e\,(K_1(x)) \cap f_e(K_2(y))$$

$$= \left(\bigcap_{f_e \in F(e)} f_e\left(K_1(x)\right)\right) \cap \left(\bigcap_{f_e \in F(e)} f_e\left(K_2(y)\right)\right)$$

$$= F(S_1) \wedge F(S_2)$$

Hence $F(S_1 \wedge S_2) \precsim F(S_1) \wedge F(S_2)$.

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