

Unsupervised Learning Approach for Detection and Localization of Structural Damage using Output-only Measurements

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Abstract

Interrogation of the vibration data collected from the sensors embedded throughout the structure without relying on a finite element model of the system for monitoring the health of structural systems has received significant attention in the recent years especially with the current advancements in sensor technology. The data-driven methods explored within this context falls into the realm of statistical pattern recognition field requiring extraction of damage detection features and a statistical decisionmaking process for identification of damage. Machine learning algorithms provide statistical means for making such decisions. In this study, an unsupervised machine learning approach, one-class support vector machine (OC-SVM), requiring training data only from the undamaged state of the structure is explored for damage detection purposes. The coefficients of the autoregressive (AR) model are extracted as damage sensitive features and used as the required training data. The trained classifier is then used with the data obtained from the same structure at different damage states for classification. Damage detection in the form of recognizing outliers or anomalies not belonging to the target class, is followed by damage localization within the given sensor resolution using statistical means. To this end, Itakura distance measuring the distance between two sets of linear predictor coefficients of the AR processes, is utilized as damage location indicator. Numerical simulations are performed on a truss and a beam structure with several damage scenarios including realistic levels of measurement noise and modeling error. Results show that the proposed approach can successfully detect existence of damage and the statistical measure shows promising performance for further localization of the damaged region.

Keywords: Structural Health Monitoring, Unsupervised Learning, Support Vector Machines, Time Series Modelling, Statistical Pattern Recognition

1. INTRODUCTION

Monitoring the health of civil engineering systems due to deterioration under normal operating conditions or after an extreme event is vital to take the necessary preventive measures to protect these systems against collapse, reducing maintenance cost and prolonging their service lives. Structural Health Monitoring (SHM) aims to assure the structural safety of civil infrastructure by evaluating the integrity of these systems and providing warning signs as soon as the condition of the structure deteriorates. Several approaches that estimate the state of the structure with different levels of refinement have been proposed in the literature. According to Rytter [1], damage characterization includes four stages: (1) damage detection, (2) damage localization, (3) damage quantification, and (4) prediction of the remaining service life.

Vibration-based damage identification is a subdiscipline of the SHM field that accomplishes these tasks through measured vibration responses of the structure. The vibrationbased damage detection can be classified in two general classes of model-based and data-driven approaches [2]. Data-driven approaches utilizing solely the recorded vibration data stand out as the more convenient and accessible alternative in regards to their ease of implementation over the model-based approaches requiring refined finite element model of the structure. Especially for complex structural systems with a large number of degrees of freedom, obtaining a refined finite element model of the structure is a difficult, computationally expensive task and from the material heterogeneity and mechanical behavior aspects, may not even be feasible or practical. Data-driven approaches also establish a model, but this is usually a statistical representation of the system. Released from the physical model requirement, data-driven damage detection depend on statistical analysis to determine the significance

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of the changes in the data and have the potential to be constructed in a fully automated manner with minimum user interaction.

Time series modeling and outlier analysis can be listed among the statistical approaches available for structural diagnosis based on collected vibration data. With these approaches, diagnosis stage usually starts by a data processing step where data collected from a sensor network deployed throughout the structure are transformed into features that are sensitive to damage. This transformation retains only the information necessary for diagnosis while discarding any other information. To this end, Worden et al. [6] performed an outlier analysis on a spring–mass–dashpot system for damage detection. Sohn et al. [3] and Sohn and Farrar [4] applied time-series modeling and utilized the residual error of the prediction model as a damage sensitive feature. Wandji [5] exploited autoregressive (AR) modeling to propose a goodness-of-fit test to distinguish damaged data. Mattson and Pandit[6] used the standard deviation of the residual of the AR model of vibration data as damage detecting feature. In another study by Nair et al.[7], the coefficients of the AR model of the ambient vibration response data was explored as damage sensitive features and damage states were identified through hypothesis testing. Kar and Mohanty [8] utilized time domain signals and diagnosed ball bearing faults by employing Kolmogorov– Smirnov (K-S) test. Distance between autoregressive moving average (ARMA) models were investigated by Zheng and Mita [9] for damage detection. Nair and Kiremidjian [10] detected damage by investigating the migration of extracted AR coefficients in which the feature vector is modeled by a Gaussian mixture model. In a later study, Gul and Catbas [11] predicted the data of the damaged structure, utilizing the autoregressive models with exogenous input (ARX) developed for the healthy structure, and proposed the difference between the predictions and the measured values as the damage feature. Roy et al. [12] explored combination of ARX model coefficients, K-S test statistical distance and the model residual error as damage sensitive features for the potential of detection and localizing damage. Entezami et al. [13] developed damage indices for damage localization that are based on damage sensitive features extracted from time series modeling of the measured vibration data.

Following the feature extraction from each data set, comparison of these features with the baseline values facilitates to arrive at a damage decision. In this context, the process can essentially be viewed as a statistical pattern recognition problem that is part of machine learning concerned with classification [14]. More specifically, this means, once these features are extracted at the initial nominal state of the structure, a machine learning approach can be employed to train a relationship between these features and the baseline state.

Despite many implementations of the time series models and machine learning techniques in the context of SHM majority of the algorithms available in the literature are custom-made for certain data sets. The lack of unsupervised techniques capable of detecting structural damage without relying on any prior knowledge about the structure's condition; the lack of strategies capable of detecting structural damage automatically in a robust manner and the lack of algorithms suitable for performing real-time detection can be listed among the obstacles several issues remain as aspects hindering the practical application of these damage detection algorithms to civil engineering structures. This study aims to address these issues and provide a technique that can be implemented in an unsupervised, automated and decentralized manner that will be suitable for real-time SHM.

For cases where training data are available only from a single condition, the baseline or the undamaged state, the unsupervised machine learning approaches can be adopted to train the decision-making algorithm. This fits very well for civil engineering applications since each structural system is unique with its own structural and dynamic properties requiring its own individual training data. It is not practical or even possible to expect that labeled training data are acquired and made available for various damage scenarios . One-class support vector machine (OC-SVM) classifier, also known as novelty detection approach, is an unsupervised learning algorithm classifying just one-class objects. It has been used successfully in many applications such as image retrieval, audio surveillance, biometric traits and more recently in SHM applications. The recent applications of machine learning methods utilized for vibration-based damage detection in civil structures are reviewed in [15, 16].

In the proposed approach, the extracted features from the AR model are utilized to train the OC-SVM. With the learned relationship, the trained classifier exploits the data obtained from the same structure at different states and arrives at a damage decision, by classifying the data as either belonging to the learned state (undamaged) or not (damaged). This completes the first stage of the damage characterization problem. For the next stage of damage localization, in the extracted feature space of AR coefficients, a statistical distance metric is employed. This metric, defined as Itakura distance, initially developed to measure the similarities between AR models of the voice segments [17]. This metric is calculated for each sensor location and the larger value is interpreted as an indication to the proximity of damage.

The proposed methodology is investigated numerically on two different structural systems: a truss and a beam type structure. Several damage scenarios are simulated including reduction in member rigidities and fracture of the members for the truss system and loss of connection rigidities for the beam model. The acceleration responses under ambient excitation data are simulated at the designated sensor locations. Modeling error and measurement noise are incorporated into the numerical work for simulating field data.

2. MATERIALS AND METHODS

2.1. Time Series Modeling for Feature Extraction

Among the time series models available for fitting a mathematical model to time series data, AR model stands out as the most widely used time series model for extracting damage sensitive system. The AR models assume the

measurements have noise, and modeling error is sampled from Gaussian distribution.

For a linear, stationary, and univariate time series, the AR model is formulated as follows:

$$
y(t) = \sum_{i=1}^{p} a_i y(t-i) + e(t)
$$
 (1)

where $y(t)$ is the measured vibration response at a single sensor at time t, $a_i = [a_1, a_2, \dots, a_p]$ represents the AR model coefficients, *p* is the order of the model, and *e(t)* is the residual error at time that corresponds to the difference between the measured and the predicted time series data obtained by the AR model [18, 19].

By fitting an AR model to time-domain response data acquired from each sensor at the nominal state of the structure, one can extract the AR parameters for the corresponding sensor location representing the healthy state. Measured data from the same set of sensors at an unknown state of the structure are also processed to extract the AR parameters fitting to that undetermined state. These parameters are compared as damage sensitive features to be compared for classification using a support machine algorithm described in the following section.

2.2. One-Class Support Vector Machine

The OC-SVM is a specialized version of the standard support vector machines with the objective of finding an optimal hyperplane in which most of the training samples are included in a minimum volume and separating and distinguishing one-class objects from all others. Objects falling outside the constructed hyperplane are considered as outliers.

Scholkopf et al. [20, 21] proposed the OC-SVM algorithm to train the classifier using only the features belonging to the target class. The decision function for the hypersphere that encloses the maximum number of training samples takes the following form

$$
f(x) = sgn\left\{\sum_{i=1}^{m} \alpha_i K(x, x_i) - \rho_i\right\}
$$
 (2)

where *m* is the number of training samples and α_i are the Lagrange multipliers obtained from optimization of the following set of equations:

$$
min_{\alpha} \left\{ \frac{1}{2} \alpha_i \alpha_j K(x_i, x_j) \right\} \tag{3}
$$

subject to

$$
0 \le \alpha_i \le \frac{1}{\nu_m} \tag{4}
$$

$$
\sum_{i}^{m} \alpha_{i} = 1 \tag{5}
$$

 ρ in equation (2) is the distance of the hypersphere from the origin. v is the percentage of the samples considered as outliers and *K*() defines the OC-SVM kernel that allows projection of samples from the original to the feature space. Among the various kernels, radial basis function is the most used kernel allowing to determine the radius of the hypersphere through the kernel parameter γ as follows:

$$
K(x, x_i) = \exp(-\gamma d(x, x_i))
$$
 (6)

such that

$$
d(x, x_i) = ||x - x_i||^2
$$
 (7)

2.3. Statistical Control Metrics: Itakura Distance Measure for Damage Localization

Following the decision on existence of structural damage, a metric implemented for damage localization is needed for completion of the damage identification process. The available literature proposes parametric-based damage indices that uses the parameters of the prediction models for identifying damage location and [13, 22, 23] or some other measures correlating locations of a sensor network with the damage features [9, 24-26]. In this study, a statistical measure defined as Itakura distance [17] is explored as a possible means for fusing the information from all sensors deployed throughout the structure and localizing damage. The theoretical basis for this statistical measure are summarized below.

Suppose that $x(t)$ and $y(t)$ are two time series corresponding to the baseline state (target class) and damaged state (outlier), respectively. Assume that the coefficients of the corresponding two AR processes of order *p* are given by

$$
a_x = \begin{bmatrix} 1 & a_{x1} & a_{x2} & \cdots & a_{xp} \end{bmatrix}
$$
 (5a)
\n
$$
a_y = \begin{bmatrix} 1 & a_{y1} & a_{y2} & \cdots & a_{yp} \end{bmatrix}
$$
 (5b)

The means square error (MSE) for the baseline process corresponding to *x*(t) is

$$
MSE_{xx} = a_x^T R_x a_x \tag{6}
$$

where R_x is the autocorrelation matrix of signal $x(t)$ of size *p*+1.

Supposing that $x(t)$ pass through the AR model parametrized by a_y , the MSE of the signal is

$$
MSE_{xy} = a_y^T R_x a_y \tag{7}
$$

The Itakura distance indicating how far the outlier state parametrized by a_v , is from the baseline state parametrized by a_x , is defined as

$$
D_{Ix,y} = \log \frac{a_y^T R_x a_y}{a_x^T R_x a_x} \tag{8}
$$

Similarly, how well the signal $y(t)$ is modeled by the ARparameters of $x(t)$, is given by

$$
D_{Iy,x} = \log \frac{a_x^T R_y a_x}{a_y^T R_y a_y} \tag{9}
$$

Combining the two distances to obtain a symmetric distance measure, one gets the Itakura distance (D_I) as

$$
D_{I} = \frac{1}{2} \left(D_{Ix,y} + D_{Iy,x} \right) \tag{10}
$$

2.4. Methodology

The proposed damage detection and localization methodology is depicted in Figure 1.

Figure 1. Damage identification methodology

The methodology starts by the training stage of the OC-SVM for the baseline (undamaged) state of the structure. The measured acceleration signals in response to ambient excitations from the structure at this state are processed including detrending and normalization procedures leading to the extraction of AR parameters. After sufficient data required for training is collected, the OC-SVM classifier is attained for each sensor location as the predictive model. The completion of the training stage is followed by the prediction stage for the data collected from the structure later at an unknown state. With the new datasets of unknown state, AR parameters are identified and these newly identified features are tested with the previously established predictive model. This comparison with the OC-SVM classifier outputs a binary decision regarding the classification of data as belonging to the baseline (undamaged) state or not. Once damage is detected damage localization stage is invoked by calculating the localization index, namely Itakura distance, for all sensor locations. The sensor with the highest value of the localization index is marked as the sensor closest to the possible damaged location.

3. NUMERICAL EVALUATION

This section presents the numerical simulations conducted on two different structures to investigate the performance of the proposed damage detection algorithm and the localization metric. The two structures, 14-degree-offreedom (DOF) truss and 24 DOF fixed-ended beam considered in this study are depicted in Figures 1(a) and 2(a), respectively. For both systems, masses are assumed to be lumped at the degrees of freedom and damping is assigned to be proportional, 2% for all modes. At the undamaged state, the members have identical rigidities.

The locations of the acceleration sensors together with the damage scenarios simulated for these structures are also depicted in these figures. Note that for each system, input motion is taken as ambient excitations acting in the vertical direction that are assumed to be unmeasured and output measurements as the acceleration responses simulated in the same direction as the input. This leads to the deployment of 7 acceleration sensors for the truss and 8 sensors for the fixed-ended beam, for output measurements as shown in Figures 1(b) and 2(b). The simulated damage scenarios in the form of loss of member and connection rigidity are summarized in Table 1 for both structural systems. The analytically computed natural frequencies of these systems for the simulation cases are determined through eigenvalue analysis and the first four modes are listed in Table 2.

Figure 1. (a) 14 DOF truss model, (b) Damage scenarios and sensor locations for input and output

scenarios and sensor locations for input and output

Configuration	TRUSS	<i>FIXED-FIXED BEAM</i>
DC1	Bar 10 - 30% loss of axial rigidity	Section 34-30% loss of flexural rigidity
DC ₂	Bar 17 - 90% loss of axial rigidity	Section 34-90% loss of flexural rigidity
DC3	Bar 19 - 90% loss of axial rigidity	Plastic hinge at joint 7
DC4	Bar $6 - 50\%$ loss of axial rigidity	Plastic hinge at joint 10

Table 1. Summary of the damage scenarios

In both cases, the input ambient excitations are modeled as Gaussian white noise processes and the output acceleration measurements are simulated with a sampling time of 0.04 sec. for a duration 300 sec. which is deemed sufficient to capture the frequency range of interest (2-25 Hz). A total of 100 simulations are performed at the nominal (healthy) state of the structures, 50% of which are used as the baseline training data for the one-class classifier. To gain a statistical sense of performance, 100 simulations for each damage scenario are examined. In the simulated acceleration measurements, sensor noise is contemplated using a random number generator with a level ranging between 2-10 % of the RMS of the response measured on DOF 1 such that

$$
SNR = \sigma_{noise}^2 / \sigma_{signal}^2 \dots \tag{13}
$$

where σ_{signal}^2 , σ_{noise}^2 are the variance of the simulated acceleration and the noise signals and *SNR* is the desired signal to noise ratio.

In order to mimic the variability in the recorded measurements due to operating conditions of existing structural systems, modeling error is also introduced to the simulations by assigning the modulus of elasticity of the members as the true values multiplied by a random scalar that is uniformly distributed between [0.9 and 1.1].

4. RESULTS AND DISCUSSION

Damage detection performance of the trained OC-SVM for the truss system and the beam in the 2-dimensional, and the 5-dimensional feature space are summarized in Tables 3 and 4, respectively. In the results presented healthy configuration is the baseline state for which training data was available and DC stands for the '*damage case'*. Examination of the data shows that, even with only two features, the damage detection performance of the OC-SVM is excellent for the truss system with the exception of a single sensor location for a specific damage scenario, namely sensor 7 for DC 2. Increasing the feature space slightly, from 2 to 5, for this specific case results in significant improvement. For the remainder of the data, the performance of the 2-dimensional versus 5-dimensional feature space data are comparable. For the fixed-ended beam simulations, although sensors close to

the damage locations detect the existence of damage, the ones further away failed to label the damage cases as '*not belonging*' to the target state.

Table 3. Percent accuracy in damage detection of the truss system using OC-SVM

Table 4. Percent accuracy in damage detection of the fixed ended beam using OC-SVM

Figure 3. Training, validation, damage data and OC-SVM decision boundary for the truss system

Figure 4. Training, validation, damage data and OC-SVM decision boundary for the fixed ended beam

Figure 5. Normalized Itakura distance (a) truss system, (b) fixed-ended beam

Although increasing the feature space leads to some improvement for these misclassifications, it is not consistently significant enough for all the simulation cases to conclude with that generalization.

For visualizing the distribution of the damaged state data in relation to the decision boundary, the 2-dimensional feature space data including the training data, the support vectors and the decision boundary generated by the trained OC-SVM for selected sensors are provided for truss and beam systems in Figures 3 and 4, respectively. Damage localization using Itakura distance measure is carried out for both systems and the normalized values of the metric are plotted in Figure 5.

Although the plots are not sharp and distinctive enough to pinpoint the damaged region, the sensor giving the largest value of 1 is considered as the region closest to damage. The localization results based on these normalized values are tabulated in Table 5. For both systems, three out of the four damage scenarios, true damage locations are included in the identified set of probable damage locations. This is an expected accuracy for a data driven methodology utilizing an unsupervised learning approach and not exploiting a finite element model or data from different damage states. With its limited information, solely data obtained at the nominal state of the structure, perfect spatial discrimination cannot always be guaranteed. Regardless, it is evident that the approach. holds great promise for addressing the most important task of damage existence.

Table 5. Damage localization results

FIXED-ENDED BEAM

5. CONCLUSIONS

Machine learning algorithms, with their capability of lending themselves easily for automation and implementation to different structural systems and sensor networks, have great potential for applications in the field of structural health monitoring. Considering the requirements of the ageing and degrading civil infrastructure, reliable detection of damage existence has the utmost importance within the damage identification problem. Although it is the first stage of the damage identification problem, it is the most important one since it precedes all other stages of locating and assessing the severity of damage and estimating the remaining service life. This study addresses this problem and presents a methodology for detection of structural damage using an unsupervised machine learning approach. The proposed approach exploits learning the undamaged state through a OC-SVM for deciding on the existence of damage and it couples this decision-making algorithm with a statistical distance measure for further localizing the damage. Simulations are performed on a beam and a truss system, two structure classes inherently different from each other, to test the robustness of the technique incorporate most of the complications that are encountered in actual applications, i.e., measurement noise on output-only measurements, modeling error, limited sensors and damages with different severity. The results demonstrate that the utilized machine is able to accurately detect damage by means of a lowdimensional feature vector obtained using the AR parameters of the output data. Damage localization by means of Itakura distance metric, include the true damage locations in the identified list of potential damage locations except for the two minor damage scenarios investigated in this study.

The proposed approach provides an unsupervised approach for damage detection that works solely on ambient vibration data collected under normal operation conditions. It overcomes the need to rely on a physical model and any apriori knowledge about the structure's condition. The strategy is capable of detecting structural damage automatically in a robust manner and operates with a localization index providing spatial information regarding damaged region within the provided sensor resolution.

The applicability of the method to damage scenarios including multiple components, changing boundary conditions and failure mechanisms include complications from the localization aspect of the damage identification problem that await future research.

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