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Robust Variable Structure Controllers for Axial Active Magnetic Bearing

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Abstract: This work focuses on robust variable structure control of a rotor-axial active magnetic bearing system. The electromagnetic force generated by active magnetic bearing is highly nonlinear characteristics. On the other hand, the magnetic force coefficient is a calculated value and its real value is not truly identified, therefore, robustness is a great importance in the operation of the active magnetic bearings system. On this works Lyapunov based three different type of variable structure controllers are proposed and experimentally tested. Robustness of the controllers were tested experimentally by creating some parametric uncertainty in the control system using an external disk mass attached to the rotor. The results of the controllers are also compared with conventional and linear robust controllers.

Keywords: *Axial active magnetic bearings, sliding mode controller, high gain robust controller, high frequency robust controller*.

1. Introduction

Active magnetic bearings (AMB) are electromechanical devices that provide noncontact support between rotor and bearing via the control of electromagnetic forces. Magnetically levitated rotors have many useful advantages like frictionless rotation. Owing to this, active magnetic bearings allow rotors to reach high rotational speeds [1-2]. However due to the nature of magnetic field, magnetically levitated rotor systems are highly nonlinear and AMBs are represented by nonlinear mathematical models [3-4]. Nevertheless they can be linearized successfully around the operating point thanks to the restricted and very small air gap between AMB stator and rotor [5]. As a result, many linear and local controllers like PID or fuzzy logic controllers [6-7] have been successfully applied to AMB systems. These controllers can offer good performance around the operating point but outside these local regions where the effects of the nonlinearities become more evident. Model based controllers are developed to increase the performance of the AMB system. Due to the magnetic saturation and existence of eddy current effects identification of exact system parameter in a magnetic bearing system is not straight forward. Therefore, robustness is a great importance for this type systems. There are available works on magnetic bearing system using robust control and linear matrix inequality design for controlling the motion of a magnetic bearing system [8-9]. Linear robust control and nonlinear adaptive backstepping control approaches have been studied by many researchers to maintain robustness in magnetic bearing systems [10-12]. Variable structure control theory has been implemented for many nonlinear processes including magnetic bearings [13-15]. One of the main features of this approach is needs to drive the error to a switching surface after which the system is in sliding mode and will not be effected by and modelling uncertainties or disturbances. However, there are two main criticisms of these controllers when they are applied to mechanical systems, first ignoring the dynamics or physical properties of the mechanical system the controllers can do no better than other controllers that disregards the dynamics. Secondly, chattering is a common problem associated with the variable structure controllers.

The remaining of the work is organized as follows: The mathematical model of an axial magnetic bearing system is given in Section 2, while the control problem formulation and error system development are stated in the control design section. Respectively the sliding mode control (SMC), high gain robust control (HGR) and high frequency robust control (HFR) are designed and experimental results are given in the rest of the paper.

2. Axial Magnetic Bearing Model

The axial active magnetic bearing produces attractive magnetic forces by the opposite coil electromagnets to limit the rotor movements in z direction. The structure of axial bearing in xOz plane is depicted in Fig.1 schematically. In this structure, a disk element is fixed to the rotor and a nominal gap z_0 exists between the opposite coils and the disk element in both side. A non-contact capacitive sensor is set to measure the axial displacement of the rotor. The aim of the control is to bring the disk element to the origin without any mechanical contact during levitation and rotation of the rotor. The parameters of the considered system are given in Table 1. Note that an external disk is possible to be fixed to the rotor to test the robustness of the controllers.

In this study, it assumed that the radial and the axial directions are completely separated. This means two radial active magnetic bearings support the rotor in the radial directions and during the control operation a complete noncontact situation is realized. As depicted in Figure 1, the magnetic force f_z is nonlinear in nature and is generated by the coils of bearing in the following form.

$$f_z = f_+ - f_- = k\left(\frac{(i_0+i_z)^2}{(z_0-z)^2} - \frac{(i_0-i_z)^2}{(z_0+z)^2}\right) \quad (1)$$

where k is a constant related to the magnetic bearing parameters. To derive the dynamical equation of the proposed system a linearized magnetic force is assumed to have the following form.

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$$m\ddot{z} = k_z z + k_i i_z \tag{2}$$

where $z(t)$, $\ddot{z}(t) \in \Re$ represent the rotor position and acceleration, respectively. Also, $i_z(t) \in \Re$ denote the control current signal. Besides $m \in \Re$ is the mass of the rotor, $k_z \in \Re$ and $k_i \in \Re$ are the constants according to displacement to force constant $k_z = 4k(i_0^2/z_0^3)$ and current to force constant $k_i = 4k(i_0/z_0^2)$ where; $k_i = \mu_0 N^2 A/4$.

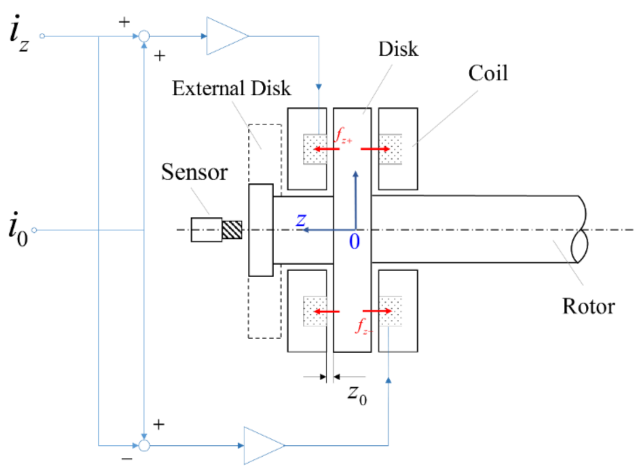


Figure 1. Schematic representation of the axial active magnetic bearing rotor system

To ease the presentation of the subsequent control development, we divide both sides of the dynamics equation by the non–zero constant k_i to obtain:

$$i_z = M\ddot{z} - Cz \tag{3}$$

where $M \triangleq m/k_i$ and $C \triangleq k_z/k_i$. The compact form of the equation can be written as

$$i_z = W\phi \tag{4}$$

where $W = [\ddot{z} \quad -z]$ and $\phi = [M \quad C]^T$.

Table 1. Parameters of axial magnetic bearings system

	Symbols	Value	Units
Mass of the rotor	m	4.821	$[mm]$
Mass of the external disk element	m_d	0.383	$[kg]$
Nominal gap	z_0	0.25	$[mm]$
Bias current	i_0	2.5	$[A]$
Number of turns in the electromagnets	N	24	$[-]$
Surface area of the electromagnets	A	419	$[mm^2]$
Magnetic permeability constant	μ_0	$4\pi 10^{-7}$	$[H/m]$

3. Design of the Controllers

In the considered axial bearing-rotor system, the objective of the control is to force the disk to the middle position with equal gaps in both sides. To this aim, define a position tracking error $e(t) \in \Re$ and its double time derivatives, denoted by $\ddot{e}(t) \in \Re$, as follows

$$e = z_d - z, \qquad \ddot{e} = \ddot{z}_d - \ddot{z} \tag{1}$$

where the desired trajectory of the rotor is $z_d = 0$. The filtered tracking error $r(t) \in \Re$ and its time derivatives for the analysis, denoted by $\dot{r}(t) \in \Re$, as follows

$$r = \dot{e} + \alpha e, \qquad \dot{r} = \ddot{e} + a\dot{e} \tag{6}$$

where, $\alpha \in \Re$ is a positive control gain. Take time derivative of filtered tracking error and multiplied by M both side of the equation

$$M\dot{r} = M\ddot{e} + M\alpha\dot{e} \tag{7}$$

Rearrange (7) by substituting (3), (5) and (6) into it as follows

$$M\dot{r} = M(\ddot{z}_d + \alpha\dot{e}) - Cz - i_z \tag{8}$$

Rewrite equation (8) as follows

$$M\dot{r} = w(t) - i_z \tag{9}$$

Design the obtained controller;

$$i_z = \hat{w}(t) + k_r r + k_i \int r dr + V_R \tag{10}$$

where $\hat{w}$ is the best estimates function of the system parameters. The selection of V_R defines the type of the controller. Note that the integral term is not consist on the variable structures controllers. However, experimental results have shown that the steady-state error occurs when levitating the rotor. Therefore, substituting the integral effect on control signal equation (10) more reliable results are obtained. The aim of this study is to apply variable structure control in the selection of V_R to have better robustness in the control system.

3.1. Sliding mode controller (SMC) design

Using equation (10), the first type controller input is defined as

$$i_{z-SMC} = \hat{w}(t) + k_{rs} r + k_{is} \int r dr + \rho_s sgn(r) \tag{11}$$

For the stability analysis, a candidate Lyapunov function (12) and its derivative (13) are defined as

$$V = \frac{1}{2} M r^2 + \frac{1}{2} k_{is} \xi^2 \tag{12}$$

$$\dot{V} = M r \dot{r} + k_{is} \xi \dot{\xi} \tag{13}$$

where $\xi = \int r dr$ and the derivative of $\dot{\xi} = r$. The final form of derivative of V is obtained by inserting equation (9) and (11) in to the equation (13)

$$\dot{V} = M\dot{r}r + k_{is}\dot{\xi} = (w(t) - \hat{w}(t) - k_{rs} r - \rho_s sgn(r) - k_{is}\xi)r + k_{is} r \xi \tag{14}$$

Since $sgn(r) = (|r|/r)$, and, $\tilde{w}(t) = w(t) - \hat{w}(t)$ the equation (14) becomes;

$$\dot{V} \leq \tilde{w}(t)|r| - k_{rs} r^2 - \rho_s |r| \tag{15}$$

Assume that there exists a $\rho_s(t) \in \Re$ such that $\rho_s(t) \geq \|\tilde{w}(t)\|$, the following upper bound can be formed;

$$\dot{V} \leq k_{rs}r^2 \quad (16)$$

Finally, the derivative of V becomes

$$\left.\begin{matrix} V > 0 \\ \dot{V} \leq 0 \end{matrix}\right\} V \in L_\infty \quad \rightarrow \quad \lim_{t\to\infty} V = V_\infty \quad (17)$$

It is clear that a global stability is maintained and all signals in the closed system are bounded.

3.2. High gain robust (HGR) controller design

The second type controller input is high gain robust controller defined as

$$i_{z-HGR} = \hat{w}(t) + k_{rg}r + k_{ig}\int rdr + \frac{\rho_g^2}{\varepsilon}r \quad (18)$$

where $\varepsilon > 0$ is a positive constant. For the stability analysis, the Lyapunov function given in equation (13) is used by replacing with the parameter k_{ig}. The derivative of V is obtained as by inserting equation (9) and (18) in to the equation (13)

$$\dot{V} = \left(w(t) - \hat{w}(t) - k_{rg}r - \frac{\rho_g^2}{\varepsilon}r - k_{ig}\xi\right)r + k_{ig}r\xi \quad (19)$$

Assume that there exists a $\rho_g(t) \in \Re$ such that $\rho_g(t) \geq \|\tilde{w}(t)\|$. And in equation (19) replace $\tilde{w}(t)$ with ρ_g and the following upper bound can be formed;

$$\dot{V} \leq -k_{rg}r^2 + \rho_g|r| - \frac{\rho_g^2}{\varepsilon}r \quad (20)$$

Rearranging equation (20);

$$\dot{V} \leq -k_{rg}r^2 + \rho_g r\left(1 - \frac{\rho_g|r|}{\varepsilon}\right) \quad (21)$$

Case-1:

where $\boldsymbol{\rho_g|r| > \varepsilon \quad \rightarrow \quad 1 - \frac{\rho_g|r|}{\varepsilon} < 0}$

Case-2:

where $\boldsymbol{\rho_g|r| \leq \varepsilon \quad \rightarrow \quad 1 - \frac{\rho_g|r|}{\varepsilon} \geq 0}$

Under these cases the derivative of the candidate Lyapunov function is transformed into the two different types:

From case-1:

$$\dot{V} \leq -k_{rg}r^2 \quad (22)$$

From case-2:

$$\dot{V} \leq -k_{rg}r^2 + \varepsilon \quad (23)$$

The worst case between *case-1* and *case-2*, the derivative of the candidate Lyapunov function with the following upper bound can be formed;

$$\dot{V} \leq -k_{rg}r^2 + \varepsilon \quad (24)$$

therefore global ultimately upper bound stability is satisfied.

3.3. High frequency robust (HFR) controller design

The third type controller input is proposed as

$$i_{z-HFR} = \hat{w}(t) + k_{rf}r + k_{if}\int rdr + \frac{\rho_f^2 r}{r|\rho_f| + \varepsilon} \quad (25)$$

Using the same candidate Lyapunov function for the stability analysis, the derivative of $V$is obtained by substituting equation (9) and (25) to the equation (13)

$$\dot{V} = \left(w(t) - \hat{w}(t) - k_{rf}r - \frac{\rho_f^2 r}{r|\rho_f| + \varepsilon} - k_{if}\xi\right)r + k_{if}r\xi \quad (26)$$

Assume that there exists a $\rho_f(t) \in \Re$ such that $\rho_f(t) \geq \|\tilde{w}(t)\|$. And in equation (26), replace the $\tilde{w}(t)$ with the ρ_f and the following upper bound can be formed;

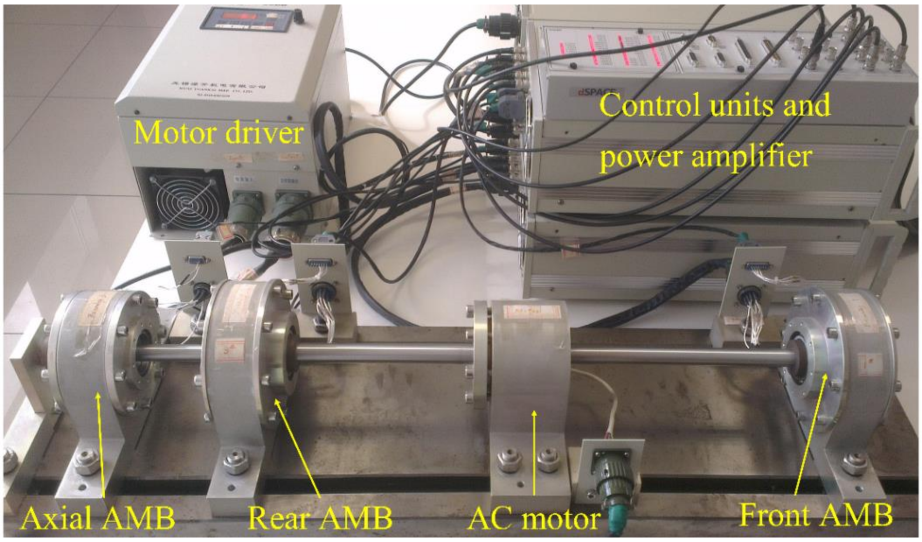
$$\dot{V} \leq -k_{rf}r^2 + \rho_f|r|\left(1 - \frac{\rho_f|r|}{r|\rho_f| + \varepsilon}\right) \quad (27)$$

Rearrange the equation and the following upper bound can be formed;

$$\dot{V} \leq -k_{rf}r^2 + \rho_f|r|\left(\frac{\varepsilon}{r|\rho_f| + \varepsilon}\right) \quad (28)$$

where $(\rho\_f\ |r|)/(\rho\_f\ |r| + \varepsilon) \leq 1$; the derivative of V becomes

$$\dot{V} \leq -k_{rf}r^2 + \varepsilon \quad (29)$$



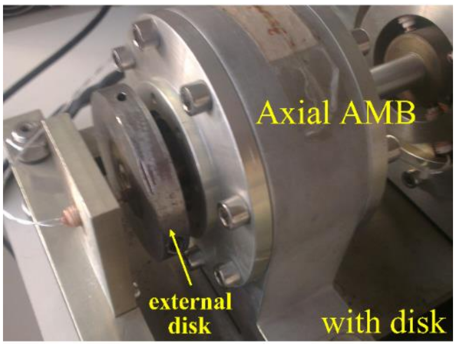
- **Figure 2.** Flexible rotor active magnetic bearing experimental setup.

therefore global ultimately upper bound stability is satisfied.

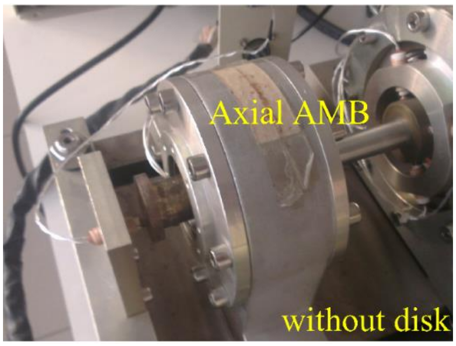
4. EXPERIMENTAL VERIFICATIONS

The flexible rotor-active magnetic bearing experimental setup used in the control design study is shown in Figure 2. The experimental setup is a five axis controlled rotor-active magnetic bearing system and the radial and the axial bearings can be controlled separately. The axial magnetic bearing is located at the one end of the rotor and limits the rotor axial movements. The air gap z_0 between the rotor and the touchdown bearing is set to 0.15 [mm].

In experiments, the controller implementations and the data acquisitions were performed using dSPACE DS1104 system. The working sample rate was selected as $T_s = 15\,kHz$ in all experiments. The best estimates of the system parameters were taken as $\varphi = [M \quad C]^T = [10000 \quad 0.3974]^T$. In experimental verifications, two cases were studied to understand the performance and robustness of the designed robust variable structure controllers. In the first case, experiments were carried out with the own weight of the rotor in the system. For the second case, an external disk mass was mounted at the end of the rotor as shown in Figure 3. After adding the disk mass, the total mass of the rotor became 5.204 [kg] and the increase in the rotor mass was about 8% percent. Adding an extra mass on the rotor imposes some parametric uncertainty to the control system and also some dynamic unbalance forces in the rotation of the rotor even in the axial direction due to imbalances in the external disk.



(a)



(b)

Figure 3. Axial magnetic bearing (a) rotor with external disk mass (b) rotor without external disk mass.

4.1. Experimental Results of Variable Structure Controllers

The variable structure controllers were tested at the rotor speed of 3000 rpm for two cases and the displacement of the rotor and the control input were observed for each cases of with and without disk. The results obtained with the sliding mode controller are shown in Fig. 4. As shown in these figures, small increases were observed in the amplitude of displacement and the control input when the external disk attached to the rotor. The parameters of sliding mode controller for both case were selected as $a_s = 10$, $k_{rs} = 1000$, $\rho_s = 0.1$ and $k_{is} = 450$.

The second results were obtained for the high gain robust controller as shown in Fig.5. For this controller, the parameters were selected as$a_g = 20$, $k_{rg} = 900$, $\rho_g = 0.0001$, $\varepsilon_g = 0.000001$ and $k_{ig} = 600$. Although increase in the amplitudes of responses with external mass were reasonable, some phase shifts were observed in the displacements and control inputs.

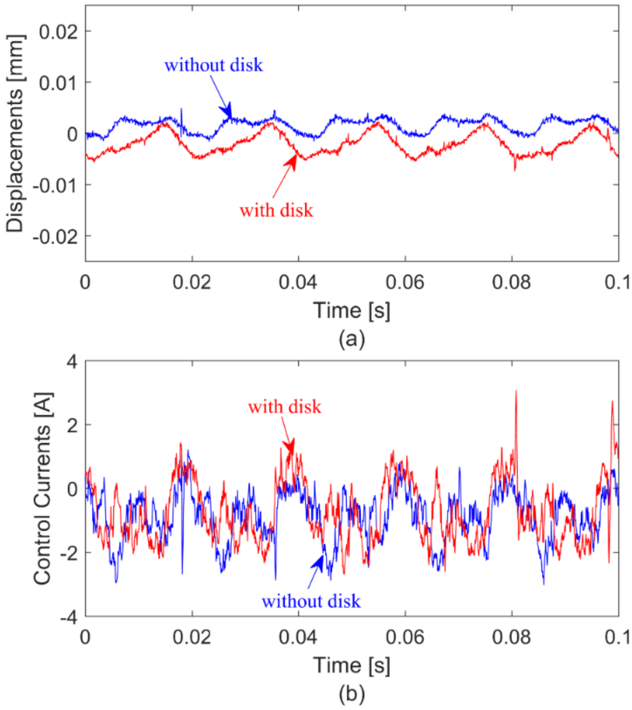


Figure 4. Sliding mode control results, (a) displacements, (b) control inputs.

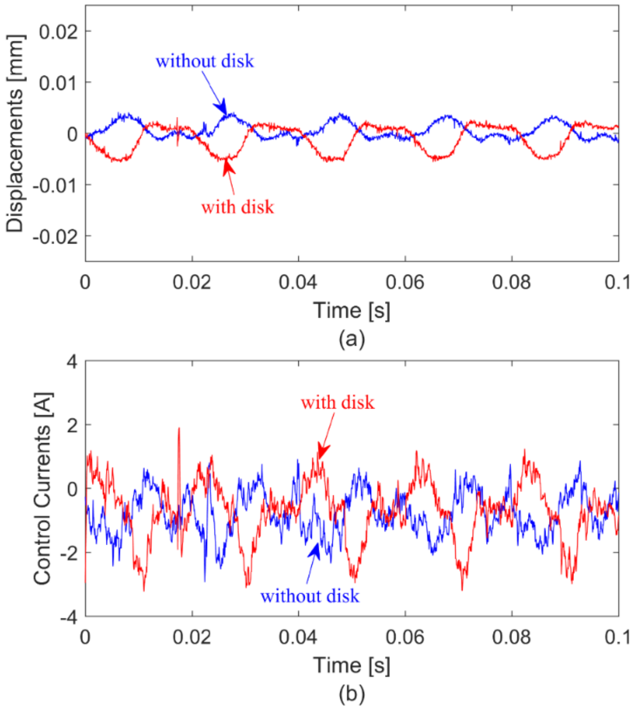


Figure 5. High gain robust control results, (a) displacements, (b) control inputs.

The high frequency robust control results were presented in Figure 6. In this controller, the parameters were $a_f = 25$, $k_{rf} = 1000$, $\rho_f = 0.001$, $\varepsilon_f = 0.000001$ and $k_{if} = 500$. Much better results were obtained in displacements and control inputs compared to other variable structure controllers as shown in Figure 6.

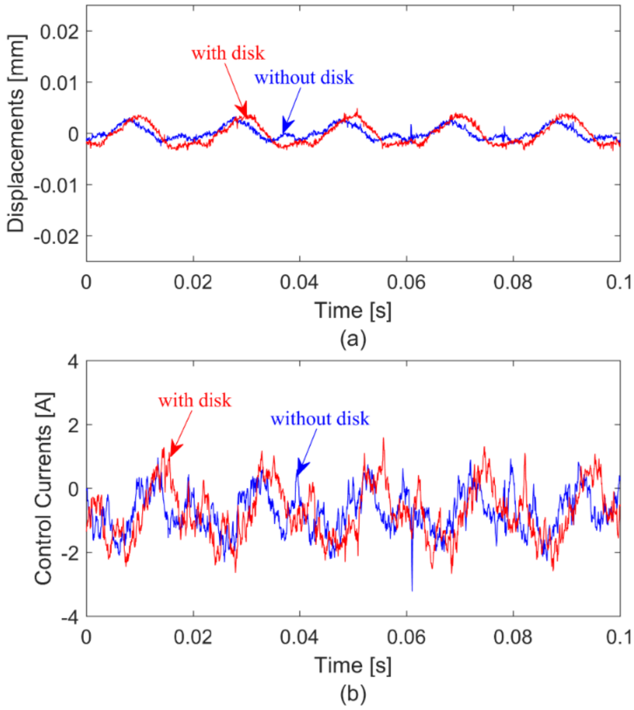


Figure 6. High frequency robust control results (a) displacements, (b) control inputs.

4.2. Comparison of Experimental Results

The results of variable structure controllers were compared with the results of conventional PID control and linear H_∞ robust controller. These controllers were tested before for the same rotor magnetic bearing system [16] and were belived to have good performances.

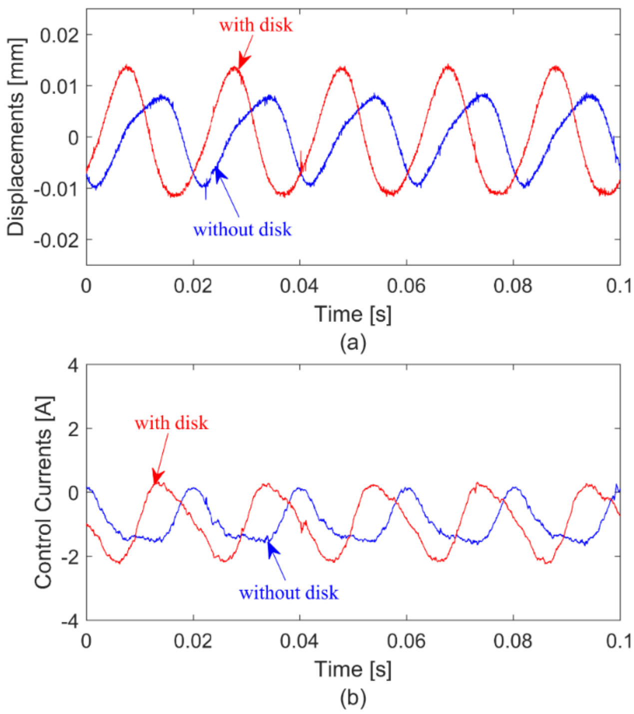


Figure 7. PID control results (a) displacements, (b) control inputs.

For the same test conditions and rotational speed of the rotor, the results obtained for PID control are shown in Figure 7. The robustness of PID control is weak againts the parameter variation. The results of linear H_∞ controller were presented in Figure 8. Since H_∞ controller was designed by considering some uncertainity models, small increases in the amplitude of the rotor were observed with the disk mass.

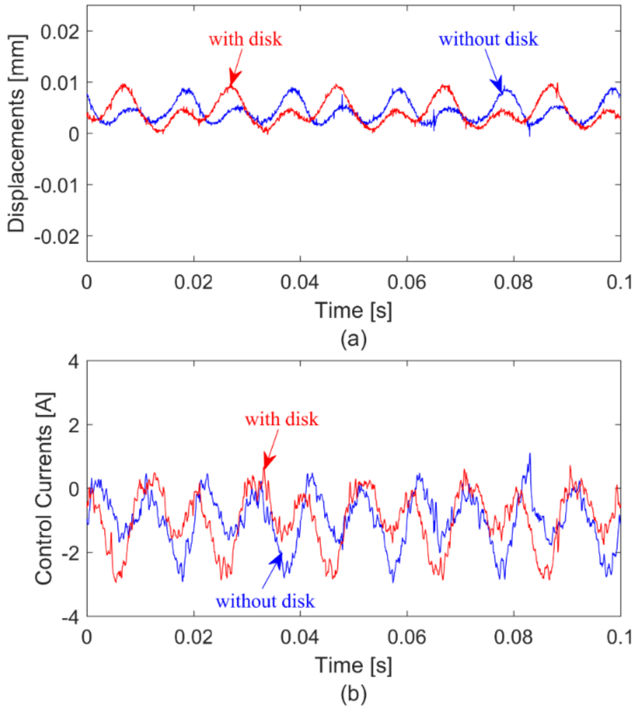


Figure 8. H_∞ control results, (a) displacements, (b) control inputs.

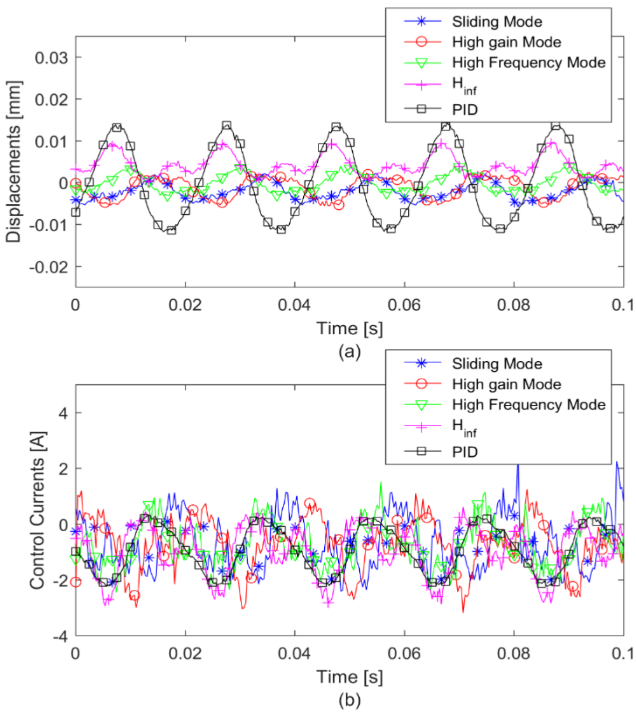


Figure 9. All tested controller results with external disk mass, (a) displacements, (b) control inputs.

- **Table 2.** RMS values for all experimental data.

		PID	H_∞	SMC	HGR	HFR
Without disk	Displacement	0.0061	0.0049	0.0022	0.0017	0.0014
	Control input	1.1023	1.3765	1.1642	1.0131	0.9862
With disk	Displacement	0.0089	0.0050	0.0026	0.0025	0.0023
	Control input	1.2426	1.4200	1.1970	1.1788	1.1650

The experimental results of the all tested controllers with external disk mass were presented in Fig.9 and without disk given in Fig.10. Also Table 2 shows RMS values of obtained experimental data. It can be detected that high frequency robust controller gives best results either in the amplitude of the rotor and the control inputs. The other two variable structure controllers have also reasonable results. The results of linear H_∞ control shows the conservativeness of the controller with large control input values but almost no increase in the amplitude of displacements but some phase shifts were observed in the displacements and control inputs.

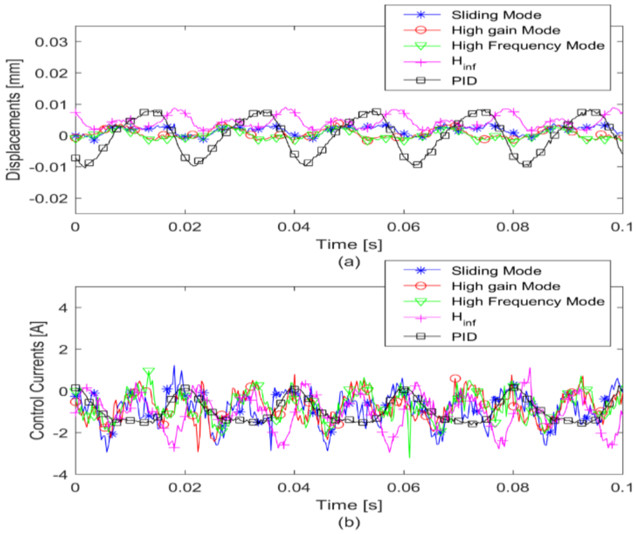


Figure 10. All tested controller results without external disk mass, (a) displacements, (b) control inputs.

5. Conclusions

In this paper, three type of robust variable structure controllers have been designed and tested experimentally for a rotor-axial active magnetic bearing system. Robustness of the controllers were tested experimentally by creating some parametric uncertainty in the control system using an external disk mass attached to the rotor. RMS values from the obtained data of the displacements and the control inputs have shown a good comparison of the controllers. The high frequency robust controller results have shown much better performance in all tested controllers.

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