



## INVESTIGATION OF PHOTONIC BAND GAPS FOR AIR-TiO<sub>2</sub> AND AIR-Te ONE DIMENSIONAL LAYERED PHOTONIC CRYSTALS

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### ABSTRACT

In this paper, we investigated photonic band gaps (PBGs) for one dimensional air-TiO<sub>2</sub> and air-Te layered photonic crystals with respect to the refractive index contrast, the relative width of layers and the angle of incidence. We analyzed that how PBGs change with the increase of refractive index contrast, the width of layers and the incident angle.

**Keywords:** Photonic crystals, Photonic band gap, Photonic band gap materials, Periodic layered media and dispersion.

## HAVA- TiO<sub>2</sub> VE HAVA- Te BİR BOYUTLU TABAKALI FOTONİK KRİSTALLERİ İÇİN FOTONİK BANT ARALIKLARININ İNCELENMESİ

### ÖZET

Bu makalede, kırılma indis farklılığına, tabakaların bağıl genişliklerine ve geliş açısına göre hava-TiO<sub>2</sub> ve hava-Te bir boyutlu tabakalı fotonik kristalleri için fotonik bant aralıklarını inceledik. Fotonik bant aralıklarının, kırılma indis farklılığının artışı, tabaka genişlikleri ve geliş açısı ile nasıl değiştiğini analiz ettik.

**Anahtar kelimeler:** Fotonik kristaller, Fotonik bant aralığı, Fotonik bant aralıklı maddeler, Periodik tabakalı ortamlar ve dispersiyon.

1. INTRODUCTION

During the last ten years, structures known as photonic crystals or photonic band gap materials have attracted increasing attention and have performed many studies in this field [1-5]. Photonic crystals are artificial structures which dielectric constant is periodically changed on length scale in one, two or three dimensions with periodicity comparable to the light wavelength [6]. As analogous to electrons in an atomic crystal, the propagation of electromagnetic waves in periodic dielectric structures can be forbidden in certain directions and within a certain frequency range [7], [8]. Because of this similarity, this frequency range, in which photonic crystals exhibit strong reflection, is called as photonic band gap (PBG) [9-10]. PBG restricts the propagation of light with certain frequencies, that is to say PBG materials can be employed to control light in a manner that traditional optics cannot [11]. Thus, propagation of light can be restrained through photonic crystals by using various materials and different geometrical parameters.

Here, we investigated how photonic band gaps vary with refractive index contrast, width of layers and incident angle, by using dispersion relation for one dimensional layered PBG structures. For this, two kinds of layered structures with periodicity in one dimension, such as air-TiO<sub>2</sub> and air-Te, were used.

2. THEORY

The basic behavior of electron waves moving within a periodic potential in solid-state physics first discussed by Kronig and Penny. When the motion of electron waves in a periodic potential considered, the formation of allowed and forbidden bands can be seen. If periodic lattice structure is replaced with the refractive indices pattern and electron waves are replaced with optical waves, the same opinion can be applicable to the case of optical radiation. The allowed and forbidden bands of the frequencies or wavelengths are obtained instead of energy bands, in this case [11].

The propagation of wave in periodic layered media was investigated by many workers [12], [13]. The simplest periodic layered medium is that medium consists of alternating layers of transparent materials with different refractive index.

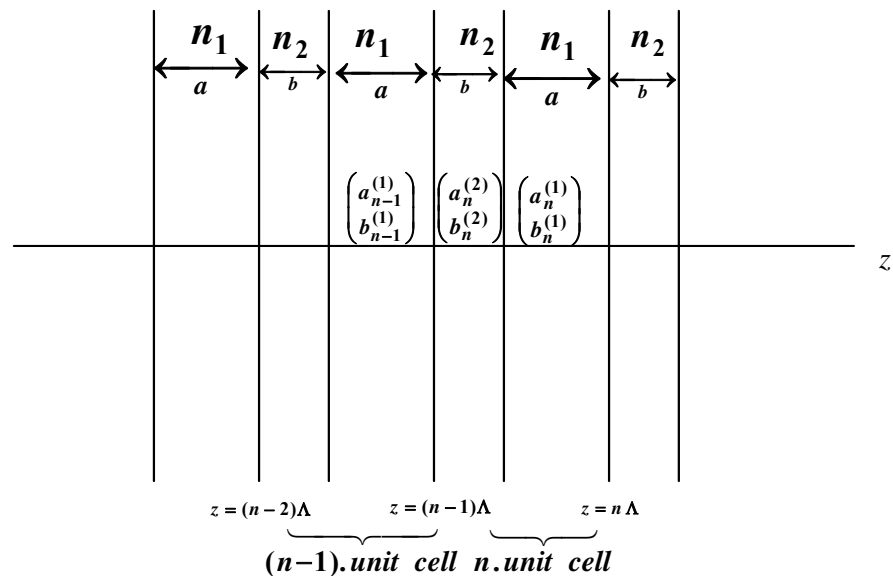


Figure 1. A schematic illustration of a periodic layered medium and the complex plane wave amplitudes associated with the n<sup>th</sup> unit cell and its adjacent layers.

The simplest periodic layered media consist of two different materials with a refractive indices profile given by

$$n(z) = \begin{cases} n_2, & 0 < z < b \\ n_1, & b < z < \Lambda \end{cases} \tag{1}$$

where

$$n(z) = n(z + \Lambda) \tag{2}$$

z axis is normal to the layer interfaces and  $\Lambda$  is period ( $\Lambda = a + b$ ). yz-plane was assumed as the propagation plane. The geometry of structure is indicated in Fig. 1.

The electric field in each homogenous layer can be expressed as a total of an incident and a reflected plane wave. The complex amplitudes of these two waves constitute the elements of a column vector. Thus, the electric field in layer  $\alpha$  ( $\alpha = 1, 2$ ) of nth unit cell can be presented by a column vector,

$$\begin{pmatrix} a_n^{(\alpha)} \\ b_n^{(\alpha)} \end{pmatrix}, \quad \alpha = 1, 2 \tag{3}$$

where  $a_n^{(\alpha)}$  and  $b_n^{(\alpha)}$  are complex amplitudes of the incident and the reflected plane waves, respectively. Consequently, the electric field distribution in the same layer can be written [11], [14]

$$E(y, z) = \left[ a_n^{(\alpha)} e^{-ik_{\alpha z}(z-n\Lambda)} + b_n^{(\alpha)} e^{ik_{\alpha z}(z-n\Lambda)} \right] e^{-ik_y y} \tag{4}$$

$$k_{\alpha z} = \left[ k_{\alpha}^2 - k_y^2 \right]^{1/2}, \quad \alpha = 1, 2 \tag{5}$$

$$k_{\alpha} = \frac{n_{\alpha} \omega}{c}, \quad k_y = k_{\alpha} \sin \theta_{\alpha} \tag{6}$$

where  $\theta_{\alpha}$  is the angle between  $\vec{k}_{\alpha}$  in layer  $\alpha$  and z -axis.

According to the Bloch theorem, the electric field vector of a normal mode in a periodic layered medium is of the form,

$$\vec{E} = \vec{E}_K(z) e^{-iKz} e^{-ik_y y} \tag{7}$$

where,  $K$  is called as the Bloch wave number and  $\vec{E}_K(z)$  is periodic with period  $\Lambda$ , that is

$$\vec{E}_K(z) = \vec{E}_K(z + \Lambda) \tag{8}$$

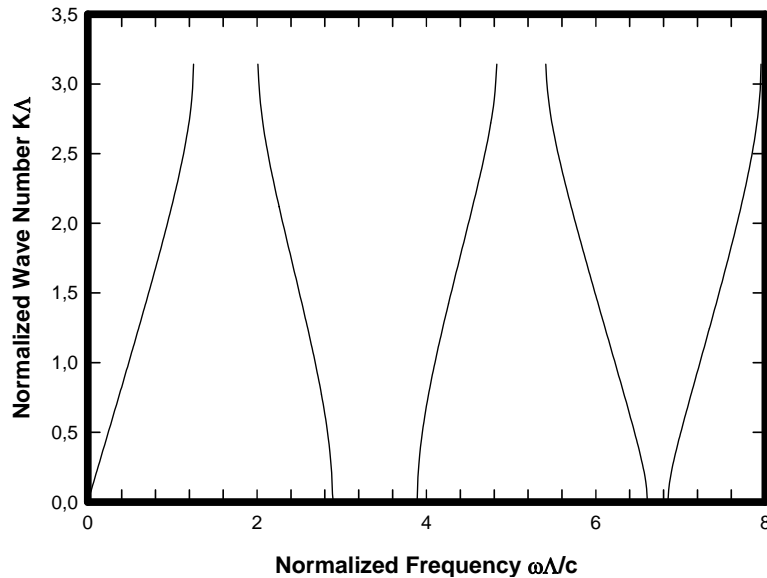
For the Bloch wave function, the dispersion relation can be written as

$$K(\omega)\Lambda = \cos^{-1} \left[ \cos(n_1 a \frac{\omega\Lambda}{c} \cos \theta_1) \cos(n_2 b \frac{\omega\Lambda}{c} \cos \theta_2) - \frac{1}{2} \left( \frac{n_1 \cos \theta_1}{n_2 \cos \theta_2} + \frac{n_2 \cos \theta_2}{n_1 \cos \theta_1} \right) \sin(n_1 a \frac{\omega\Lambda}{c} \cos \theta_1) \sin(n_2 b \frac{\omega\Lambda}{c} \cos \theta_2) \right] \quad (9)$$

where  $a = p\Lambda$ ,  $b = q\Lambda$ .

### 3. RESULTS AND DISCUSSION

Two structures have been considered for numerical calculations. The first is air-Titania layered structure with refractive indices  $n_1 = 1$  and  $n_2 = 2.72$ , and the other is air-Tellurium layered structure with refractive indices  $n_1 = 1$  and  $n_2 = 4.9$ . For these structures,  $a = b = 0.50\Lambda$  and  $\theta_1 = 0$  (normal incidence) have been accepted as the initial case. Because the  $\Lambda$  period is arbitrary, the result obtained is valid for arbitrary wavelengths and PBG occurs provided that  $\lambda \cong \Lambda$  [15]. For both structures, photonic band gaps were calculated by varying the layer widths. The variation of photonic band gaps for air-TiO<sub>2</sub> also investigated by changing the angle of incidence. By using the dispersion relation (9), the  $K\Lambda$  curves versus  $\omega\Lambda / c$  were plotted.



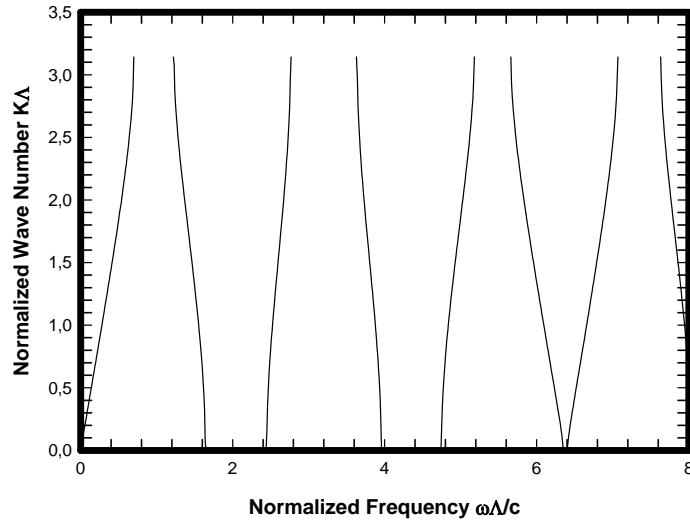
**Figure 2.**  $K\Lambda$  curves versus  $\omega\Lambda / c$ , for  $n_1 = 1, n_2 = 2.72$ ,  $a = b = 0.50\Lambda$  and  $\theta_1 = 0^\circ$ .

From Figures (2) and (3), it is observed that the increase in the refractive index contrast increases the number of allowed bands while the bandwidths decrease.

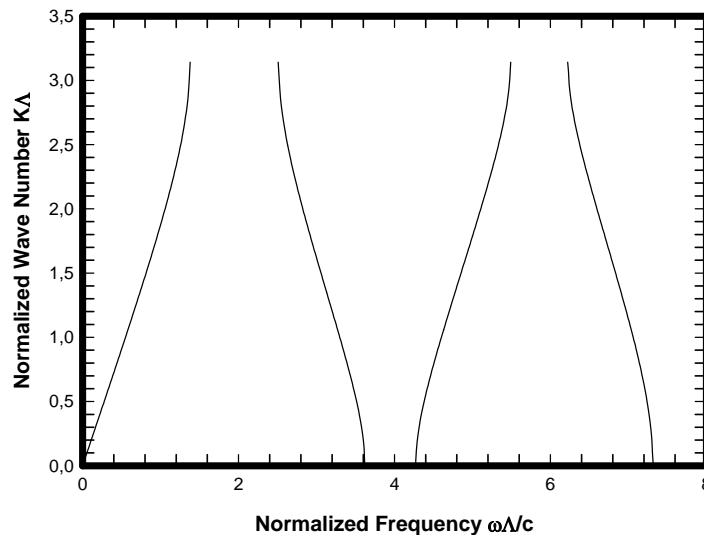
For air- TiO<sub>2</sub>, Figures (2) and (4) show that, when  $a > b$ , the bandwidths increase while the number of allowed bands decreases (except for fourth band). From Figure (3), it is observed that the same case is valid for air- Te (except for sixth band).

From Figures (2) and (5), when  $a < b$ , the bandwidths decrease while the number of allowed bands increases and moving towards the lower  $\omega\Lambda / c$  ranges, for air- TiO<sub>2</sub>.

For air-Te structures, from Figures (3) and (6), when  $a < b$ , the regular increase or decrease has not been seen at width of allowed bands. However, the allowed bands travel towards the lower  $\omega\Lambda / c$  ranges. Finally, for air-  $\text{TiO}_2$ , the dispersion relation is investigated at the angle of incidence  $\theta_1 = 30^\circ, 60^\circ$  and  $80^\circ$ . For the incident angle  $\theta_1 = 30^\circ$ , the bandwidths increase while the number of allowed bands decreases. For the incident angle  $\theta_1 = 60^\circ$ , the number of allowed bands again increases and the bandwidths decrease. For the incident angle

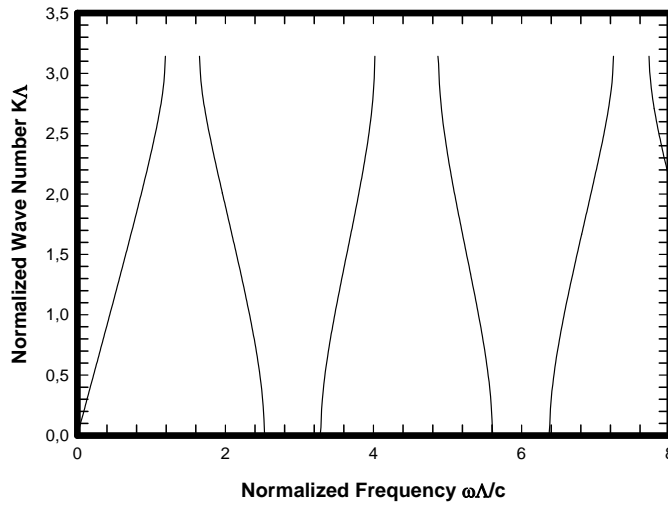


**Figure 3.**  $K\Lambda$  curves versus  $\omega\Lambda / c$ , for  $n_1 = 1, n_2 = 4.9$ ,  $a = b = 0.50\Lambda$  and  $\theta_1 = 0^\circ$ .

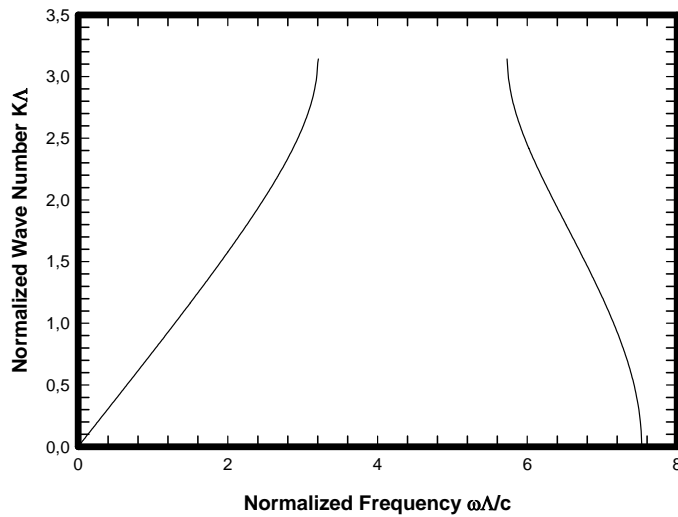


**Figure 4.**  $K\Lambda$  curves versus  $\omega\Lambda / c$ , for  $n_1 = 1, n_2 = 2.72$ ,  $a = 0.65\Lambda, b = 0.35\Lambda$  and  $\theta_1 = 0^\circ$ .

$\theta_1 = 80^\circ$ , the number of allowed bands again decreases. The all of this information can be observed from Figures (7), (8) and (9).

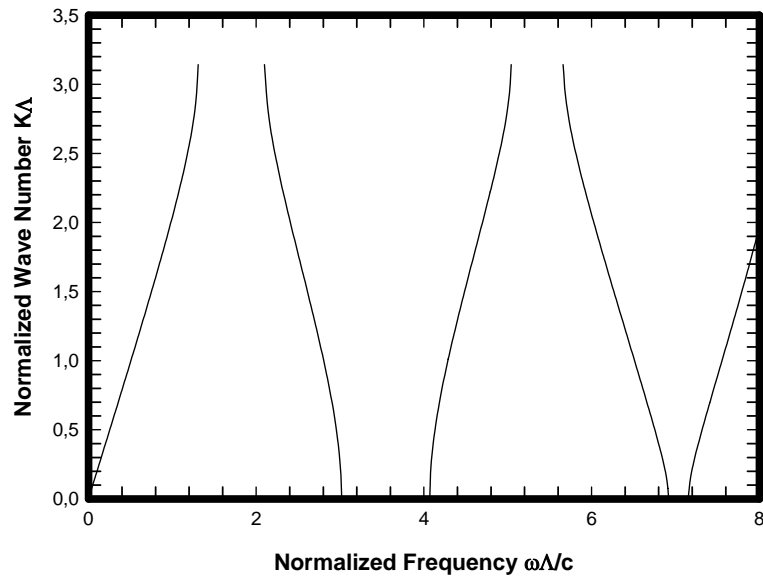


**Figure 5.**  $K\Lambda$  curves versus  $\omega\Lambda/c$ , for  $n_1 = 1, n_2 = 2.72$ ,  $a = 0.35\Lambda$ ,  $b = 0.65\Lambda$  and  $\theta_1 = 0^\circ$ .

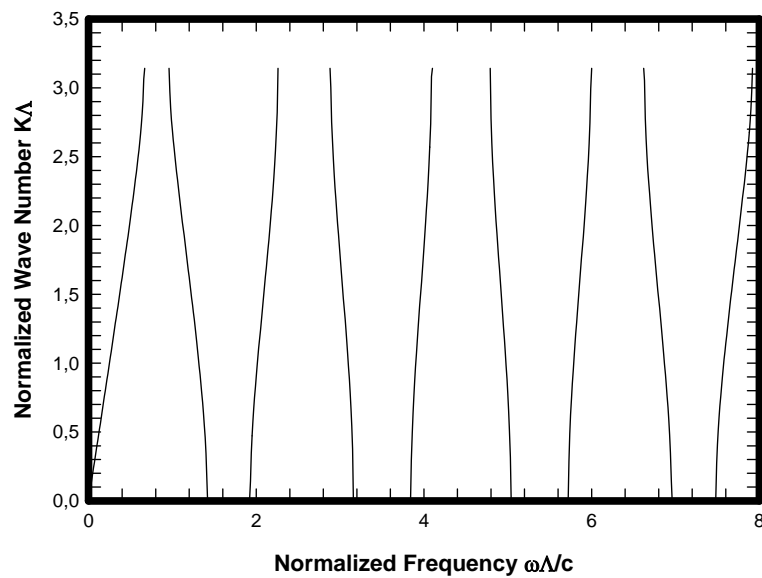


**Figure 6.**  $K\Lambda$  curves versus  $\omega\Lambda/c$ , for  $n_1 = 1, n_2 = 4.9$ ,  $a = 0.65\Lambda$ ,  $b = 0.35\Lambda$  and  $\theta_1 = 0^\circ$ .

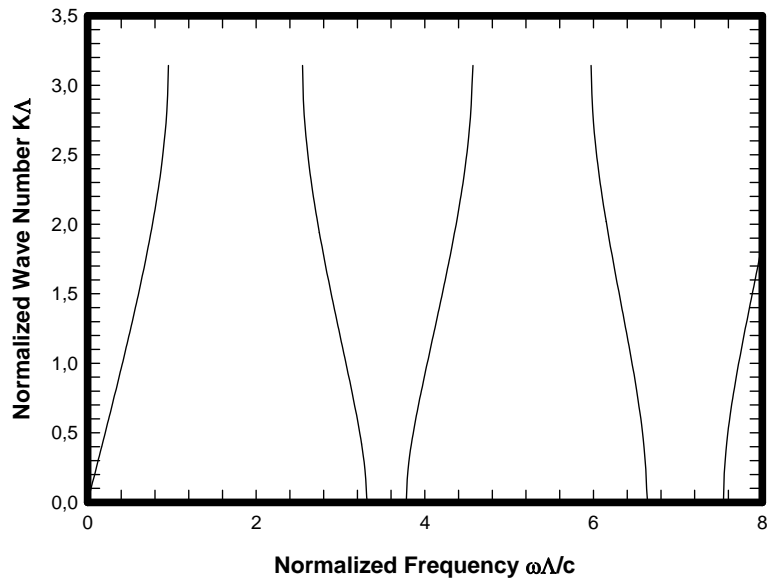
As a result, for the one dimensional periodic layered photonic crystals, the number of allowed and forbidden bands, the bandwidths and  $\omega\Lambda/c$  ranges associated with these bands can be changed by varying the layer widths, the refractive indices and the angle of incidence. Thus, the forbidden or allowed bands are constituted at desired frequency ranges by changing these parameters.



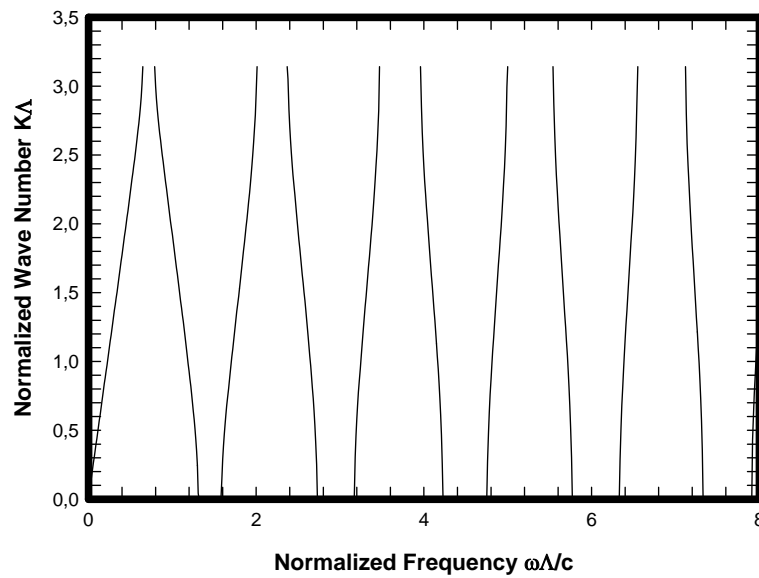
**Figure 7.**  $K\Lambda$  curves versus  $\omega\Lambda/c$ , for  $n_1 = 1, n_2 = 4.9$ ,  
 $a = 0.35\Lambda, b = 0.65\Lambda$  and  $\theta_1 = 0^\circ$ .



**Figure 8.**  $K\Lambda$  curves versus  $\omega\Lambda/c$ , for  $n_1 = 1, n_2 = 2.72$ ,  
 $a = b = 0.50\Lambda$  and  $\theta_1 = 30^\circ$ .



**Figure 9.**  $K\Lambda$  curves versus  $\omega\Lambda/c$ , for  $n_1 = 1, n_2 = 2.72$ ,  $a = b = 0.50\Lambda$  and  $\theta_1 = 60^\circ$ .



**Figure 10.**  $K\Lambda$  curves versus  $\omega\Lambda/c$ , for  $n_1 = 1, n_2 = 2.72$ ,  $a = b = 0.50\Lambda$  and  $\theta_1 = 80^\circ$ .



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