

CALCULATION OF THE OPTIMAL BURN-IN TIME USING DIFERENT CRITERIA

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Abstract: Burn-in is the test process that is used to minimize the warranty cost of a product by screening out of the defective products with high infant mortality through prior operation for a period of time before sale. However, the burn-in test also has a cost associated with it and uses some portion of the life of the product. Therefore in the literature of burn-in minimization of the total expected cost and maximization of the reliability of the product are two criteria that are used for the calculation of the optimal burn-in time. In this paper calculation of the optimal burn-in time is illustrated for a product whose lifetime distribution is mixed Weibull.

Keywords: Optimal burn-in time, mixed Weibull distribution, expected cost, mean residual life, delivered reliability

OPTİMUM AŞIRI ISINMA SÜRESİNİN FARKLI KRİTERLERE GÖRE HESAPLANMASI

Özet: Aşırı ısınma, bir ürünün garanti maliyetini düşürmek için hatalı ürünlerin satımdan önce elenmesini amaçlayan bir testtir. Aynı zamanda aşırı ısınma test sürecinin de bir maliyeti vardır ve ürün ömrünün bir kısmı test süreci içerisinde kullanılmaktadır. Buna göre optimum aşırı ısınma süresinin hesaplanmasında kullanılan kriterler beklenen toplam maliyetin en aza indirilmesi ve ürün güvenilirliğinin en yükseğe çıkarılmasıdır. Bu makalede bu kriterler kullanılarak ömür dağılımı karma Weibull dağılımı olan bir ürün için optimum aşırı ısınma süresi hesaplanacaktır.

Anahtar kelimeler: Optimum aşırı ısınma testi süresi, karma Weibull dağılımı, beklenen maliyet, ortalama geri kalan ömür, teslim güvenilirliği

1. Introduction

Burn-in is the test process that is used to minimize the warranty cost of a product by screening out of the defective products with high infant mortality through prior operation for a period of time before sale. During this period, refereed to as *burn-in time*, the failure of the product can be dealt with in a far less costly manner. As a result, burn-in can reduce warranty cost. However, the burn-in process also has a cost associated with it and uses some portion of the life of the product. Therefore in the literature of burn-in, minimization of the total expected cost and maximization of the reliability of the product are two criteria that are used for the calculation of the optimal burn-in time.

A balk of the literature on burn-in uses expected cost as a criterion. Genadis (1996) minimizes the expected cost for the case where products that failed during burn-in and warranty period are minimally repaired. Mi (1996) considers the case where products that are failed during burn-in are completely repaired and products that are failed during warranty period are replaced by the new burned-in products. Mi (1997) minimizes the expected cost function under two different warranty policies, failure-free policy and rebate policy. Under failure-free policy a product that fails during warranty period is either fixed or replaced by the manufacturer. Under rebate policy for each failure during the warranty period the customer is refunded an amount of money which is assumed to be a decreasing function of

time. To find the optimal burn-in time, Yun, Lee, and Ferreira (2002) minimize the expected cost function under cumulative free warranty where for a lot of products, if the total service time of all the items purchased is less than the warranty period, manufacturer replaces the failed item until the total service time of the failed items plus the service time of currently used item equals to the warranty time.

Several authors investigate the burn-in time and maintenance schedule that minimizes the cost of burn-in and maintenance. Mi (1994) finds the optimal burn-in time and maintenance schedule for a repairable component under two maintenance policies, block replacement and age replacement. Under block replacement a component is replaced at failure and at times kT , where $k = 1, 2, \dots$ and T is fixed. Under age replacement a components is replaced at age T and at failures which ever occurs first. Mi (1994) assumes that products that are failed during burn-in is completely repaired. Cha (2000) shows that under block replacement policy if products that are failed during burn-in is minimally repaired the long run average cost is less than the complete repair case. Cha (2001) considers two types of failure for a product. Type 1 failure can be removed by a minimal repair or complete repair where as Type 2 failure can only be removed by a complete repair. He proposes two burn-in procedures with block replacement warranty policy. In Procedure 1 regardless of the type of failure the product is completely repaired upon a failure and subjected to burn-in after repair. In Procedure 2 Type 1 failures are removed by minimal repair and Type 2 failures are removed by complete repair.

Reliability criterion is represented in two ways, maximization of the mean residual life function and maximization of the delivered reliability (residual-life). Definitions of the mean residual life and the delivered reliability are given in section 3 and 4 respectively. Whitbeck and Leemis (1989) calculate the burn-in time for a product by maximizing the system mean residual life (MRL). Launer (1993) and Mi (1994) used delivered reliability as a criterion for the calculation of the optimal burn-in time.

In this paper calculation of the optimal burn-in time is illustrated for a product whose lifetime distribution is mixed Weibull. In Section 2 mixture lifetime models is reviewed. In Section 3 the definition of the mean residual life function is given. In Section 4 the definition of the delivered reliability is given. In Section 5 the expected cost function that is used in this paper is introduced. In Section 6 the optimal burn-in time is calculated using burn-in criteria for the same example. Finally in Section 7 conclusion of this paper is given.

2. Mixture Lifetime Models

The results of life tests of electronic and mechanical products exhibited infant mortality of some portion of the products. This observation lead to the idea that the population of products is heterogeneous and is composed of sub-populations of defected (weak) products and non-defected (strong) products where defects cause the early failures. This notion of a mixture of weak and strong products was made popular by Jensen and Petersen (1982). Lifetime of a randomly selected product from a heterogeneous population is represented through a mixture lifetime model.

Usually, lifetimes of weak and strong sub-populations are assumed to be from the same family of distribution. Let $f_w(t | \Theta_w)$ and $f_s(t | \Theta_s)$ represent the probability density function (PDF) of the lifetime of the products from the weak and the strong sub-populations respectively where Θ_w is parameter vector of f_w and Θ_s is the parameter vector of f_s . Then PDF of the lifetime of a randomly selected product from the whole population is the mixture distribution and it is calculated as

$$f(t | \Theta) = \pi f_w(t | \Theta_w) + (1 - \pi) f_s(t | \Theta_s) \quad (1)$$

where $\Theta = (\Theta_w, \Theta_s)$ and π is the proportion of the weak products in whole population.

From (1), the reliability function of the mixture is calculated as

$$R(t | \Theta) = \pi R_w(t | \Theta_w) + (1 - \pi) R_s(t | \Theta_s) \quad (2)$$

where $R_w(t | \Theta_w)$ and $R_s(t | \Theta_s)$ are the reliability functions of the weak and strong sub-populations respectively. Since reliability function of the mixture population is a convex combination of the reliability function of the weak and the strong populations, reliability function of the mixture is bounded by the reliability function of the weak and the strong sub-populations.

Given $q = \pi R_w(t | \Theta_w) / [\pi R_w(t | \Theta_w) + (1 - \pi) R_s(t | \Theta_s)]$, the failure rate function of the mixture is calculated as

$$h(t | \Theta) = q h_w(t | \Theta_w) + (1 - q) h_s(t | \Theta_s) \quad (3)$$

where $h_w(t | \Theta_w)$ and $h_s(t | \Theta_s)$ are the failure rate functions of the weak and the strong sub-populations respectively. Since failure rate function of the mixture population is a convex combination of the failure rate function of the weak and the strong sub-populations, failure rate function of the mixture is bounded by the failure rate function of the weak and the strong sub-populations.

Often mixture of distributions results a bathtub shaped failure rate function. A bathtub shaped failure rate function has three parts. In the first part called infant mortality period, failure rate is high and decreases as the weak products fail. In the second part called useful life period failure rate stays constant and in the last part called wear-out period failure rate increases due to aging of the product. A formal definition of a bathtub (upside-down bathtub) shaped function $g(t)$ given as

$$g(t) \left\{ \begin{array}{ll} \text{strictly decreases (increases),} & \text{for } 0 \leq t \leq t_1 ; \\ \text{is a constant,} & \text{for } t_1 \leq t \leq t_2 ; \\ \text{strictly increases (decreases),} & \text{for } t \geq t_2 \end{array} \right. , \quad (4)$$

where $0 \leq t_1 \leq t_2$ and the domain of the function is $[0, \infty)$. t_1 and t_2 are called first and second change points of the failure rate function.

Burn-in is useful for the products that have a bathtub shaped failure rate function since burn-in eliminates the first part of the failure rate function so that customers receive the products when they reached the constant portion of the failure rate function. Elimination of the infant mortality significantly reduces the cost of field failures.

The Weibull family of distributions is widely used as a lifetime model because it can accommodate constant, decreasing, and increasing failure rate functions. In this paper, the mixed Weibull distribution is used to model the lifetime of a randomly selected product from a batch of products that is composed of defected (weak) and non defected (strong) products. The PDF of the mixed Weibull model is given as

$$f(t | \Theta) = \pi \left(\frac{\beta_w}{\theta_w} \right) \left(\frac{t}{\theta_w} \right)^{\beta_w - 1} e^{-\left(\frac{t}{\theta_w} \right)^{\beta_w}} + (1 - \pi) \left(\frac{\beta_s}{\theta_s} \right) \left(\frac{t}{\theta_s} \right)^{\beta_s - 1} e^{-\left(\frac{t}{\theta_s} \right)^{\beta_s}} \quad (5)$$

where π is the mixture parameter, β_w and θ_w are the shape and scale parameters of the lifetime distribution of the

weak subpopulation respectively, β_s and θ_s are the shape and scale parameters of the lifetime distribution of the strong subpopulation respectively and $\Theta = (\pi, \beta_w, \theta_w, \beta_s, \theta_s)$ is the parameter vector of the mixed Weibull distribution.

3. Mean Residual Life

Mean Residual life (MRL) of a product is the expected lifetime of the product given that it survives the burn-in. MRL is used as a criterion of burn-in instead of delivered reliability when a mission time (warranty time) is not specified for a product and the product is required to live for a very long time. MRL is calculated as

$$m(t_b | \Theta) = E[T - t_b | T > t_b, \Theta] = \frac{\int_{t_b}^{\infty} R(t | \Theta) dt}{R(t_b | \Theta)} \quad (6)$$

where T is the lifetime of the product, Θ is the parameter vector of the life time distribution, and $R(t | \Theta)$ is the reliability function of the product. Given the parameter vector Θ , the MRL is a function of burn-in time. According to MRL criterion the optimal burn-in time, t_b^* is such that

$$t_b^* = \max_{t_b \geq 0} m(t_b | \Theta) \quad (7)$$

4. Delivered Reliability

Delivered reliability is the conditional reliability of a product for a mission time given that the product survived the burn-in. Given the mission time t_m and the parameters of the lifetime distribution, delivered reliability is a function of burn-in time t_b . It is calculated as

$$R^{(D)}(t_m | t_b, \Theta) = P(T > t_m + t_b | T > t_b) = \frac{P(T > t_m + t_b)}{P(T > t_b)} = \frac{R(t_m + t_b | \Theta)}{R(t_b | \Theta)} \quad (8)$$

According to delivered reliability criterion the optimal burn-in time, t_b^* is such that

$$t_b^* = \max_{t_b \geq 0} R^{(D)}(t_m | t_b, \Theta) \quad (9)$$

5. The Expected Total Cost Function

The cost of burn-in process is composed of the cost of burn-in test process and the cost of warranty claims. The cost of burn-in test is composed of fixed set up cost of burn-in, a variable per product per unit time cost and a cost of repair and/or scrap due to failures during burn-in period. The cost of burn-in process increases in burn-in time. The cost of warranty claims depends on the warranty contract and it is composed of cost of repair and/or replacement due to failure in the field. The cost of warranty claims decreases in burn-in time. Given parameters of the lifetime distribution and the mission time, cost of burn-in is a function of burn-in time. According to cost criterion the optimal burn-in time, t_b^* is such that

$$t_b^* = \max_{t_b \geq 0} Cost(t_b | t_m, \Theta) \quad (10)$$

Depending on the warranty contract and treatment of failed products, there are several cost functions for a burn-in process. In this paper the cost function of Perlstein, Jarvis and Mazzuchi (2001) is used. According to this cost function products in a batch of size N are burned-in simultaneously. Products that are failed during burn-in and during warranty time are scrapped. A product can fail during burn-in or during warranty time or it can survive the warranty time. Under the assumption that failures times of products are conditionally independent, number of possible outcomes for products has a multinomial distribution. Given that $F(t | \Theta)$ is the cumulative distribution function (CDF) of the products, parameters of the multinomial distribution are N , $F(t_b | \Theta)$, $[F(t_b + t_m | \Theta) - F(t_b | \Theta)]$, and $R(t_b + t_m | \Theta)$. Letting C_0 be the fixed set up cost of burn-in, C_1 be the per product per unit time cost of burn-in, C_2 be the scrap cost of a product that is failed during burn-in, and C_3 be the field replacement cost of a product that is failed during warranty time, the expected cost function given as

$$E[Cost(t_b | \Theta)] = C_0 + C_1 N t_b + C_2 N F(t_b | \Theta) + C_3 N [F(t_b + t_m | \Theta) - F(t_b | \Theta)] \quad (11)$$

In this cost function the quantity $C_1 N t_b$ represents the variable cost of burning-in the products in the batch simultaneously. The quantity $N F(t_b | \Theta)$ is the expected number of products that are failed during burn-in so that the quantity $C_2 N F(t_b | \Theta)$ represents the expected cost of failures that are occurred during burn-in. In the same way the quantity $N [F(t_b + t_m | \Theta) - F(t_b | \Theta)]$ is the expected number of products that are failed during warranty period so that the quantity $C_3 N [F(t_b + t_m | \Theta) - F(t_b | \Theta)]$ represents the expected cost of failures during warranty period.

6. Example

It is known from the previous life test that a product has a mixed Weibull lifetime distribution with parameters $\pi = 0.2$, $\beta_w = 1.2$, $\beta_s = 1.5$, $\theta_w = 25$, $\theta_s = 1450$. The PDF, reliability, and failure rate function for this model are presented in Figure 1.

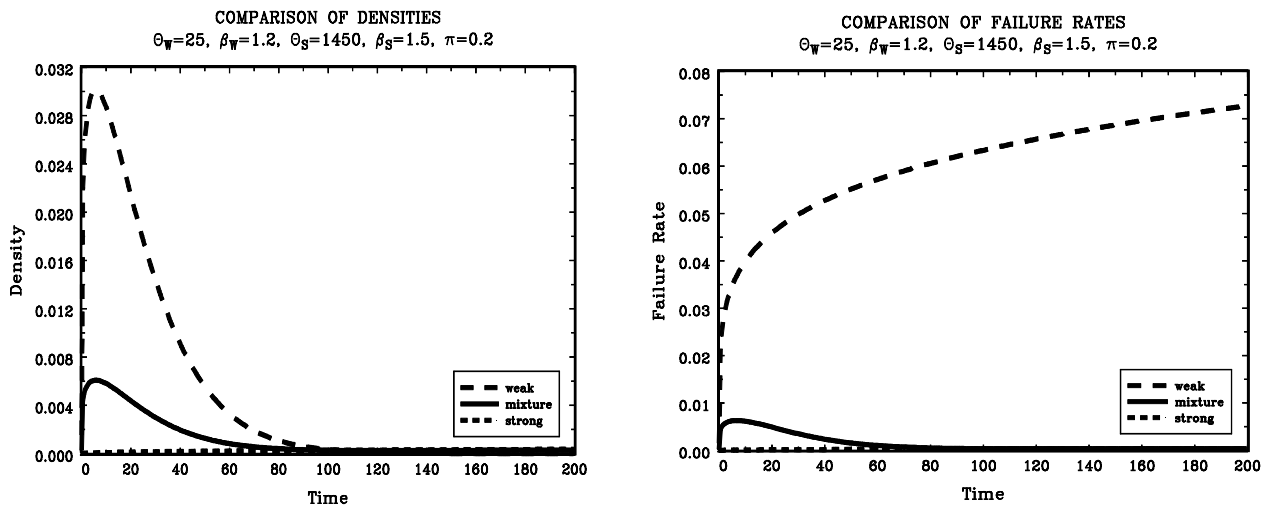


Figure 1. PDF, Reliability and failure rate function of the product.

It can be observed from Figure 1 that the lifetime distributions of the weak and the strong sub-populations do not overlap much. In Figure 1 the failure rate function of the mixture population has a modified bathtub shape and it is bounded by the failure rate function of the weak and the strong sub-populations. A modified bathtub shape function has an increasing part at the beginning creating a hump at the function. The shape of the failure rate function suggests that burn-in will improve the reliability of the product. By burning-in the product the increasing and the

decreasing part of the failure rate function can be eliminated and product can be delivered to the customer when it reaches the minimum of the failure rate function.

For this product, the optimal burn-in time will be calculated using the criteria mentioned in section 3, 4, and 5.

6.1 Mean Residual Life

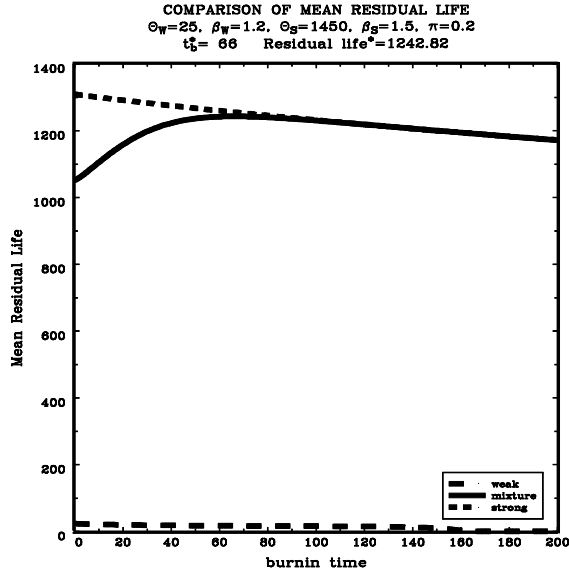


Figure 2. MRL as a function of burn-in time.

The mean residual life as a function of burn-in time for the above product is presented in Figure 2 below. The mean residual life function of the mixture population has an upside down bathtub shape suggesting that burn-in will improve the mean residual life of the product. Maximum value of the of mean residual life 1242.82 time units is achieved at the burn-in time 66 time units. Therefore optimum burn-in time t_b^* equals to 66 time units. By burning-in the product for 66 time units manufacturer will deliver the product to customers with the maximum mean residual life. It can be also observed from Figure 2 that the MRL of the mixture population is bounded by the MRL of the weak and the strong sub-populations.

6.2 Delivered Reliability

Delivered reliability functions of the same product for mission times 10, 100, 500 and 1000 time units are presented in Figure 3 below. It can be observed from the Figure 3 that the delivered reliability of the mixture has an upside down bathtub shape for all mission times suggesting that the burn-in will improve the delivered reliability of the product. By burning-in the product the increasing part of the delivered reliability function will be eliminated and the product will be delivered to the customer when delivered reliability reaches the maximum level.

The maximum levels of the delivered reliability values are calculated as 1, 0.97, 0.78, and 0.53 for the mission times 10, 100, 500, and 1000 time units respectively. The optimal burn-in times resulting the maximum levels of the reliability values are 119, 94, 72, and 64 time units respectively. Notice that as mission time increases, optimal burn-in time decreases. Delivered reliability of the mixture is bounded by the delivered reliability of the weak and the strong sub-populations for all the mission times since

$$R^{(D)}(t_m | t_b, \Theta) = qR_w^{(D)}(t_m | t_b, \Theta) + (1 - q)R_s^{(D)}(t_m | t_b, \Theta) \quad (12)$$

where $q = \pi R_w^{(D)}(t | \Theta_w) / [\pi R_w^{(D)}(t | \Theta_w) + (1 - \pi)R_s^{(D)}(t | \Theta_s)]$.

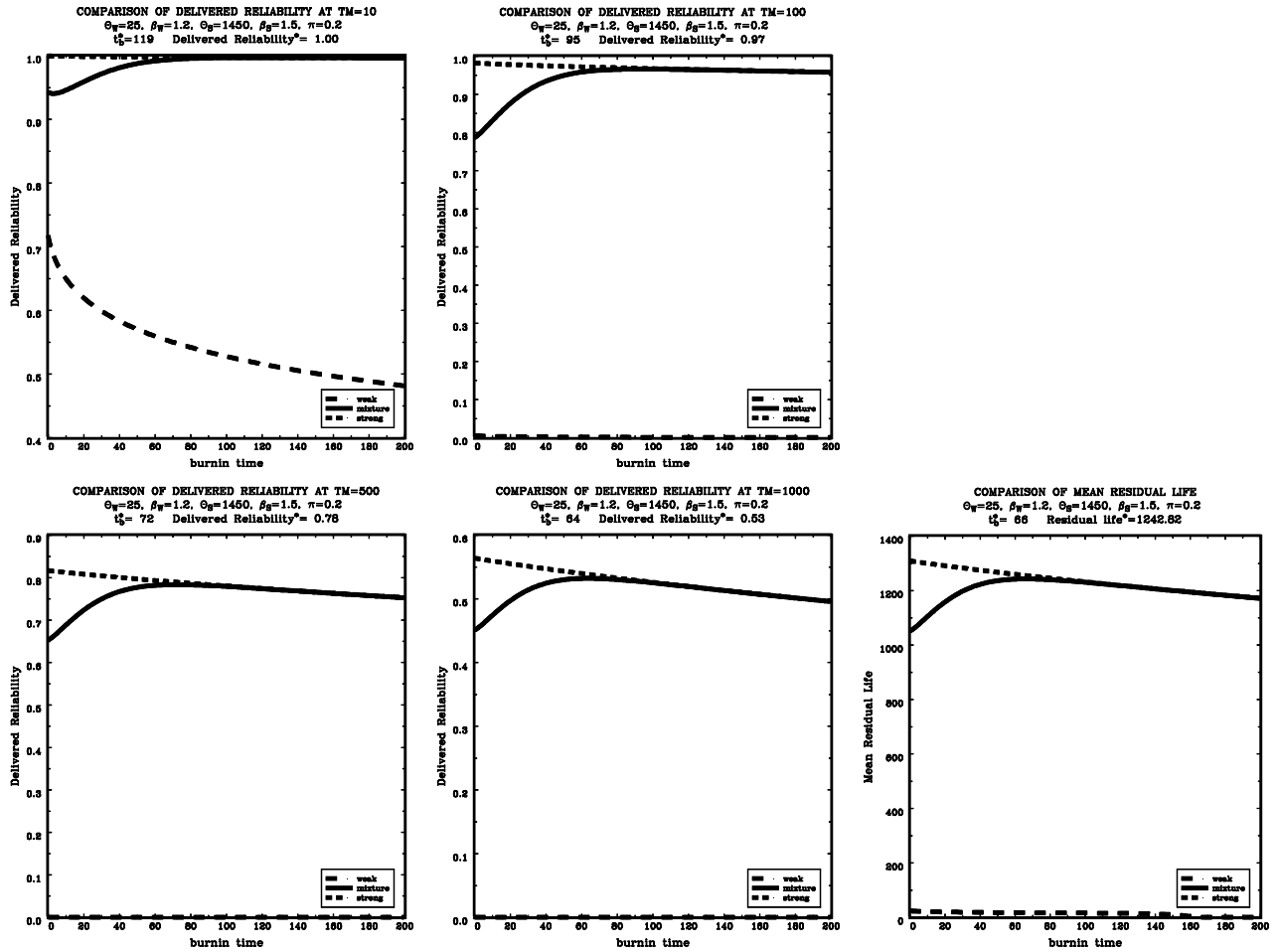


Figure 3. Delivered reliability as a function of burn-in time, for mission times 10, 100, 500 and 1000 time units.

6.3 The Total Expected Cost

For the product considered above, the cost parameters are given as $C_0 = \$15$, $C_1 = \$3$, $C_2 = \$300$ and $C_3 = \$1500$ and $N = 50$. The expected cost functions for the various mission times are calculated using (11) and presented in Figure 4. From Figure 4, it can be seen that for the mission times 10, 100, 500, and 1000, the burn-in times that minimize the expected cost function for the mixture population are 0, 31, 30, and 30 respectively. Note that even though optimum burn-in times are equal for mission times 500 and 1000, expected cost increases by 66.6% for the mission time 1000 since expected number of failures during mission time 1000 increases by 94.3%. Notice that, expected cost of the mixture is bounded by the expected cost of the weak and the strong products for all mission times. Notice, however, that for mission times 10 and 100, the expected cost of the weak subpopulation is consistently greater than the expected cost of the strong subpopulation, whereas for mission times 500 and 1000 the expected cost of the weak subpopulation is greater than the expected cost of the strong subpopulation for relatively small values of burn-in times. As burn-in time increases, the expected cost of strong subpopulation becomes greater than the expected cost of the weak subpopulation.

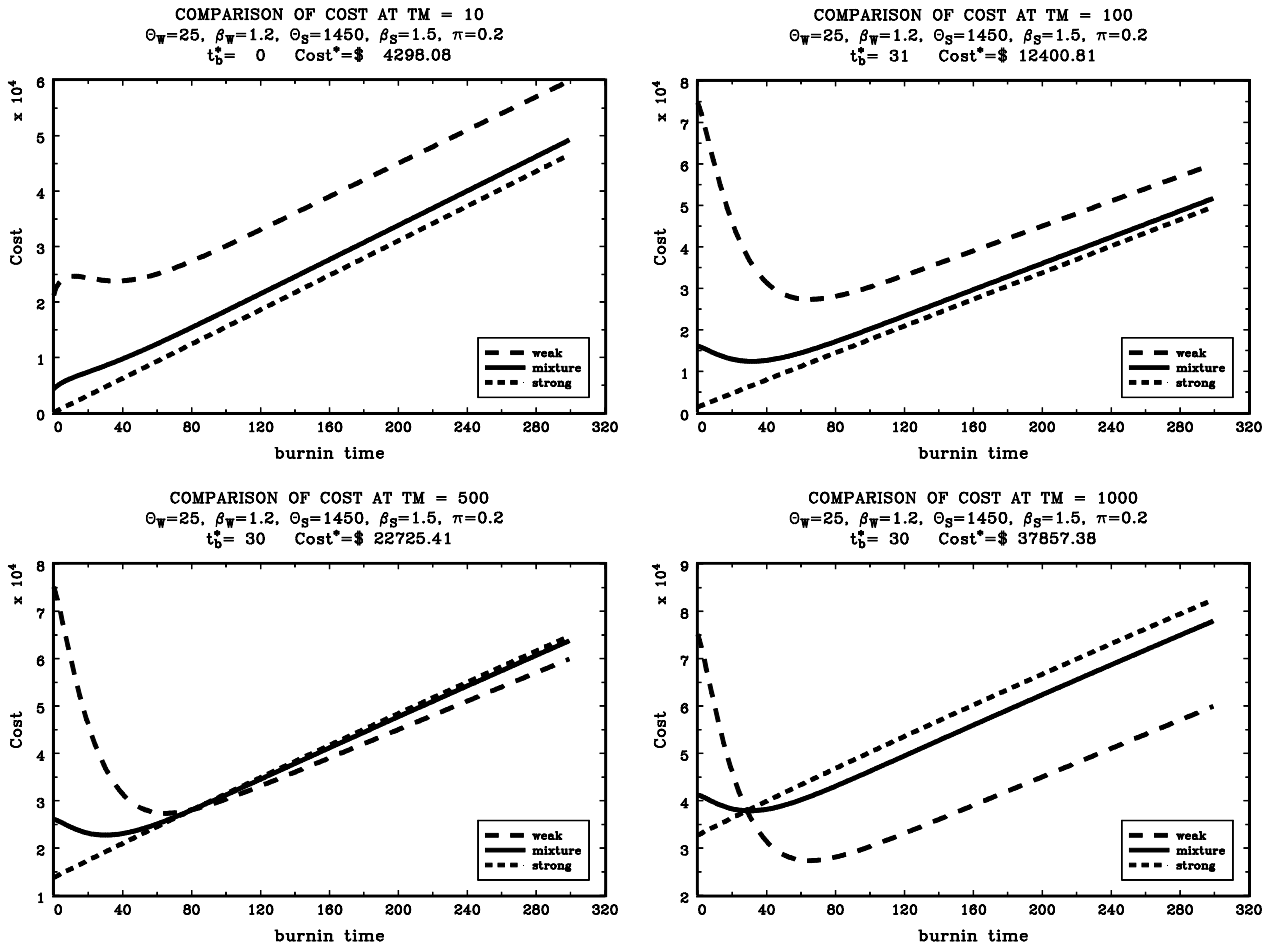


Figure 4. Expected cost as a function of burn-in time for the mission times 10, 100, 500, and 1000 hr.

7. Conclusion

The optimal burn-in times and resulting optimal values for different criteria are given in Table1. Burn-in times that minimize the cost are much less than the burn-in times that maximizes the delivered reliability. Delivered reliability resulting from the cost optimum burn-in times for mission times 10, 100, 500, 1000 are 0.943, 0.914, 0.750, and 0.514 respectively. These values are less than the maximum delivered reliability achieved using the delivered reliability criteria so that a manufacturer has to decide how much she/he is willing to pay to increase the delivered reliability.

Table 1. The optimal burn-in times and the optimal values for different criteria.

	<i>Mission time</i>							
	10		100		500		1000	
<i>Criteria</i>	t_b^*	Opt. Value	t_b^*	Opt. Value	t_b^*	Opt. Value	t_b^*	Opt. Value
<i>Cost</i>	0	\$4298.08	31	\$102400.81	30	\$22725.41	30	\$37857.38
$R^{(D)}$	119	1.00	94	0.97	72	0.78	64	0.53
<i>MRL</i>	66	1242.82	66	1242.82	66	1242.82	66	1242.82

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