*Erciyes Üniversitesi Fen Bilimleri Enstitüsü Dergisi,* **17** (1-2), 1-8, 2001

# SHAPED BEAM PATTERN SYNTHESIS OF LINEAR ANTENNA ARRAYS BY USING INTELLIGENT OPTIMIZATION TECHNIQUES: COSECANT PATTERN EXAMPLE

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**Abstract**: In this work, the synthesis of shaped beam pattern of linear antenna arrays is presented. Three intelligent optimization techniques, genetic, simulated annealing and tabu search, are used to determine the excitations of array elements. Examples of the cosecant pattern with the restricted sidelobe levels are given to illustrate the performance of the algorithms.

**Key Words:** Antenna array pattern synthesis, beam shaping, genetic algorithm, simulated annealing, tabu search.

# ZEKİ OPTİMİZASYON TEKNİKLERİ KULLANARAK LİNEER ANTEN DİZİLERİNİN ŞEKİLLENDİRİLMİŞ DEMET DİYAGRAM SENTEZİ: KOSEKANT DİYAGRAMI ÖRNEĞİ

**Özet:** Bu çalışmada, lineer anten dizilerinin şekillendirilmiş demet diyagram sentezi sunulmuştur. Genetik, simulated annealing ve tabu araştırma algoritmaları gibi zeki optimizasyon teknikleri, dizi elemanlarının uyarımlarını belirlemek için kullanılmıştır. Algoritmaların performansını göstermek için, yan demet seviyeleri kısıtlanmış kosekant diyagram örnekleri verilmiştir.

**Anahtar Kelimeler:** Anten dizi diyagram sentezi, demet şekillendirme, genetik algoritma, simulated annealing, tabu araştırma.

## 1. Introduction

Due to their capacity for beam shaping, beam steering, directivity and high gain, antenna arrays are one of the most versatile classes of radiators. In antenna array pattern synthesis, in general, the main concern is to determine a set of excitations which produce a desired pattern. The problem of determining the excitation distribution has been generally classified into three categories [1]. The first is the pattern synthesis problem to produce a pattern having nulls in desired directions [2]. The second is to produce a pattern with a narrow beam and low sidelobes [3, 4]. The third is to produce a shaped beam pattern. For this class of the pattern synthesis, techniques such as the Woodward-Lawson synthesis [5], the method of Fourier series [1], and the Orchard-Elliot method [6] have been shown to be effective techniques, each with its own benefits and limitations [1, 7].

To solve the antenna array pattern synthesis problems, among a number of optimization procedures, the artificial intelligence techniques such as genetic, simulated annealing and tabu search algorithms owing to their simplicity, flexibility and accuracy have received much attention in recent years [8-17]. Genetic algorithm (GA) [18] is a search technique based on an abstract model of Darwinian evolution. Solutions are represented by fixed length strings over some alphabet ("gene" alphabet). Each string can be thought of as a "chromosome". The value of the solution represents the fitness of the chromosome. Survival of the fittest principle is then applied to create a new generation with slow increase of average fitness. So, genetic algorithms also have the facility of allowing some weak members to survive in the solution pool, but always have mechanisms for favoring fitter solutions. Simulated annealing (SA) technique [19,20] is essentially a local search, in which a move to an inferior solution is allowed with a probability, according to some Boltzmann-type distribution, that decreases as the process progresses. So there will always be a chance that a solution with a less good value would be retained in preference to a better solution. Simulated annealing shows that inclusion of "downhill" moves in a neighborhood search algorithm is a good idea. Tabu search (TS) algorithm [21,22] has been developed to be an effective and efficient scheme for combinatorial optimization that combines a hill-climbing search strategy based on a set of elementary moves and a heuristics to avoid to stops at sub-optimal points and the occurrence of cycles. One characteristic of the tabu search is that it finds good near-optimal solutions early in the optimization run.

The search techniques mentioned above are the probabilistic search techniques that are simple and easily be implemented without any gradient calculation. In comparison with the conventional gradient-based search procedures, these techniques can also produce flexible solutions for a given design problem. Because of these fascinating features, the GA, SA and TS are used to determine the element excitations of a linear antenna array to synthesis a desired cosecant pattern. It was observed that all the techniques used here are capable of synthesizing the array pattern with some desired properties. In previous works [17, 23-26], we successfully used these intelligent optimization techniques to solve some antenna problems efficiently.

## 2. Intelligent Optimization Techniques

In this section, the intelligent optimization algorithms used in this study for the shaped beam pattern synthesis of a linear antenna array are described briefly.

### 2.1 Genetic Algorithm (GA)

A basic GA [18] consists of five components. These are a random number generator, a “fitness” evaluation unit and genetic operators for “reproduction”, “crossover” and “mutation” operations.

The initial population required at the start of the algorithm is a set of number strings generated by the random number generator. Each string is a representation of a solution of the optimization problem being addressed. Binary strings are commonly employed. Associated with each string is a fitness value as computed by the evaluation unit. A fitness value is a measure of the goodness of the solution that it represents. The aim of the genetic operators is to transform the set of the strings into the sets with higher fitness values.

The reproduction operator performs a natural selection function known as “seeded selection”. Individual strings are copied from one set (representing a generation of solutions) to the next according to their fitness value, i.e. the greater the probability of a string being selected for the next generation.

The crossover operator chooses pairs of strings at random and produces new pairs. The simplest crossover operation is to cut the original parent strings at a randomly selected point and exchange their tails. The number of crossover operations is governed by a crossover rate.

The mutation operator randomly mutates or reverses the values of bits in a string. The number of mutation operations is determined by a mutation rate.

A phase of the algorithm consists of applying the evaluation, reproduction, crossover and mutation operations. A new generation of solutions is produced with each phase of the algorithm.

### 2.2 Simulated Annealing (SA)

The SA [19,20] is a heuristic algorithm for solving combinatorial optimization problems. It is based on a local search procedure, and can be viewed as a control strategy for the underlying heuristic search. The SA has been shown to be a powerful stochastic search method applicable to a wide range of problems.

The basic idea in SA is to track a path in the feasible solution space of the given optimization problem. Starting with a valid solution, SA repeatedly generates succeeding solutions using the local search procedure. Some of them are accepted and some are rejected, according to a predefined acceptance rule. The acceptance rule is motivated by an analogy with annealing processes in metallurgy. In the beginning of the optimization process the main control parameter- the temperature- is high and decreases until no improvement of the current solution is attainable. Starting with an arbitrary solution, every improvement is accepted. Deteriorations of the objective function are accepted according to the Boltzmann probability $e^{-\Delta C/T}$, where $\Delta C$ and T are the difference between the result of the current solution and the result of the next solution, and the temperature, respectively.

After some iterations of the local search procedure, the temperature is decreased and the optimization continues on a new temperature level. The best solution found during the optimization is the output of the algorithm after the system is frozen, i.e. no improvements can be found.

### 2.3 Tabu Search (TS)

The TS [21,22] has a flexible memory to keep the information about the past steps of the search and uses it to create and exploit the new solutions in the search space. It has the ability of finding global optimum of a multimodal search space. The process with which the tabu search overcomes the local optimality problem is based on an evaluation function that chooses the highest evaluation neighbour at each iteration. The best admissible neighbour means the highest evaluation neighbour in the neighbourhood of the current solution in terms of the objective value and tabu restrictions. The highest evaluation function selects the neighbour which produces the most improvement or the least disimprovement in the objective function. A tabu list is employed to store the characteristics of accepted neighbours so that these characteristics can be used to classify certain neighbours as tabu (i.e. to be avoided) in later iterations. In other words, tabu list determines which neighbours may be reached by a move from the current solution.

A simple tabu search algorithm consists of three main strategies: forbidding strategy, freeing strategy and short term strategy. Forbidding strategy controls what goes into the tabu list. Forbidding strategy is employed to avoid cycling problem by forbidding certain moves (tabu). The freeing strategy controls what goes out of the tabu list, and when. This strategy erases their tabu restrictions so that they can be reconsidered in any future search. Short term strategy manages the interplay between the forbidding and freeing strategies to select trial solutions. Setting the tabu status to a move is sometimes too restrictive. When a tabu move leads the search to a promising region, then the tabu status has to be overridden. An aspiration criterion is used to override the tabu status of a solution if this solution is good enough and sufficient to prevent cycling. While the aspiration criteria has a role in guiding the search process, tabu conditions has a role in the constraining the search space. A solution is acceptable if tabu restrictions are satisfied. However, a tabu solution is also assumed acceptable if the aspiration criteria apply regardless of the tabu status.

Tabu restrictions used here are based on the recency and frequency memory storing the information about the past steps of the search. The recency of a move is the difference between the current iteration count and the last iteration count at which that move was made. The recency-based memory prevents cycles of length less than or equal to a predetermined number of iterations from occurring in the trajectory. The frequency measure is the count of changes of the move. The frequency-based memory keeps the number of change of solution vector elements. Details on these optimization algorithms and their applications to some engineering problems can be found in [27].

## 3. Formulation

If the array element excitations are conjugate-symmetrical about the center of the linear array, the far field array factor of this array with an even number (2N) of uniformly spaced isotropic elements can be written as:

$$F(u) = 2 \sum_{k=1}^{N} a_k \cos\left[\frac{2\pi}{\lambda} d_k u + \delta_k\right] \tag{1}$$

here, $u=\sin(\theta)$ where $\theta$ is the scanning angle from broadside, $d_k$ is the distance between position of the $k^{th}$ element and the array center, and $a_k$ and $\delta_k$ are amplitude and phase weights of the $k^{th}$ element, respectively. To synthesis the shaped beam pattern of a linear antenna array, element amplitude ($a_k$) and phase ($\delta_k$) weights are determined by the GA, SA and TS. In the optimization process, we also consider the maximum sidelobe level in the sidelobe region by including the weighting functions $W_1$ and $W_2$ in the cost function given below.

$$C = W_1(u)\ \left|F_o(u) - F_d(u)\right| + W_2(u)\ \left|MSLL_o(u) - MSLL_d(u)\right| \tag{2}$$

Here, $F_o$, $F_d$, $MSLL_o$ and $MSLL_d$, are, respectively, the obtained pattern, the desired one, the obtained maximum sidelobe level and the desired maximum sidelobe level. To obtain the desired pattern with the constrained sidelobe levels, the cost function given by eq. (2) will be minimized by the GA, SA and TS.

## 4. Numerical Results

In order to illustrate the capabilities of the intelligent optimization techniques for the beam shaping of linear antenna array patterns, two examples of cosecant pattern have been carried out. In the optimization, a linear antenna array with one-half wavelength spaced 20 isotropic elements was used and the element excitations of this array were determined. The dynamic range ratio ($|a_{max}/a_{min}|$) was chosen to be less than 8. The all calculations took almost 5-10 minutes on a personal computer with a Pentium III processor running at 750 MHz.

In the first example, the pattern with a cosecant beam in the sector $u \in [0.1 – 0.5]$ and no sidelobe higher than -25 dB was chosen. In order to obtain this pattern, the following parameter values of the cost function given in (2) are used.

$$F_d(u) = \begin{cases} \operatorname{cosecant}(u) & \text{for} \quad 0.1 \le u \le 0.5 \\ 0 & \quad\quad\ \text{elsewhere} \end{cases} \tag{3}$$

$$W_1(u) = \begin{cases} 10 & \text{for} \quad 0.1 \le u \le 0.5 \\ 1 & \quad\quad\ \text{elsewhere} \end{cases} \tag{4}$$

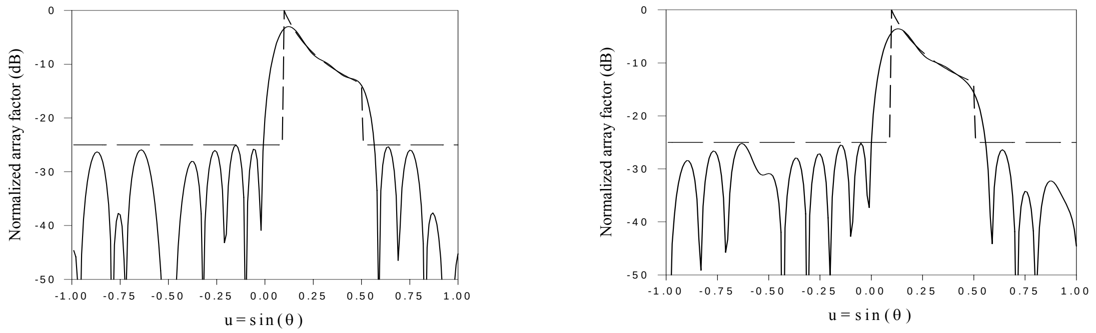
$$\mathrm{MSLL_d(u)} = -25 \quad \text{for} \quad u \le 0 \quad \text{and} \quad u \ge 0.6 \tag{5}$$

$$\mathrm{W_2(u)} = \begin{cases} 1000 & \text{if } \mathrm{MSLL_o(u)} > \mathrm{MSLL_d(u)} \\ 1 & \text{if } \mathrm{MSLL_o(u)} \le \mathrm{MSLL_d(u)} \end{cases} \tag{6}$$

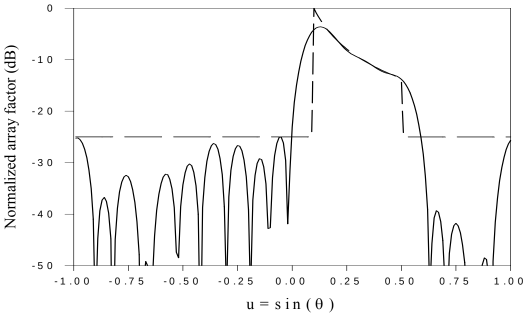
It is assumed that the sidelobe regions start with a distance of 0.1 $u$ from both the left and right side of the shaped region. Because of this assumption, $MSLL_d(u)$ and $MSLL_o(u)$ are only defined in the ranges of $-1 \le u \le 0$ and $0.6 \le u \le 1$. In Figure 1, the cosecant patterns obtained by the GA, SA and TS are illustrated. As can be seen from Figure 1, all the algorithms used here are capable of synthesizing the shaped beam pattern with a good performance for both the shaped region and the sidelobe region. In order further to inspect the versatility of the optimization algorithms, in the second example, it is aimed to obtain the pattern having the values of $MSLL_d(u)$ as given below while the other design parameters are the same as those of the first example.

$$\mathrm{MSLL_d(u)} = \begin{cases} -30 & \text{for} \quad -0.5 \ge u \ge -1 \\ -25 & \text{for} \quad 0 \ge u > -0.5 \\ -35 & \text{for} \quad 1 \ge u \ge 0.6 \end{cases} \tag{7}$$

The obtained patterns for these values of the $MSLL_d(u)$ are shown in Figure 2. It is clear from the Figure 2 that there is no sidelobe that exceeds the specified values given in eq. (7) with respect to their specified regions, and the shaped regions also have a good performance. The required element amplitudes and the phases computed by GA, SA and TS for Figures 1 and 2 are listed in Table 1. Since the element amplitudes have even-symmetry around the array center, the number of attenuators to be used is N for an array of 2N elements. It should also be noted that the number of phase shifters is 2N, but the number of controllers for the phase shifters is N due to odd symmetry property of the element phases.

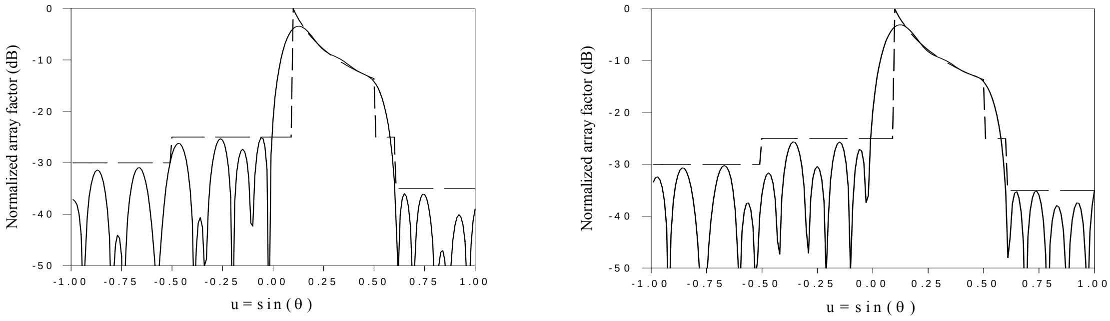


(a) (b)



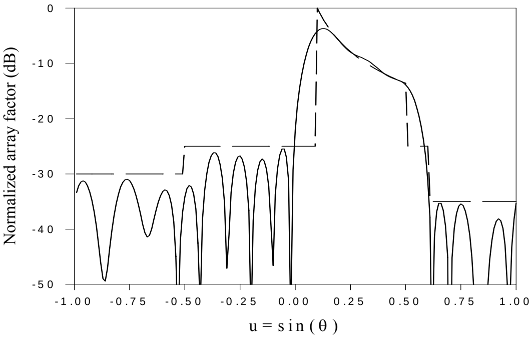
(c)

Figure 1. Envelope of the desired cosecant pattern (dashed line) and normalized array factors (solid line) obtained by (a) GA, (b) SA and (c) TS.



(a)

(b)



(c)

Figure 2. Envelope of the desired cosecant pattern with the restricted sidelobe levels (dashed line) and normalized array factors (solid line) obtained by (a) GA, (b) SA and (c) TS

Even though the patterns obtained from three algorithms are in good agreement with the desired patterns, when the run times of three optimization techniques are compared with each other for this pattern synthesis problem, we found that the GA and SA are the fastest and the slowest algorithms, respectively. Because the algorithms used in this work have good accuracy and require no complicated mathematical functions, they can be very useful to antenna engineers. The potentiality of the algorithms are that any parameter of interest (directivity, beam width, etc.) can easily be considered by including it in the cost function. Using these algorithms with a personal computer, one can calculate accurately the array element excitations to obtain a desired pattern with the desired properties. It is worth mentioning that, although the algorithms proposed here are implemented to constrained synthesis of a linear array with isotropic half-wavelength spaced elements, it is not limited to this case. These can easily be implemented to nonisotropic-elements antenna arrays with different geometries for the design of various array patterns.

Table 1. Element amplitudes and phases (in degree) obtained by using GA, SA and TS

| | Figure 1 | | | | | | Figure 2 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | (a)-GA | | (b)-SA | | (c)-TS | | (a)-GA | | (b)-SA | | (c)-TS | |
| k | $a_k$ | $\delta_k$ | $a_k$ | $\delta_k$ | $a_k$ | $\delta_k$ | $a_k$ | $\delta_k$ | $a_k$ | $\delta_k$ | $a_k$ | $\delta_k$ |
| ±1 | 2.04 | 157.25 | 1.86 | 158.38 | 1.95 | 160.43 | 1.98 | 157.82 | 2.07 | 158.88 | 2.00 | 160.43 |
| ±2 | 1.65 | 123.13 | 1.64 | 121.33 | 1.62 | 114.59 | 1.72 | 117.03 | 1.65 | 121.62 | 1.60 | 114.57 |
| ±3 | 1.09 | 85.13 | 1.03 | 82.68 | 0.98 | 83.00 | 1.05 | 82.51 | 1.03 | 84.2 | 1.04 | 84.40 |
| ±4 | 0.67 | 74.12 | 0.61 | 59.28 | 0.62 | 72.82 | 0.57 | 65.64 | 0.65 | 75.24 | 0.58 | 66.47 |
| ±5 | 0.49 | 82.81 | 0.52 | 73.51 | 0.50 | 74.46 | 0.45 | 70.10 | 0.53 | 79.78 | 0.49 | 77.26 |
| ±6 | 0.52 | 67.99 | 0.41 | 67.21 | 0.56 | 59.24 | 0.48 | 65.48 | 0.57 | 61.39 | 0.50 | 58.81 |
| ±7 | 0.57 | 19.39 | 0.57 | 15.96 | 0.35 | 14.38 | 0.42 | 31.80 | 0.41 | 29.04 | 0.35 | 21.07 |
| ±8 | 0.27 | 24.39 | 0.27 | 14.35 | 0.27 | 17.35 | 0.28 | 14.39 | 0.29 | 16.12 | 0.28 | 14.97 |
| ±9 | 0.26 | 42.36 | 0.25 | 15.72 | 0.26 | 16.58 | 0.38 | 14.76 | 0.26 | 26.5 | 0.28 | 33.08 |
| ±10 | 0.33 | 27.65 | 0.27 | 27.32 | 0.28 | 34.06 | 0.26 | 28.82 | 0.31 | 32.5 | 0.27 | 32.74 |

## 5. Conclusions

In this work, three robust intelligent optimization techniques, GA, SA, and TS, were successfully used to determine the both the element amplitudes and the phases of a linear antenna array for the synthesis of shaped beam pattern. Numerical results illustrated show that the techniques are capable of synthesizing the desired array pattern with the restricted sidelobe levels and dynamic range ratio. Advantages of the techniques are easiness of implementation, accuracy and flexibility.

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